Titan Atmospheric Density Reconstruction Using Cassini Guidance, Navigation, and Control Data

Siamak Sarani†
Jet Propulsion Laboratory, California Institute of Technology

Titan, the largest moon of Saturn, is the only known moon in the Solar System with a significant atmosphere. The variation of Titan's atmospheric density with altitude is the focus of many past and present science investigations. During a low-altitude Titan flyby, attitude control thrusters on the Cassini spacecraft are fired to overcome the torque imparted on the spacecraft due to the Titan atmosphere, as well as to slew the spacecraft in order to meet the pointing needs of the science instruments. This paper describes a novel methodology that uses guidance, navigation, and control data to estimate the external torque imparted on the spacecraft due to the Titan atmosphere. The Titan atmospheric density is determined accordingly. The methodology described has already been used to reconstruct the Titan atmospheric density for all low-altitude Titan flybys of the Cassini spacecraft in 2004–2008 and most part of 2009. It will be used on all remaining low-altitude Titan flybys in the extended missions of Cassini spacecraft. With some modifications, the described methodology could also be used to reconstruct the Titan atmosphere density using data to be collected by a planned Titan Orbiter.

Nomenclature

A_{Proj}	=	spacecraft projected area, m^2
C_D	=	drag coefficient
CM_x, CM_y, CM_z	=	X, Y, and Z components of the spacecraft center of mass, m
C_{RWA}^{SC}	=	transformation matrix from reaction wheel frame to spacecraft's fixed frame
h	=	Titan flyby altitude, km
h_{TCA}	=	altitude of closest approach for Titan flyby, km
h_s	=	scale height, km
$\vec{H}_{RWA}$	=	total angular momentum vector of the three reaction wheels, Nms
I	=	spacecraft's inertia tensor, kgm^2
$\vec{r}_{CM}$	=	position vector for the spacecraft's center of mass relative to the origin of the spacecraft coordinate, m
$\vec{r}_{CP}$	=	position vector for the spacecraft's center of pressure relative to the origin of the spacecraft coordinate, m
t	=	time, s
t_{TCA}	=	time of closest approach for Titan flyby, s
$\vec{T}_{Atm}$	=	drag torque imparted on spacecraft due to Titan atmosphere, Nm
$\vec{T}_{GG}$	=	gravity-gradient torque imparted by Titan on Cassini, Nm
$\vec{T}_{Mag}$	=	magnetic torque imparted on spacecraft due to Saturn/Titan magnetic fields, Nm
$\vec{T}_{RTG}$	=	torque imparted on the spacecraft due to radioisotope thermoelectric generator (RTG) thermal radiation, Nm
$\vec{T}_{RWA}$	=	reaction torque imparted on spacecraft by three reaction wheels, Nm
$\vec{T}_{Sol}$	=	solar radiation torque imparted on spacecraft, Nm
$\vec{T}_{Thruster}$	=	torque imparted on spacecraft due to RCS thruster firings, Nm
$\hat{u}_V$	=	unit vector of spacecraft velocity vector as expressed in a spacecraft body frame
V	=	magnitude of the body-fixed spacecraft velocity, m/s
$\vec{X}$	=	coefficient vector for accumulated angular momentum fitting function
x_1, x_2, x_3, x_4	=	components of coefficient vector for accumulated angular momentum fitting function
X, Y, Z	=	spacecraft body axes

† Senior Member of Technical Staff, Guidance and Control Section, Jet Propulsion Laboratory, Mail Stop 230-104, 4800 Oak Grove Drive, Pasadena, California 91109-8099, USA. Siamak.Sarani@jpl.nasa.gov

$\vec{\varepsilon}$	=	vector sum of the torques due to all possibly existing thruster leaks and all other un-modeled torques, Nm
ε_L	=	altitude-dependent, lower-bound dispersion in the best fit density, %
ε_U	=	altitude-dependent, upper-bound dispersion in the best fit density, %
ρ	=	Titan atmospheric density, kg/m^3
$\rho_{Best\text{-}Fit}$	=	best-fit density as a function of altitude, kg/m^3
ρ_{HASI}	=	density at 1000 km reported by the Huygens Atmospheric Structure Instrument, kg/m^3
ρ_{TCA}	=	atmospheric density at closest approach for Titan flyby, kg/m^3
σ	=	standard deviation
τ	=	a dummy variable representing time, s
$\vec{T}_{\varepsilon}$	=	vector sum of external torques imparted on spacecraft but not accounted for in $\vec{T}_{Thruster}$, $\vec{T}_{RWA}$, and $\vec{T}_{Atm}$, Nm
$\vec{\omega}$	=	spacecraft's angular rate vector, rad/s
$\dot{\vec{\omega}}$	=	spacecraft's angular acceleration vector, rad/s^2

Cassini Continues Making Exciting Discoveries

Cassini, the most sophisticated interplanetary spacecraft ever launched, proceeds to conduct further scientific investigations of Saturn and its rings, magnetosphere, icy satellites, and Titan, the Saturn's largest moon[1] (Fig. 1). Cassini continues making new and exciting discoveries in the ringed world. The international Cassini-Huygens mission to Saturn and its moons has truly become an envoy of the humanity on the most fascinating planetary system within our solar system. This mission is a cooperative project of NASA, the European Space Agency (ESA) and the Italian Space Agency[1]. Jet Propulsion Laboratory (JPL), a division of the California Institute of Technology in Pasadena, manages the Cassini-Huygens mission for NASA's Science Mission Directorate, Washington. This successful, well-publicized mission has already accomplished its 4-year prime mission and is more than half way through its 2-year extended, Equinox mission. After the completion of the Equinox mission around September 2010, Cassini will likely start its few-year extended, Solstice mission. Besides official NASA and ESA websites, there are now public "Wikipedia" websites available for Cassini-Huygens mission and the moon Titan.

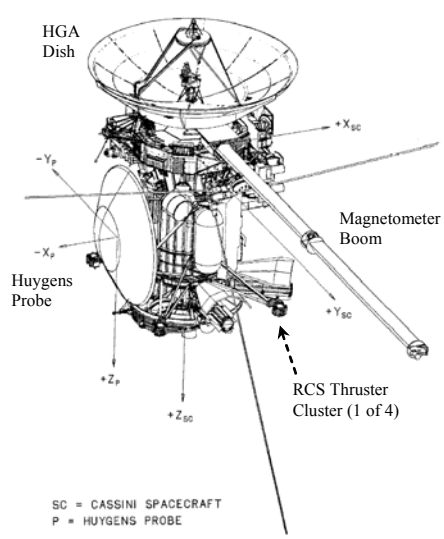

Fig. 1 Cassini orbiter with attached Huygens probe

This paper extends on the methodology reported by this author in Reference 1, which uses the Cassini guidance, navigation, and control data to reconstruct the external atmospheric torque imparted on the spacecraft and the Titan atmospheric density, accordingly. The effectiveness of this technique has been proven and demonstrated in 31 past low-altitude flybys of Titan thus far. For technical details of this

technique, the reader is encouraged to consult Reference 1. An overview of the design and flight performance of the Cassini Attitude Control System is described in Reference 2.

Lifting the Veil on Titan

Titan continues to baffle scientists. Being the largest moon of Saturn and the second largest moon in the Solar System, Titan possesses a significant atmosphere (Figs. 2 and 3), an order of magnitude thicker than Earth's, and great many atmospheric and surface features, such as winds, lightning activity, dunes, canals and canyons (Figs. 4 and 5), and lakes or seas. Having a radius of 2575 km, Titan is larger than both Mercury and Pluto, yet smaller than Jupiter's moon Ganymede. Titan orbits Saturn at a distance of 1.222 million kilometers, taking 15.95 days to complete one revolution. Titan average distance from Sun is 1,429 million km (9.52 AU). Its year is 29.5 Earth year and its density is 1880 kg/m^3. Titan's surface gravity is 1.35 m/s^2 and the escape velocity is from Titan's gravity is 2.64 km/s.

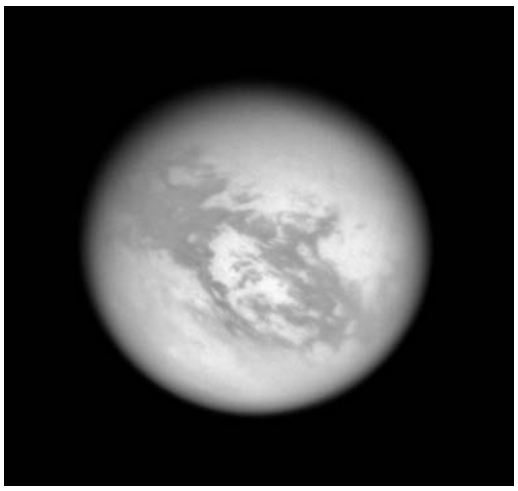

Fig. 2 The amazing Titan with significant atmosphere, courtesy of NASA JPL

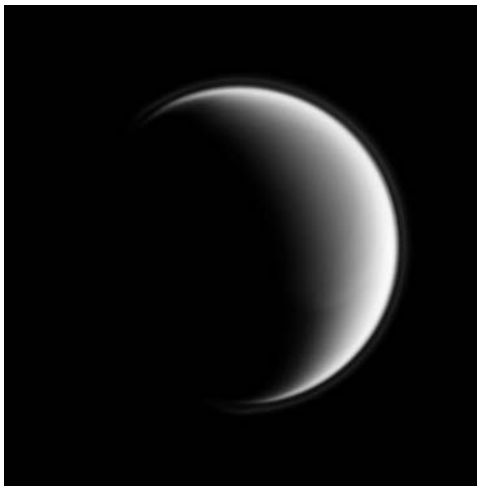

Fig. 3 Titan's atmospheric halo, courtesy of NASA JPL

Except for some clouds, Earth's surface is visible from space. But on Titan, a thick haze extending up to approximately 3,000 kilometers above the surface obscures the entire surface from optical observations. The variation of Titan's atmospheric density with altitude has always been the focus of many past and present science investigations. Scientists have discovered that the organic haze in the atmosphere of Titan is similar to the haze in early Earth's air, i.e. the same haze that may have helped nourish life on our planet. Haze particles form high in Titan's stratosphere where sunlight can penetrate to break up the methane. There are features on Titan which are distinguished by their albedo, such as Xanadu (bright) and Shangri-La (dark).

Titan's atmosphere is highly stratified. With regard to the composition and pressure, Titan is a celestial body with the most Earth-like atmosphere. However, Titan is not like an Earth in deep freeze and the bulk composition of the two celestial bodies are totally different. Despite the similarities, Titan's atmosphere is very different from Earth's with respect to chemistry. Composition of Titan's atmosphere is overwhelmingly nitrogen (95% near surface and 98% in stratosphere), 4.9% methane at the surface near equator, approximately 0.1% hydrogen in lower atmosphere, and a large list of nitriles and oxygen compounds, cyanide, other hydrocarbon species and Argon with parts per million and parts per billion densities[3,4]. Titan's atmospheric mean molecular weight was measured to be 28.6 amu, which is near the mass of nitrogen (28 amu). The most commonly accepted theory of the origin of nitrogen in Titan atmosphere is from the ammonia hydrates and methane clathrates on Titan. A hydrate is a molecule such as ammonia bonded to water ice, Clathrates, on the other hand, are lattice-like ice structures usually made up of water ice, which creates cavities that can be filled with gas molecules[1].

