Dependence of Adaptive Cross-Correlation Algorithm Performance on the Extended-Scene Image Quality

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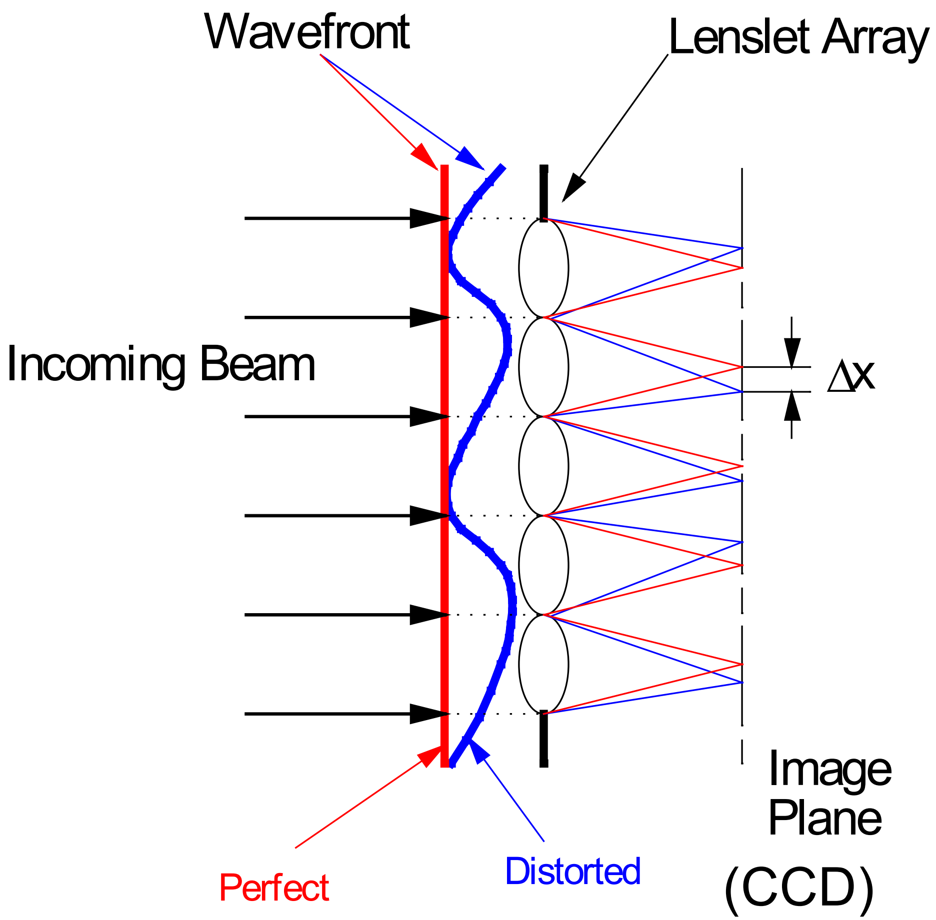
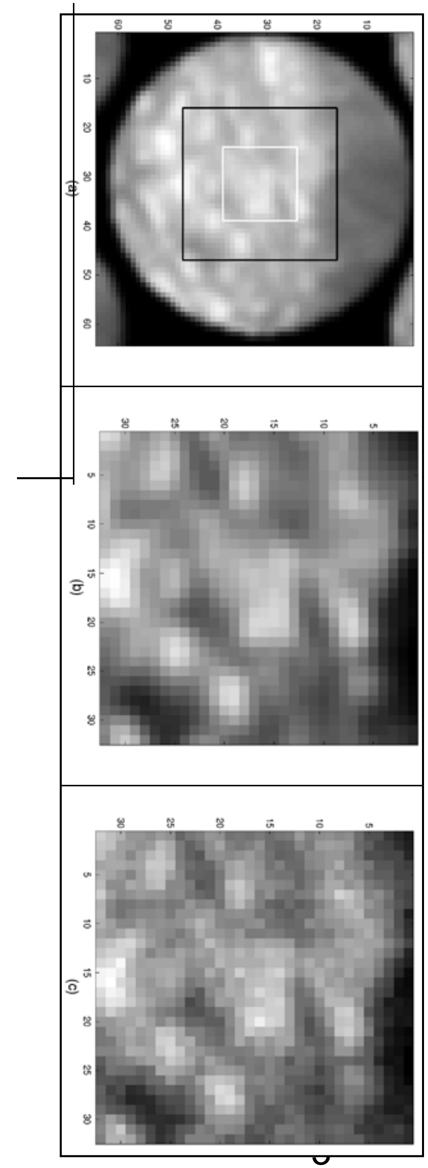
Principle of Conventional SH-WFS

- A Shack-Hartmann sensor places a lenslet array at a plane conjugate to the WF error source
- Each sub-aperture lenslet samples the WF in the corresponding patch of the WF
- When observing a star, the image is an array of spots, each of which is a sub-aperture PSF
  - Δx is proportional to local wavefront tilt
  - Wavefront-sensing → Finding Δx for all sub-images
  - Use centroiding (center-of-mass) method to find Δx



Extended-Scene S–H WFS

(a) (b) (c)



- The Shack-Hartmann Camera produces images as the convolution of the conventional image (limited by a tight field stop) with a regular grid of subaperture PSFs, as above
- Each subaperture is much blurrier than the main image, as its diffraction limit is defined by the subaperture, not the full aperture

- Subaperture image shown at right at full size (64x64)
- ACC algorithm finds the central 32x32 box, and then identifies the multi-pixel shift of the features in the inner 16x16 cell with respect to a reference subaperture
- The subaperture-to-subaperture cell shifts give a measure of subaperture tilt

Adaptive Cross-Correlation (ACC) Algorithm — How it Works

- Property of Fourier-transform:
  - Shift in time-domain ←→ Linear-phase in frequency-domain
  - In Fourier optics, t → (x,y) and f → (u,v)

- Fourier-transform pair—Shown as one-dimensional for simplicity:

$$s(x) \leftrightarrow \hat{s}(u)$$

$$s(x-\Delta x) \leftrightarrow \hat{s}(u)\,\mathrm{e}^{-j2\pi\Delta x u}$$

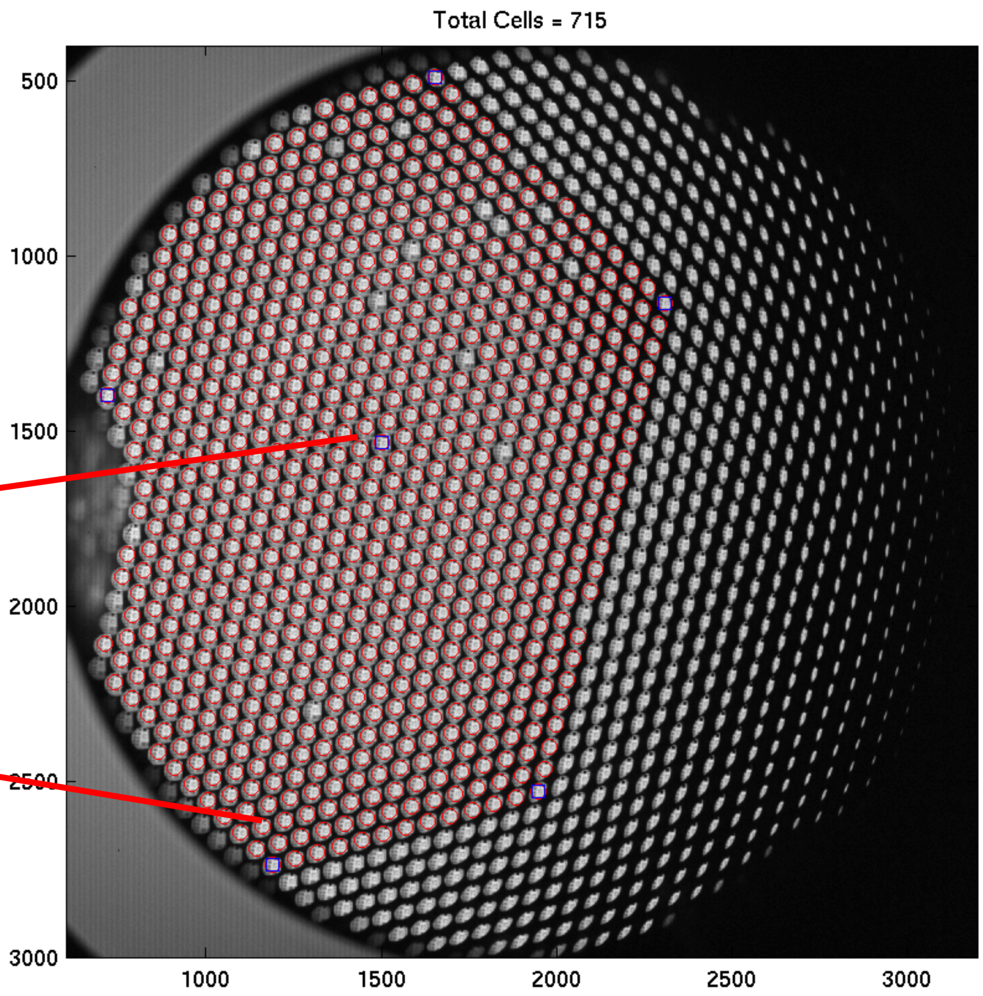
ACC Algorithm — How it Works (con.)

