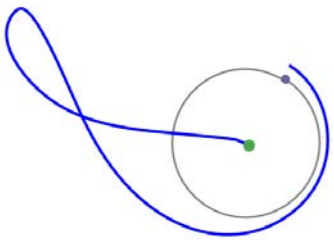
Monthly Variations of Low-Energy Ballistic Transfers to Lunar Halo Orbits



Jeffrey S. Parker

Jet Propulsion Laboratory
California Institute of Technology

3 Aug 2010

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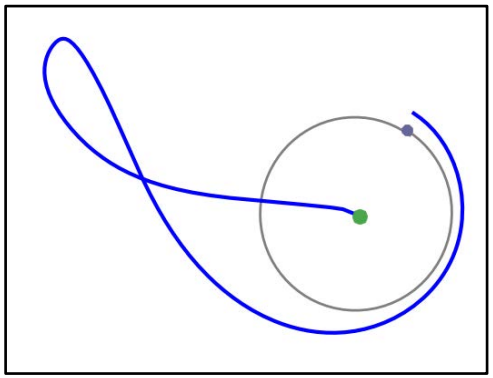
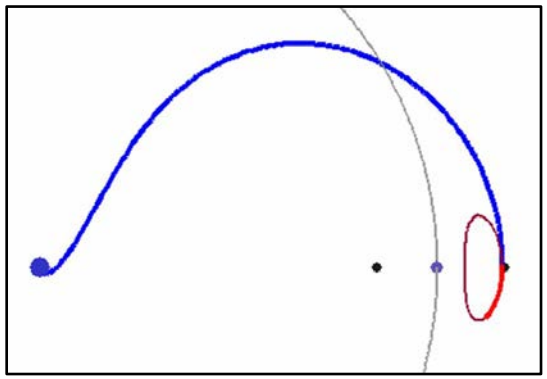
Motivation for Research

- Low-energy transfers: a cost-effective method of transferring satellites and cargo to the Moon.
  - Low-energy transfers between the Earth and Moon take advantage of the Sun's gravity to boost the spacecraft's energy.
  - They require 15 – 20% less ΔV, compared to direct transfers to the same lunar halo orbit.
  - As much as 35 – 40% more payload mass compared to a direct transfer to a halo orbit, given a standard launch vehicle and standard assumptions about the payload.
- Artemis is currently taking advantage of two such low-energy transfers.
- GRAIL is expecting to launch onto a low-energy lunar transfer next year.

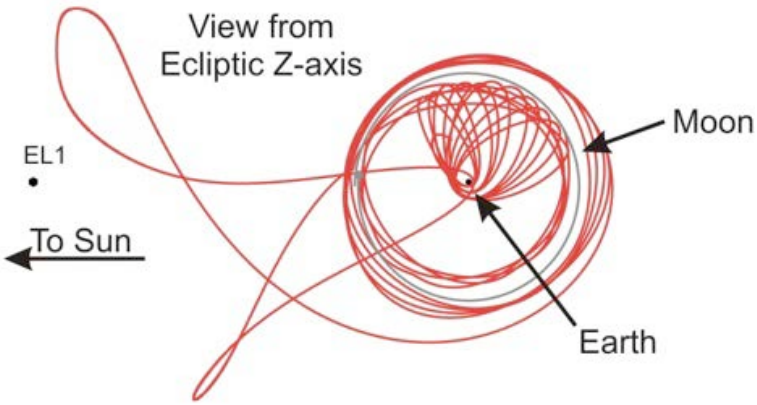




Motivation for Research

- Low-energy transfers: difficulties
  - No analytical methods have been found to build low-energy transfers!
  - Time-intensive work to design one transfer. Impractical to design every contingency situation and/or extended mission.
    - Missed maneuvers
    - Missions of opportunities
    - Extended missions

- On-going research:
  - Mapping out the trade-space of low-energy transfers
  - Development of a rapid-design tool for building low-energy transfers.



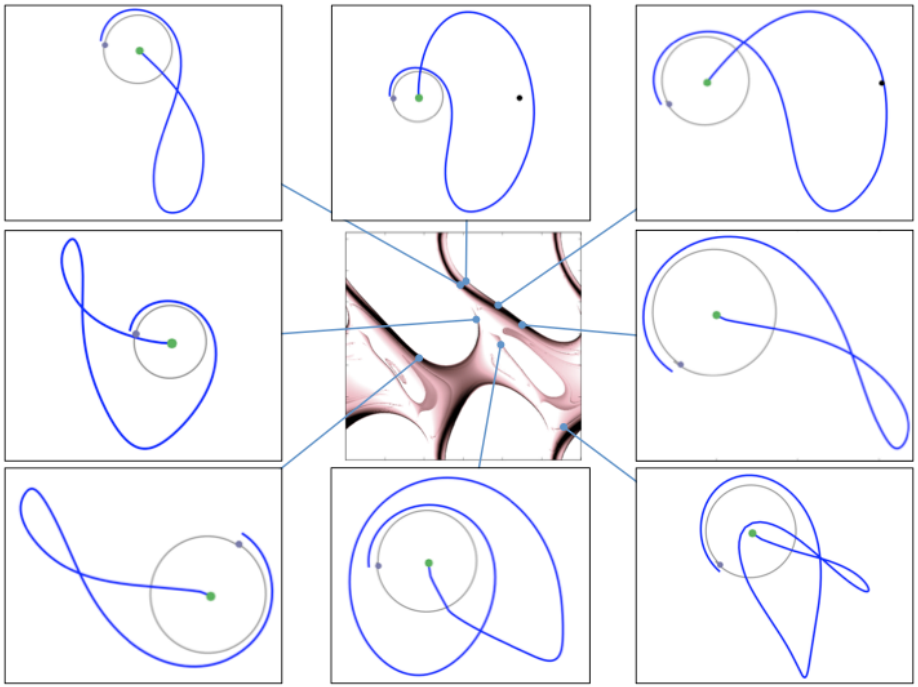
Artemis' complex lunar transfer

Motivation for this Paper

- Previous work has identified numerous families of low-energy transfers that exist in a particular month.

- Issues with the real solar system:
  - Earth's equator is tilted relative to the ecliptic
  - Moon's orbit is not coplanar with the Earth's orbit
  - The Earth and Moon are in elliptical orbits.

- Questions:
  - How do these transfers vary from month to month?
  - Do some exist only in a few months of the year?
  - Can we predict these variations?



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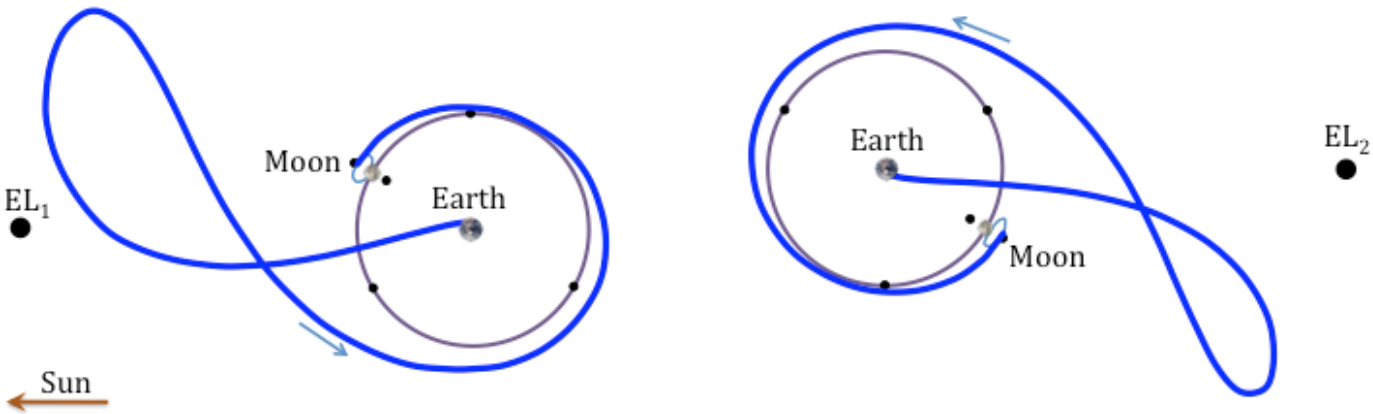
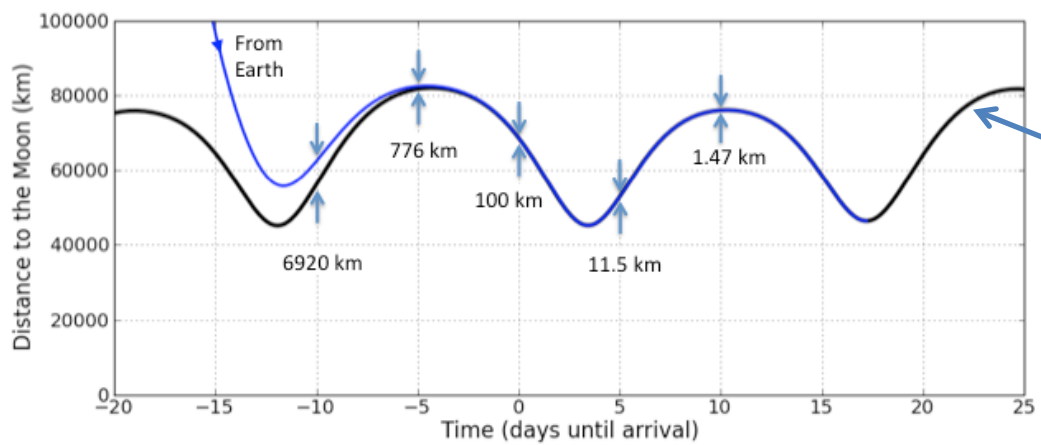
Monthly Variations of Low-Energy Ballistic Transfers to Lunar Halo Orbits

- Background
  - Models and Methodology
  - Map of the state space of a full month's worth of low-energy lunar transfers
- Survey of the state space of a year's worth of low-energy lunar transfers
- Filter the data to explore the variations in the parameters of simple, direct low-energy transfers
- Explore annual variations
- Discussion and Conclusions