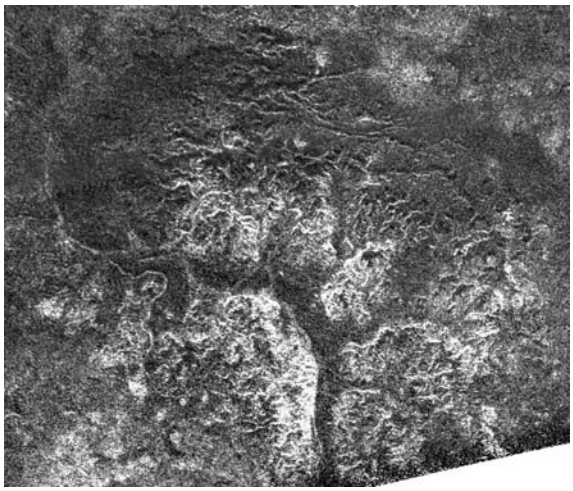

Fig. 4 Southern canyons of Titan, courtesy of NASA JPL

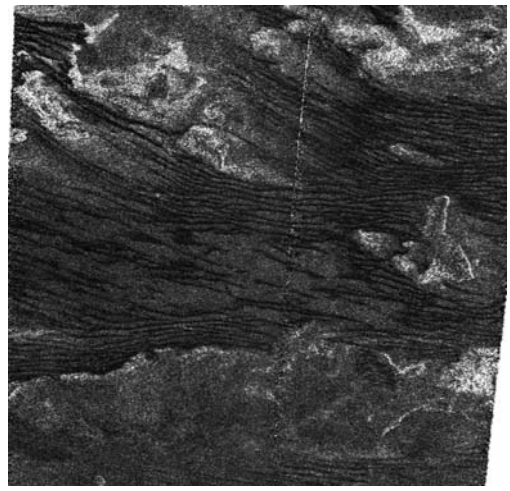

Fig. 5 The edge of Titan dunes, courtesy of NASA JPL

The atmospheric pressure at the surface of Titan is 1.5 bar, that is 50% greater than the pressure at Earth's surface, and the temperature at Titan's surface is around 94 K (or -179 ºC)[3,4]. At this temperature, water ice is as hard as rock on Earth, and because of this low temperature, unlike on Earth, liquid water and water vapor are not generally present. However, just like rock on Earth, the water ice may be soft or molten in the deep interior of Titan. The realization that methane and ethane could both be liquid on Titan has always led scientists to the idea that substantial lakes or oceans should exist on Titan, but in July 2006, the Radar imaging data from Cassini provided convincing evidence for the presence of large bodies of liquid methane in northern hemisphere of Titan (Fig. 6). These lakes are the strong evidence that Titan's surface and atmosphere have an active hydrological cycle and in this cycle, lakes are filled through methane rainfall or intersect with a subsurface layer saturated with liquid methane.

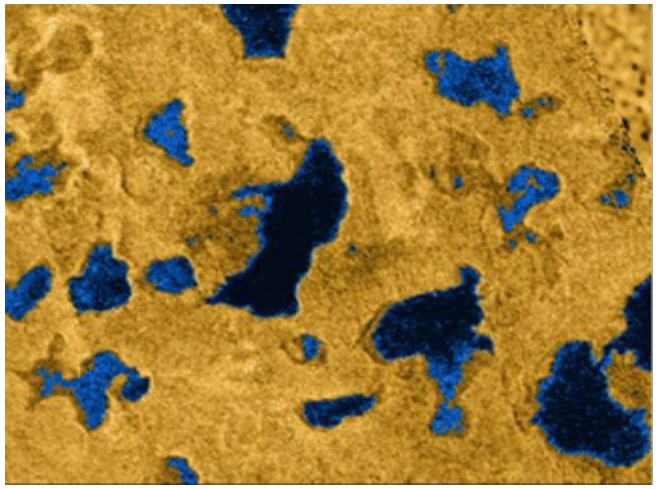

Fig. 6 Evidence of seas at the surface of Titan as seen by radar, courtesy of NASA JPL

Clouds on Titan have been observed regularly both by Cassini and also by ground observations (Fig. 7). A giant cloud half the size of the United States has recently been imaged by the Cassini Radar Team. The cloud might be responsible for the methane, ethane, and other organics that fill the lakes on Titan. When some part of these organics evaporates, it contributes to the formation of these clouds. Ground-based observations show this Titan cloud system comes and goes with the seasons. A season on Titan lasts approximately seven Earth years. Based on the global circulation models, it seems that such

cloud activity can last about 25 Earth years before almost vanishing for four to five years, and then appearing again for 25 years.

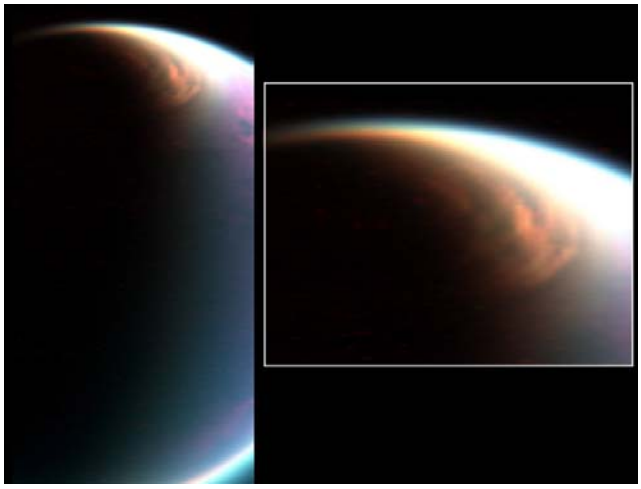

Fig. 7 Covering almost the entire north pole of Titan, this cloud extends down to 60º north latitude and is roughly 1,490 miles in diameter, courtesy of NASA JPL

Atmospheric Density Reconstruction for Titan Flybys under Thruster Control

Cassini has executed numerous low-altitude flybys of Titan after arrival at Saturn in June 2004. These flybys have emphasized different science objectives, such as the Ion and Neutral Mass Spectrometer (INMS) observation[3,5], radar mapping, and radio sciences, and remote sensing. The main objective of Cassini Titan flybys is to determine the makeup and behavior of Titan's atmosphere and its isotope ratios, ionosphere, and the moon's surface features. The prime mission of Cassini-Huygens started in July 2004 and ended in July 2008. It included 25 low-altitude, targeted flybys ranging from 950 to 1200 km. A low-altitude flyby is the one with Titan closest approach (TCA) altitudes less than 1300 km. The first flyby of Titan by Cassini occurred in October 2004 and was named Titan-A flyby. The TCA altitude of this flyby was 1174 km while the Huygens probe was still attached to Cassini. The last low-altitude Titan flyby of the prime mission was Titan-43 performed in May 2008 with the TCA altitude of 1001 km. The Equinox extended mission of Cassini started soon after Titan-43 flyby in July 2008 but the first low-altitude Titan flyby of this mission, Titan-46, occurred on November 3, 2008. Titan-46 was a radio science occultation observation with the TCA altitude of 1105 km. At the time of writing this paper, the last executed low-altitude Titan flyby has been the 955-km Titan-57 flyby.

In order to maintain a stable attitude during the course of Cassini mission, two distinct control systems are used: the attitude controller using the Reaction Control System (RCS) thrusters, and the attitude controller using the reaction wheels[2]. The attitude controller using thrusters is a bang-off-bang control system. The RCS system consists of sixteen (eight primary or A-branch and eight backup or B-branch) hydrazine thrusters, placed in four clusters, with a thrust range of 0.5-1.0 N[6,7]. A diagram of thruster locations and thrust directions is depicted in Fig. 8. Pointing controls about the S/C's X and Y-axis are performed using four Z-facing thrusters, while controls about the Z-axis are performed using four Y-facing thrusters[6,7].

The Monopropellant Tank Assembly (MTA) of Cassini was recharged on April 10, 2006, and this caused an increase of approximately 50% in thrust magnitudes of the primary and backup RCS thrusters. The rise time constant of the thruster pulses remained unchanged at 20 ms after the recharge event, whereas the tail-off time constant dropped from approximately 65 ms to 43 ms.

On March 12, 2009, Cassini successfully swapped to the backup RCS thrusters. Shortly after the swap, the nominal operation of the B-branch RCS thrusters was verified by Cassini Attitude and Articulation Control Subsystem (AACS) team, and the thrust magnitudes of the B-branch thrusters were reconstructed by the author using the telemetry collected during a set of reaction wheel biases and RCS-based slews. Once again, the rise time constant remained unchanged after the swap, but the tail-off time constant dropped. Not accounting for the larger thrust magnitude of B-branch thrusters compared to A-

branch, the X-axis control authority with B-branch thrusters is approximately 4% larger than that generated by A-branch thrusters while the Y and Z-axis control authorities are identical.

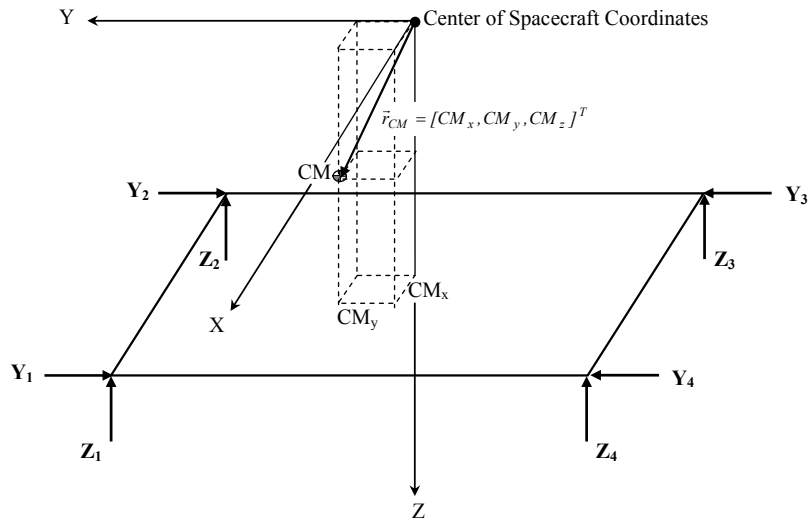

Fig. 8 Cassini RCS thruster configuration.

The atmospheric torque imparted on Cassini during a low-altitude Titan flyby is larger than it can be handled by reaction wheel control system. Therefore, RCS thrusters control the spacecraft during such flybys. The thrusters are fired to overcome the atmospheric torque imparted on the spacecraft due to the Titan atmosphere as well as to slew the spacecraft in inertial space to meet the pointing needs of the science instruments such as the INMS and radar[1,2]. Obviously, the denser Titan's atmosphere is, the more thruster firings will be required. Therefore, thruster firing telemetry data could be used to estimate the three per-axis torques imparted on the spacecraft due to the Titan atmosphere. Since there is a well-defined relationship between the atmospheric torque imparted on the spacecraft and Titan's atmospheric density, the estimated torque can be used to reconstruct the Titan atmospheric density.