- In JPL testbed, only those cells marked with red-circle are used:
  - $r(x)$ = reference cell
  - $s(x)$ = test cells

- In ideal case:

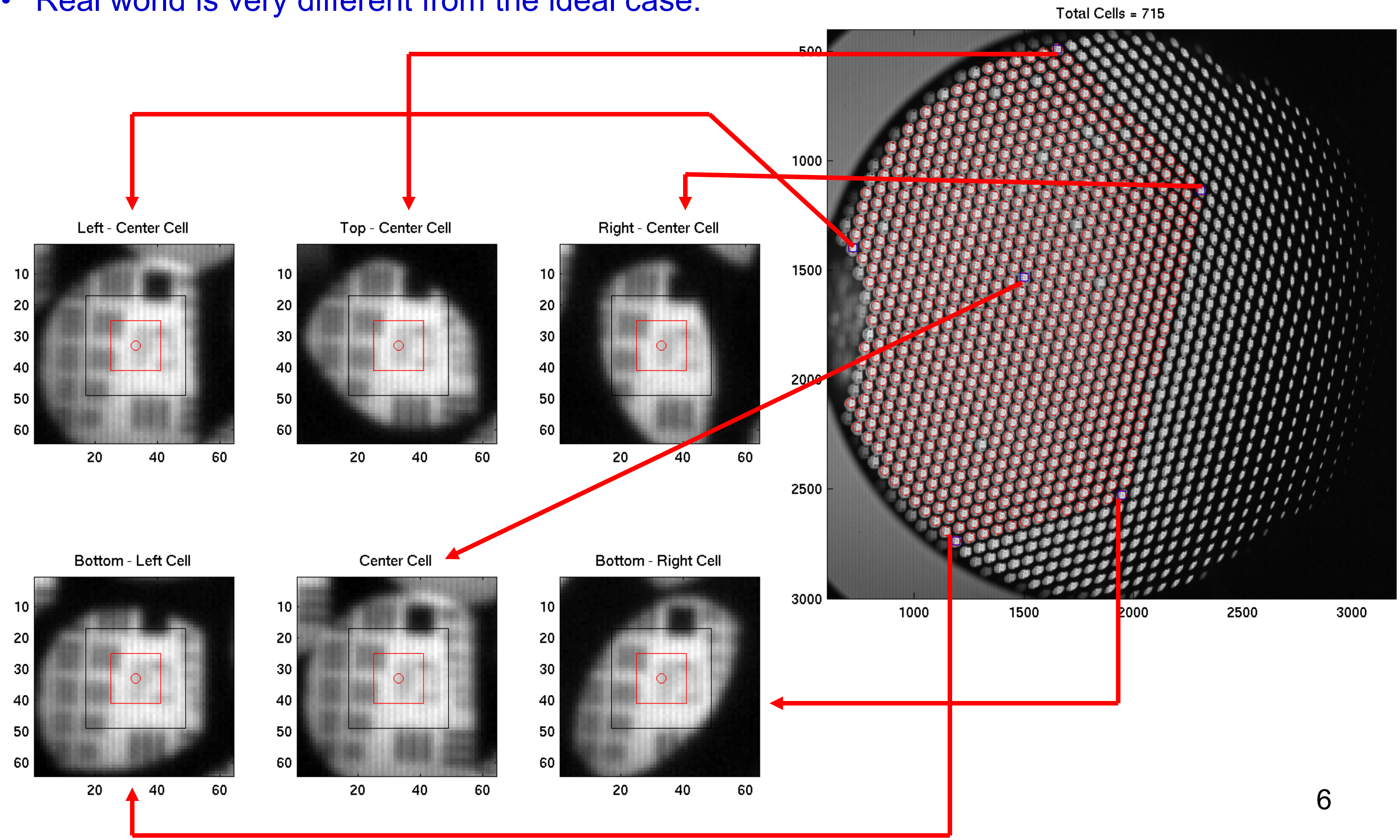
$$r(x) \leftrightarrow \hat{r}(u)$$

$$s(x) = r(x - \Delta x) \leftrightarrow \hat{r}(u)\, e^{-j2\pi\Delta x u}$$



ACC Algorithm — How it Works (con.)

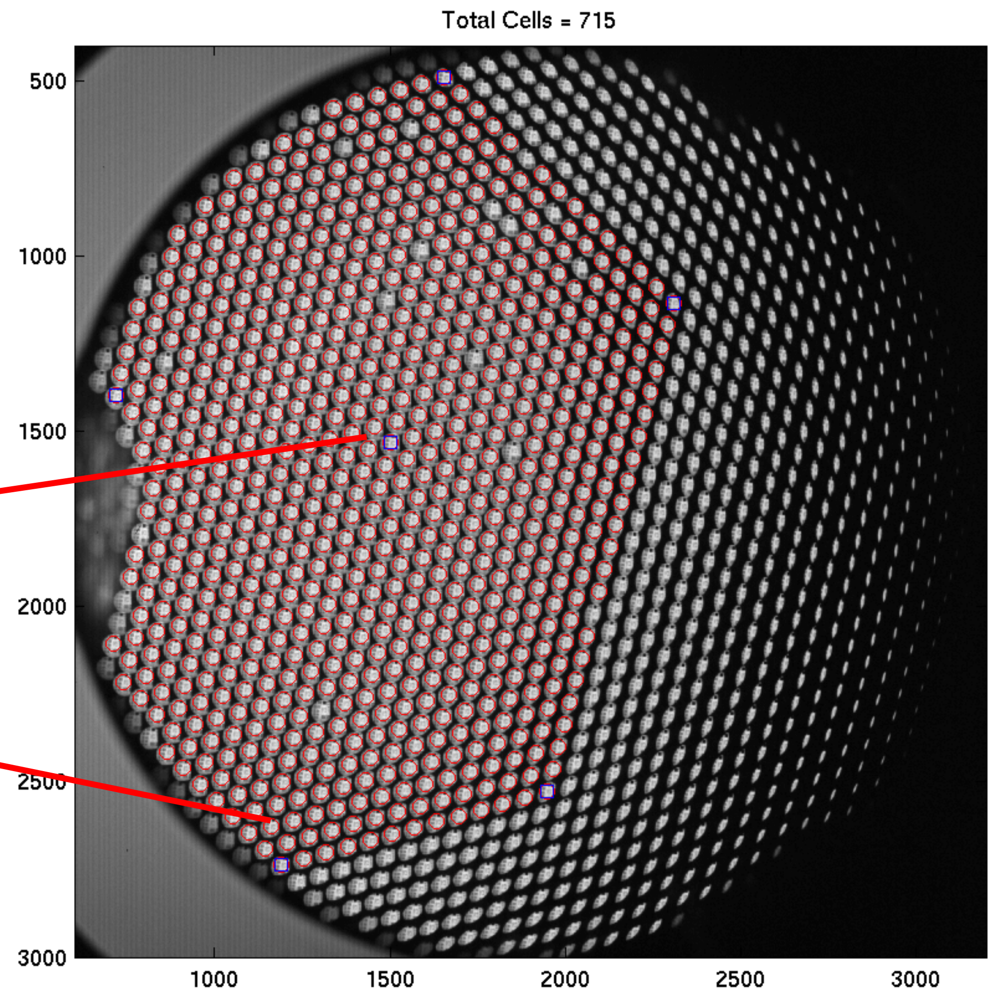
- Black square = usable sub-image (cell)
- Real world is very different from the ideal case:



ACC Algorithm — How it Works (con.)

- Following illustration was made in one-dimension only. In reality, everything is 2-dimensional: (x,y) ←→ (u,v)

- In real world:



$$r(x) \leftrightarrow \hat{r}(u)$$

$$s(x) \neq r(x - \Delta x)$$

$$s(x) \leftrightarrow \hat{s}(u)$$

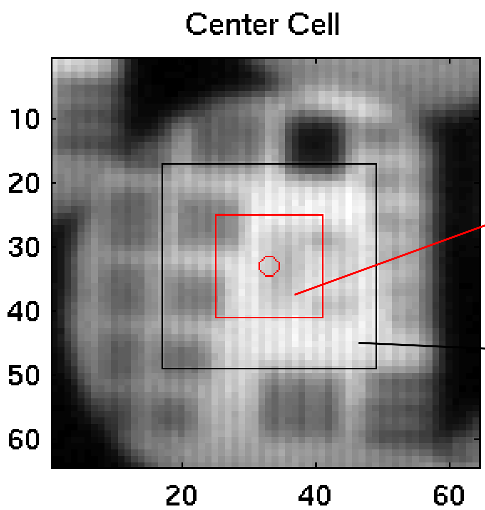
$$\hat{c}(u) = \hat{r}^*(u)\hat{s}(u) = |\hat{c}(u)|e^{j2\pi\varphi(u)}$$

$$\varphi(u) = \Delta x u + \varphi'(u)$$

Linear phase

Cross-correlation function

ACC Algorithm — How it Works (con.)



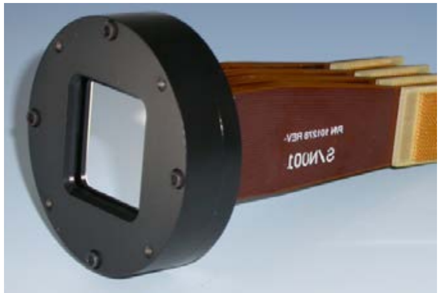
16x16 maxtrix $r(x,y)$ or $s(x,y)$:
Used to find $(\Delta x, \Delta y)$ from CC-function

32x32 maxtrix $S(x,y)$:
Used to shift $s(x,y)$ by $(-\Delta x, -\Delta y)$ to match it with $r(x,y)$

- Advantages of using smaller dimensions for $r(x,y)$ & $s(x,y)$:
  - Avoids wrap-around error when performing sub-image multi-pixels shifting
  - Makes the ACC calculations much faster
  - Increases the WFS dynamic range