Background: Low-Energy BLTs

- Low-energy ballistic lunar transfers to lunar halo orbits

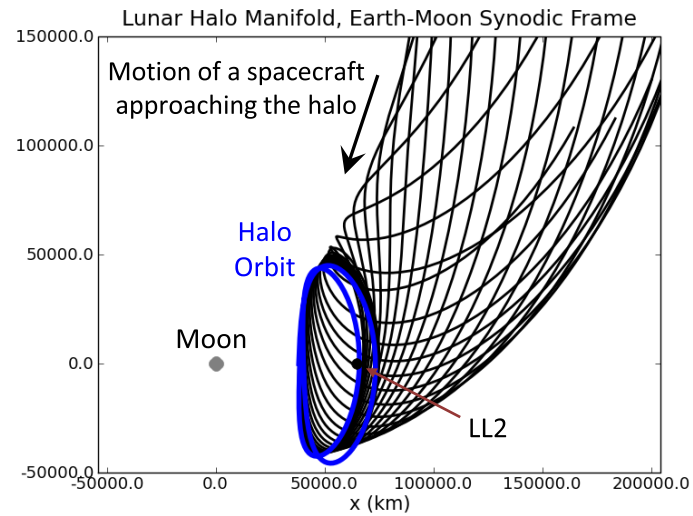


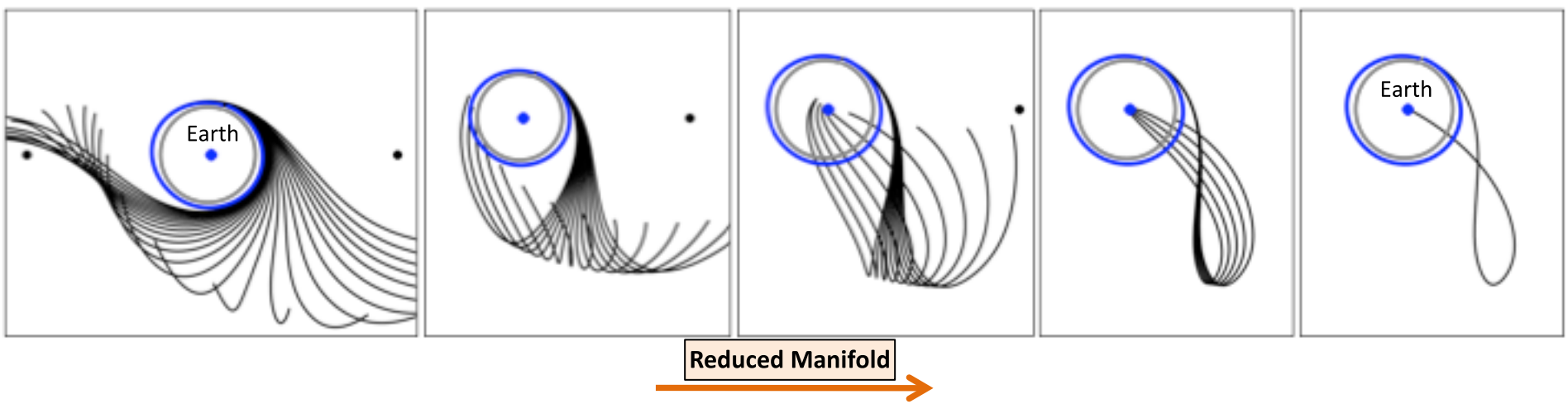


Lunar Halo Orbit

Background: Dynamical Systems Methods

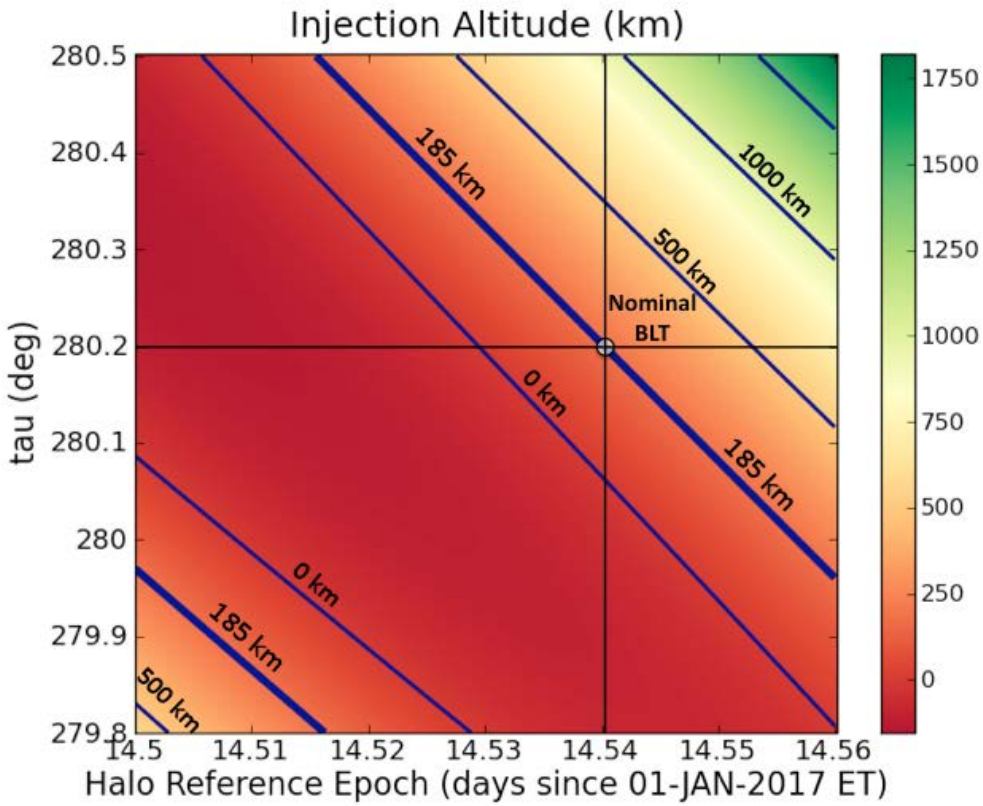
1. Build the target orbit
   - E.g., Southern LL_2 halo orbit with A_z = 30,000 km
   - Convert it to DE421 model with reference epoch of 1/1/2017
2. Generate the orbit's stable manifold
3. Track each trajectory's perigee altitude
4. Identify those trajectories that encounter the Earth
   - Desirable altitude: 185 km

