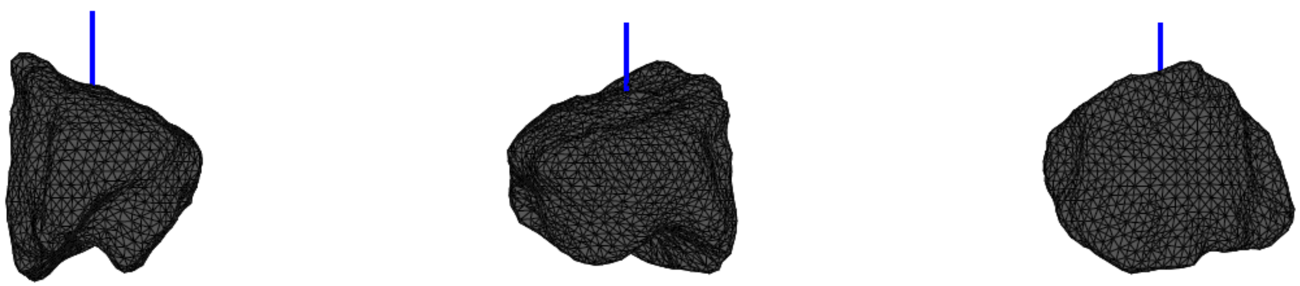
Identification of Non-Chaotic Terminator Orbits near 6489 Golevka



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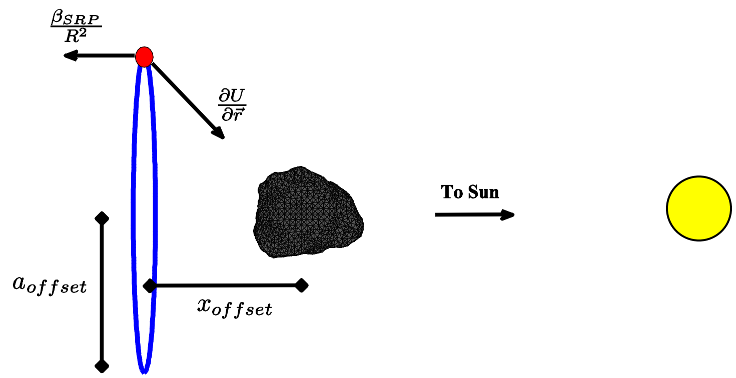
Terminator Orbits

- Terminator orbits are a class of orbits known to exhibit stable behavior when solar radiation pressure (SRP) is a significant perturbation to the orbit dynamics.
  - Very applicable near small asteroids and comets (roughly less than 10 km diameter), where gravity and SRP often are of equal order of magnitude

- Here, a procedure is applied for assessing the *long-term* stability properties of terminator orbits near a *specific* small body of interest.
  - Demonstration here uses a model of asteroid 6489 Golevka
  - EOM include effects of solar gravity, solar radiation pressure, eccentric small-body orbit, arbitrary small-body gravitational potential, and arbitrary (but constant) small-body rotation pole.

Geometry of a Terminator Orbit



- Mission applications
  - Beacon or relay missions
  - Missions that require extended loiter or hibernation time
  - Missions concerned about an extended “safe” or planetary protection
  - High-precision gravity estimation