The Cassini AACS team implements a methodology, developed by this author, to reconstruct the atmospheric density for all low-altitude Titan flybys under RCS control using the Cassini guidance, navigation, and control data. The details of this methodology are thoroughly explained in Reference 1, and only the underlying principles are summarized in this paper. In addition, another technique is developed by this author, as presented in this paper, to reconstruct the atmospheric density for Titan flybys under reaction wheel control with TCA altitudes low enough to "detect" the presence of atmosphere through changes in reaction wheel spin rates (for flyby altitudes in the range of 1300-1400 km). In both techniques, first the per-axis Titan atmospheric torque is determined, and then the density (as a function of altitude) is reconstructed from the atmospheric torque. The author has also created special purpose ground software tools to automate the implementation of these techniques. The effectiveness of this approach has been confirmed via numerous simulations and has been applied on data collected from 31 low-altitude Titan flybys, as listed in Table 1.

In order to estimate Titan's atmospheric density for a given low-altitude Titan flyby, the atmospheric torque imparted on the spacecraft must first be estimated. The rotational motion of the spacecraft during a Titan flyby is governed by the Euler equation expressed in spacecraft body-fixed coordinate system[1]:

$$I\dot{\vec{\omega}} + \vec{\omega} \times (I\vec{\omega} + \vec{H}_{RWA}) = \vec{T}_{Thruster} + \vec{T}_{RWA} + \vec{T}_{Atm} + \vec{T}_{\varepsilon} \tag{1}$$

Both $\vec{H}_{RWA}$ and $\vec{T}_{RWA}$ are zero and are dropped out from Eq. 1. This is because at least half an hour before TCA, a transition is made from reaction wheel control to RCS control. All the wheel rates are zero while the spacecraft is under the influence of Titan atmosphere[1].

The vector sum of all other external torques on the spacecraft, denoted by $\vec{T}_{\varepsilon}$, is defined as[1]:

$$\vec{T}_{\varepsilon} = \underbrace{\vec{T}_{GG} + \vec{T}_{RTG} + \vec{T}_{Sol} + \vec{T}_{Mag} + \vec{\varepsilon}}_{\approx 0} \tag{2}$$

The predominant term in $\vec{T}_{\varepsilon}$ is the gravity-gradient torque. The magnitude of gravity-gradient torque is a function of both spacecraft attitude and its distance from the celestial body, in this case the Titan. Gravity-gradient torque for low-altitude Titan flybys is in the range of m-Nm. This term is dropped from Eq. 1 for simplicity because dropping it only introduces an error of less than 0.1%. For Titan flybys under reaction wheel control, $\vec{T}_{\varepsilon}$ is not dropped from Euler equation, as seen in the next section.

The torque imparted on the spacecraft due to the Titan atmospheric density, denoted by $\vec{T}_{Atm}$, is the unknown quantity to be estimated in Eq. 3. This equation is obtained from Eq. 1 after dropping the negligible terms.

$$I\dot{\vec{\omega}} + \vec{\omega} \times I\vec{\omega} = \vec{T}_{Thruster} + \vec{T}_{Atm} \tag{3}$$

It should also be noted that torque exerted on the spacecraft due to thruster firings, $\vec{T}_{Thruster}$, is not available directly from the flight software and should be reconstructed.

Integrating Eq. 3 and solving for the accumulated angular momentum due only to atmospheric torque yields[1]:

$$\int_0^t \vec{T}_{Atm}\, d\tau = \int_0^t \left(I\dot{\vec{\omega}} + \vec{\omega} \times I\vec{\omega} - \vec{T}_{Thruster} \right) d\tau = I\left[\vec{\omega}(t) - \vec{\omega}(0)\right] + \int_0^t \vec{\omega} \times I\vec{\omega}\, d\tau - \int_0^t \vec{T}_{Thruster}\, d\tau \tag{4}$$

All quantities on the right-hand-side of this equation are available from either telemetry or ground-estimated values of spacecraft parameters. Specifically, the estimated rate vector $\vec{\omega}(t)$ is available via telemetry from the onboard attitude estimator, an extended Kalman-Bucy filter. The accumulated on-times for the eight primary RCS thrusters are available via telemetry from the onboard Propulsion Manager, and the spacecraft's inertia tensor and center of mass location are estimated via ground tools.

To resolve the issue of dealing with noisy accumulated angular momentum vector due to the atmospheric torque, the key components of this vector (as functions of time) are "curve-fit" to a sum of two hyperbolic tangent functions of time, using the Nelder-Mead multi-dimensional unconstrained nonlinear minimization technique[8]. The fit function is then smoothly differentiated the respective components of the atmospheric torque vector[1].

The Titan atmospheric density is related to the torque imparted to the spacecraft. The density (as a function of altitude) is determined using[1]:

$$\vec{T}_{Atm} = \frac{1}{2} C_D\, \rho V^2\, A_{Proj}\, \hat{u}_V \times (\vec{r}_{CP} - \vec{r}_{CM}) \tag{5}$$

where ρ is the Titan atmospheric mass density in kg/m^3, which is a function of time. The Titan atmospheric density is also a function of the flyby altitude. Only one axis of the atmospheric torque is essentially required to determine the density from Eq. 5. However, in order to minimize the effect of the slight error in the knowledge of the spacecraft's aero-center location, two best components of the atmospheric torque vector, e.g. Y and Z, are used to determine the density. The weighted average of the two density profiles, computed from the selected torque components, is determined based on their respective least-squares fit error[1]. The 3σ uncertainties in estimated atmospheric torque and density using this technique are better than 12% and 15%, respectively[1].

The isothermal model of density versus altitude suggests the following equation:

$$ln\left(\frac{\rho}{\rho_{TCA}}\right) = \left(\frac{-1}{h_s}\right)\left(h - h_{TCA}\right) \tag{6}$$

where the parameter h_s is the scale height in km. In other words, the density doubles for every $h_s(ln2)$ km decrease in flyby altitude, where ln is the natural logarithm[1].

Executed on June 22, 2009, the Titan-57 flyby has been the last Titan swing-by completed by Cassini at the time of writing this paper. The B-branch thruster magnitudes, estimated by the Cassini Propulsion team, are approximately 0.77 N at the time of this flyby. This flyby was an INMS-radar joint observation with a TCA altitude of 955 km at 42º south latitude and with the highest observed RCS thruster duty cycle. The peak duty cycle of Y2-Y4 thruster pair is 69%, as shown in Figs. 9 and 10. The duty cycle of Z1, Z3, and Z4 thrusters is 32%, 5%, and 37%, respectively. The duty cycle Z2 thruster and the Y1-Y3 thruster pair are negligible. The most recent AACS flight software build (A8.7.7) has been uploaded on June 1, 2009, and Titan-57 is the first low-altitude flyby executed with this software version on-board Cassini. Near TCA, the atmospheric force was along spacecraft's +X axis, the spacecraft's projected area of was approximately 18.4 m^2, and the spacecraft's velocity magnitude relative to Titan was 5.99 km/s.

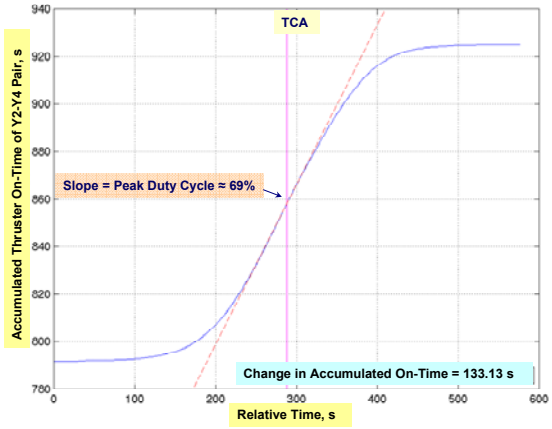

Fig. 9 Accumulated on-time of Y2-Y4 RCS thruster pair for Titan-57

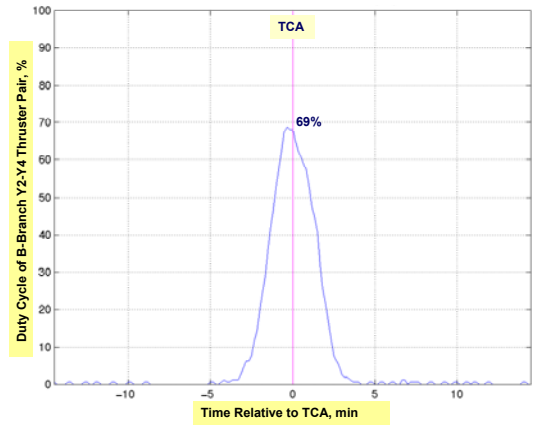

Fig. 10 Duty cycle of the B-branch Y2-Y4 RCS thruster pair for Titan-57

Figs. 11 and 12 depict the curve-fit Z and Y components of the accumulated angular momentum due to atmospheric torque for Titan-57. The fitting function used for both components during the time interval of TCA ± 15 minutes is of the form[1]:

$$-x_1\left\{1+tanh\left(10^{-4}x_2\left(t-t_{TCA}\right)\right)\right\}-x_3\left\{1+tanh\left(10^{-4}x_4\left(t-t_{TCA}\right)\right)\right\} \qquad (7)$$

where $\vec{X}=\left[x_1,x_2,x_3,x_4\right]^T$ is the coefficient vector for fitting function. For the smoothed-out Z-axis accumulated angular momentum: $\vec{X}=\left[3010.807,\ 82.395,\ -2883{,}071,\ 81.305\right]^T$, and the total Z-axis accumulated angular momentum during the time interval of TCA ± 15 min. is -255.5 Nms. For the smoothed-out Y-axis accumulated angular momentum: $\vec{X}=\left[598.577,\ 78.118,\ -545.317,\ 75.206\right]^T$, and the total Y-axis accumulated angular momentum during the same time is -106.5 Nms.

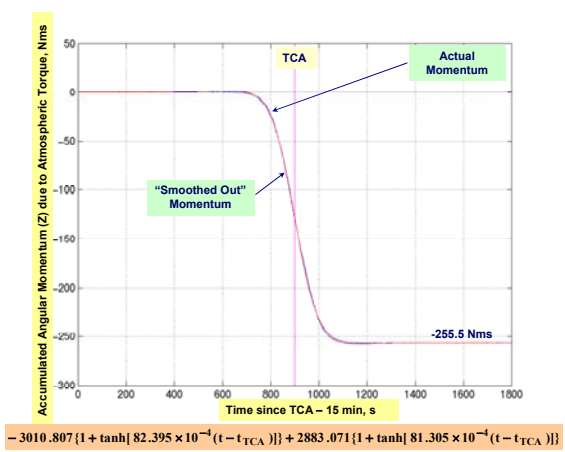

Fig. 11 The Z-axis accumulated angular momentum due to atmospheric torque for Titan-57

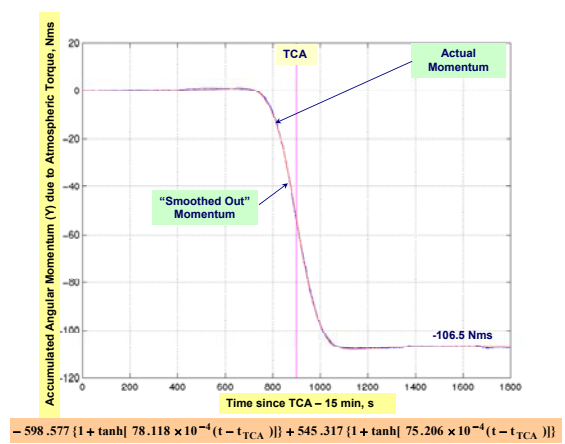

Fig. 12 The Y-axis accumulated angular momentum due to atmospheric torque for Titan-57

The reconstructed Z and Y-axis atmospheric torque for Titan-57 flyby at TCA are -1.37 Nm and -0.58 Nm, respectively, as shown in Figs. 13 and 14.