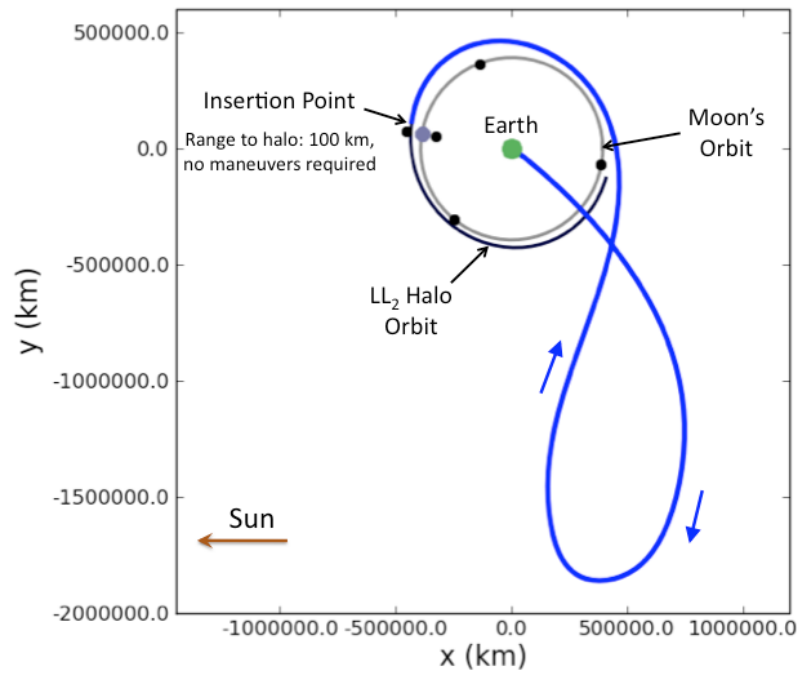




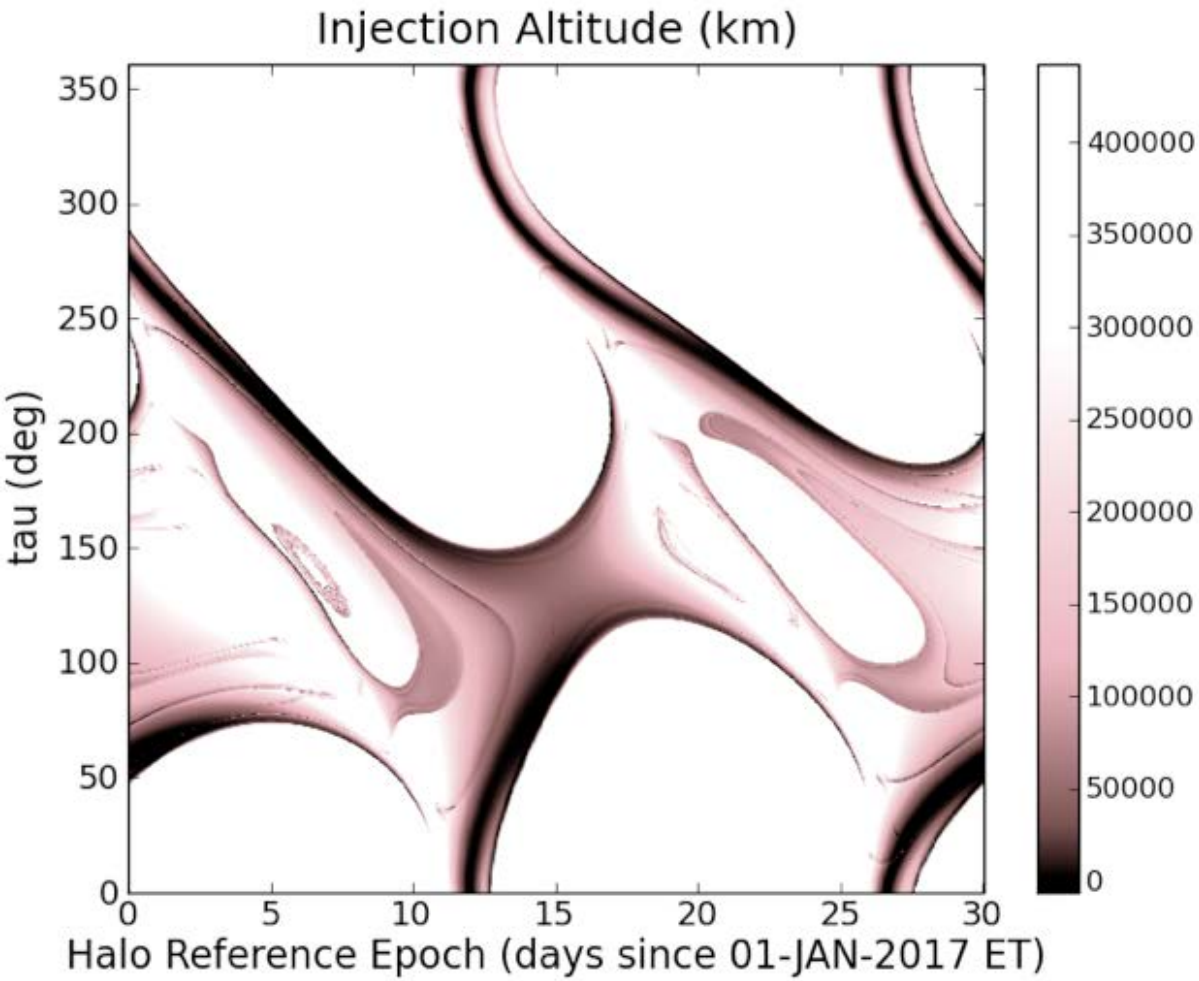
Background: Dynamical Systems Methods

5. Vary the orbit's reference epoch to search for all trajectories that exist in a month to the same target orbit.

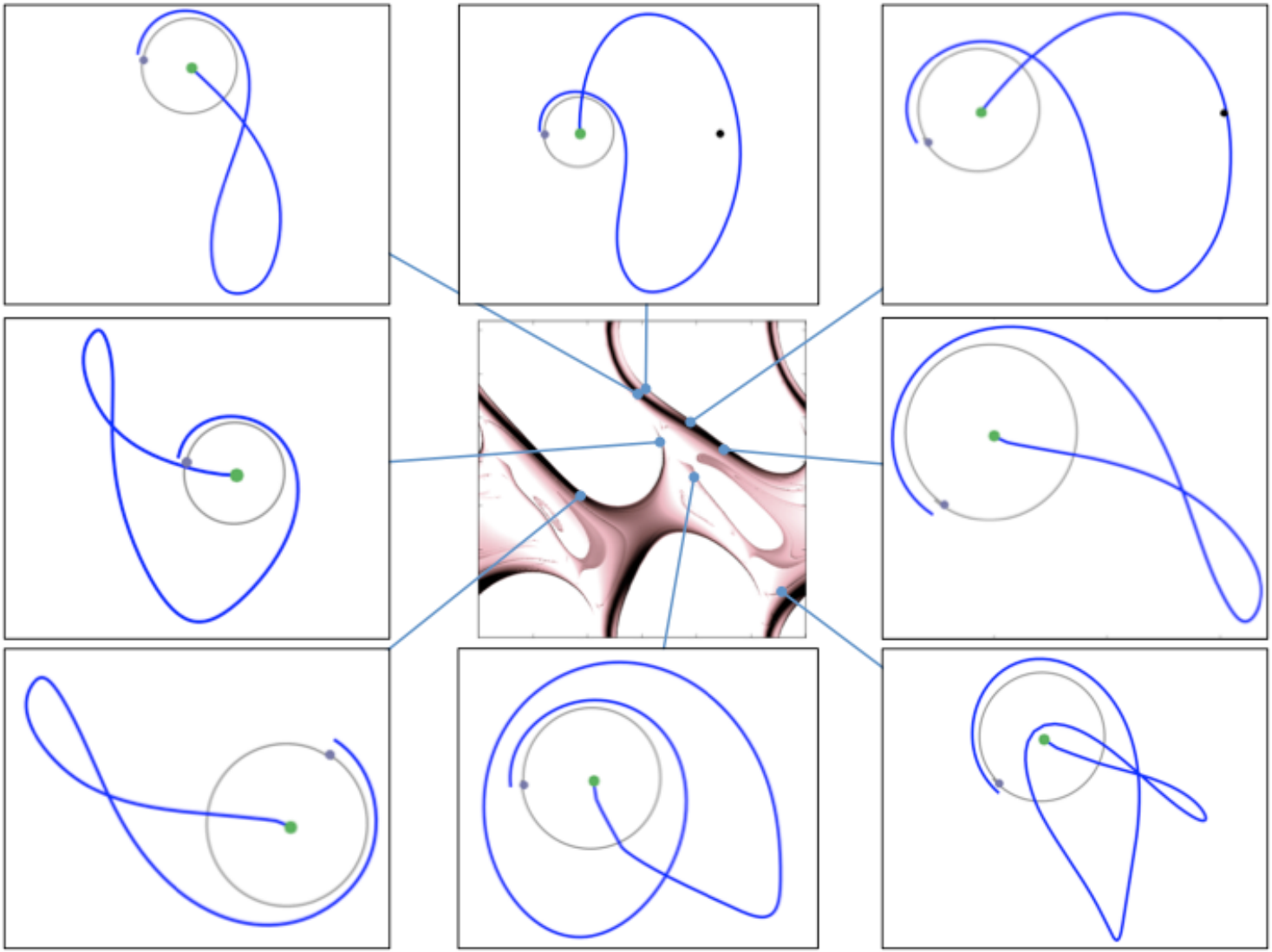




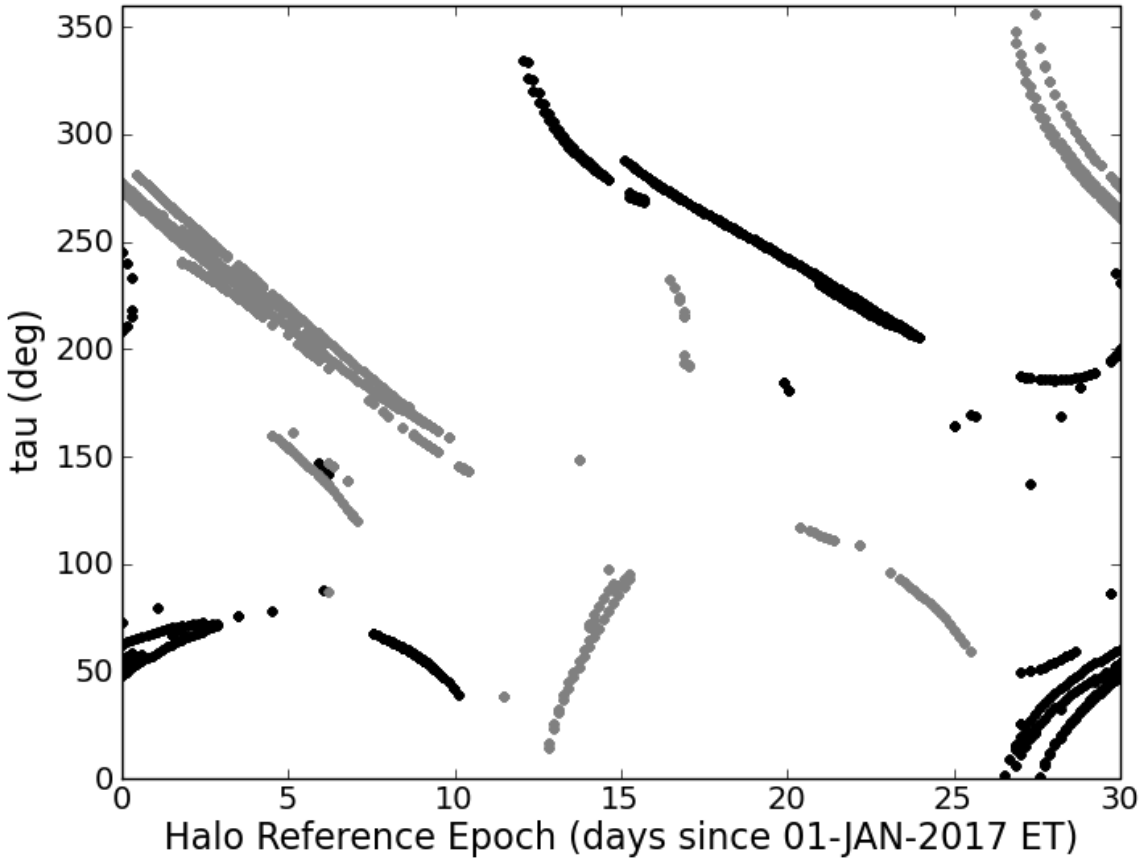
BLT Maps


Injection Altitude (km)
tau (deg)
Halo Reference Epoch (days since 01-JAN-2017 ET)
0
50
100
150
200
250
300
350
0
5
10
15
20
25
30
400000
350000
300000
250000
200000
150000
100000
50000
0

BLT Maps



Tracing Families of BLTs



Identifying all points where h_p = 185 km

Black dots: EL_2 transfers
Gray dots: EL_1 transfers

Varieties of BLTs

#	Δ Reference Epoch* (days)	τ (deg)	EL_1 / EL_2	C3 (km^2/s^2)	Transfer Δt (days)	# Earth Flybys	# Lunar Flybys	Injection Inclination (deg) Equatorial	Injection Inclination (deg) Ecliptic
1	12.060302	334.519	2	−0.2902	133.76	0	0	23.225	28.192
2	12.211055	333.736	2	−0.3457	132.91	0	0	131.701	151.274
3	14.170854	283.655	2	−0.3444	118.36	0	0	51.319	69.045
4	15.226131	271.069	2	−0.4944	108.76	0	1	32.329	51.431
5	15.829146	279.419	2	−0.4296	171.62	0	0	85.326	103.860
6	20.351759	238.347	2	−0.6556	130.11	0	0	115.737	137.694
7	20.351759	239.232	2	−0.5856	145.20	0	0	21.877	22.738
8	22.311558	221.171	2	−0.6904	137.51	0	1	35.973	13.527
9	23.819095	206.901	2	−0.7153	129.17	0	0	22.180	10.275
10	20.050251	180.970	2	−1.8533	171.79	0	1	97.684	92.972
11	25.025126	164.113	2	−1.9222	146.35	0	1	20.490	4.271
12	27.286432	137.373	2	−2.0307	176.72	0	2	38.302	36.809
13	28.190955	168.405	2	−2.0880	122.46	2	2	19.325	30.359
14	28.040201	185.608	2	−1.0318	145.08	0	1	34.251	11.315
15	28.040201	185.630	2	−1.6144	145.75	0	2	103.995	126.244
16	0.000000	55.325	2	−0.9032	179.35	2	1	143.590	121.792
17	0.150754	63.382	2	−0.6429	97.90	0	0	23.372	0.836
18	0.452261	54.781	2	−0.6608	132.55	0	0	145.538	168.969
19	1.507538	66.990	2	−1.1266	113.39	0	1	166.454	144.152
20	8.592965	59.539	2	−0.8393	178.32	0	1	99.214	87.676
21	8.592965	59.962	2	−0.6791	165.37	0	0	14.732	20.434
22	6.030151	144.580	2	−0.6940	170.11	0	3	23.140	17.669
23	27.889447	53.118	2	−0.9637	140.22	1	2	11.452	28.632
24	28.040201	15.470	2	−0.4261	172.37	0	1	27.743	40.712
25	28.190955	34.787	2	−0.5891	105.30	0	0	148.336	171.495
26	28.341709	43.756	2	−0.5740	96.55	0	0	20.962	3.797

Varieties of BLTs

#	Δ Reference Epoch* (days)	τ (deg)	EL_1 / EL_2	C3 (km^2/s^2)	Transfer Δt (days)	# Earth Flybys	# Lunar Flybys	Injection Inclination (deg) Equatorial	Injection Inclination (deg) Ecliptic
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3	14.170854	283.655	2	−0.3444	118.36	0	0	51.319	69.045
4	15.226131	271.069	2	−0.4944	108.76	0	1	32.329	51.431
5	15.829146	[illegible]	[illegible]	−0.4296	171.62	0	0	85.326	103.860
6	20.351759	[illegible]	[illegible]	−0.6556	130.11	0	0	115.737	137.694
7	20.351759	[illegible]	[illegible]	−0.5856	145.20	0	0	21.877	22.738
8	22.311558	221.171	2	−0.6904	137.51	0	1	35.973	13.527
9	23.819095	206.901	2	−0.7153	129.17	0	0	22.180	10.275
10	20.050251	180.970	2	−1.8533	171.79	0	1	97.684	92.972
11	25.025126	164.113	2	−1.9222	146.35	0	1	20.490	4.271
12	27.286432	137.373	2	−2.0307	176.72	0	2	38.302	36.809
13	28.190955	168.405	2	−2.0880	122.46	2	2	19.325	30.359
14	28.040201	185.608	2	−1.0318	145.08	0	1	34.251	11.315
15	28.040201	185.630	2	−1.6144	145.75	0	2	103.995	126.244
16	0.000000	55.325	2	−0.9032	179.35	2	1	143.590	121.792
17	0.150754	63.382	2	−0.6429	97.90	0	0	23.372	0.836
18	0.452261	54.781	2	−0.6608	132.55	0	0	145.538	168.969
19	1.507538	66.990	2	−1.1266	113.39	0	1	166.454	144.152
20	8.592965	59.539	2	−0.8393	178.32	0	1	99.214	87.676
21	8.592965	59.962	2	−0.6791	165.37	0	0	14.732	20.434
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25	28.190955	34.787	2	−0.5891	105.30	0	0	148.336	171.495
26	28.341709	43.756	2	−0.5740	96.55	0	0	20.962	3.797

Low C3

Varieties of BLTs

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9	23.819095	206.901	2	−0.7153	129.			22.180	10.275
10	20.050251	180.970	2	−1.8533	171.7			97.684	92.972
11	25.025126	164.113	2	−1.9222	146.3			20.490	4.271
12	27.286432	137.373	2	−2.0307	176.7			38.302	36.809
13	28.190955	168.405	2	−2.0880	122.46	2	2	19.325	30.359
14	28.040201	185.608	2	−1.0318	145.08	0	1	34.251	11.315
15	28.040201	185.630	2	−1.6144	145.75	0	2	103.995	126.244
16	0.000000	55.325	2	−0.9032	179.35	2	1	143.590	121.792
17	0.150754	63.382	2	−0.6429	97.90	0	0	23.372	0.836
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~28.5° Injection Inclination

Varieties of BLTs

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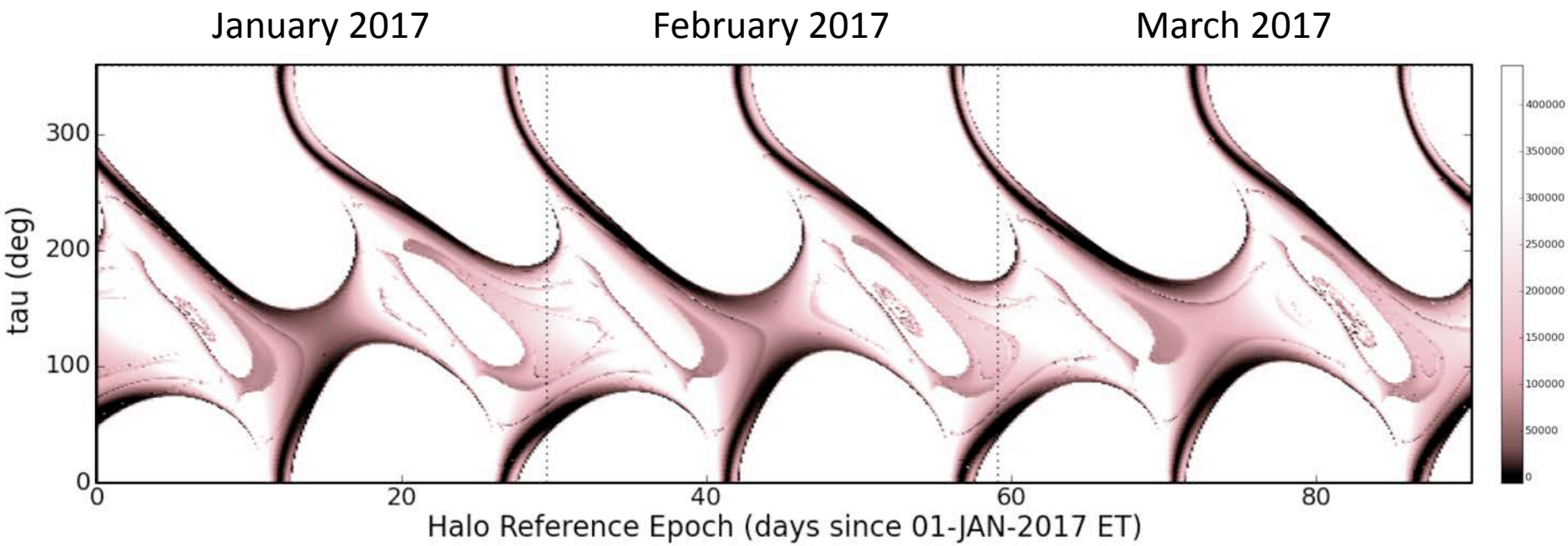
22	6.030151	144.580	2	−0.6940	170.11	0	3	23.140	17.669
23	27.889447	53.118	2	−0.9637	140.22	1	2	11.452	28.632
24	28.040201	15.470	2	−0.4261	172.37	0	1	27.743	40.712
25	28.190955	34.787	2	−0.5891	105.30	0	0	148.336	171.495
26	28.341709	43.756	2	−0.5740	96.55	0	0	20.962	3.797

Short Transfer Time

BLT Maps Varying over Time

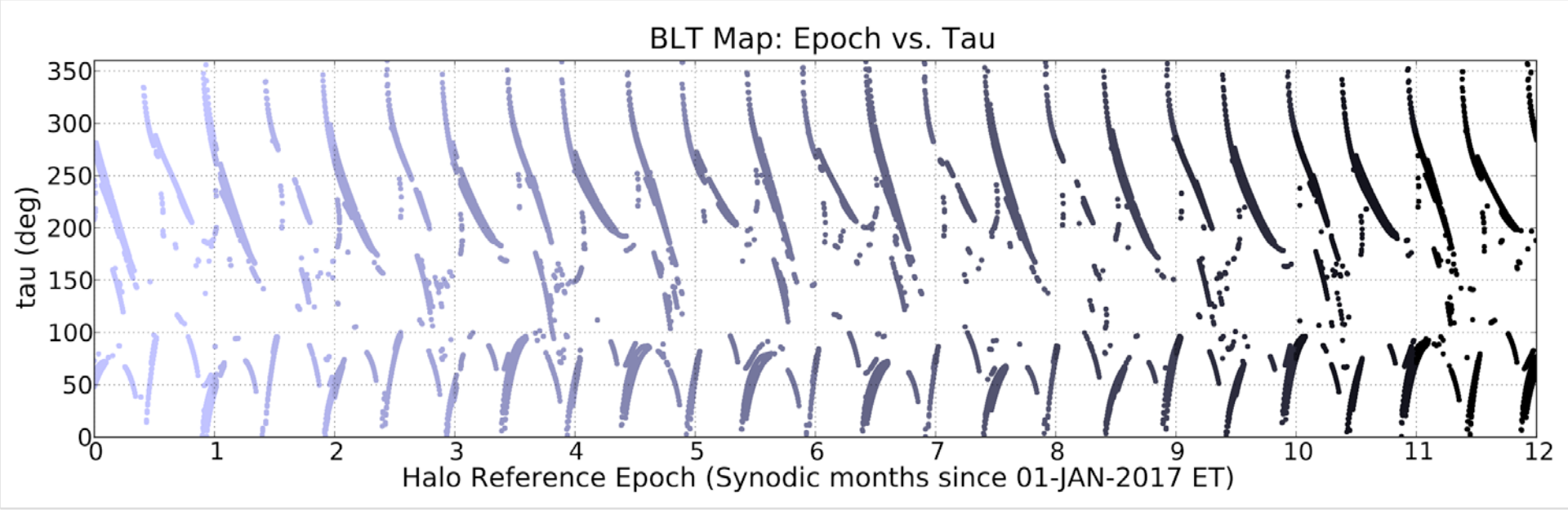


Many dominant features repeat each month, although there are small variations.