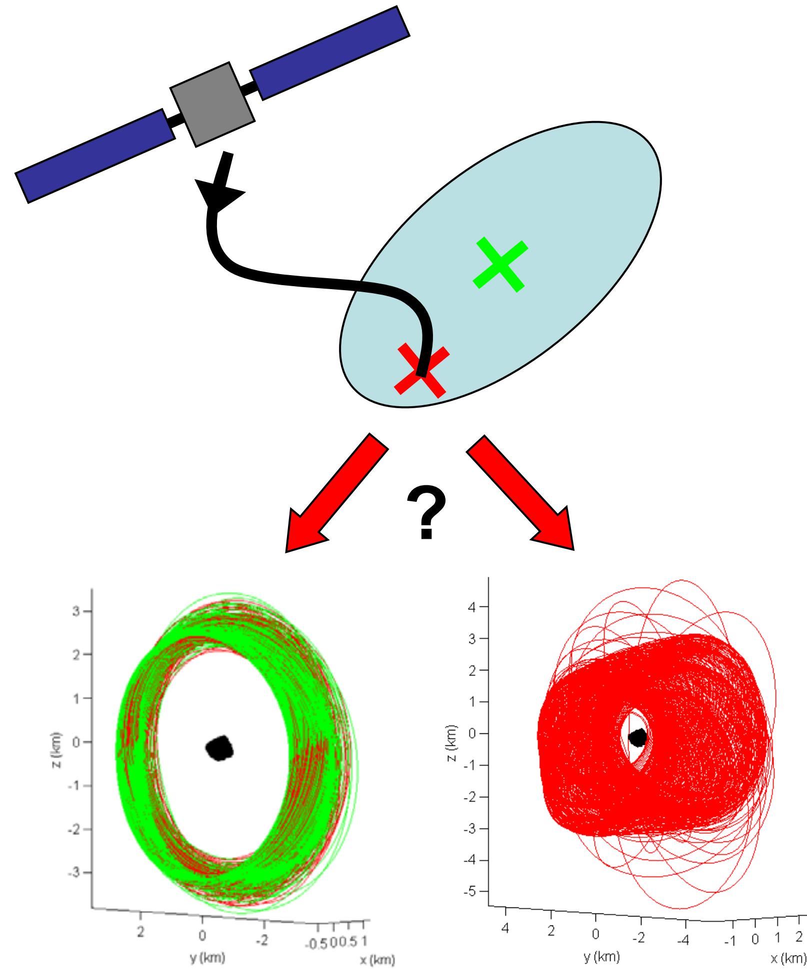
- Other applications
  - Indentification of stable dust, impact ejecta, and moon orbits

Regular vs. Chaotic Motion

- Desired characteristics of long-term stable orbits
  - Trajectory behavior is robust with respect to uncertainty in initial state and system parameters
  - Does not impact the small body
  - Does not escape the small body vicinity

- We develop a procedure for identifying such orbits through use of periodic orbit and chaoticity analyses.
  - The methodology can also be used to identify other types of small-body orbits with similar characteristics.

- *Chaos* in dynamical systems can be defined (loosely) as an extreme sensitivity to initial conditions.
  - Orbits that evolve very differently for different states in the insertion uncertainty ellipsoid are undesirable. These are *CHAOTIC* orbits. Chaotic orbits have a non-zero probability of impacting or escaping the small body.
  - It is desirable for the orbits resulting from initial uncertainty distribution exhibit behavior similar to the target orbit. Such orbits exhibit *REGULAR* or *LONG-TERM STABLE* motion.



Regular Motion **Chaotic Motion**

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How do we find regular motion??

- Trajectories in an integrable dynamical system, like the two-body equations of motion, all exhibit regular motion
- Perturbing this dynamical system results in chaotic dynamics; however, KAM theory says that regions of regular motion may persist in the vicinity of some periodic orbits.
  - Perturbations here include solar effects and irregular gravity field

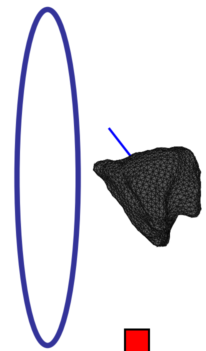
  ➔ *Look for regular motion in the vicinity of periodic orbits!*

- We hypothesized that long-term stable (i.e., regular) motion may be found near periodic terminator orbits in the Hill dynamics!

- Step 1: Identify periodic orbits
  - We look in the autonomous Hill three-body equations of motion (with SRP)
  - These orbits have appropriate timescales for a study of spacecraft dynamics.

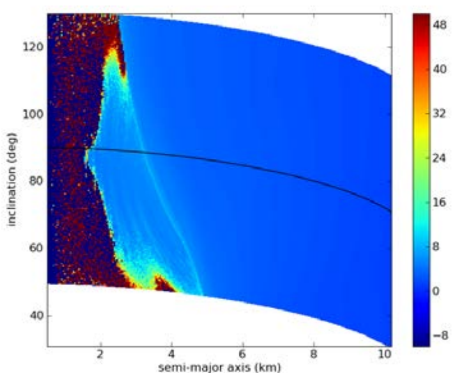
- Step 2: Use higher-fidelity dynamical model and measure chaoticity of trajectories near the periodic orbits
  - Add an irregular small-body gravity field and an eccentric small-body orbit around the Sun.
  - Integrate nearby dynamics and compute the *Fast Lyapunov Indicator* (FLI) measure of chaoticity