Fig. 15 depicts the reconstructed density profile as a strong function of altitude for Titan-57 using both the reconstructed Z and Y-axis atmospheric torques. Fig. 16 shows the same density in log scale.

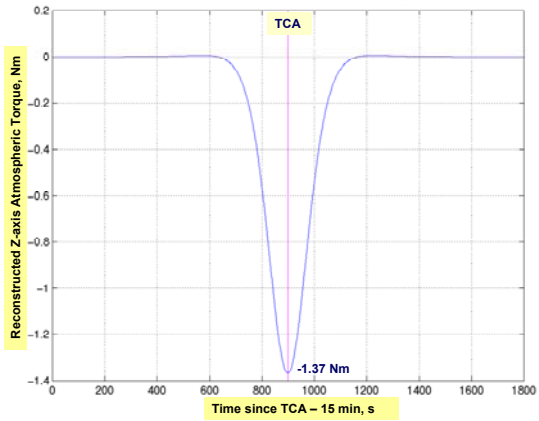

Fig. 13 Reconstructed Z-axis atmospheric torque for Titan-57

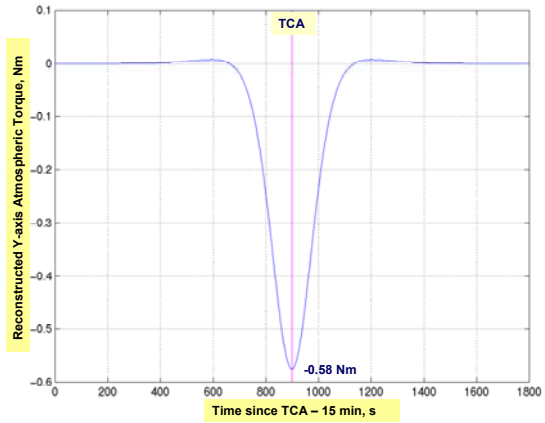

Fig. 14 Reconstructed Y-axis atmospheric torque for Titan-57

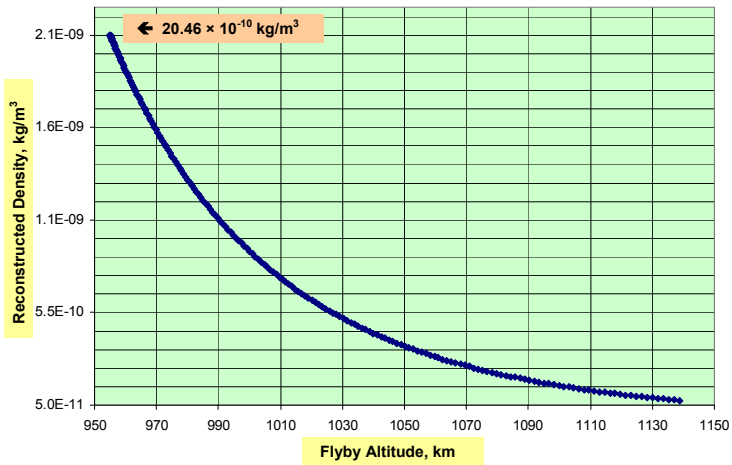

Fig. 15 Reconstructed density as a function of flyby altitude for Titan-57

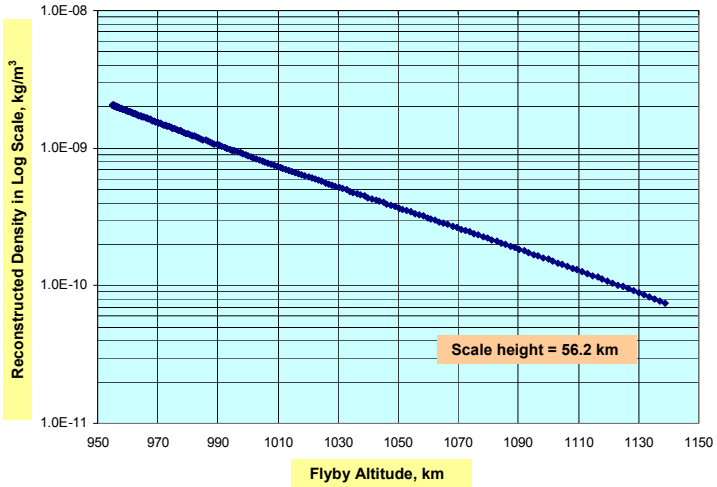

Fig. 16 Reconstructed density in log scale versus altitude for Titan-57

The reconstructed density at TCA was 20.46 × 10^{-10} kg/m^3, and the density at 1000 km altitude was 8.80 × 10^{-10} kg/m^3. The scale height (h_s) for Titan-57 flyby is 56.2 km. In other words, the density doubles for every 39 km decrease in flyby altitude.

The 950-km Titan-16 flyby has by far been the closest Cassini flyby of Titan. Reference 1 discusses this Titan flyby in more details. When the reconstructed density versus altitude profiles of Titan-16 and 57 flybys, separated by 35 months, are superimposed, as shown in Fig. 17, it is noted how closely the two profiles compare to each other. This shows the accuracy of the technique used for density reconstruction by AACS.

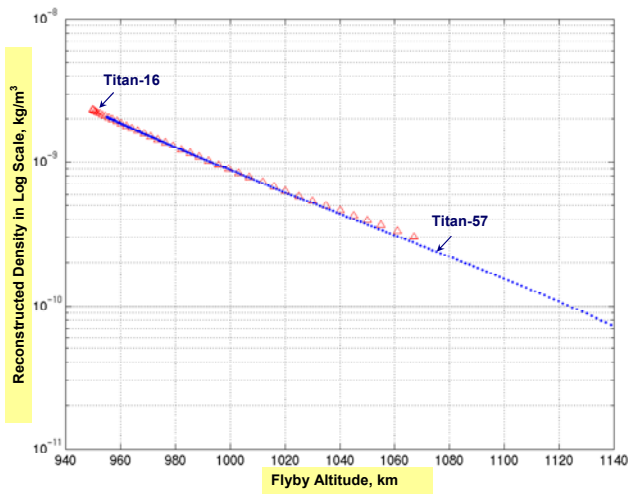

Fig. 17 Comparison of the reconstructed density of the two lowest-altitude Titan flybys

The low-altitude Titan flybys for which the atmospheric density is reconstructed by AACS to date are listed in Table 1. Northern latitudes are indicated as being positive and the southern latitudes as negative. For various reasons, such as lack of sufficient telemetry data, the density was not reconstructed for 2 low-altitude Titan flybys, Titan-B and Titan-37, of the prime mission and one low-altitude flyby, Titan-46, of Equinox extended mission. Titan-B was a 1192-km flyby[1] that took place on December 13, 2004 and was dedicated to a scientific observation using the Ultraviolet Imaging Spectrometer Subsystem (UVIS). Titan-37, an INMS observation, was a 999-km flyby that was executed on November 19, 2007. Titan-46 was a radio science occultation observation with the TCA altitude of 1105 km, which occurred on November 3, 2008. Titan-A and B were both flown with the Huygens probe still attached to Cassini orbiter[1]. The rest of the flybys have occurred after the probe release on December 24, 2004. Among all low-altitude Titan flybys flow thus far, the 950-km Titan-16 flyby has been the closest Cassini flyby of Titan and the 955-km Titan-57 flyby has experienced the highest thruster duty cycle.

Table 1 Reconstructed atmospheric density of low-altitude Titan flybys

Titan Flyby	Date	Altitude, km	Latitude, °	Estimated Thruster Magnitude, N	Peak Thruster Duty Cycle, %	Thrusters with Highest Duty Cycle	Density at TCA, 10^{-10} kg/m^3
Titan-A[a]	10/26/04	1174	+39	0.700	6	Y2-Y4	2.04
Titan-5	4/16/05	1028	+74	0.675	20	Y2-Y4	6.36
Titan-7	9/7/05	1075	-67	0.667	14	Y2-Y4	4.13
Titan-16[b]	7/22/06	950	+85	0.991	62	Y2-Y4	23.34
Titan-17	9/7/06	1000	+23	0.991	19	Y2-Y4	7.62
Titan-18	9/23/06	960	+71	0.991	42	Y2-Y4	16.78
Titan-19	10/9/06	980	+61	0.982	28	Y2-Y4	10.60
Titan-20	10/25/06	1030	+8	0.982	44	Z4	6.70
Titan-21	12/12/06	1000	+44	0.967	31	Y2-Y4	11.10
Titan-23	1/13/07	1000	+31	0.963	28	Y2-Y4	10.64
Titan-25	2/22/07	1000	+31	0.936	26	Y2-Y4	8.24

Titan-26	3/10/07	980	+32	0.936	35	Y2-Y4	11.49
Titan-27	3/26/07	1010	+41	0.940[c]	36	Z1	8.51
Titan-28	4/10/07	990	+51	0.940	41	Y2-Y4	12.61
Titan-29	4/26/07	980	+59	0.925	51	Y2-Y4	16.34
Titan-30	5/12/07	960	+69	0.921	54	Y2-Y4	17.69
Titan-32	6/13/07	965	+84	0.918	54	Y2-Y4	16.77
Titan-36	10/2/07	973	-60	0.900	33	Y2-Y4	10.59
Titan-39	12/20/07	970	-70	0.866	46	Y2-Y4	13.67
Titan-40	1/5/08	1014	-12	0.886	28	Y2-Y4	8.09
Titan-41	2/22/08	1000	-34	0.874	41	Y2-Y4	10.44
Titan-42	3/25/08	999	-27	0.865	27	Y2-Y4	8.33
Titan-43[d]	5/13/08	1001	+17	0.865	25	Y2-Y4	7.70
Titan-47	11/19/08	1023	-22	0.800	(16,17)	(Z3,Z4)	3.07
Titan-48	12/5/08	960	-10	0.790	42	Y2-Y4	13.29
Titan-49	12/21/08	970	-44	0.800	37	Y2-Y4	13.05
Titan-50	2/7/09	967	-34	0.776	48	Y2-Y4	14.15
Titan-51[e]	3/27/09	963	-31	0.79	44	Y2-Y4	13.81
Titan-55	5/21/09	966	-22	0.78	44	Y2-Y4	13.30
Titan-56	6/6/09	968	-32	0.78	34	Y2-Y4	10.81
Titan-57	6/22/09	955	-42	0.77	69[f]	Y2-Y4	20.46

[a] *The first low-altitude Titan flyby and the only flyby in the list with the Huygens probe still attached to Cassini orbiter.*
[b] *By far the lowest altitude flown by Cassini and also the first low-altitude flyby after the MTA recharge.*
[c] *The small increase in the thrust magnitude, as compared to Titan-26, is due to a slight improvement in the RCS thrust estimation method by Cassini Propulsion Team.*
[d] *The last low-altitude Titan flyby of Cassini prime mission. The dashed line marks the end of Cassini prime mission.*
[e] *The first low-altitude Titan flyby using B-branch RCS thrusters.*
[f] *By far the highest thruster duty cycle observed during a low-altitude Titan flyby.*

Fig. 18 shows the reconstructed density profile as a function of flyby altitude for 31 past low-altitude Titan flybys. Fig. 19 depicts the same but on log scale.