Complex features observed in the map are far more sensitive.

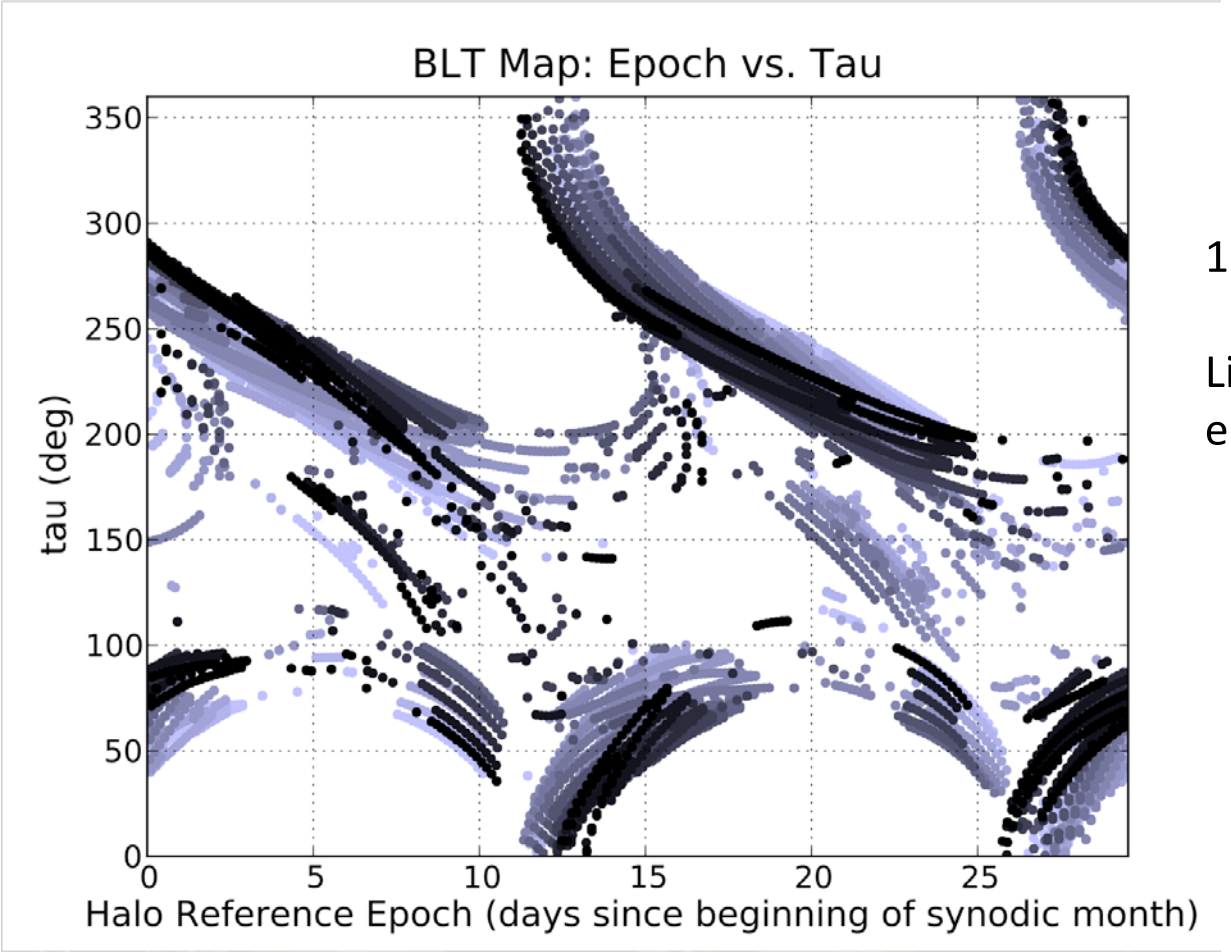
BLT Maps Varying over Time

Tracking the locations of 185-km perigees throughout the state space year.
Many features repeat each month of the year. Others do not.



Jan Mar May July Sept Nov

Tracking Families over 12 Months



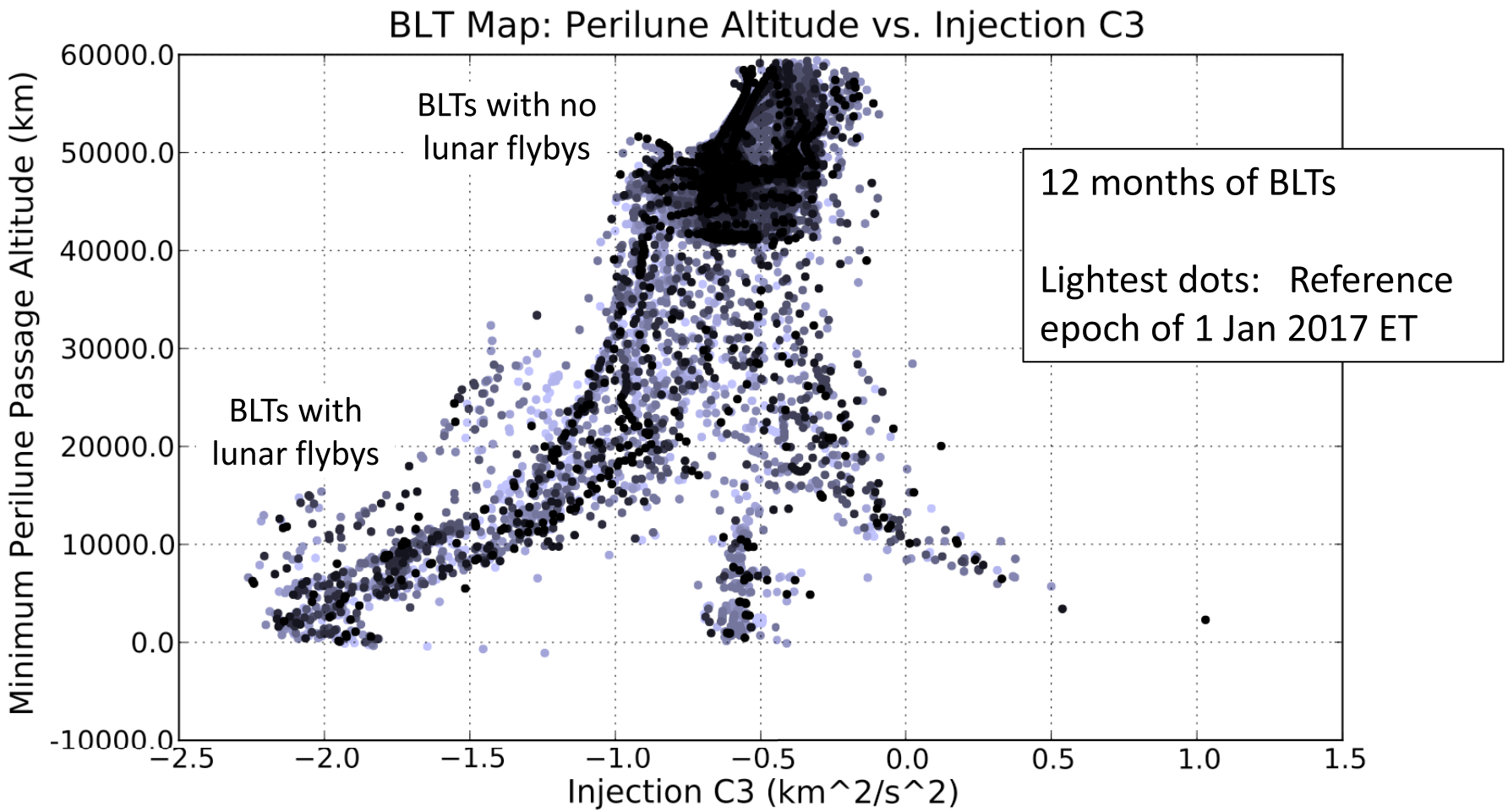
12 months of BLTs

Lightest dots: Reference epoch of 1 Jan 2017 ET

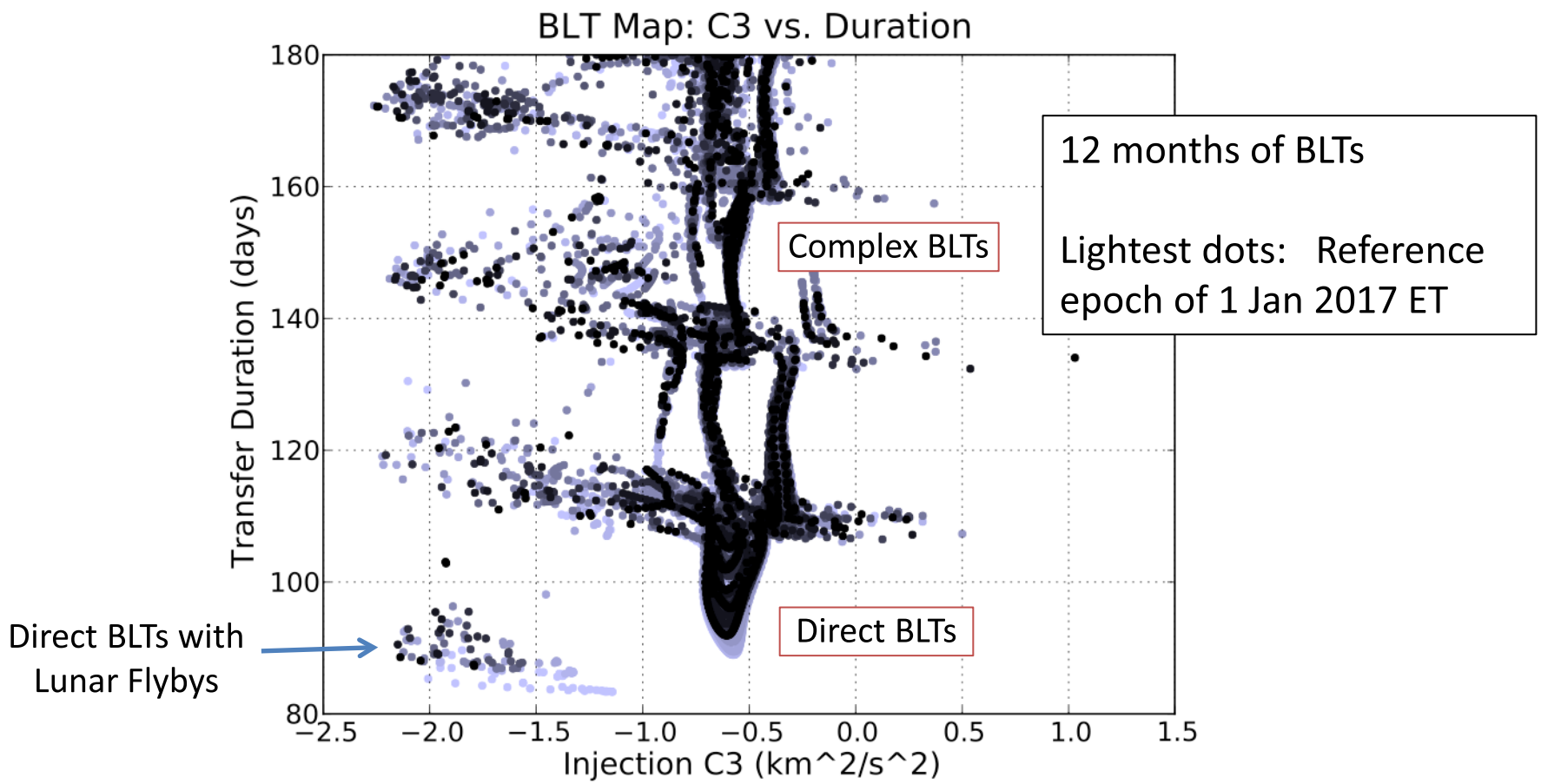
Surveys of Monthly Variations

- We are now interested in surveying these transfers.
- Identify trends in the data, including secular trends, monthly cycles, and annual cycles.
- The goal: Predict the characteristics of low-energy transfers in any given month
  - Benefits mission planning
  - Develop rapid design tools to generate such transfers quickly