- Step 3: Plot FLI values to distinguish between regular and chaotic motion.
  - Gives overview of the available dynamics



High-Fidelity

Dynamics

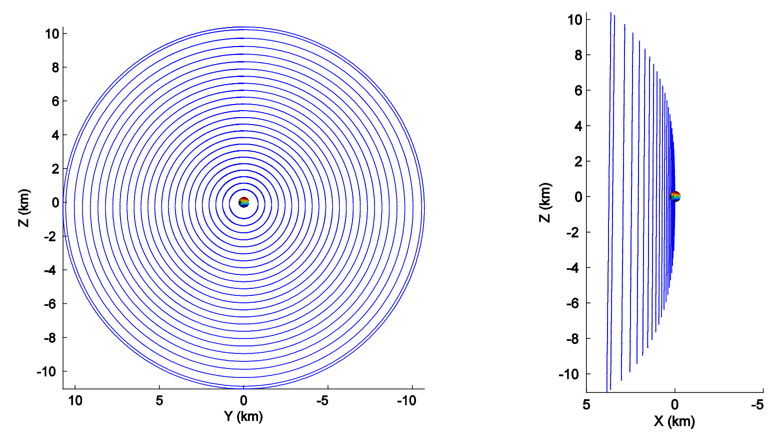


Periodic Orbits near Golevka

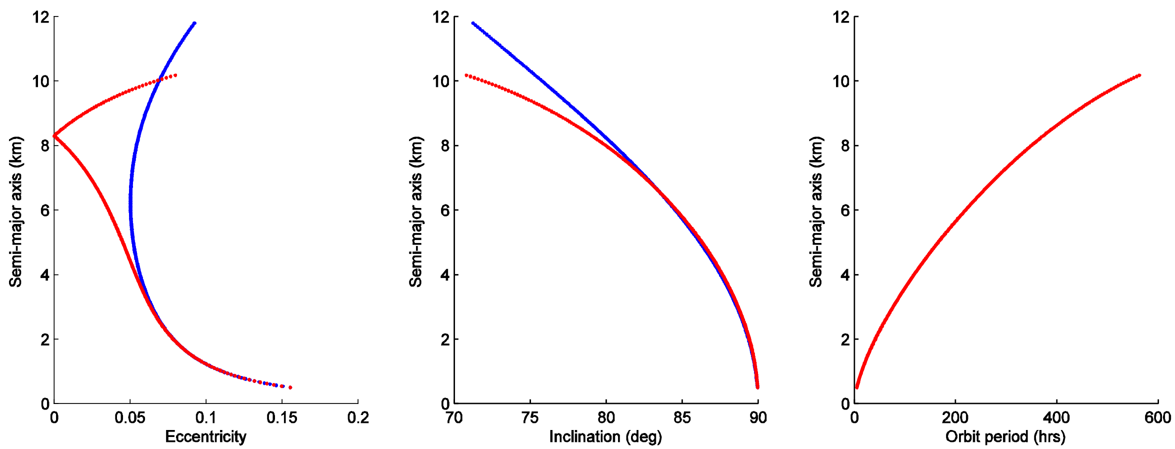
- A continuous family of periodic terminator orbits can be identified in the autonomous Hill 3BP with a flat-plate SRP model.
  - Equations parameterized by: SRP strength (depends on s/c mass and area), heliocentric orbit, and small-body GM.
  - Family can be parameterized by semi-major axis or Jacobi constant

- A numerical differential correction and continuation approach of the discrete dynamics on a Poincare surface is used to find these orbits.

Periodic Orbits near Golevka



Periodic Terminator Orbit Elements at Golevka



Numerically computed initial state   Mean orbit element solution

Measuring Chaoticity

- Chaotic motion is characterized by an exponential divergence of adjacent initial states.
- We can measure the rate of divergence using the *Fast Lyapunov Indicator*

$$FLI = \sup_{\tau \leq T} \ln ||\Psi(\tau, t_0)||_n$$

- The matrix ψ is a "*fundamental matrix*" obtained by integrating the variational equations.
  - The State Transition Matrix could be used for ψ
- The FLI is increases monotonically.
- The FLI permits characterization of the dynamics with a finite integration interval T
  - In a given interval T, not all chaotic motion will make itself known (e.g., high order resonances). T must be tuned to a duration consistent with the dynamics of interest.
- Other chaoticity indicators include the Maximum Lyapunov Exponent (MLE) and the Lyapunov Characteristic Exponent (LCE).