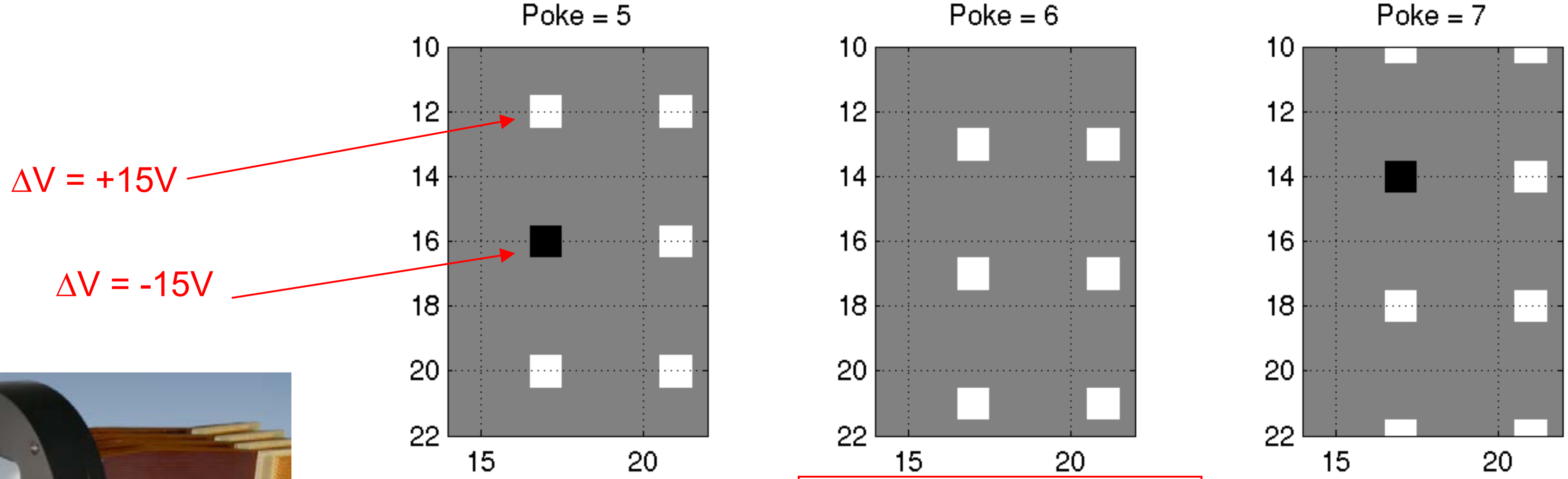
- To shift $S(x,y)$ by $(-\Delta x,-\Delta y)$:
  - Obtain $S(u,v)$ by FFT → $S(u,v)\exp[-j2\pi(-\Delta xu-\Delta yv)]$ → (by IFFT) $S(x+\Delta x, y+\Delta y)$

Examples: Point-Source Spot Image Analyzed with ACC

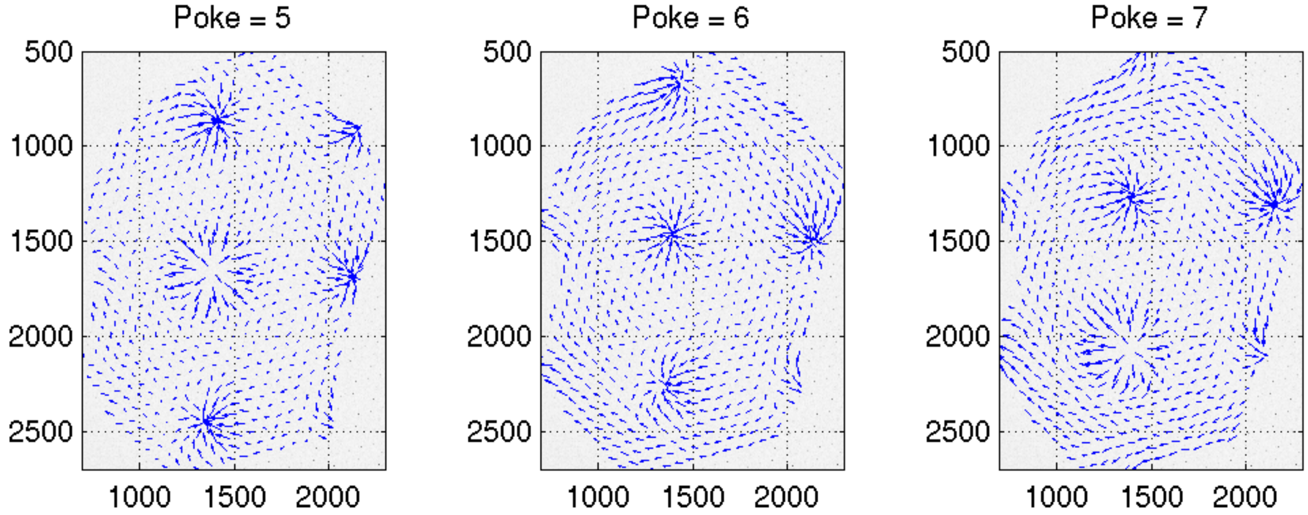


Deformable-Mirror (DM)

DM Poke Patterns



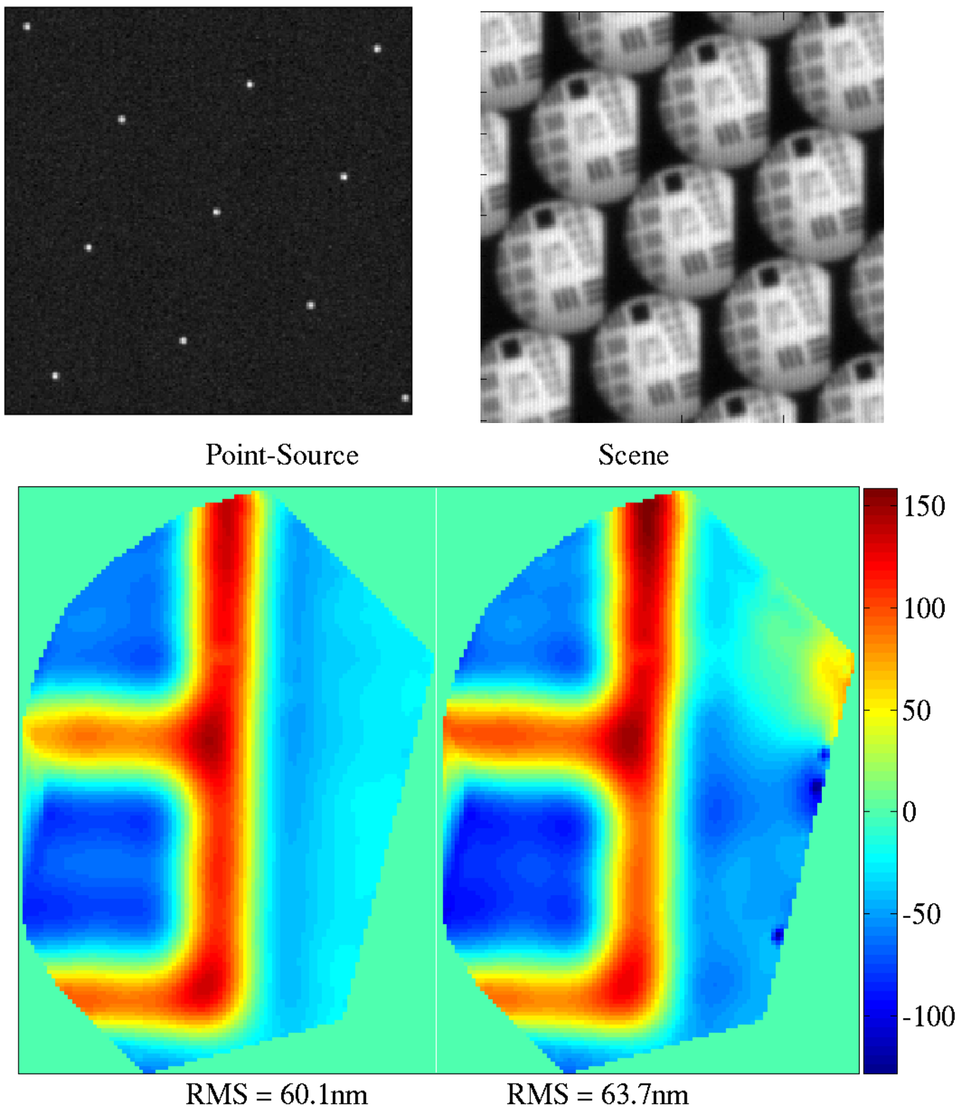
Offset Diagrams



Extended Scene versus Point-Source

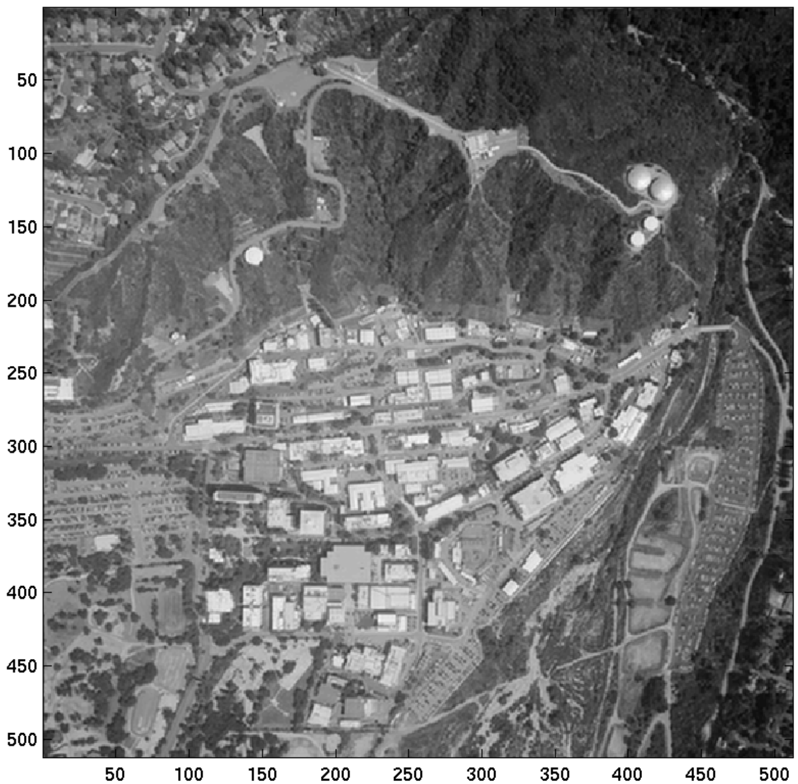
- Same poke patterns are used in both cases, but the measurement were done on different days.