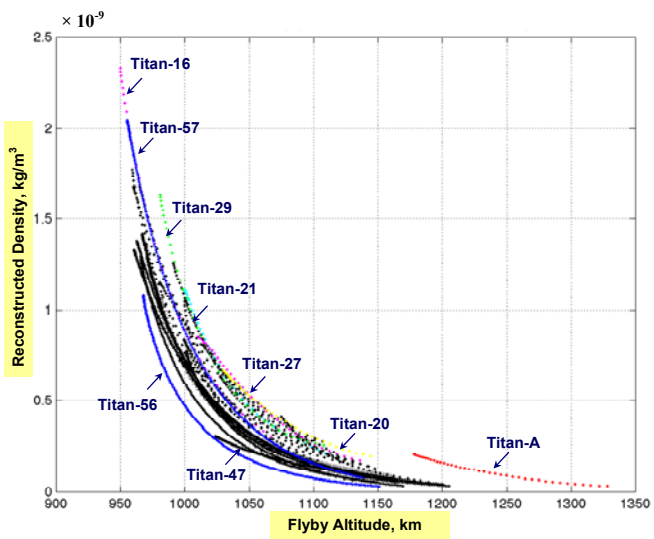

Fig. 18 Reconstructed density as a function of altitude for past low-altitude Titan flybys by Cassini

As it can be observed from Fig. 19, Titan-56 has by far had the lowest density profile among all low-altitude flybys listed in Table 1, and the density profiles of Titan-20, 21, 27, and 29 are among the highest in the list. The density profile of Titan-47 tends to stay outside the general trend seen in Fig. 19. For this 1023-km flyby, dedicated to Visual and Infrared Mapping Spectrometer science observation, the A-branch thrusters Z3 and Z4 had the highest duty cycles and not the Y2-Y4 pair.

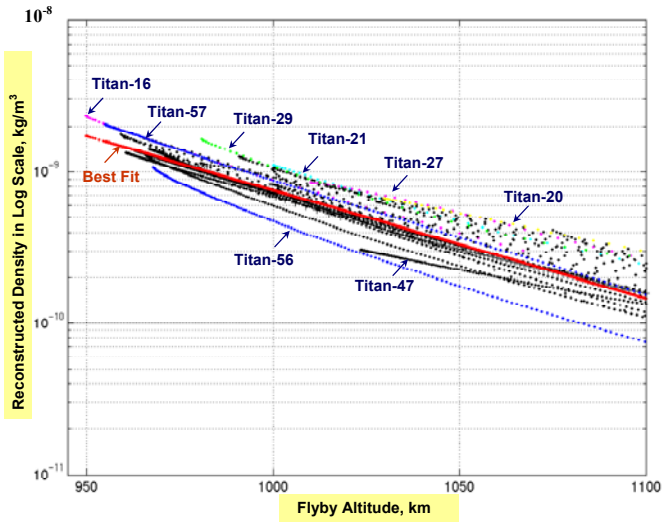

Fig. 19 Density versus altitude in log scale for altitude range of 950 to 1100 km and the best fit

An engineering model for Titan atmospheric density is developed for the altitude range of 950-1100 km using the least square fit to the natural log of density as a function of altitude for Titan flybys listed in Table 1, as shown in Fig. 20, and computing the dispersion. The density profiles of all Titan flybys listed in Table 1 lie inside the highlighted region shown in Fig. 20. The equations of the upper and lower boundaries are shown in this figure.

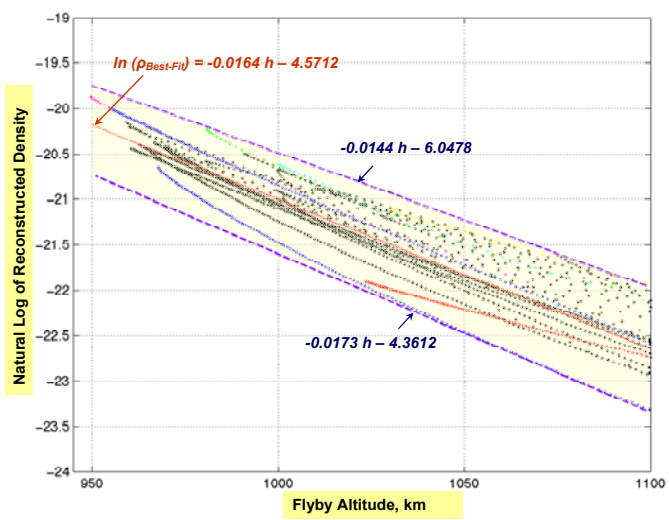

Fig. 20 Log of density versus altitude, the least-square fit, and the upper and lower bounds

From the least square fit equation and the equations associated with the upper and lower boundaries of the highlighted region, shown in Fig. 20, the following two equations are derived. The combination of Eqs. 8 and 9 form the current engineering model for Titan atmospheric densities for the flyby altitude range of 950-1100 km. In these equations, h is the flyby altitude and the scale height for the engineering model is 60.86 km.

$$\rho_{Best-Fit} = 17.20 \times 10^{-10} e^{-\left(\frac{h-950}{60.86}\right)} \begin{pmatrix} +\varepsilon_U(h)\% \\ -\varepsilon_L(h)\% \end{pmatrix} \quad (3\sigma) \qquad (950\ km \le h \le 1100\ km) \qquad (8)$$

where,

$$\varepsilon_U(h) = 22.8394 e^{0.00201\,h} - 100, \text{ and } \varepsilon_L(h) = 100 - 123.3577 e^{-0.00082\,h} \quad (950\ km \le h \le 1100\ km) \qquad (9)$$

Using Eqs. 8 and 9, the best fit density is 17.20×10^{-10} kg/m^3 (+54.1%, -43.4%, 3σ) at 950 km altitude. The reconstructed peak density for Titan-16 flyby is 35.7% higher than the best fit density at the same altitude. Also, the best fit density is 1.46×10^{-10} kg/m^3 (+108.4%, -49.9%, 3σ) at 1100 km altitude.

The engineering Titan atmospheric density model is of interest to the mission operations engineers and perhaps to planetary scientists, as well. The mission operations engineers must have a good estimate of the dependency of Titan's atmospheric density with respect to the altitude. This knowledge is important because an unexpectedly high density profile has the potential to tumble the spacecraft out of control during a low-altitude Titan flyby[1]. It also has the potential to cause thermal threat to the health and safety of some of the instruments onboard the spacecraft[1].

The Huygens probe was separated from Cassini on December 25, 2004[1], and it proceeded to successfully land on the surface of Titan near the Xanadu region on January 14, 2005. A multi-sensor package was on board this probe that measured the physical quantities characterizing the atmosphere of Titan during the probe descent and on the surface of the moon. The primary task of this package, called the Huygens Atmospheric Structure Instrument (HASI) was to determine the atmospheric physical structure by measuring the upper atmosphere and stratospheric density, pressure and temperature as well as the pressure and temperature in the troposphere[9,10,11]. Huygens Team inferred the atmospheric structure of Titan on the basis of measurements taken during entry phase and while the probe was descending under the parachutes. The atmosphere was first detected at an altitude of approximately 1500 km, when it exceeded the sensitivity threshold of the on-board accelerometer[9]. The density of the upper atmosphere was derived from the probe deceleration due to aerodynamic drag force. The derived density profile is shown in Fig. 21, with a comparison of the model obtained from the re-analysis of Voyager 1 data[9]. According to HASI, the density at 1000 km altitude[9] was 9.9×10^{-10} kg/m^3. The density reported by HASI was very comparable to the densities reconstructed by Cassini AACS team, especially for the earlier low-altitude Titan flybys. On the other hand, the densities reported buy the INMS team are generally 3 to 4 times smaller than the densities reconstructed by the AACS. By far, much discussion and investigation has failed to resolve this discrepancy. The aerothermodynamics branch of NASA Langley Research Center performed an independent verification[12] of the atmospheric torques imparted on Cassini reconstructed and reported by this author for two low-altitude Titan flybys, Titan-A with Huygens probe attached to Cassini and Titan-5 with probe released. The torques resulting from the free molecular and direct simulation Monte Carlo analyses agreed, within 15%, with torques reported by the Cassini AACS team[12]. It was therefore concluded that the assumptions made in the original analysis performed by AACS team were reasonable, and the root cause of the discrepancy between the atmospheric density derived by AACS and INMS teams had to lie elsewhere[12]. Is it possible that certain unknown particles exist in Titan's atmosphere, for which the INMS and the Cosmic Dust Analyzer instruments are not designed to detect?

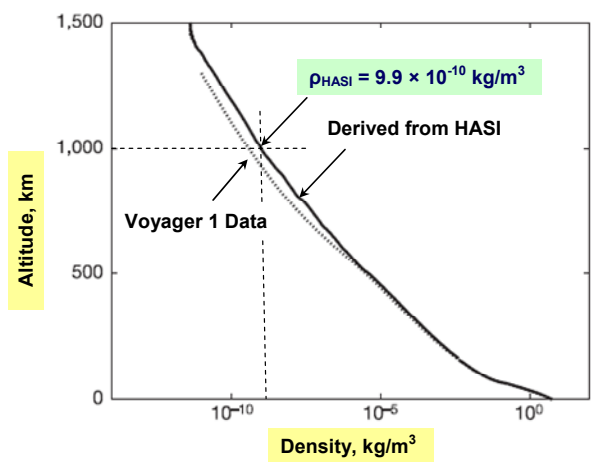

Fig. 21 The atmospheric density profile of Titan as measured by HASI.

Among the flybys listed in Table 1, the TCA altitude of 24 of them is less than or equal to 1000 km. Table 2 lists the density at 1000 km altitude of these 24 flybys.

Table 2 Reconstructed density at 1000 km

Titan Flyby	Date	Days Since Titan-16 Flyby	Latitude, °	Density at 1000 km, 10^{-10} kg/m^3
Titan-16	7/22/06	0	+85	8.98
Titan-17	9/7/06	47	+23	7.62
Titan-18	9/23/06	63	+71	8.97
Titan-19	10/9/06	79	+61	6.87
Titan-21	12/12/06	143	+44	11.10
Titan-23	1/13/07	175	+31	10.64
Titan-25	2/22/07	215	+31	8.24
Titan-26	3/10/07	231	+32	9.30
Titan-28	4/10/07	262	+51	10.60
Titan-29	4/26/07	278	+59	10.85
Titan-30	5/12/07	294	+69	7.82
Titan-32	6/13/07	326	+84	9.21
Titan-36	10/2/07	437	-60	6.77
Titan-39	12/20/07	516	-70	8.14
Titan-41	2/22/08	580	-34	10.24
Titan-42	3/25/08	612	-27	8.26
Titan-43	5/13/08	661	+17	7.70
Titan-48	12/5/08	867	-10	6.86
Titan-49	12/21/08	883	-44	7.26
Titan-50	2/7/09	931	-34	7.06
Titan-51	3/27/09	979	-31	7.21
Titan-55	5/21/09	1034	-22	5.92
Titan-56	6/6/09	1050	-32	4.71
Titan-57	6/22/09	1066	-42	8.80

It was originally thought that the densities at equal altitudes may have be a latitude dependency. When densities at 1000 km for the set of 24 flybys listed in Table 2 were plotted against the flyby latitudes near the 1000 km altitude, as shown in Fig. 22, no apparent trend was observed. Titan-21 and Titan-56 flybys have the highest and lowest densities at 1000 km, respectively. The mean and standard deviation of densities listed in Table 2 are 8.30×10^{-10} and 1.64×10^{-10} kg/m^3, respectively. The best fit density, using Eqs. 8 and 9, is 7.56×10^{-10} kg/m^3 (+70.5%, -45.7%, 3σ) at 1000 km altitude. Both the mean and the best fit densities are shown in Fig. 22. The difference between the density reported by HASI at 1000 km, denoted by ρ_{HASI} and equal to 9.9×10^{-10} kg/m^3, and this mean density is less than the standard deviation.