C3 vs. Perilune Altitude



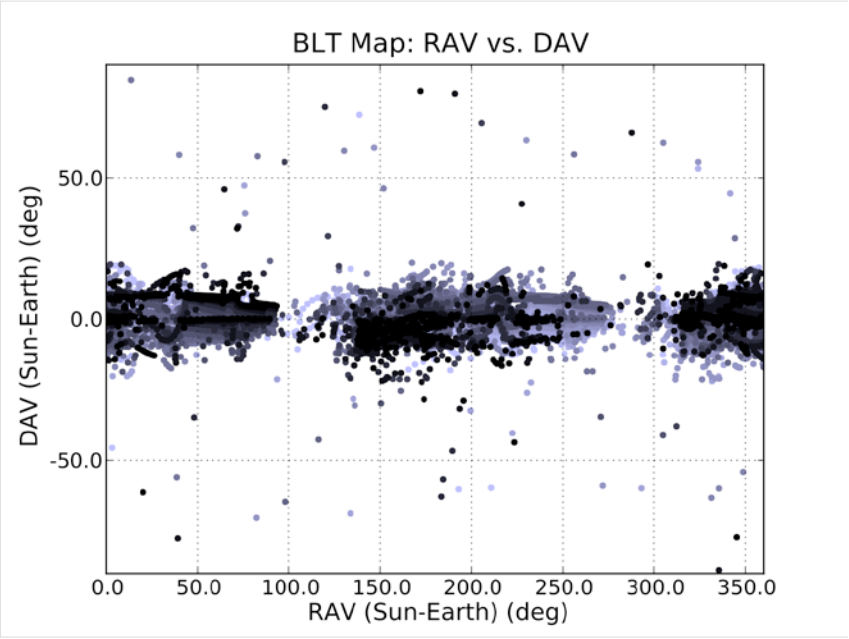
C3 vs. Transfer Duration

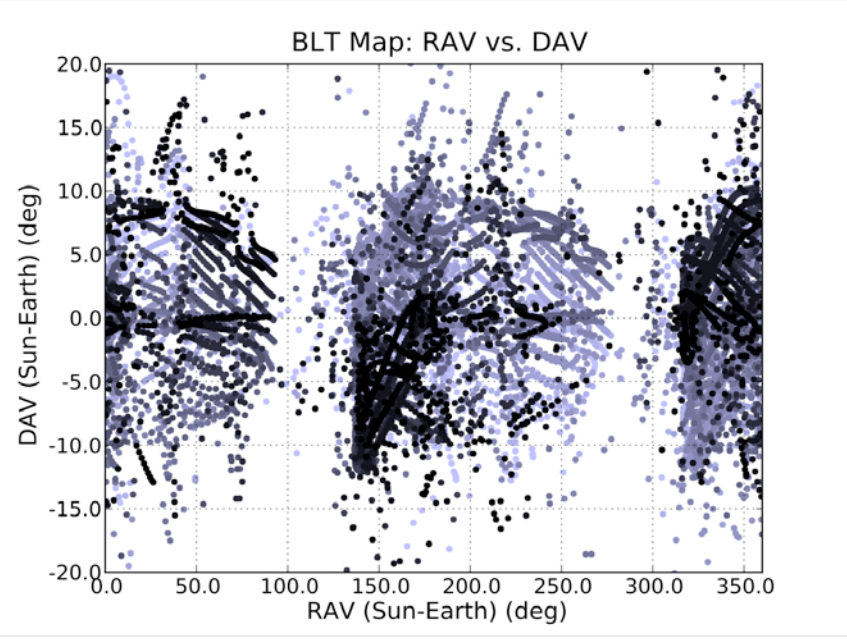


RAV vs. DAV

GRAIL is implementing RAV/DAV targeting, since it is launching into a high orbit about the Earth, rather than the conventional RLA/DLA.

RAV = Right Ascension of the Apogee Vector
DAV = Declination of the Apogee Vector

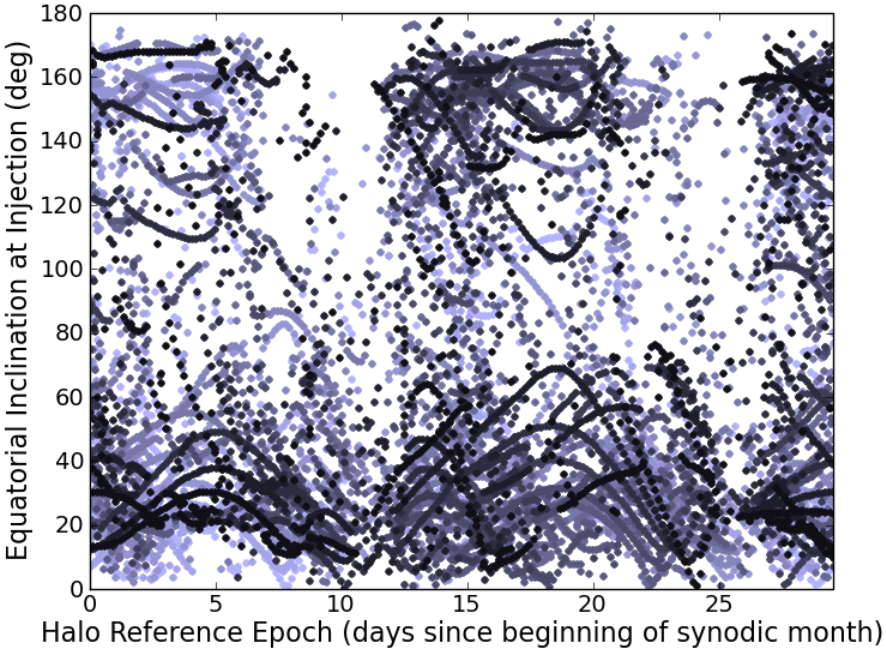




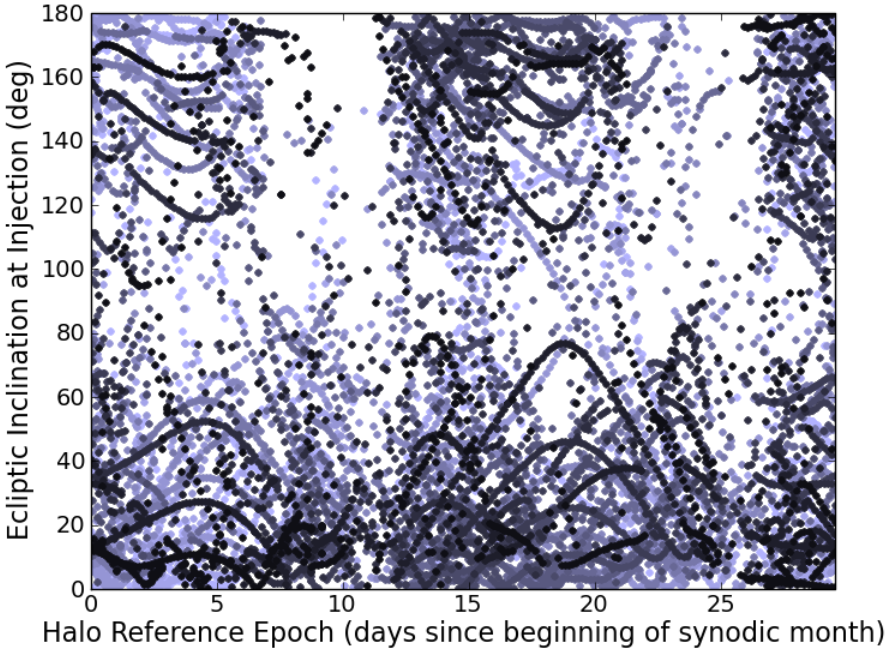
The majority of transfers depart the Earth very near the ecliptic plane.

Injection Inclination

Equatorial Inclination



Ecliptic Inclination



BLTs depart from nearly any inclination.

Similar BLTs have very different injection inclinations from month to month.

Filtering the Results

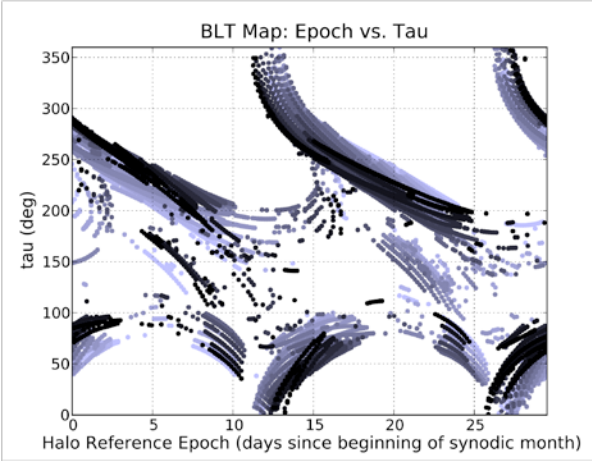
- Trends while considering all transfers identified in a year:
  - Typical transfers require injection C3 values near -0.6 km^2/s^2.
  - Transfers that fly by the Moon during the outbound segment may require less C3 (-2.0 km^2/s^2) or more (0.5 km^2/s^2), depending on the geometry.
  - Departure DAV values are typically in the range $-10° \leq DAV \leq 10°$ relative to the ecliptic.
  - Simple, direct BLTs require 90 – 110 days of transfer duration.
  - No identified trends in the departure inclination – can be any value.

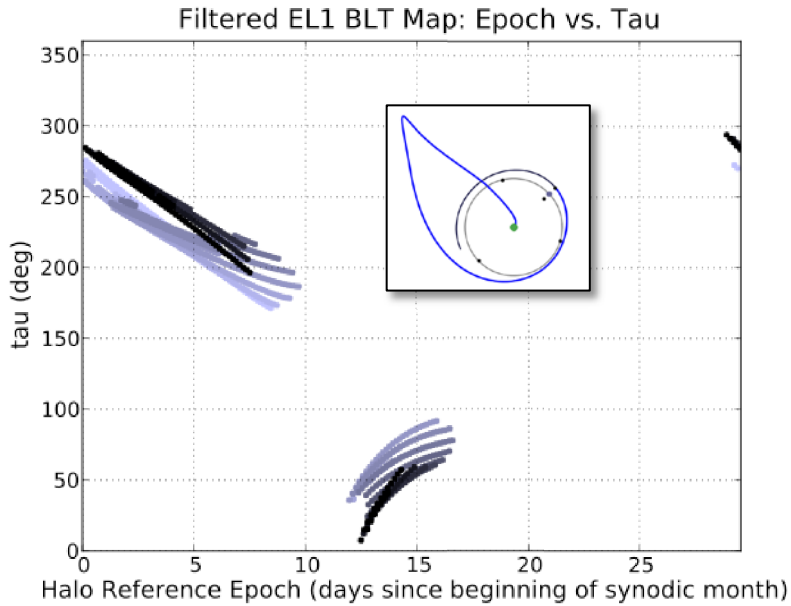
- To achieve more information, we need to filter the set and observe how a particular family varies over time
  - Maximum transfer duration: 105 days [short transfers]
  - Minimum perigee altitude after TLI: 50,000 km [no Earth phasing orbits]
  - Minimum perilune altitude: 20,000 km [no lunar flybys]

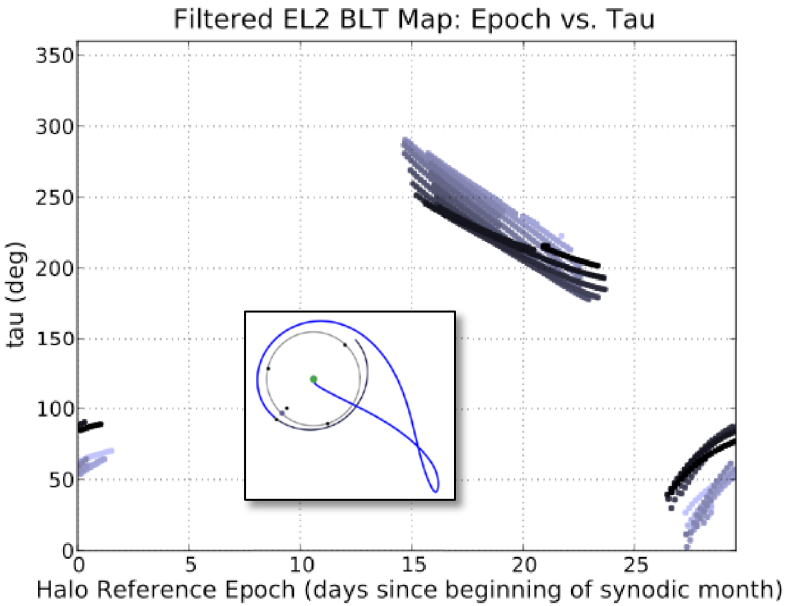
Filtered BLT Map

Simple, Direct, quick BLTs

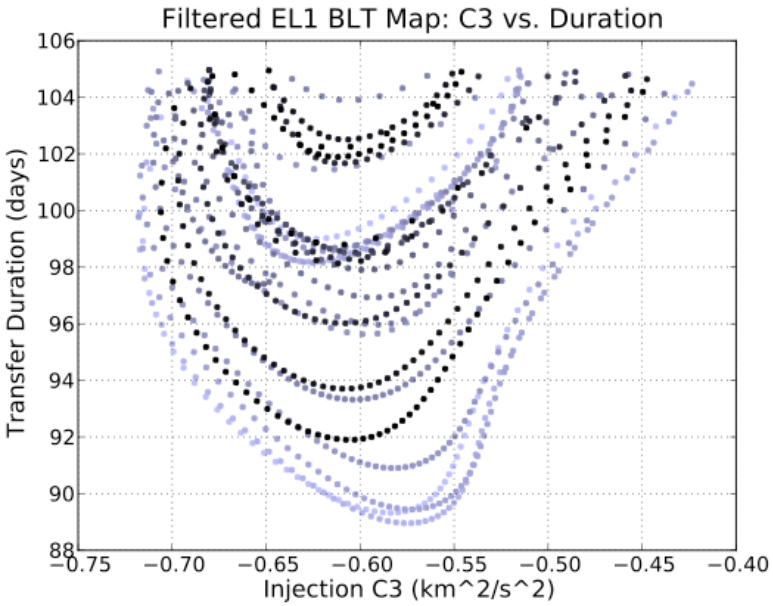
No flybys
No phasing orbits
Duration < 105 days