- There are some differences in light path and actuator registration for point-source and extended scene, which is partially responsible for difference in OPD results.

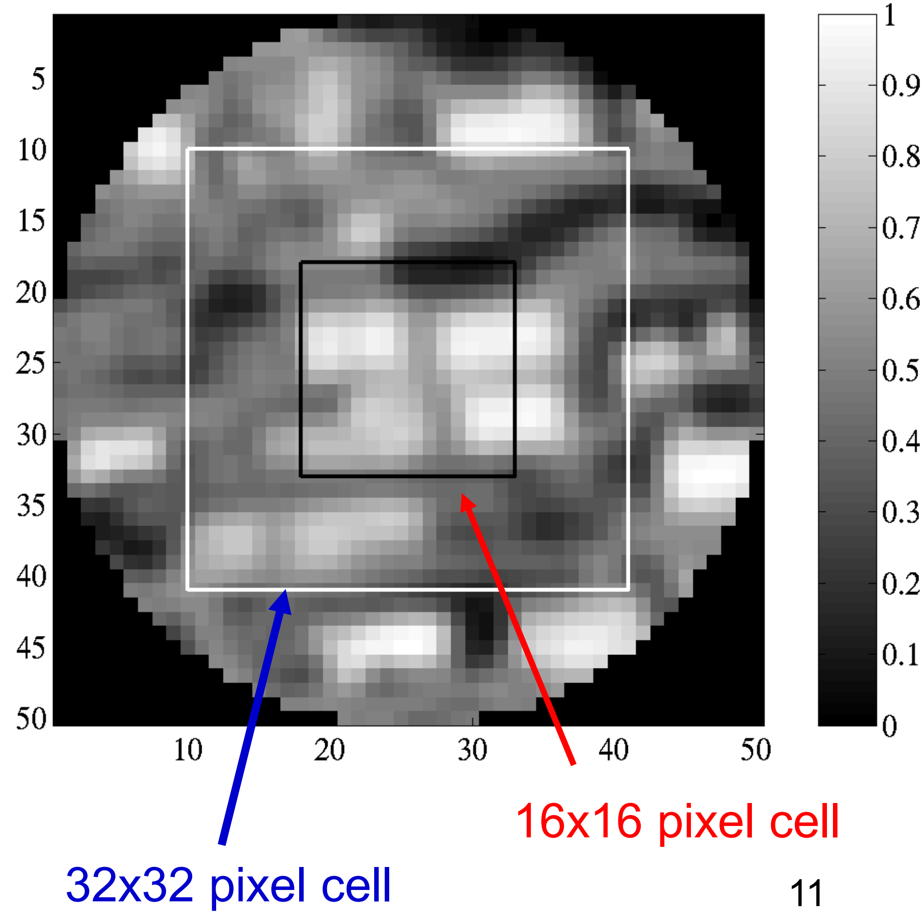


Sub-Image for Studying the Effects of Illumination Level & Background

Original Scene



One Sub-Image



How Noise is Simulated?

Normalize the sub-image image G(m,n):

$$0 \leq G(m,n) \leq 1$$

Include photon level (γ_s) and background (γ_b) ,

I_0 = 50,000 e/pix

$$S_e(m,n) = I_0\left[\, \gamma_b + \gamma_s G(m,n) \,\right]$$

Add noise: ***N = Poisson + Readout + Dark-current***

Readout = 40 e/pix, Dark-current = 125 e/sec/pix

Exposure time = 0.1 sec

Readout & Dark-current → White Gaussian noise

$$S_{en}(m,n) = S_e(m,n) + N[m,n,S_e(m,n)]$$

Digitize the image for a 12-bit SH camera:

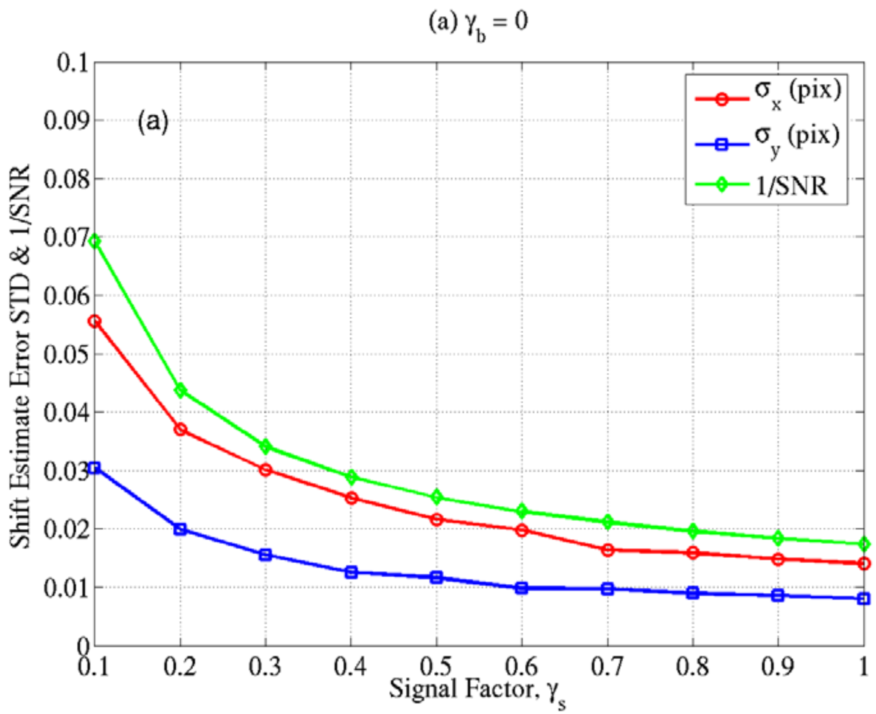
$$S_{dnf} = round\left[\, S_e \times 4095 / I_0 \,\right]$$
$$S_{dn} = round\left[\, S_{en} \times 4095 / I_0 \,\right]$$

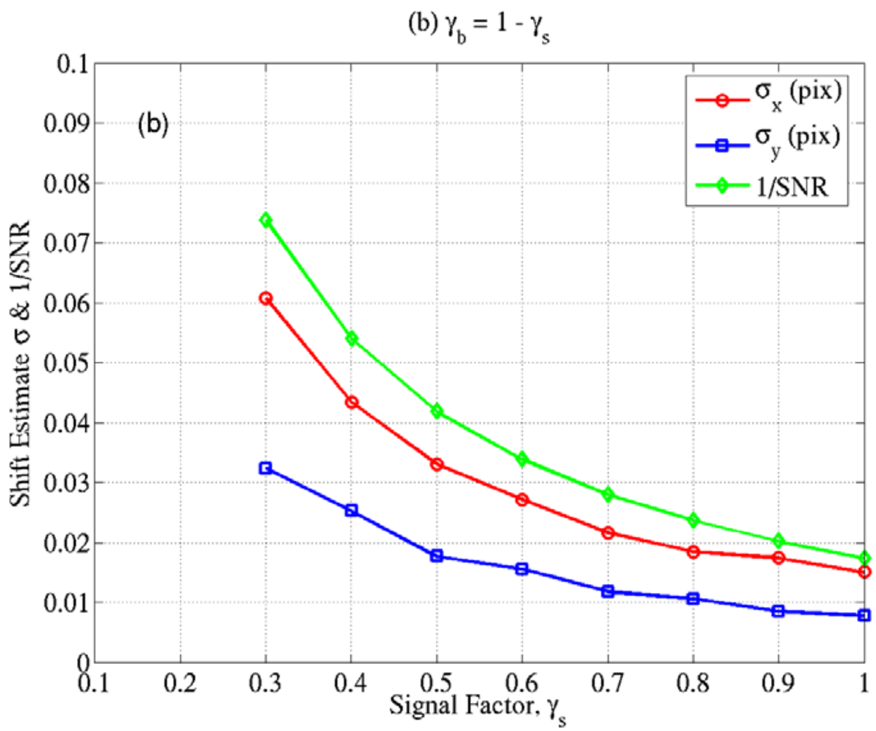
SNR calculated from (stdev = standard deviation)

$$SNR = \frac{stdev(S_{dnf})}{stdev(S_{dn} - S_{dnf})}$$

Effects of Illumination Level & Background—Zero-Shift Case

- Obtained from a single cell. Each data point was obtained from 500 noise realizations.
- The $\gamma_s = \gamma_b = 0.5$ produces the same result as $g_b = 0$ & $\gamma_s = 0.25$





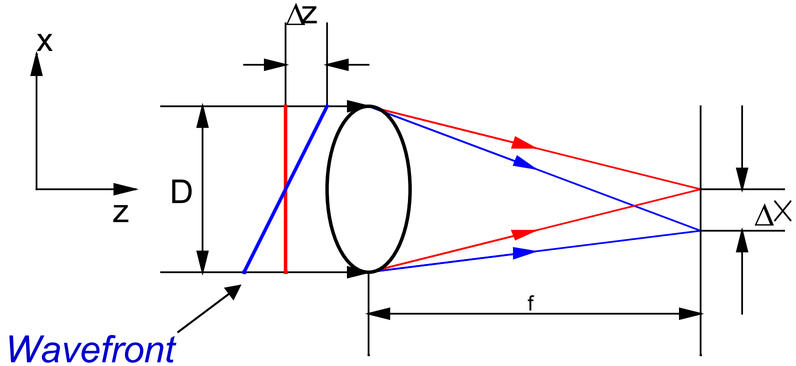
Effects of Illumination Level & Background—Zero-Shift Case