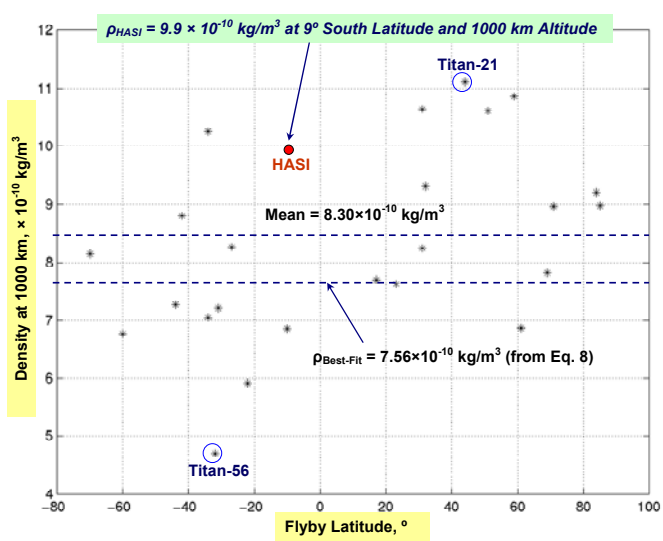

Fig. 22 Density at 1000 km versus the flyby latitude

When densities at 1000 km for the set of 24 flybys were plotted against time, specifically the number of days since Titan-16 flyby, as shown in Fig. 23, a clear trend was observed. The density has generally decreased with time during the interval of 35 months starting from 7/22/06 and ending on 6/22/09. The possible explanation for this trend is the effect of solar magnetic activity cycle on Titan's atmospheric density.

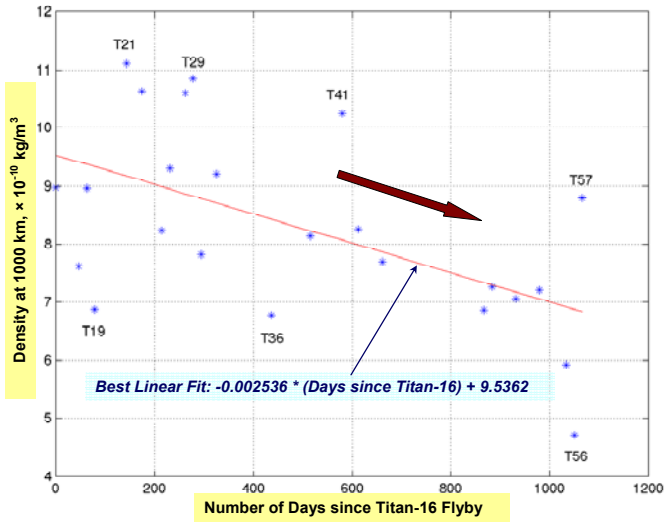

Fig. 23 Time-dependency of density possible due to the decrease in solar cycle

Solar minimums and maximums are the two extremes of the Sun's eleven-year magnetic activity cycle. The solar minimum is the period of least solar activity in the solar cycle of the sun. During this period, sunspot, solar flare, and corona activities diminish, and often do not occur for days at a time. The solar cycle has been observed to have a considerable effect on Earth's atmosphere, and could have similar effect on Titan's atmosphere. Fig. 24 depicts the solar cycle prediction by the National Oceanic and Atmospheric Administration panel. The panel agrees that we are now experiencing solar minimum-like conditions, and predicts that the solar maximum will occur sometime in 2013. During the period of 2004 through 2009, the solar magnetic activities were diminishing, and Titan atmosphere was generally becoming colder and less dense.

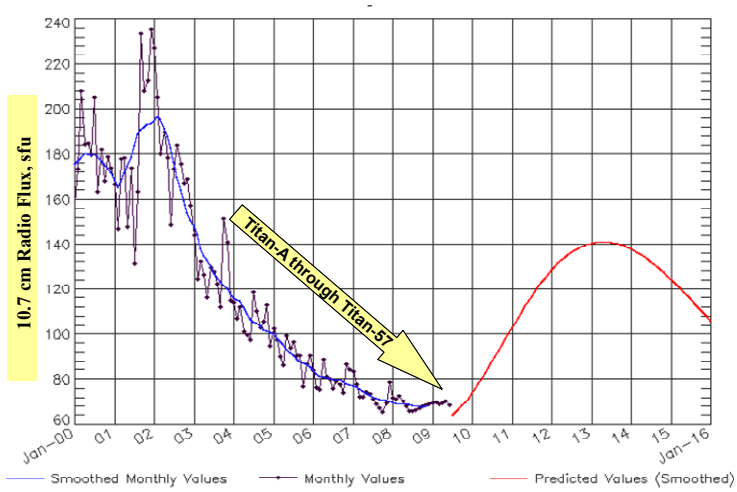

Fig. 24 Solar cycle prediction, courtesy of the National Oceanic and Atmospheric Administration and the Space Weather Prediction Center.

Density Reconstruction for Flybys under Reaction Wheel Control

Cassini has a set of three fixed reaction wheels that are oriented such that their spin axes make equal angles with the spacecraft's Z-axis. Each wheel is considered a Reaction Wheel Assembly (RWA), and the three fixed reaction wheels are named RWA 1, 2, and 3, respectively[2]. In addition, Cassini has a backup, articulatable reaction wheel (RWA 4). Currently, RWA 3 is out of commission and the spin axis of RWA 4 is co-aligned with RWA 3. At Launch, RWA-4 was aligned parallel with reaction RWA-1. On July 11, 2003, the platform was articulated to align the backup reaction wheel with reaction RWA-3, because the bearings of RWA-3 had developed occasional excessive frictional torque[2]. Each wheel has a wheel spin axis, with one direction designated as the positive spin direction. Location and orientation of Cassini reaction wheels are shown in Fig. 25[2].

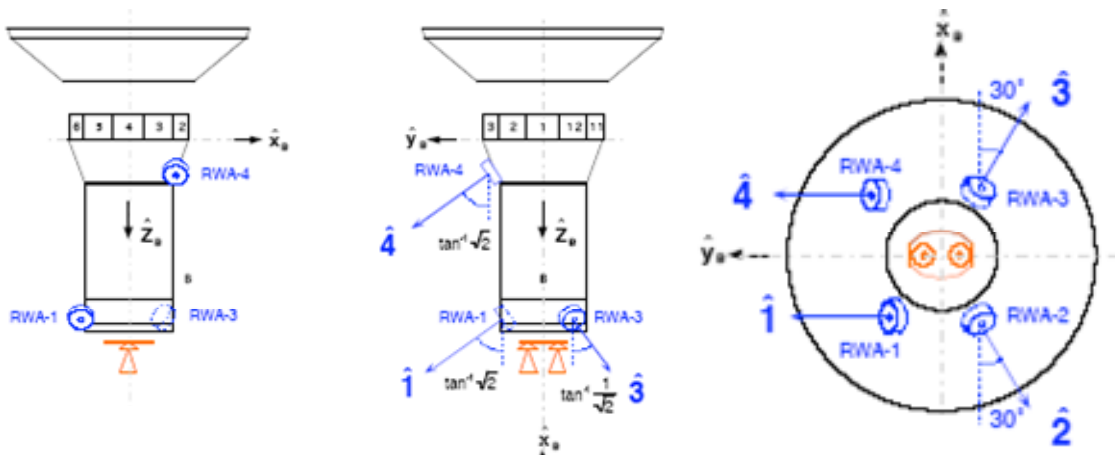

Fig. 25 Locations and orientation of Cassini reaction wheels

The reaction wheel control system is a proportional-plus-derivative controller. This control system uses an algorithm that determines the desired torque on the spacecraft, and this torque vector is projected along each wheel axis to determine the contribution of each wheel[2]. The result is then negated to determine the commanded torque on each wheel. The torque of the wheel on the spacecraft is equal and opposite to the torque applied to the wheel itself.

The RWAs are used primarily for attitude control when precise and stable pointing of a science instrument is required during an observation. The advantages of using reaction wheels over thrusters are the conservation of hydrazine and the absence of unwanted delta-V imparted on the spacecraft. Typically, Cassini is under reaction wheel control during high-altitude Titan flybys. The closest approach altitude must be high enough so that the atmospheric torque imparted on the spacecraft is well within reaction wheel control authority. In the absence of science slews near TCA, Titan flybys with TCA altitudes of 1300 km or higher can be handled by reaction wheel control system. Thus far, Cassini has performed two flybys under RWA control with TCA altitudes of approximately 1300 km. They are Titan-22 on December 28, 2006 and Titan-38 on December 5, 2007, respectively. The atmospheric torque encountered at TCA for both of these Titan flybys are within the control authority of the RWA motors.

While under reaction wheel control, e.g. during Titan-22 and 38 flybys, thruster firings are inhibited, and the RWA spin rate telemetry data are used to estimate the three per-axis torques imparted on the spacecraft due to the Titan atmosphere. The angular momenta imparted on the spacecraft due to the atmospheric torque are entirely absorbed by wheels, and the change in RWA spin rates is used to estimate the atmospheric torque. Since there is a well-defined relationship between the atmospheric torque imparted on the spacecraft and Titan's atmospheric density, the estimated torque can be used to reconstruct the Titan atmospheric density. A ground software tool has been developed by the author to reconstruct the Titan atmospheric density for Titan-22, 38, and future Titan flybys under reaction wheel control with TCA altitudes in the range of 1300-1400 km, that is, with sensible peak density. The effectiveness of the approach used in this ground software has also been confirmed via numerous simulations.

Once again, we start with the Euler equation expressed in spacecraft body-fixed coordinate system (Eq. 1). Integrating and re-arranging that equation yields:

$$\int_0^t \vec{T}_{Atm}\, d\tau = \int_0^t \left\{ I\dot{\vec{\omega}} + \vec{\omega} \times (I\vec{\omega} + \vec{H}_{RWA}) - \vec{T}_{RWA} \right\} d\tau - \int_0^t \vec{T}_{GG}\, d\tau \tag{10}$$

The accumulated angular momentum due to the gravity-gradient torque, $\int_0^t \vec{T}_{GG}\, d\tau$, is maintained in the equation this time around in order to increase the accuracy of atmospheric torque reconstruction. Once again, all quantities on the right-hand-side of Eq. 10 are available from either telemetry, at a certain allowable sampling rate, or the ground-estimated values of spacecraft parameters. The estimated rate $\vec{\omega}(t)$ is available via telemetry at a frequency of 0.25 Hz. The RWA spin rates as a function of time for all 3 prime RWAs are available via telemetry from the onboard FSW named the Reaction Wheel Manager at a frequency of 0.25 Hz. The inertia of each RWA is measured accurately before launch, and the spacecraft's inertia tensor and center of mass location are all estimated via ground tools.