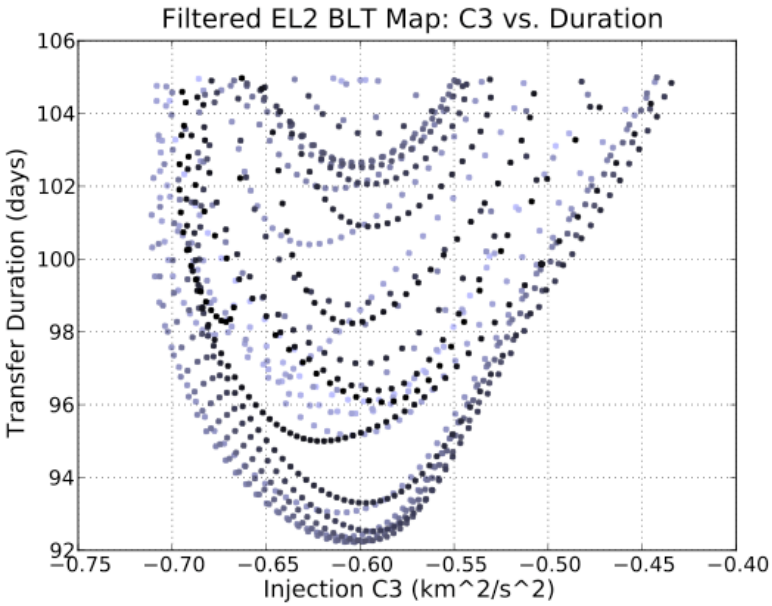






Filtered, C3 vs. Duration



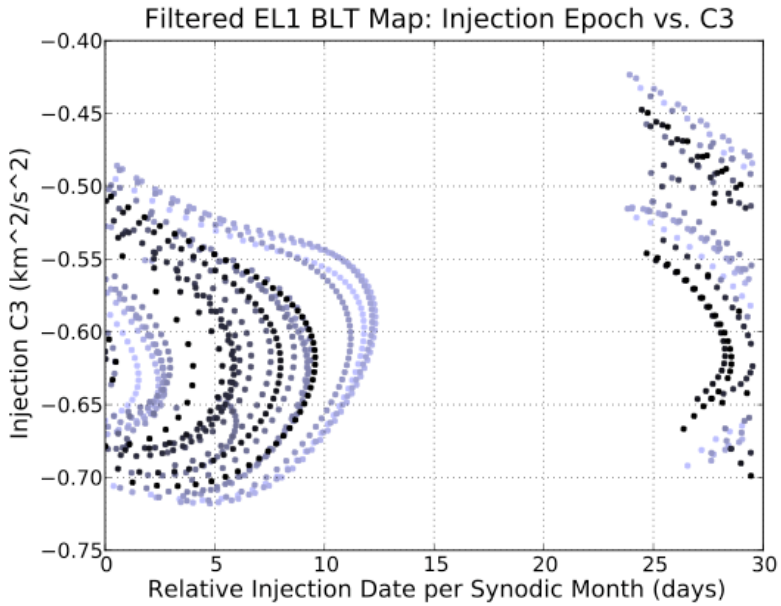


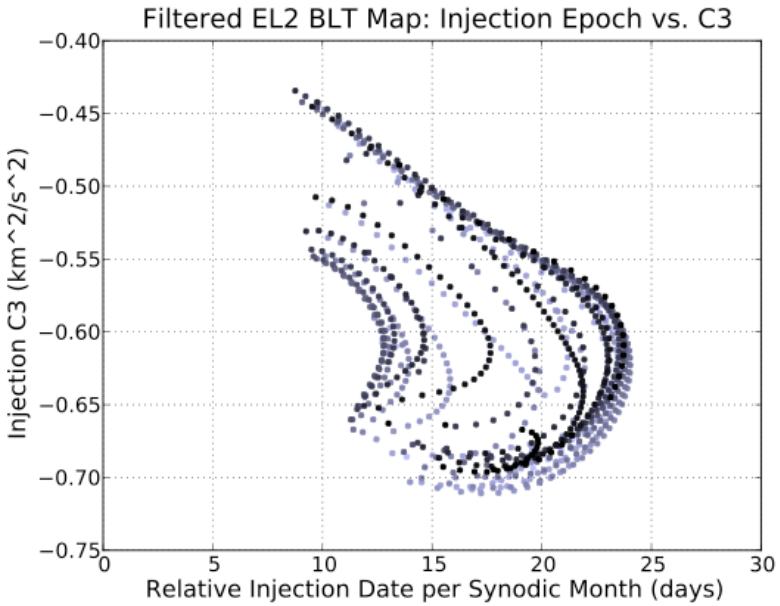
C3 = -0.6 ± 0.15 km^2/s^2

Transfer Duration > 89 days

(between LEO and 100 km from the target halo orbit)

Filtered, Injection Epoch vs. C3

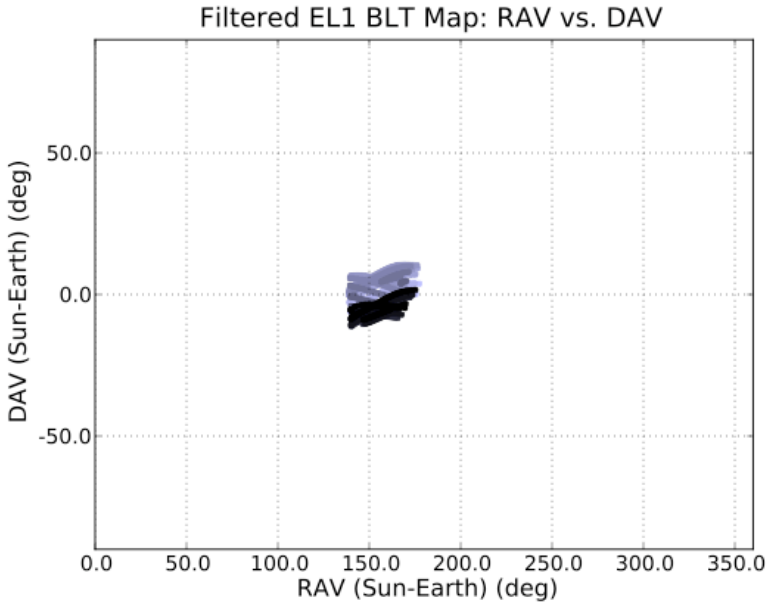


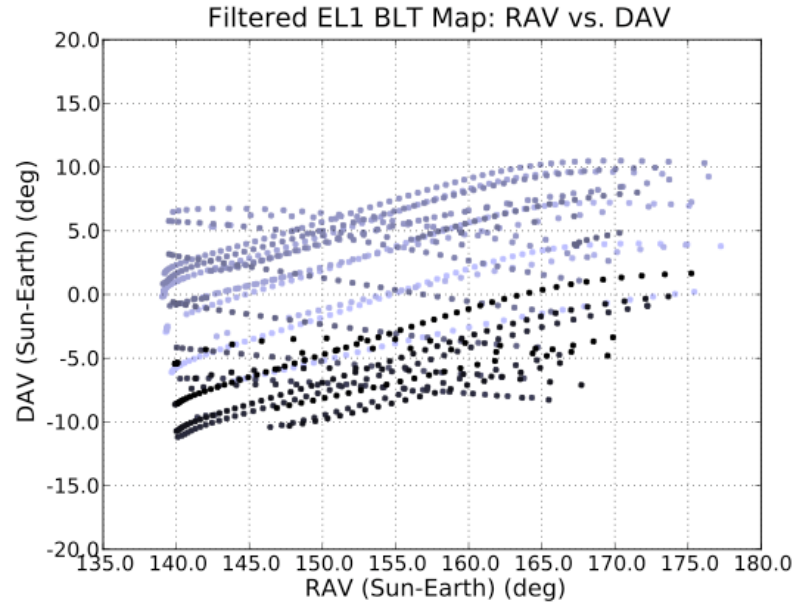


There are about 15 days per month that one can launch on a direct BLT through EL_1 or EL_2 only.

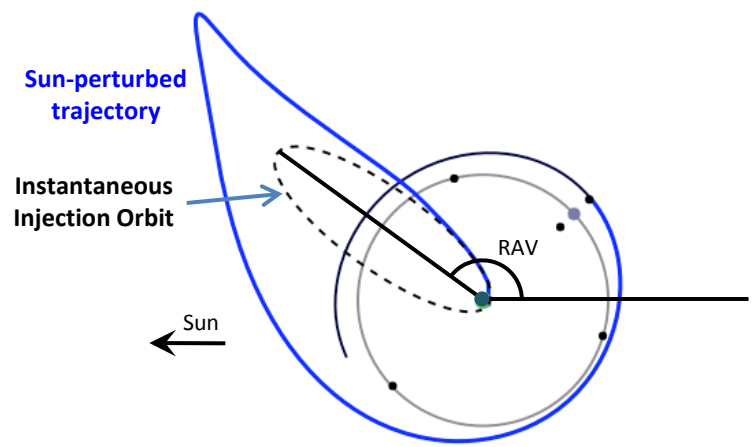
Longer transfers will extend this launch period.

Filtered, RAV vs. DAV (EL_1)

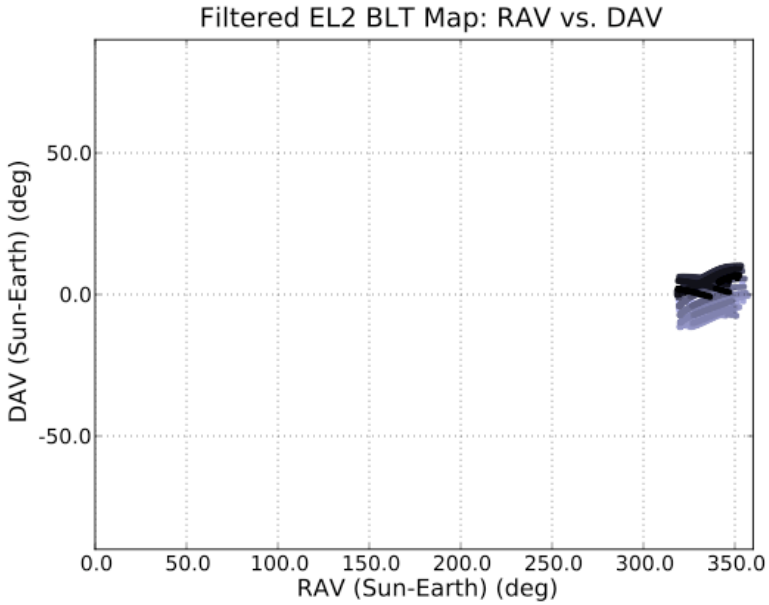


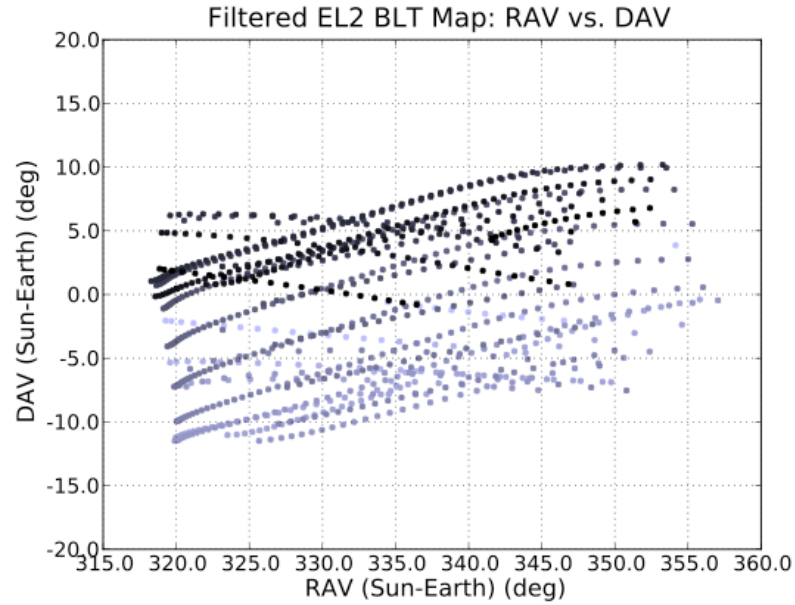


RAV and DAV are very tightly defined in the Sun-Earth frame!