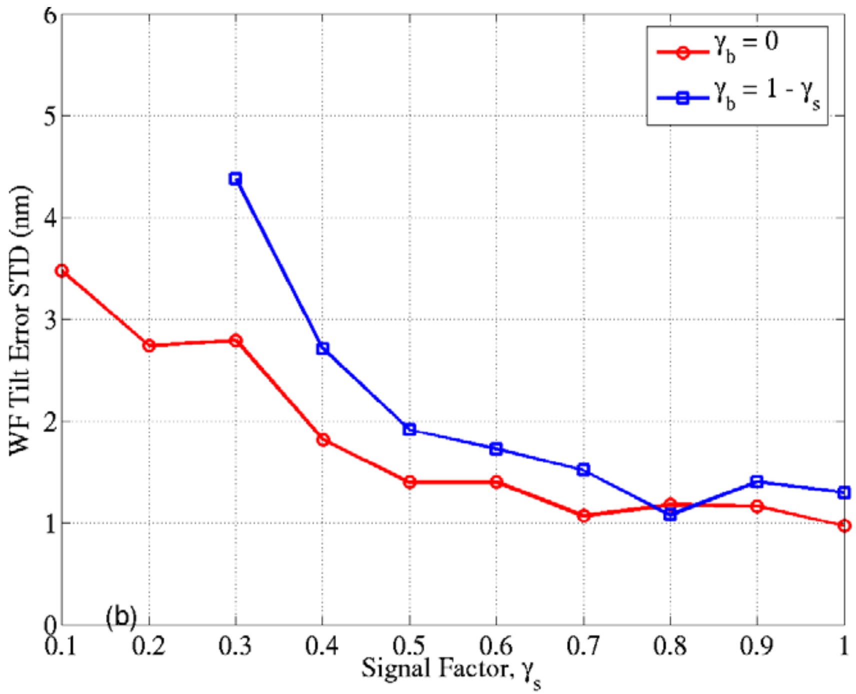
Hexagonal lenslet pitch = 300μm

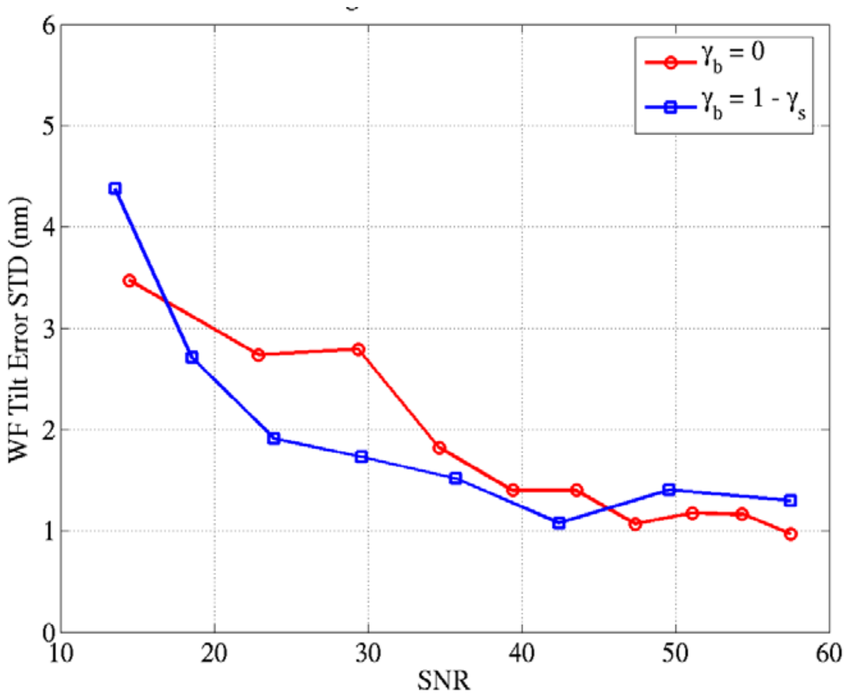
F/# = 25.3

λ = 650 nm

Pixel size = 9 μm

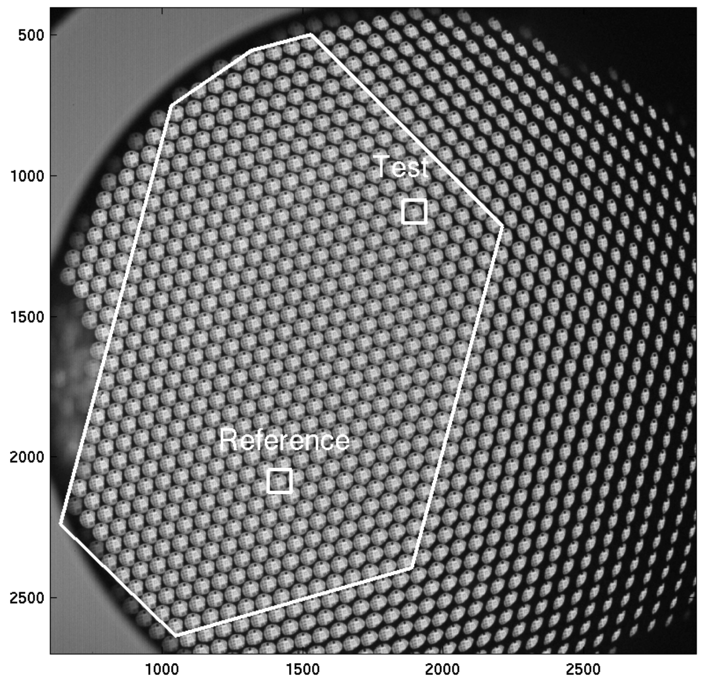




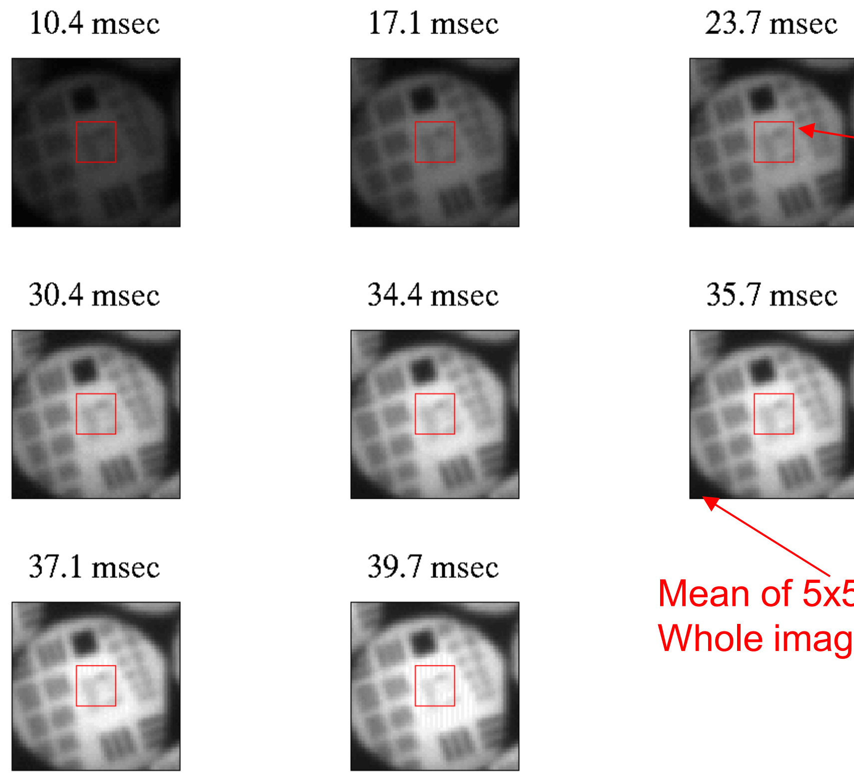


Effects of Illumination Level & Background — Real Data

SH camera image taken at JPL



8 Frames were taken with different integration times



16x16 pix cell

Mean of 5x5 pix area =
Whole image background

Sub-image parameters were obtained from the 16x16 pix cells, $s_i(m,n)$, as follows:

Background: γ_b = Min[si(m,n)]

Illumination Level: γ_s = Max[si(m,n)] – Min[si(m,n)] = Max(si(m,n)] - γ_b

Parameters of Measured Extended-Scene Sub-Images

- Sub-image parameters were obtained from 400 cells having a size of 16x16 pix

  Si(m,n) = 16x16 pix cell

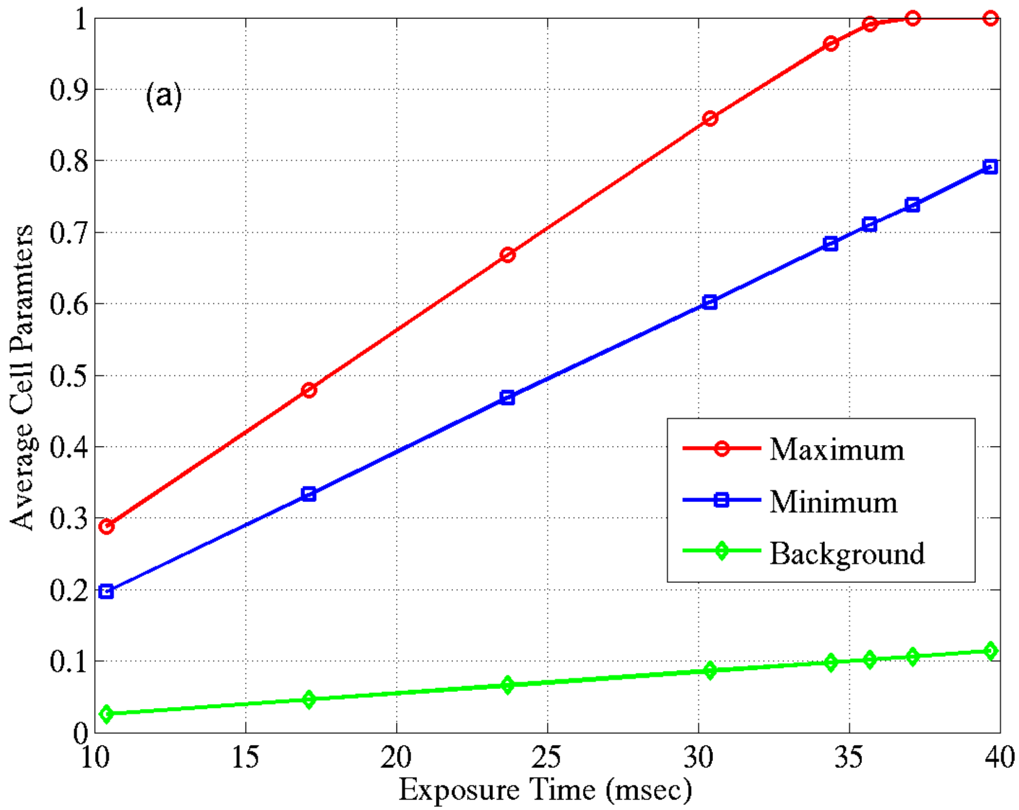
  Background: γ_b = Min[si(m,n)]