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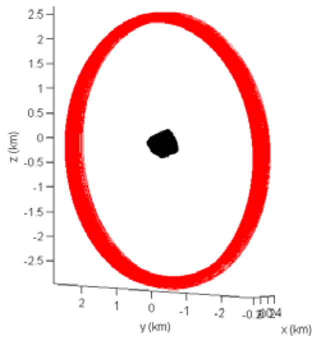
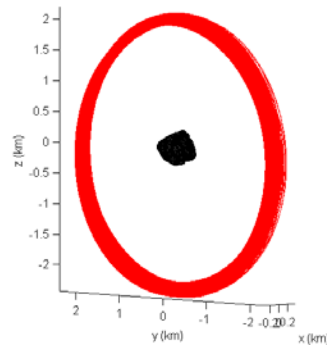
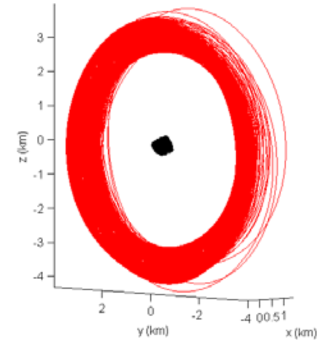
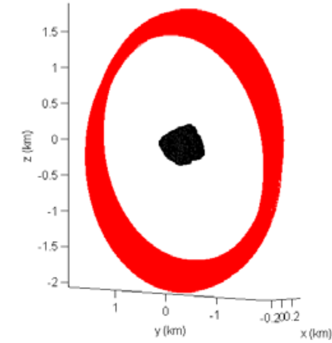
Sample Terminator Trajectories

Regular Motion:

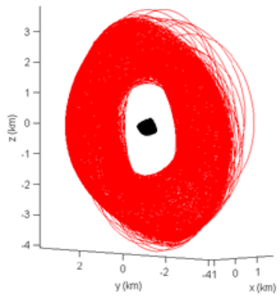




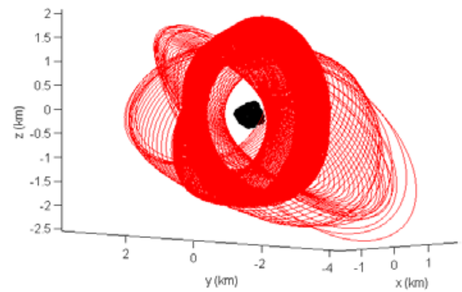




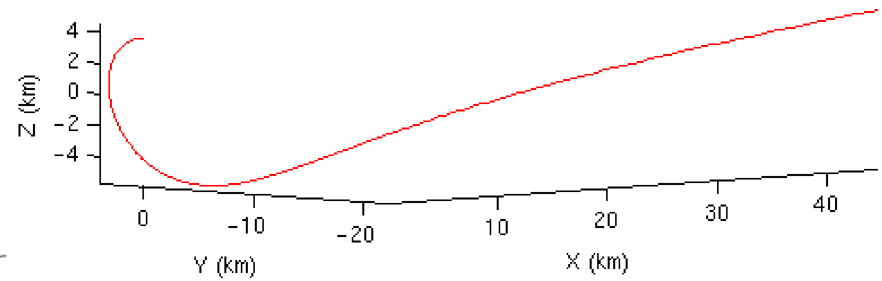
Less-Regular



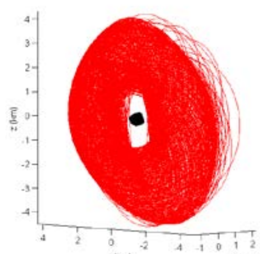
Impact

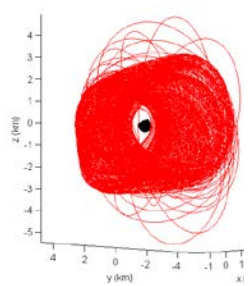


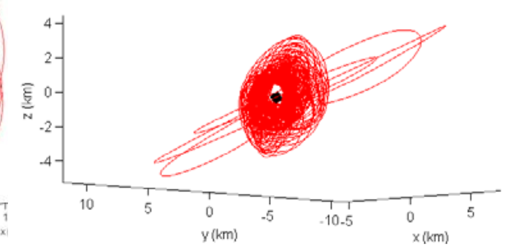
Escape

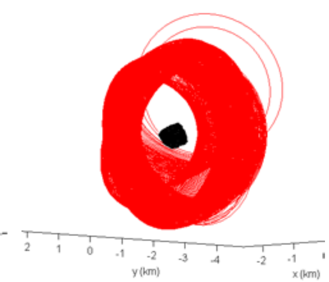


Chaotic Motion:









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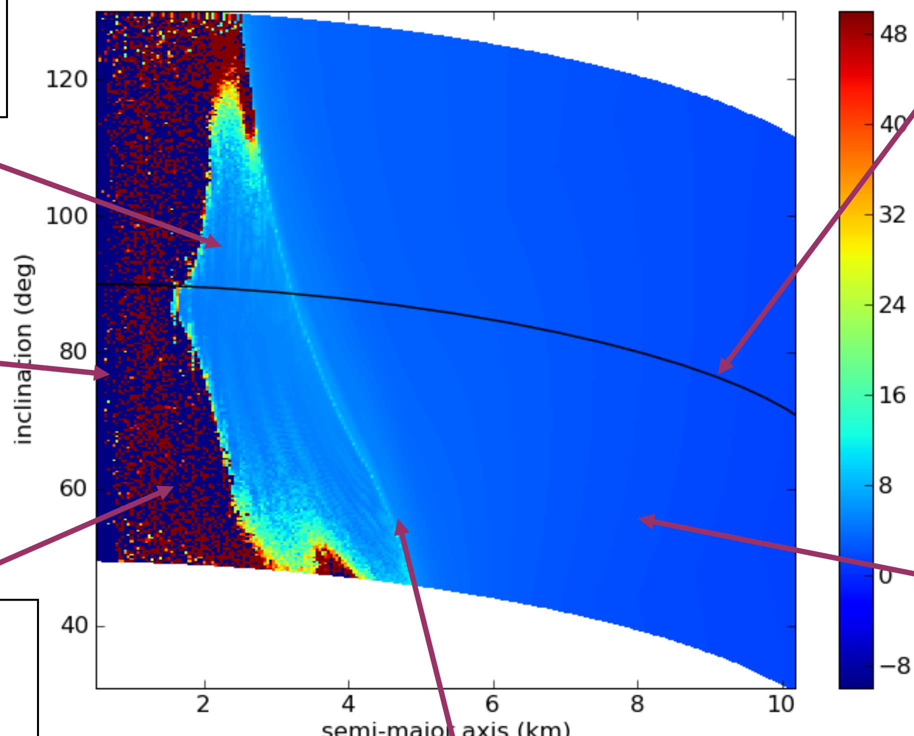
Anatomy of a Chaoticity Map

- A “*chaoticity map*” can be plotted by computing the FLI on a range of initial conditions.
  - Here, 2-D maps are created by varying one orbit element for each periodic orbit found.