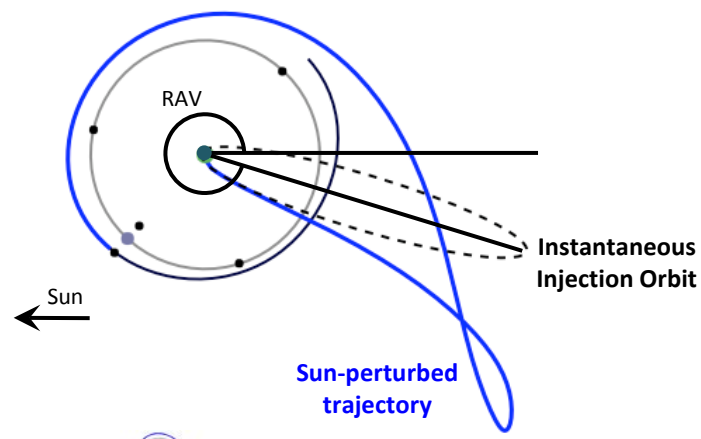


Filtered, RAV vs. DAV (EL_2)

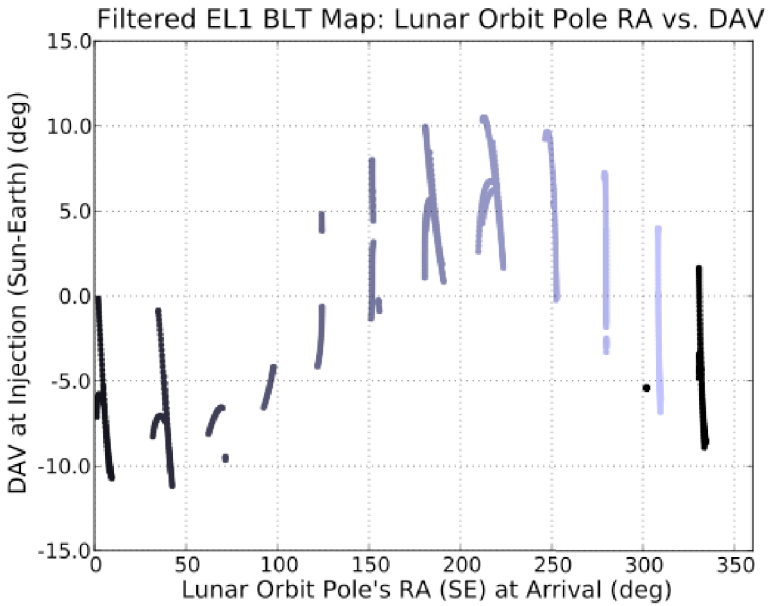


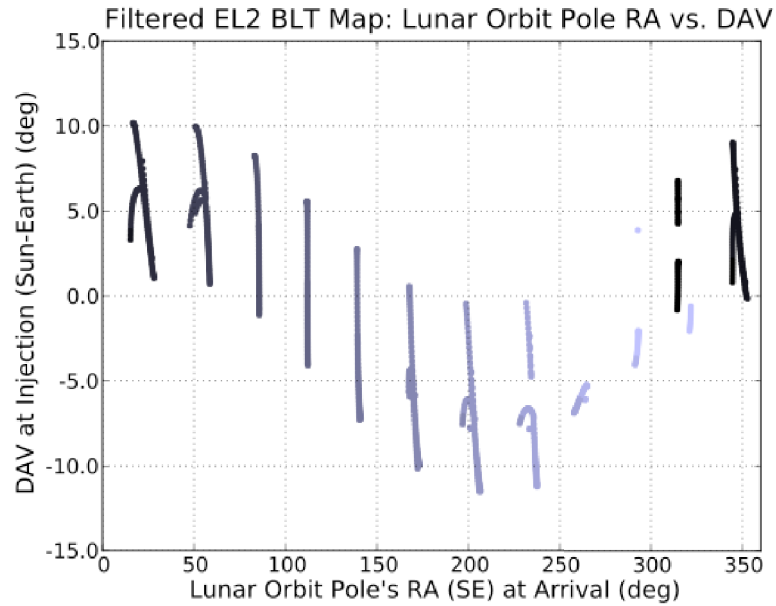


RAV and DAV are very tightly defined in the Sun-Earth frame!



Filtered, Lunar Orbit Pole RA vs. DAV

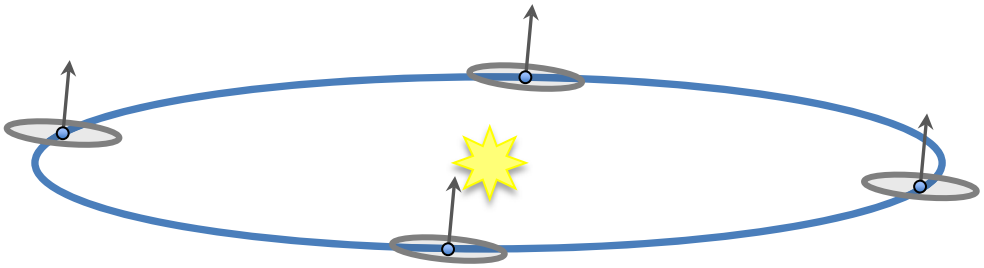




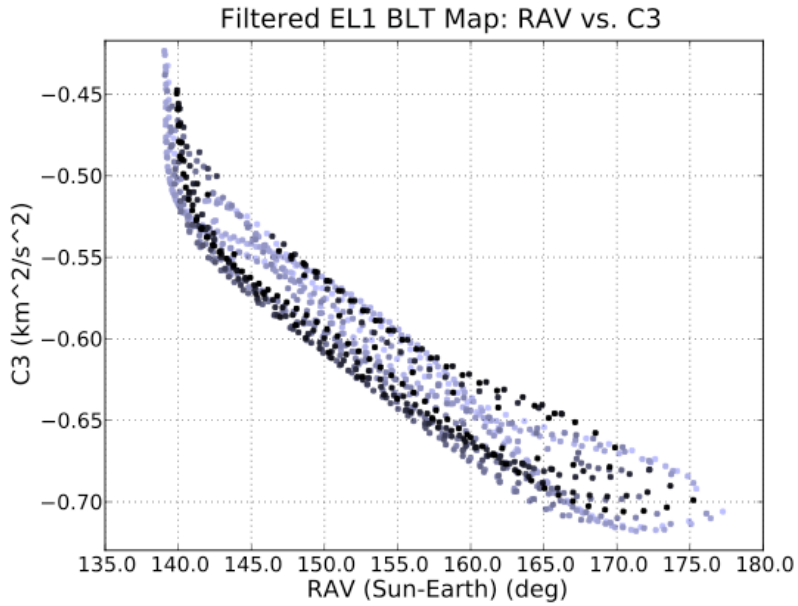
Mean inclination of the Moon's orbit relative to the ecliptic: 5.14°

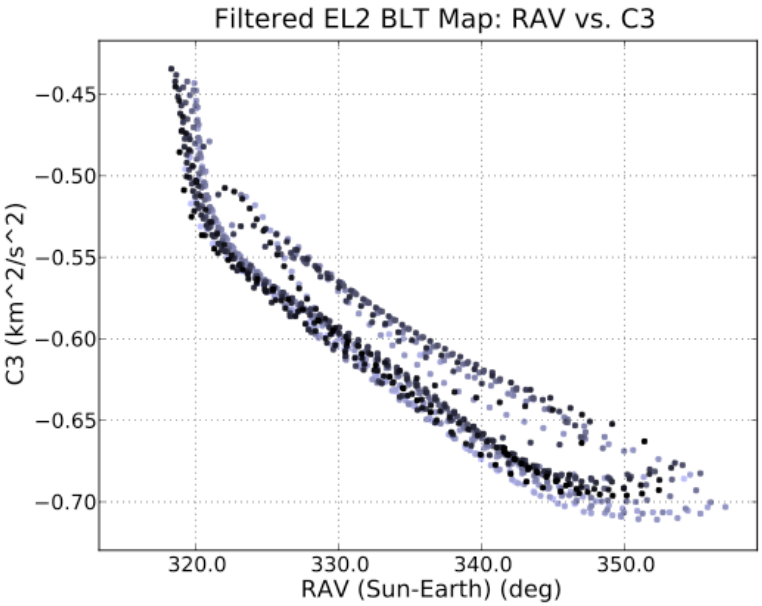
Measure the right ascension of the Moon's orbit pole vector in the Sun-Earth frame.

Reduces the uncertainty in an initial guess of DAV from ±10° to ±5°



Filtered, RAV vs. C3

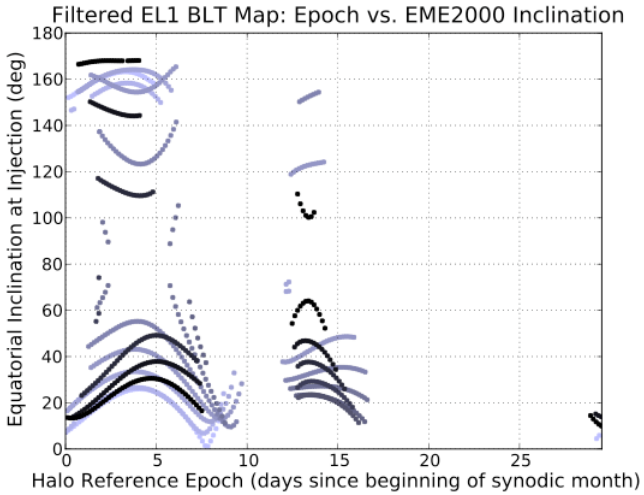


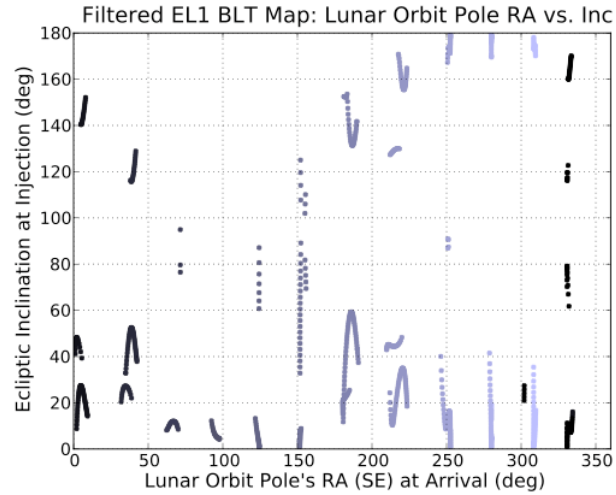


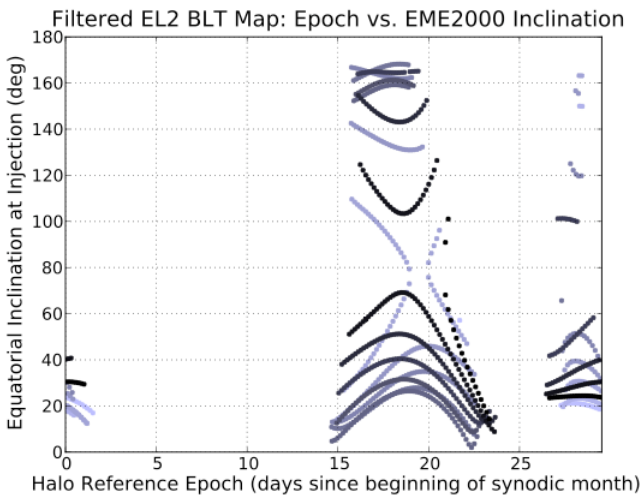
There is a clear relationship between the injection RAV and the injection C3.

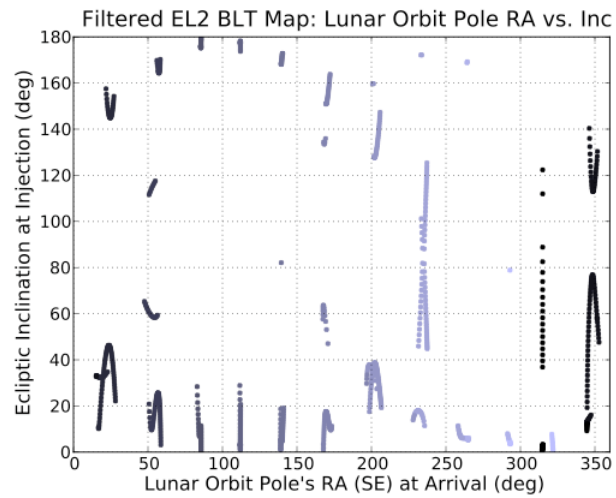
To first order, one finds that by increasing RAV by 5°, one can reduce C3 by ~0.05 km^2/s^2.

Filtered, Injection Inclination









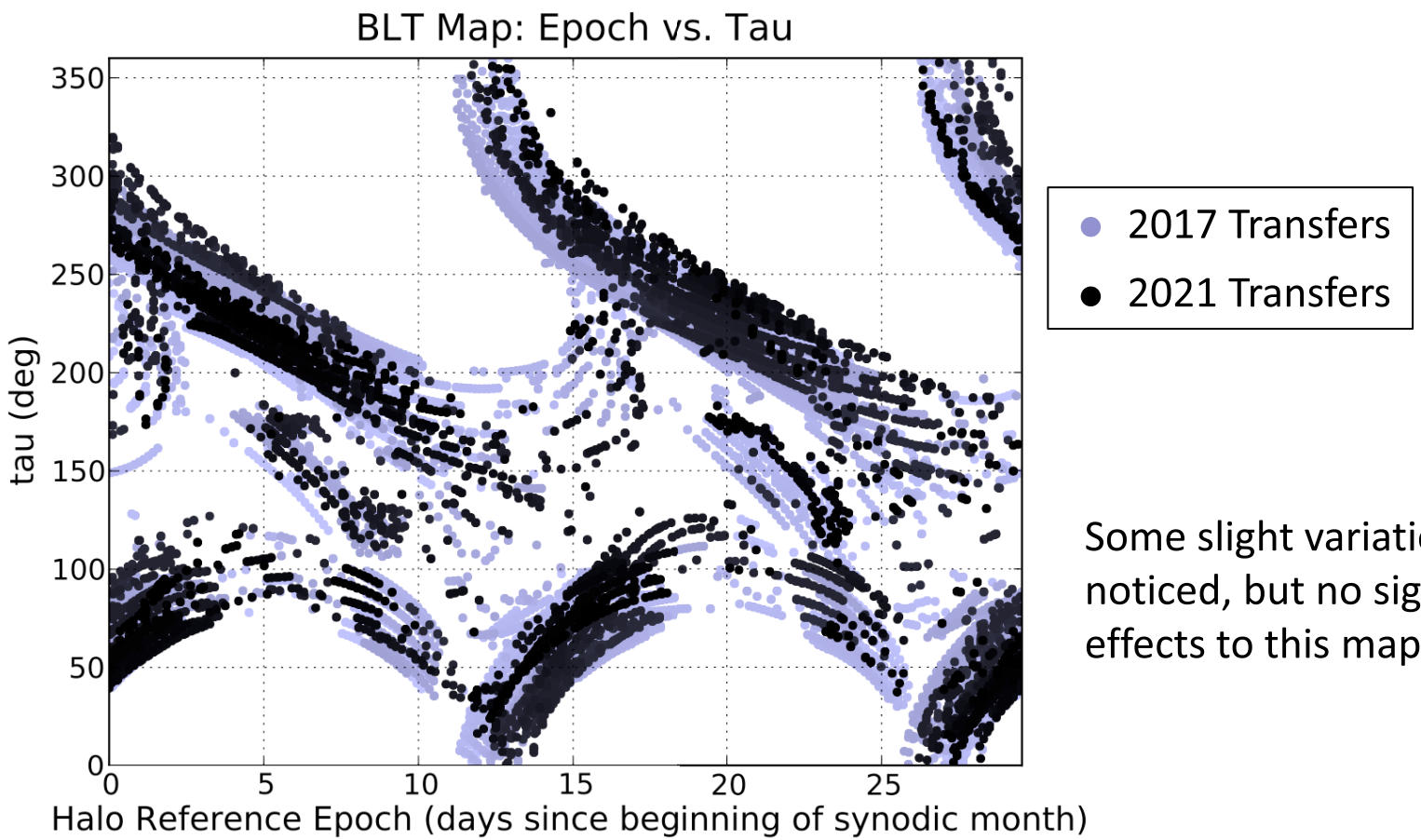
Monthly Trends

- Trends while considering all transfers identified in a year:
  - Typical transfers require injection C3 values near -0.6 km^2/s^2.
  - Transfers that fly by the Moon during the outbound segment may require less C3 (-2.0 km^2/s^2) or more (0.5 km^2/s^2), depending on the geometry.
  - Departure DAV values are typically in the range -10° ≤ DAV ≤ 10° relative to the ecliptic.
  - Simple, direct BLTs require 90 – 110 days of transfer duration.
  - No identified trends in the departure inclination – can be any value.