  Illumination Level: γ_s = Max[si(m,n)] – Min[si(m,n)] = Max(si(m,n)] - γ_b

- From the graph on the right:

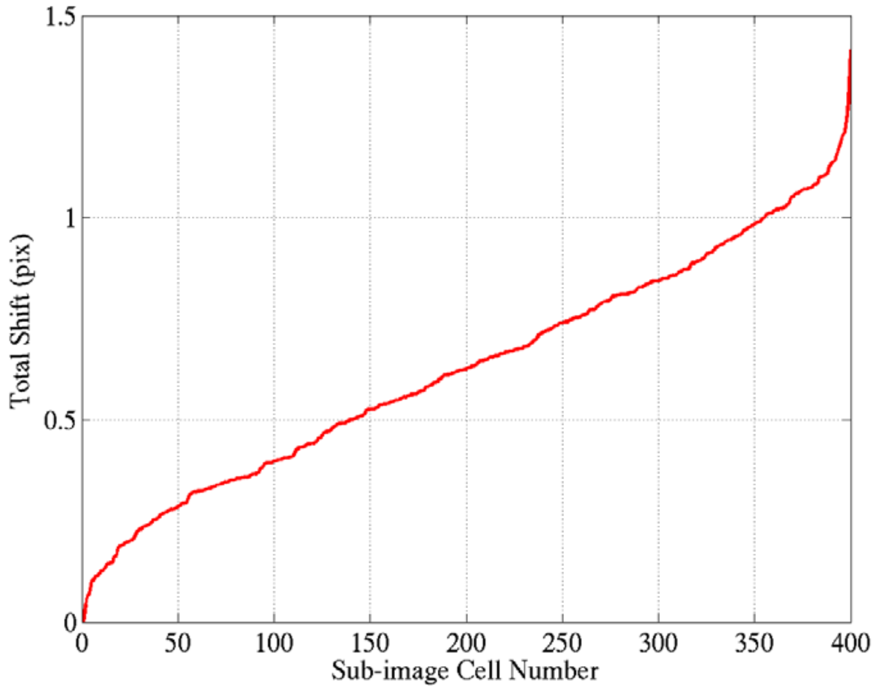
$$\gamma_b = 2.54\gamma_s - 0.04$$

- Will use the shift estimate @ 35.7 msec as a reference

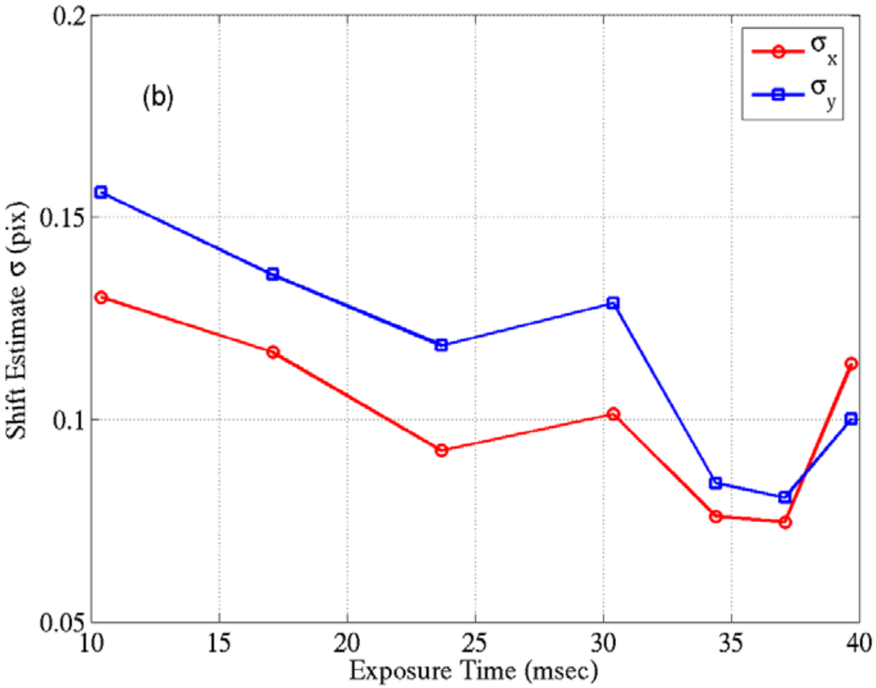


Shift Estimate Error versus Exposure Time

Sorted image shifts of 400 cells relative to their reference (exposure time = 39.7 msec)



Shift estimate error STD (σ) versus exposure time



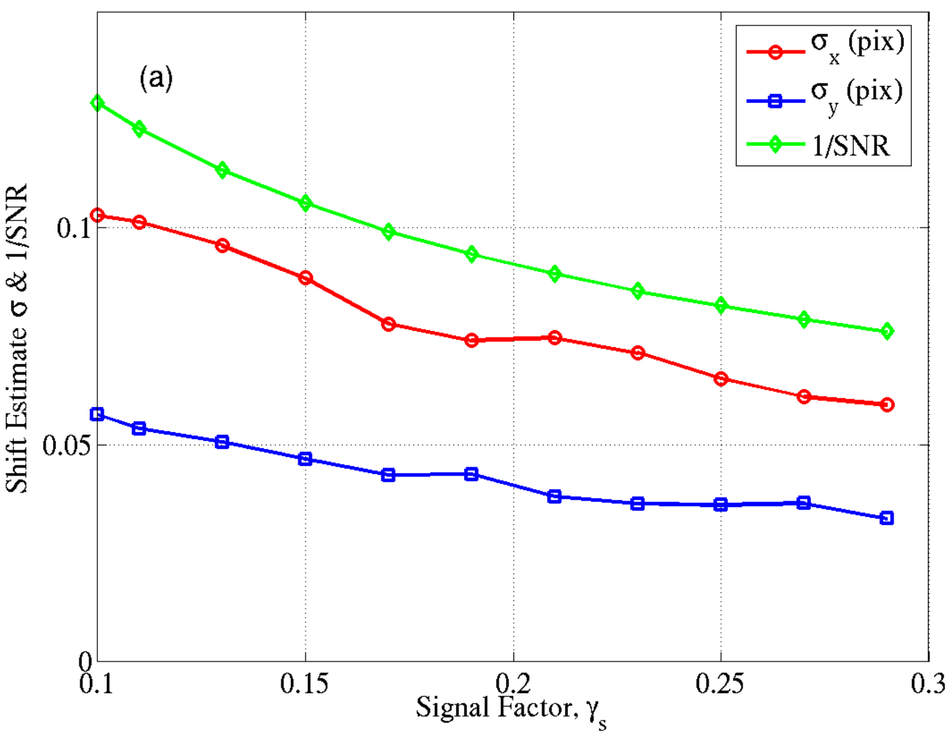
Shift estimate error (right-plot) = (Shift estimate @ T = Ti) – (Shift estimate @ T = 35.7 msec)

Effects of Illumination Level & Background

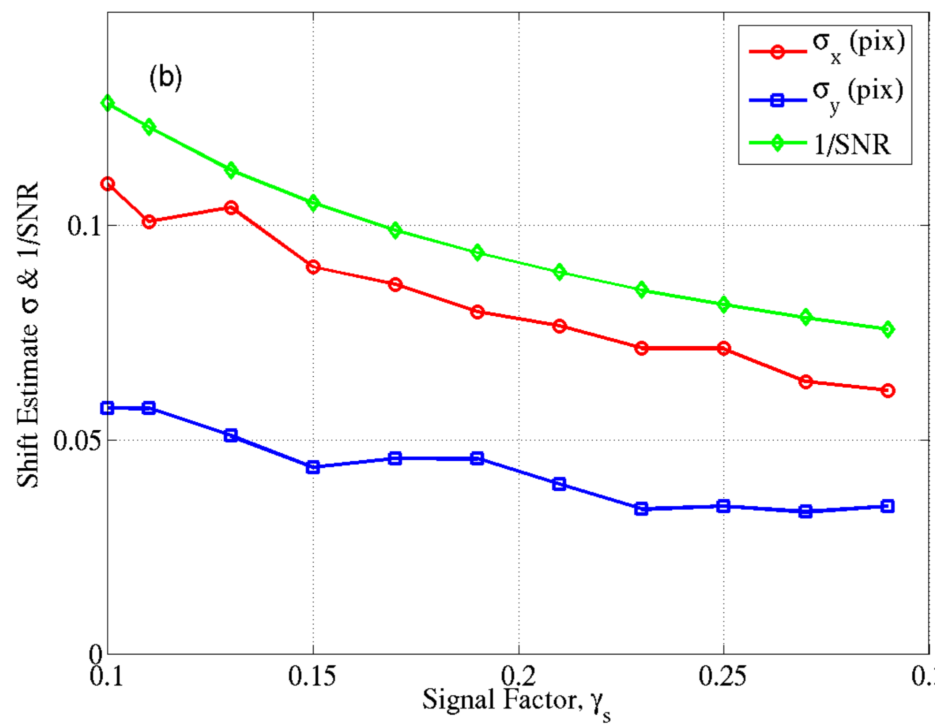
- Obtained from a single cell. Each data point was obtained from 500 noise realizations.

$$\gamma_b = 2.54\gamma_s - 0.04$$

Actual Shift Δx = 0 pix (Zero-shift case)