__Regular Motion__
- Smooth Variation in the FLI
- Relatively low FLI values

__Nominally Impacting__
- Shown in dark blue

__Chaotic Motion__
- Abrupt Variation in the FLI
- Relatively large FLI values
- Some FLI values off the scale

__Escape Boundary__

__Periodic Orbit Solutions__
- Black line through the center

__Escaping__
- Smoothly varying FLI value
- Beyond the escape boundary

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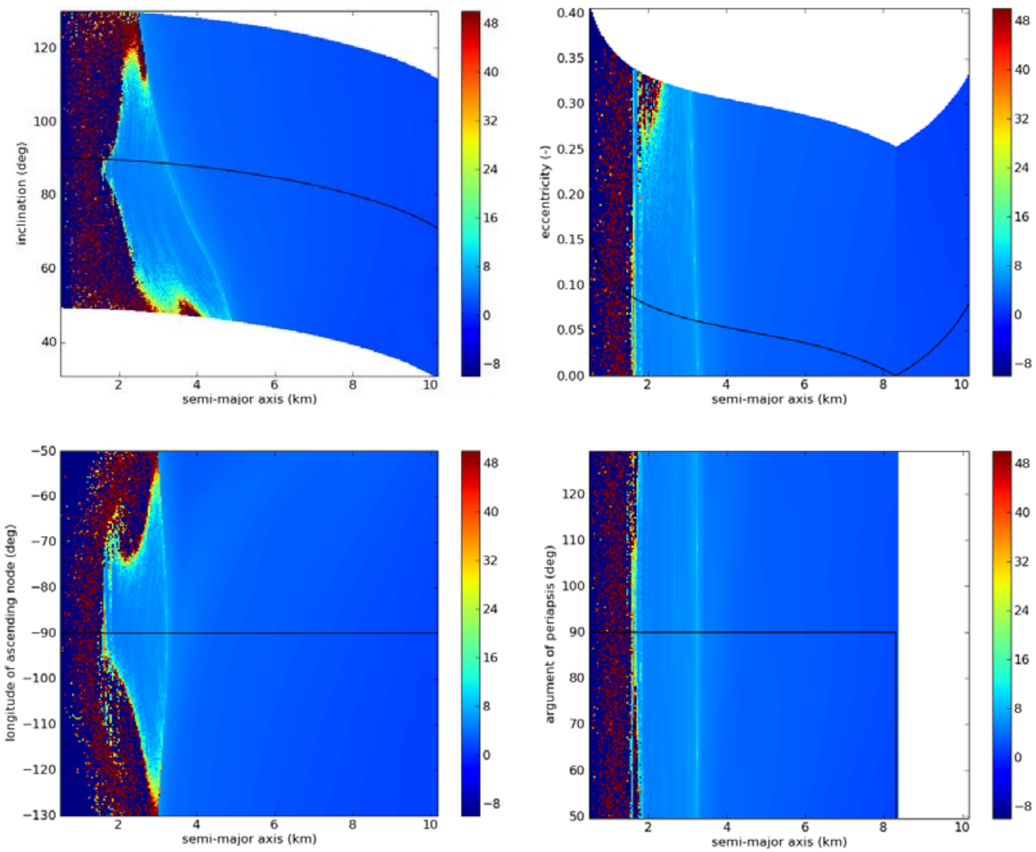
Study of Terminator Orbits near 6489 Golevka

- Our procedure for finding long-term stable terminator orbits can be applied to any small body/spacecraft combination.
  - Here, 6489 Golevka has been chosen as an example. The orbiting spacecraft is 800 kg with a 40 m^2 area.



A region of regular motion must be 6-D! Regular behavior must exist for a given semi-major axis in all maps!

Map characteristics vary with: spacecraft mass-to-area ratio, integration time, which orbit element is varied, small-body properties, etc.

Variations in FLI value due to parameter variations can also be mapped

Comparison with Existing Terminator Orbit Analysis Tools

- This approach to studying the characteristics of terminator orbits is complementary to results from the existing analytical tools.
  - Quick search and mission characteristics provided by analytical tools
  - Periodic orbit and chaoticity analysis can provide mission refinement and detail for a small body of interest

- Key contributions:
  - Existing methods are general – this method is specific
  - Application of analytical results are limited by inherent assumptions – Domain of applicability is very broad for this method
  - Can provide information when analytical assumptions do not apply
  - Provides new information on resonant phenomenon and the extent of the stable region of motion

- Nothing is free though…. This method is numerically intensive!