The 3-axis angular momenta in RWA frame are transformed to the spacecraft's fixed frame via the transformation matrix given by Eq. 11[2]. In this equation, the first two columns (for RWA-1 and 2) are determined pre-launch, and the last column (for RWA-4) is estimated after the RWA-4 articulation event in 2003[2].

$$C_{RWA}^{SC} = \begin{bmatrix} 0 & -\frac{1}{\sqrt{2}} & 0.713318 \\ \sqrt{\frac{2}{3}} & -\frac{1}{\sqrt{6}} & -0.402252 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0.573909 \end{bmatrix} \tag{11}$$

The ground software first determines the accumulated angular momentum vector profile due to the atmospheric torque using Eq. 10. In order to resolve the problem with noisy data, two best components of the accumulated angular momentum vector are first optimally curve-fit using the Nelder-Mead multi-dimensional unconstrained nonlinear minimization technique. The fit function is then smoothly differentiated to yield the respective components of the atmospheric drag torque imparted on the spacecraft as a function of time. The ground software then proceeds to determine the atmospheric density profile as a function of time and Titan flyby altitude using the knowledge of spacecraft's center of mass, projected area, and the aero-center along with the estimated drag coefficient in a free molecular flow field and the spacecraft's Titan-relative range and velocity data.

In this paper, the result of density reconstruction for Titan-22 is further explored through the subsequent plots of key telemetry data and reconstructed parameters. Similar technique is used for Titan-38, and the reconstructed density at TCA for both of these Titan flybys is 1.4×10^{-11} kg/m^3. Dedicated to the radio science, Titan-22 is the first of the two flybys on reaction wheels with TCA altitude of about 1300 km. The spacecraft rates are very quiescent throughout the flyby, even in the presence of external disturbances. The RWAs have operated outside the undesirable region of ±300 rpm before, during, and after the flyby. The operational time duration each wheel spends inside this "low-rpm" region (±300 rpm) must be minimized. The reason why this is the case is explained in Reference 2. In general, Cassini operation is required to limit the low-rpm dwell time of each wheel to be less than 12,000 hours for the four-year prime mission. The RWA spin rates for Titan-22 flyby are sown in Fig. 26.

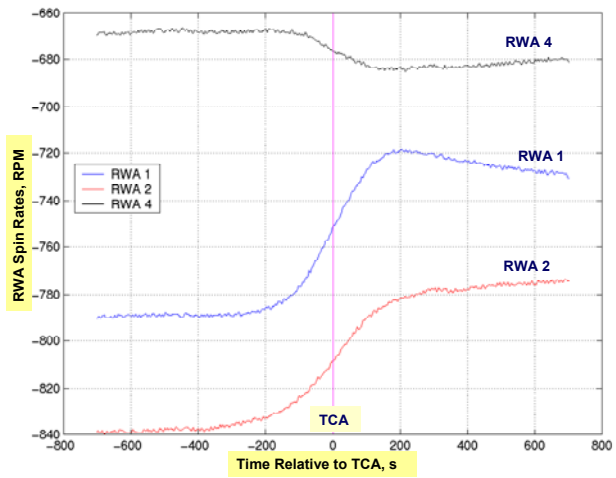

Fig. 26 RWA spin rate as a function of time relative to TCA for Titan-22 flyby

The change in RWA spin rates due to external torque of $\vec{T}_{Atm} + \vec{T}_{GG}$ imparted on the spacecraft is +72 RPM for RWA-1, +64 RPM for RWA-2, and -15 RPM for RWA-4, as shown in Fig. 26. The gravity-gradient torque estimate and its integral are shown in Figs. 27 and 28, respectively.

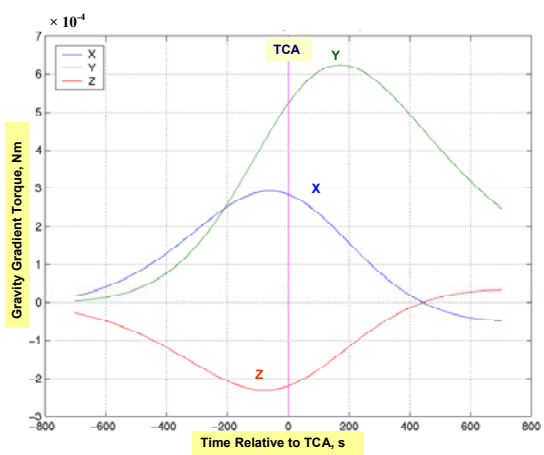

Fig. 27 Gravity-gradient torque estimate for Titan-22

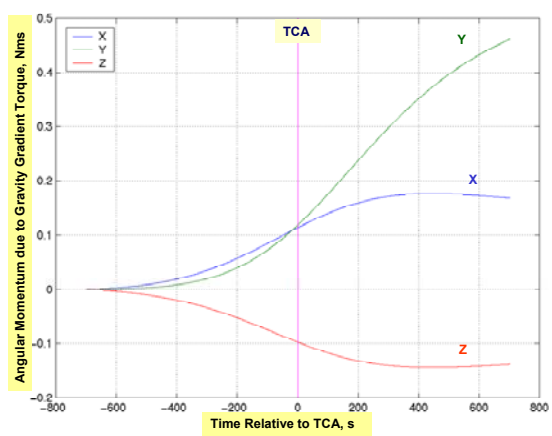

Fig. 28 Accumulated angular momentum due to the gravity-gradient torque for Titan-22

The accumulated angular momentum in spacecraft fixed body frame due to the atmospheric drag torque is depicted in Fig 29, and the curve-fit X and Z axes angular momenta due to the atmospheric drag torque are captured in Figs. 30 and 31, respectively. Eq. 7 is used again for curve-fitting during the time interval of TCA ± 700 s. For the smoothed-out X-axis accumulated angular momentum: $\vec{X} = [0.3866,\ 79.6408,\ 0.1548,\ 34.8476]^T$, and for the smoothed-out Z-axis accumulated angular momentum: $\vec{X} = [-0.5837,\ 70.4944,\ -0.0932,\ 5.9760]^T$.

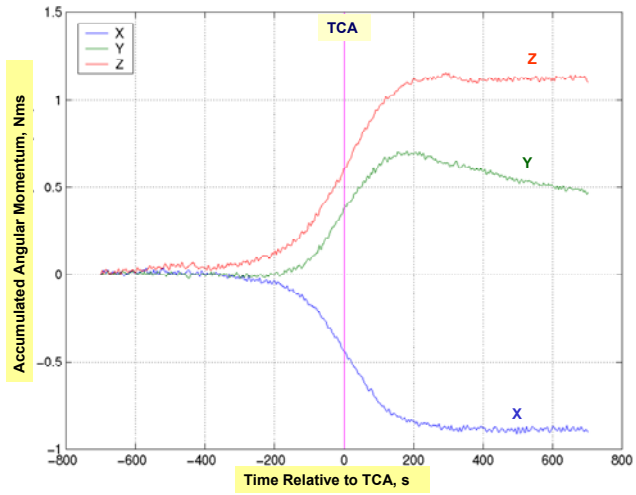

Fig. 29 Accumulated angular momentum in spacecraft body frame due to the atmospheric drag torque for Titan-22

The reconstructed X-axis and Z-axis atmospheric torques are shown in Fig. 32. Figs. 33 and 34 show the reconstructed atmospheric density as a function of time since TCA and Titan-22 flyby altitude, respectively.

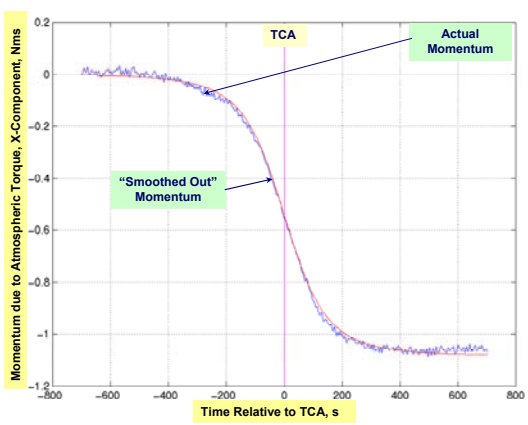

Fig. 30 Smoothed-out X-axis angular momentum due to atmospheric drag torque for Titan-22

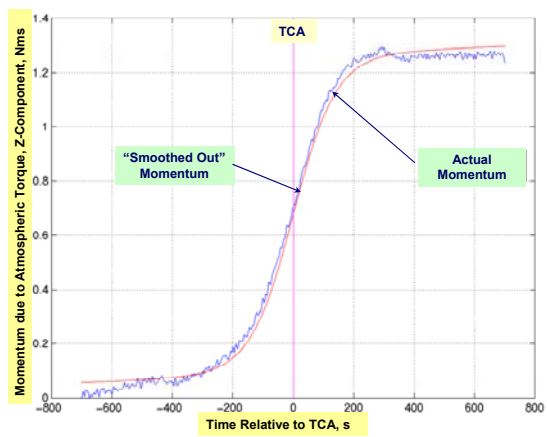

Fig. 31 Smoothed-out Z-axis angular momentum due to atmospheric drag torque for Titan-22

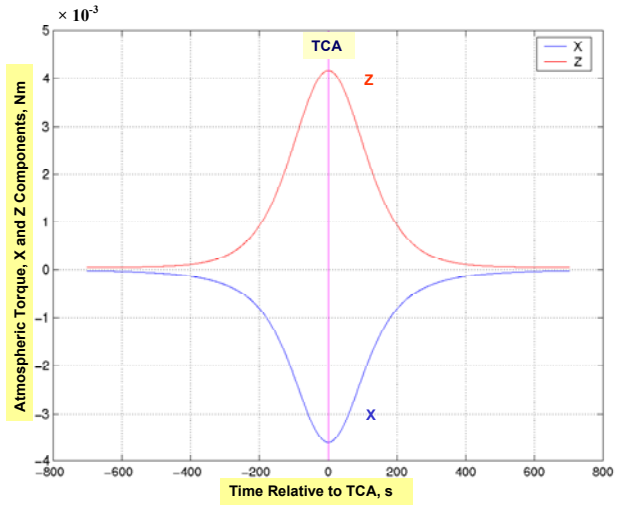

Fig. 32 Reconstructed X and Z-axis atmospheric torques for Titan-22

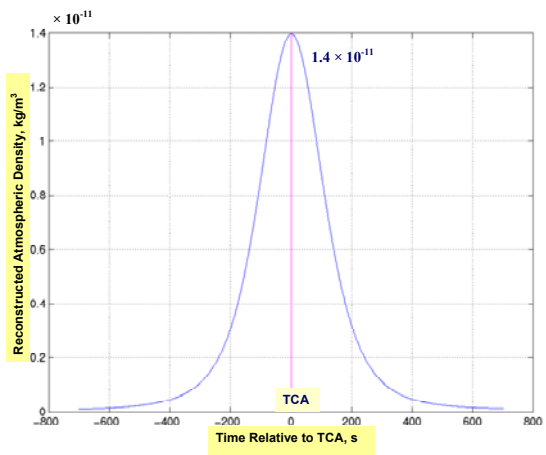

Fig. 33 Reconstructed atmospheric density versus time since TCA for Titan-22

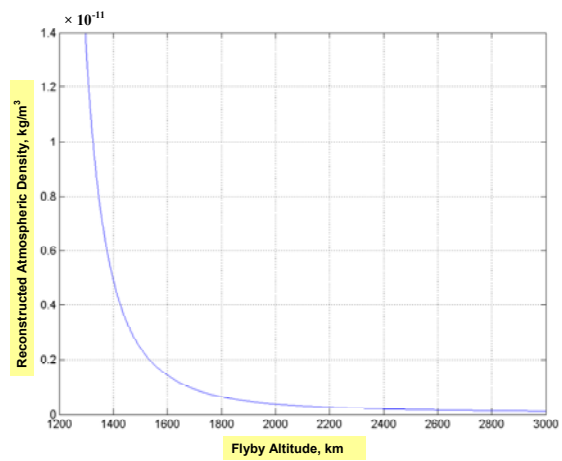

Fig. 34 Reconstructed atmospheric density as a function of flyby altitude for Titan-22