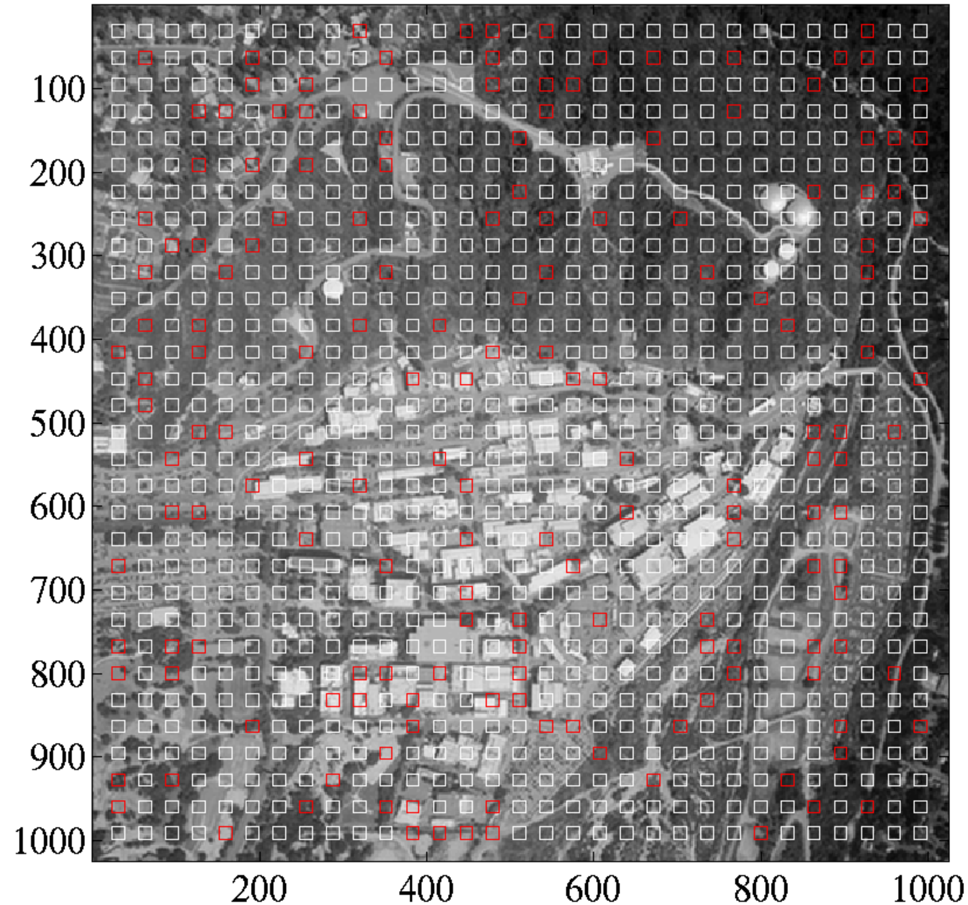
Actual Shift Δx = 3 pix



Scene Content: Good or Bad?

- Produced xxx cells of 32x32 pix from one satellite image.
- Used each original cell as the reference, and its shifted version as a test cell. Shift Δx = 5 pix.
- Obtained shift estimates using ACC
- The "red" cells worked, but the "white" did not

Red: Good Cells when Max-Δx = 5pix. Total = 167



- Tried to correlate "good" and "bad" cells with the following image quality parameters:
  - Mean-Squared Error (MSI)
  - Modified Fisher-Information (MFI)
  - RMS-Contrast
  - Image Sharpness
  - Visibility or Contrast (Fourier-domain)
- But no correlation was found.

Image Quality Failsafe Test

1. Choose a reference and a test cells from 2 well separated locations
2. Shift test cell Δx = -5, -4, …, +5 pix and Δy = -5, -4, …, +5 pix, respectively
3. Estimate the above shifts using ACC algorithm
4. Accept the image if the maximum shift estimate error is less than a pre-determined value. For example, 0.05 pix. Otherwise reject it.

SH Camera Image

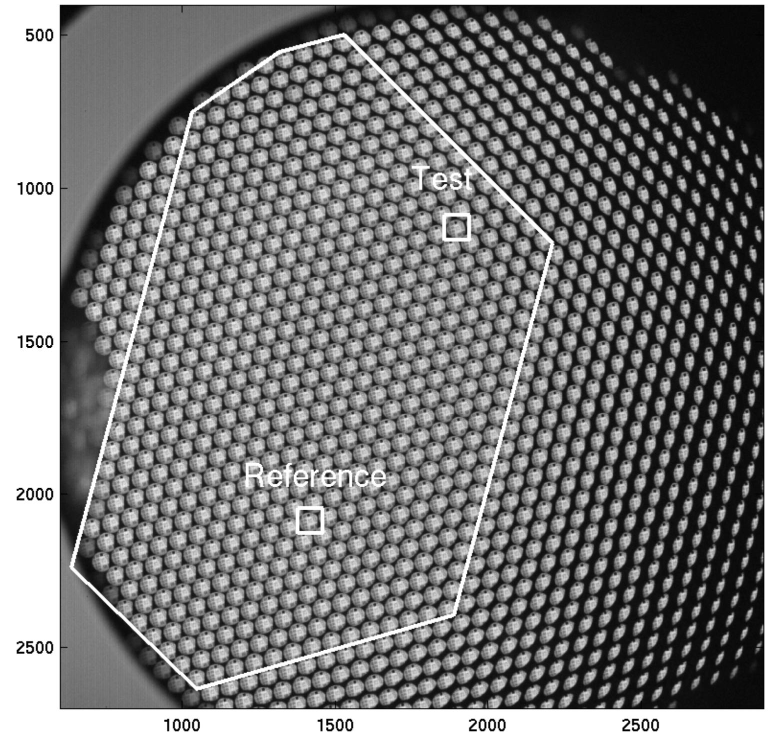
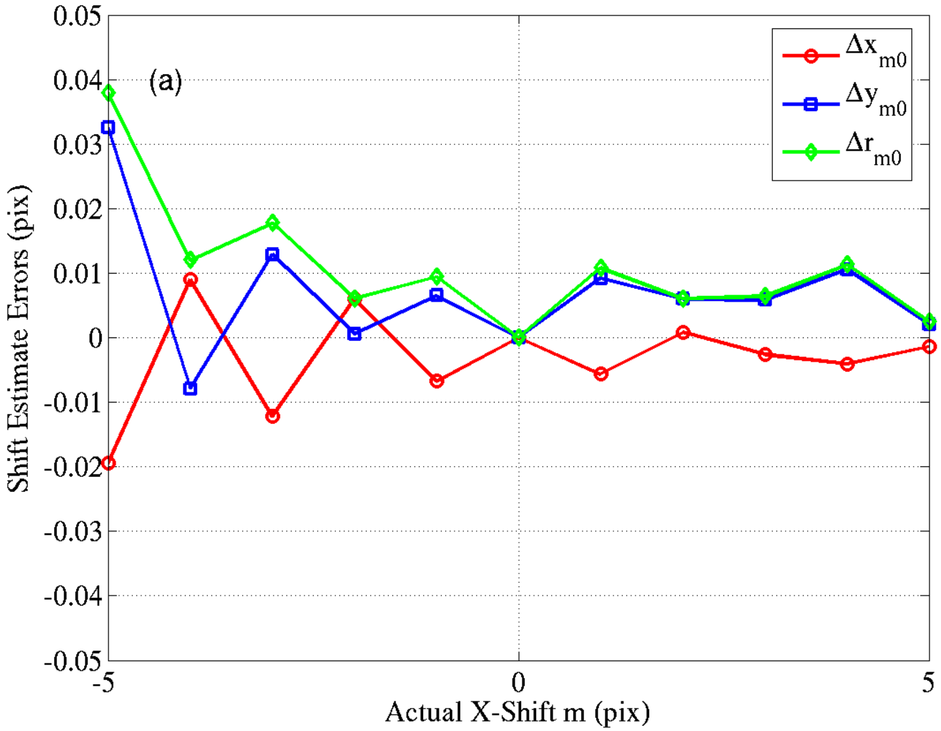


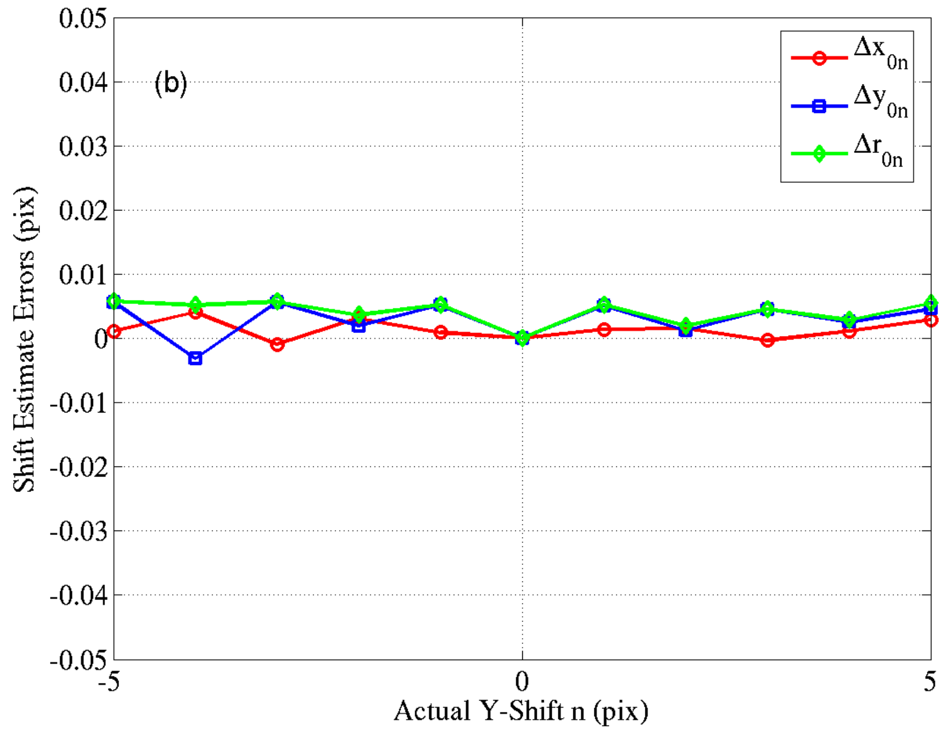
Image Quality Failsafe Test: An Example

- For the reference and the test cells shown in the previous page
- If the acceptance criterion is Δr = 0.05 pix in both directions, then this image is acceptable