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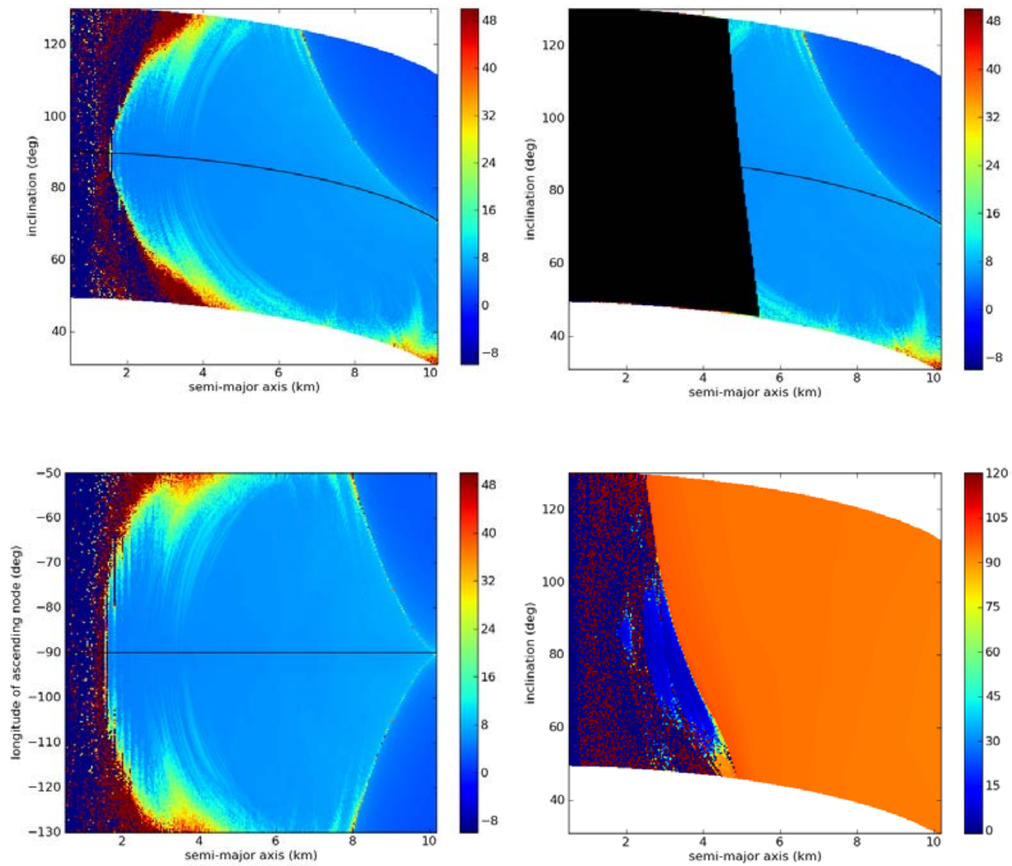
Comparison with Analytical Results for Golevka

- Maximum terminator orbit size
  - Using the escape Jacobi energy to determine maximum terminator size is found to be overly conservative for Golevka.
  - Numerically continuing the periodic orbit family is found to be an excellent way of identifying the maximum terminator orbit size.
- Minimum terminator orbit size
  - Guidance of 1.5 times the 1:1 resonance did not consider SRP… which can be significant for bodies of this size!
  - We find the minimum to vary with time and orbit elements. At best, the minimum size is found to be about 3 times the 1:1 resonance radius.
- Discussion of stability region
  - Linear stability does not give any information about the extent of the stable region.
  - Chaoticity analysis provides a quantitative measure of the size of the domain of stable motion around the periodic solution.
  - Further, chaoticity analysis identifies terminator orbits that become unstable after multiple revolutions around the Sun due to resonances.



Conclusions

- A method has been presented for assessing the long-term stability characteristics of terminator orbits near small bodies.
  - Periodic orbits are computed numerically to serve as the “backbone” of the stable terminator search space.
  - Dynamics for states adjacent to the periodic orbits are characterized using the Fast Lyapunov Indicator of chaoticity and compiled in chaoticity maps.
  - Method permits inclusion of the SRP, solar gravity, and irregular small-body gravity effects for the specific environment of interest.
- This method of analysis complements existing findings.
  - After general characteristics are given by quick analytical methods, this method can help in generating a more detailed trajectory design.
  - This method provides information for situations where assumptions in the analytical results do not hold.
  - This method assesses the size of the region of regular/stable motion and identifies destabilizing resonances.
- If interested, please attend the presentation of our companion paper on Thursday morning!

Periodic Orbit Analysis: Bifurcations

- A bit of an aside:
  - Analysis of periodic orbit families allows bifurcating families to be identified.
  - Below is an example of a periodic orbit of a family that bifurcates from the main terminator orbit family. It repeats itself every 4 revolutions around the body.
  - For lack of space, this paper doesn't cover these orbits as much as we intended. Description and analysis of the stability properties of these results will appear in future work.