Applying the Density Reconstruction Methodology to Future Titan Flybys

Cassini successfully continues its Equinox extended mission and prepares for an additional set of 9 targeted low-altitude flybys of Titan with TCA altitudes ranging from 880 km to 1300 km, as summarized in Table 3, before ending the Equinox mission. The reason why Titan flyby numbers are sometimes out of sequence in this and subsequent table is because only the low-altitude flybys are listed in the table, and all of those flybys will be executed while Cassini is under the RCS control. The author will continue to reconstruct Titan atmospheric density for these flybys using the same technique described in this paper.

Table 3 Remaining planned low-altitude Titan flybys of Cassini Equinox mission

Titan Flyby	Date	Science Observation	TCA Altitude, km	TCA Latitude, °
Titan-58	7/8/09	Radar/UVIS	965	-52
Titan-59	7/24/09	INMS	955	-62
Titan-60	8/9/09	Radar	970	-72
Titan-61	8/25/09	Radar	970	-19
Titan-62	10/12/09	UVIS	1300	-62
Titan-64	12/28/09	INMS/Radar	955	+82
Titan-65	1/12/10	INMS/Radar	1073	-82

Titan-70	6/21/10	Minimum Torque Attitude[a]	880	+84
Titan-71	7/7/10	INMS	1005	-56

[a]*The lowest altitude Titan flyby with an attitude that guarantees minimum atmospheric torque imparted on Cassini.*

The TCA altitude of Titan-70 flyby is 880 km and this is the lowest altitude ever to flyby inside Titan's atmosphere during the entire mission of Cassini, both prime and extended. This flyby is safe from the attitude control and thermal viewpoints, and there is no danger of tumbling out of control near TCA. The minimum torque attitude for Titan-70 is an attitude that minimizes the atmospheric torque imparted on Cassini near TCA because near the TCA the atmospheric force exerted on Cassini and the $\vec{r}_{CP} - \vec{r}_{CM}$ vector become approximately parallel and their cross product becomes nearly zero[1]. This flybys has been assessed by the author to be quite robust against the knowledge uncertainties in key parameters such as the RCS thrust magnitudes and the aero-center and center of mass locations, the contribution of the bending of spacecraft's appendages to the change in aero-center location, and the perturbations in spacecraft's attitude due to the dead-banding motion. Since Titan-70 will be a minimum torque attitude flyby, the atmospheric density could not be reconstructed using the technique described in this paper.

To explain how Cassini will be pointing near TCA during Titan-70 flyby, which is a minimum torque attitude for Cassini, let's assume a pointing attitude for which the unit vector along spacecraft's $\vec{r}_{CP} - \vec{r}_{CM}$ points into the spacecraft's velocity direction or the ram direction, which is opposite to the direction of molecular flow, while the spacecraft's –X axis is pointing towards the center of Titan. This is essentially equivalent to pointing spacecraft's +Y axis into the ram direction and +Z to Titan with 57.3º, 0, and -2.5º angular offset about X, Y, and Z axes, respectively. Now, Cassini's pointing attitude near TCA for Titan-70 flyby is a +20º offset from the above attitude about Cassini's ram direction. This is necessary to avoid the exposure of Cassini's Star Tracker to the Sun throughout the flyby. At this attitude, near TCA, the spacecraft's center of mass will be behind its aero-center, and a lift force is generated. However, in the free molecular flow regime associated with this Titan flyby, the ratio of lift to drag should be bounded by approximately 1/3.

Cassini Solstice extended mission will officially start on October 1, 2010 and will last for approximately 7 years. A total of 23 low-altitude Titan flybys, numbered Titan-83 through Titan-126, with TCA altitude ranging from 955 km to 1236 km are being planned to be executed in this mission, as shown in Table 4, encompassing INMS, radar, radio science, and optical remote sensing observations. The design of this mission is now in the process of being finalized and the altitudes for Titan flybys in this mission may be subject to small tweaks. Once again, the technique described in this paper will be used to reconstruct the atmospheric density for all these flybys.

Table 4 Planned low-altitude Titan flybys of Solstice mission

Titan Flyby	Date	TCA Altitude, km
Titan-83	5/22/12	955
Titan-84	6/7/12	959
Titan-85	7/24/12	1012
Titan-86	9/26/12	956
Titan-87	11/13/12	973
Titan-88	11/29/13	1014
Titan-91	5/23/13	970
Titan-92	7/10/13	964
Titan-95	10/14/14	961
Titan-98	2/2/14	1236
Titan-100	4/7/14	963
Titan-104	8/21/14	964
Titan-106	10/24/14	1013
Titan-107	12/10/14	980
Titan-108	1/11/15	970
Titan-109	2/12/15	1200
Titan-113	9/28/15	1036
Titan-117	2/16/16	1018

Titan-118	4/4/16	990
Titan-119	5/6/16	971
Titan-120	6/7/16	975
Titan-121	7/25/16	976
Titan-126	4/22/17	979

Summary and Conclusions

Cassini continues its successful mission conducting various scientific investigations of Saturn and its rings, icy and rocky satellites, and the mysterious moon, Titan. Cassini's four-year prime mission has ended in 2008, but this fantastic and sophisticated spacecraft is extending its mission by 9 years and has already completed its first year of Equinox mission. Titan has a thick and highly stratified atmosphere overwhelmingly composed of nitrogen. It also has great many atmospheric and surface features, and its surface and atmosphere have an active hydrological cycle. This makes Titan an important celestial body to study.

The density of Titan as a function of the altitude from the surface of Titan is of profound interest to planetary scientists and spacecraft operations engineers. Unexpectedly large density profile, and the respective large atmospheric torque profile, could potentially cause Cassini to tumble out of control during a low-altitude Titan flyby. Hence, an accurate estimate of density profile as a function of altitude is required to allow mission planners to select safe flyby altitudes and to allow navigation engineers to accurately predict the delta-V associated with those flybys.

The novel methodologies reported in this paper are used to reconstruct the Titan atmospheric density as a function of altitude using Cassini's guidance, navigation, and control data. The effectiveness of these techniques has been demonstrated in more than 30 past flybys of Titan under both thruster and reaction wheel control. Density estimates collected from these low-altitude Titan flybys have provided and will continue to provide planetary scientists with valuable data to better understand the density structure of Titan. These techniques could potentially be modified to reconstruct the Titan atmospheric density for future NASA Titan Orbiter projects as well as determining the atmospheric or plume density of other planets and moons.

Acknowledgements

The work described in this paper was carried out by Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration. Reference to any specific commercial product, process, or service by trade name, trademark, manufacturer or otherwise, does not constitute or imply its endorsement by the United States Government or the Jet Propulsion Laboratory, California Institute of Technology. The methodologies and the ground software tools described in this paper have been reported to the Patent Counsel Office of Jet Propulsion Laboratory for potential patent application. The author is indebted to Dr. Allan Y. Lee, the Project Element Manager of the Cassini Attitude and Articulation Control Subsystem team, for his expert discussions and critiques. The inputs and helpful suggestions from my Navigation, AACS Uplink, Downlink, Propulsion, and Mission Planning colleagues, as well as the Cassini scientists are greatly appreciated. The author is also grateful to Cassini management for its guidance and support. Additional thanks go to Sima Lisman, the Guidance and Control Systems Engineering Group Supervisor, and Dr. Allan Lee for improving on the technical accuracy of this paper through their careful proofreading. Any remaining factual or interpretation errors are of course the author's own responsibility. Finally, the author dedicates this paper to the remembrance of his recently deceased colleague and friend, Dr. Randy Lin.

References

[1]Sarani, S., "A Novel Methodology for Reconstruction of Titan Atmospheric Density Using Cassini Guidance, Navigation, and Control Data," *Proceedings of the AIAA Guidance, Navigation, and Control Conference*, Hilton Head, South Carolina, August 20–23, 2007.

[2]Lee, A. Y. and Hanover, G. "Cassini Spacecraft Attitude Control System Flight Performance," *Proceedings of the AIAA Guidance, Navigation, and Control Conference*, San Francisco, California, August 15-18, 2005.

[3]Cassini Reveals Titan, *Science* Volume 308, No. 5724, 2005, pp. 978-992.

[4]Waite, J. H., et al., "Ion Neutral Mass Spectrometer Results from the First Flyby of Titan, " *Science* 308, 2005, pp. 982-986.

[5]Jaffe, L. and Herrell, L., "Cassini Huygens Science Instruments, Spacecraft, and Mission," *Journal of Spacecraft and Rockets* Vol. 34, No. 4, 1997, pp., 521. 509.

[6]Sarani, S., "Assessment of Per-Axis Thruster Control Authority of Cassini Spacecraft for Low Altitude Titan Flybys," Paper AAS 04-275, *14th AAS/AIAA Space Flight Mechanics Conference*, Maui, Hawaii, February 8-12, 2004, pp. 4-6.

[7]Barber, T.J. and Cowley, R.T., "Initial Cassini Propulsion System In-Flight Characterization," AIAA-2002-4152. *38th AIAA/ASME/SAE/ASEE Joint Propulsion Conference and Exhibit*, Indianapolis, Indiana, 2002.

[8]Nelder, J. A. and Mead, R. A., "Simplex Method for Function Minimization," *Computer Journal*, Vol. 7, No. 4, 1965, pp. 308-313.

[9]Fulchignoni M. et al., "In situ measurements of the physical characteristics of Titan's environment," *Nature* Vol. 438, 2005, pp., 785-786.

[10]Fulchignon, M., et al., "The Characterisation of Titan's Atmospheric Physical Properties by Huygens Atmospheric Structure Instrument (HASI)," *Space Science Reviews*, No. 104, 2002, pp. 397-434.

[11]Kazeminejad, B., et al., "Temperature variations in Titan's upper atmosphere: Impact on Cassini/Huygens," *Annales Geophysicae*, No. 23, pp. 1186-1187.

[12]Liechty, D.S., "Cassini-Huygens Aerodynamics with Comparison to Flight," NASA Technical Report, Aerothermodynamics Branch, NASA Langley Research Center, 2006.