Shifting the test cell in the x-direction



Shifting the test cell in the y-direction

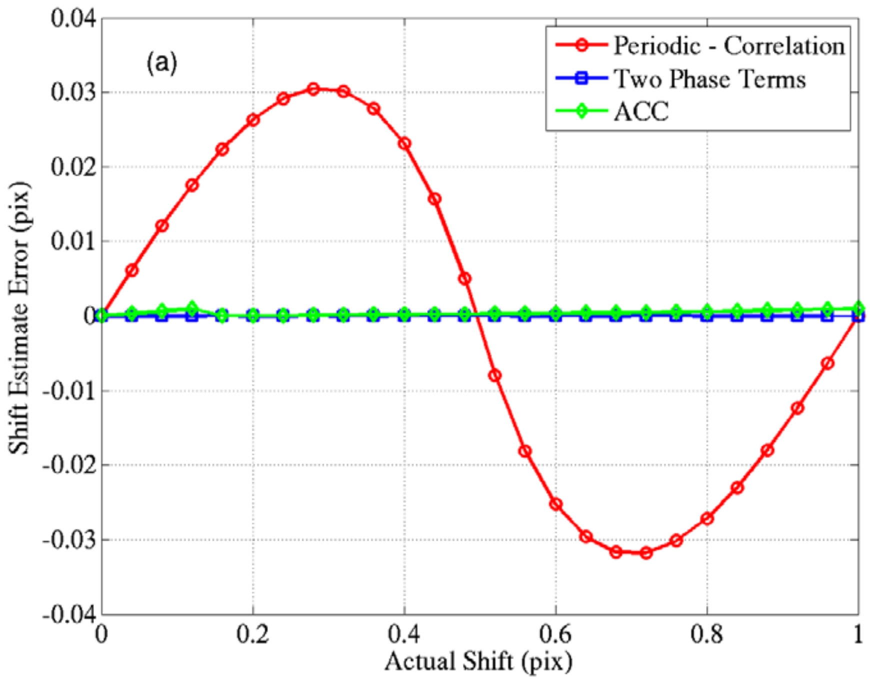


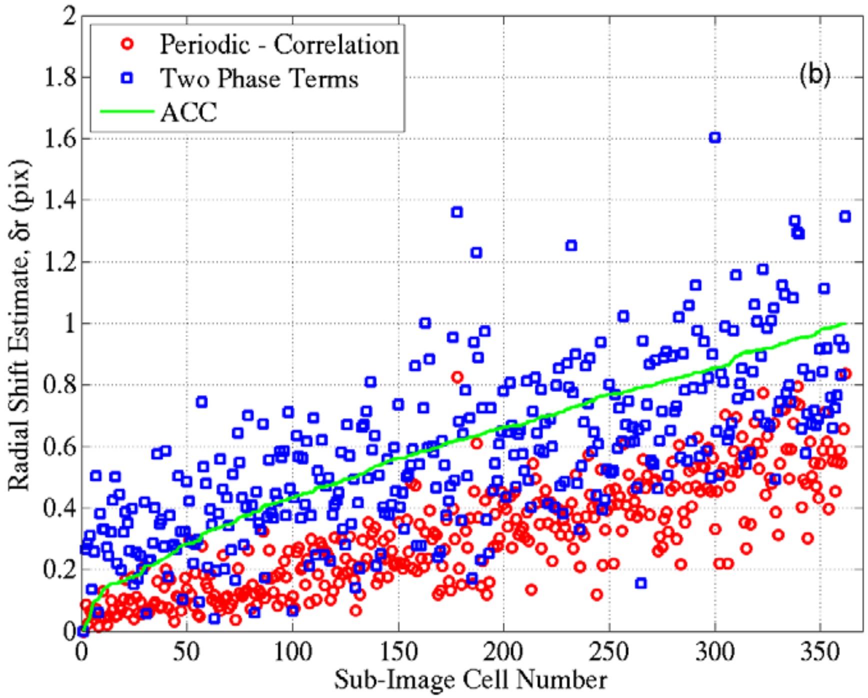
Comparison of Different Approaches

- "Periodic-Correlation" by Lisa Poyneer (2003)
- "Two Phase Terms" by Knutsson & Peterson (2005)
- Only those cells whose ACC shift estimates are < 1.0 pix are chosen on the right-plot
- "Periodic-corr." under-estimates the shift, and "Two Phase Terms" exhibits large deviation from ACC

To convince ourselves; from single cell

From the testbed data measured with T = 35.7 msec

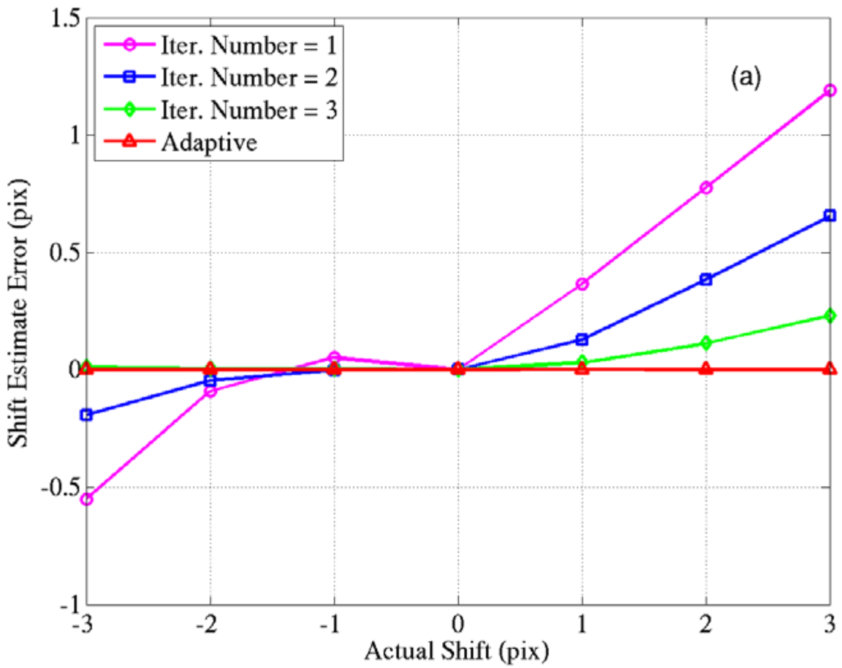




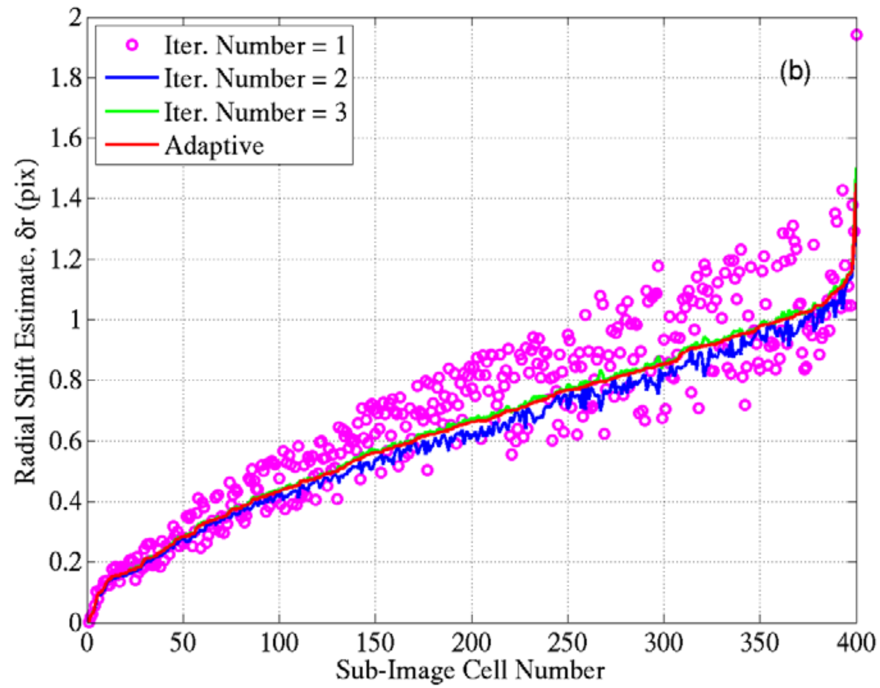
Comparison of Different Approaches (cont.)

- For JPL testbed data, a phase slope-fitting iteration number of 2 works much better than iteration = 1, and iteration = 3 produces fairly accurate estimates

From a single cell



From the testbed data measured with T = 35.7 msec



Summary

- We have studied the effects of extended scene image illumination level, background, noise and scene content on the shift estimate accuracy
- In the zero-shift case
  - Shift estimate error is proportional to 1/SNR
  - When background = 0, shift estimate error < 0.05 pix in both directions when illumination >15% full-well
  - The $\gamma_b = \gamma_s = \gamma_0$ and $\gamma_b = 0$, $\gamma_s = \gamma_0/2$ gives similar results
  - The results of the measured image data are comparable to predictions

- When we analyzed the JPL testbed image data (with low quality),
  - "Periodic-Correlation" method under-estimated the image shifts, and "Two Phase Terms" approach gave results that randomly deviate by large amounts from those obtained with the ACC algorithm
  - The ACC algorithm worked well with phase slope-fitting iteration number of 2 or greater, but not with iteration = 1

- We have proposed a highly reliable image-quality failsafe criterion for the ACC algorithm