- Trends relevant to **simple, quick BLTs**:
  - C3 = -0.6 ± 0.15 km^2/s^2
  - The injection RAV and DAV values are very tightly defined, relative to SE system.
    - DAV = 0.0° ± 10° in general, but knowledge of the Moon's orbit reduces the variability to ±5°
    - RAV = 140° – 175° for EL_1 transfers
    - RAV = 320° – 355° for EL_2 transfers
  - Inverse relationship between C3 and RAV, constraining the search space further.
  - Complex relationship between departure conditions and departure inclination.

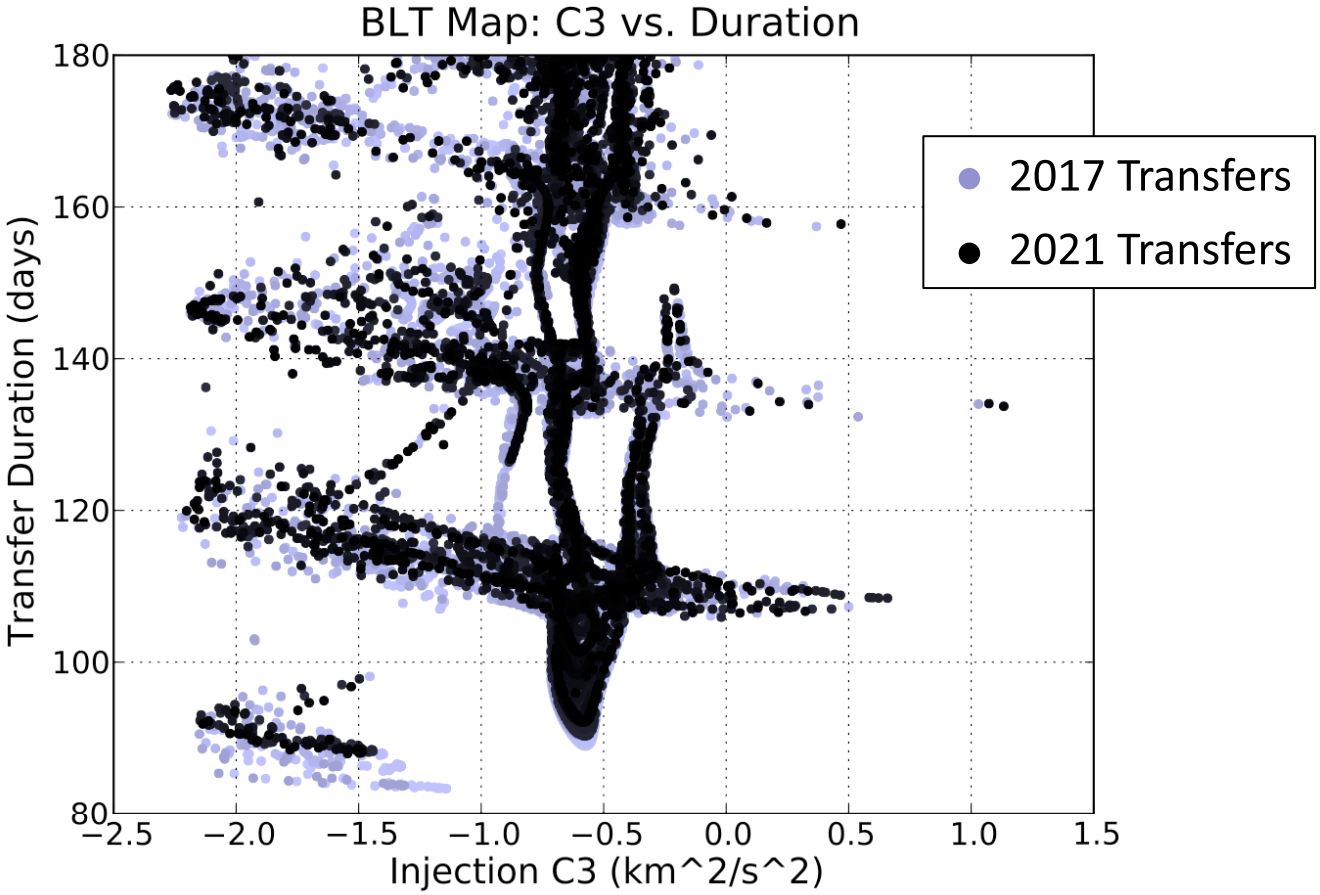
- Annual Trends?

2017 vs. 2021, BLT Map

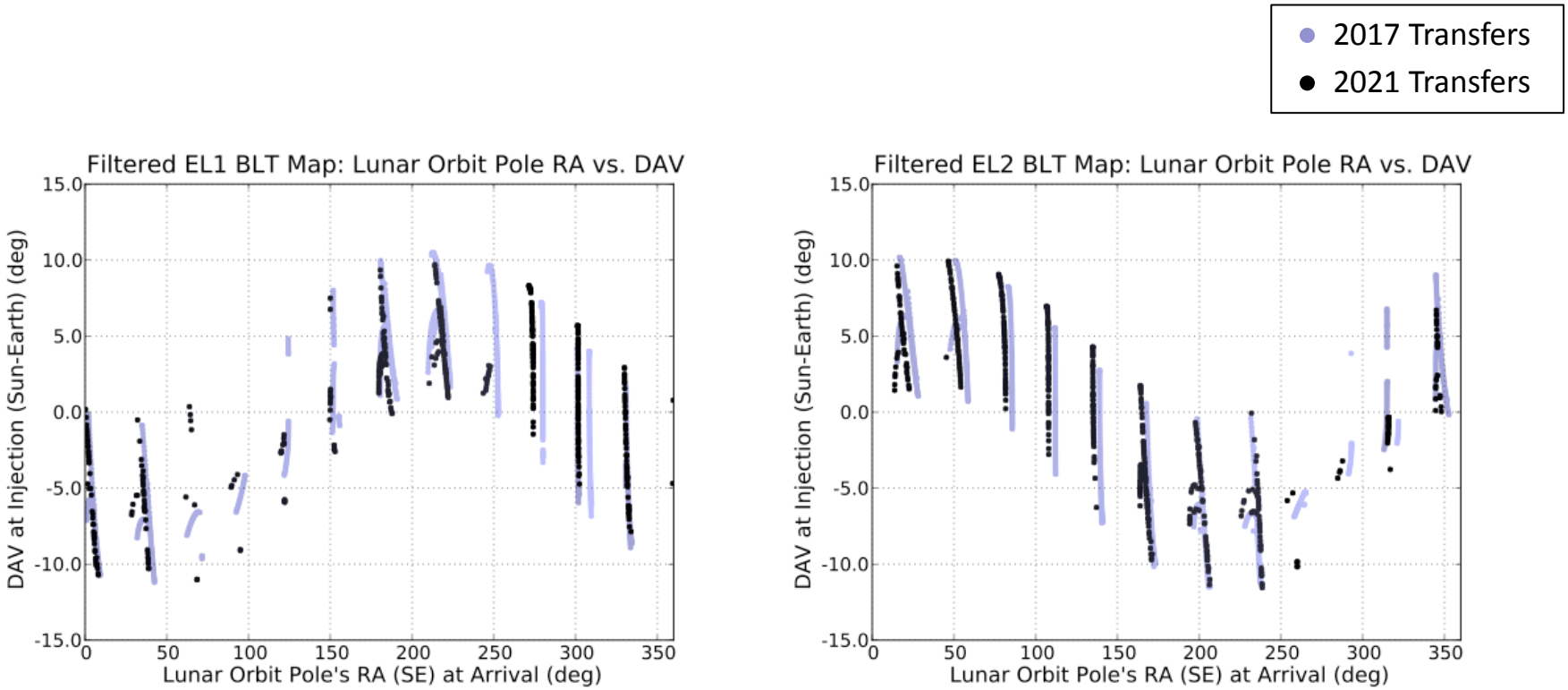


Some slight variations noticed, but no significant effects to this map.

2017 vs. 2021, C3 vs. Duration

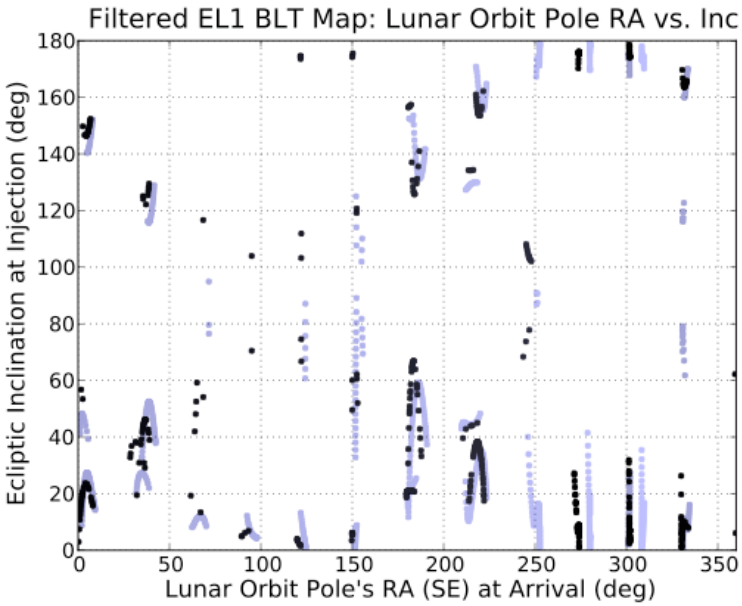


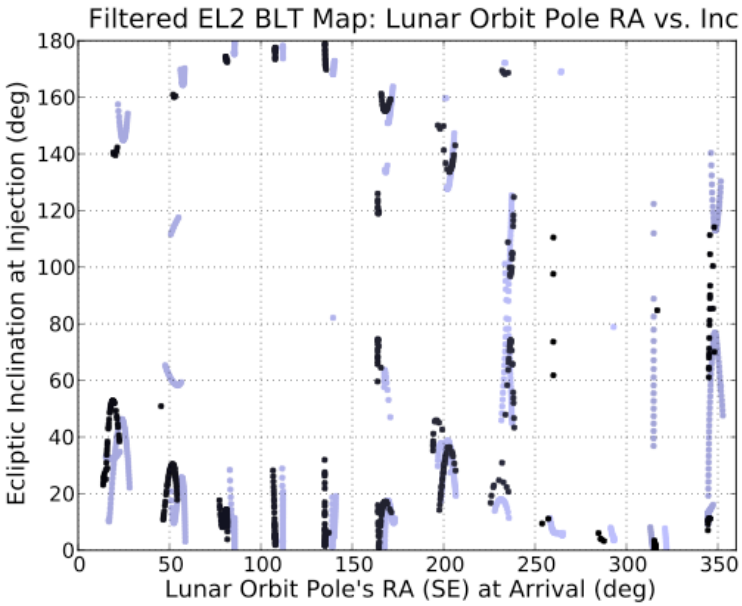
2017 vs. 2021, Lunar Orbit Pole RA vs. DAV



The same relationship is observed in 2017 and 2021 between the lunar orbit pole's RA and the DAV of injection.

2017 vs. 2021, Lunar Orbit Pole RA vs. Ecliptic Injection Inclination





Survey Results

- Trends while considering all transfers identified in a year:
  - Typical transfers require injection C3 values near -0.6 km^2/s^2.
  - Transfers that fly by the Moon during the outbound segment may require less C3 (-2.0 km^2/s^2) or more (0.5 km^2/s^2), depending on the geometry.
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    - DAV = 0.0° ± 10° in general, but knowledge of the Moon's orbit reduces the variability to ±5°
    - RAV = 140° – 175° for EL_1 transfers
    - RAV = 320° – 355° for EL_2 transfers
  - Inverse relationship between C3 and RAV, constraining the search space further.
  - Complex relationship between departure conditions and departure inclination.

- Annual Trends:
  - No significant trends observed.
  - Subtle variations

Discussion

- Many of the trends identified correspond to simple, direct transfers only. Other trends exist for other families of low-energy transfers.

- All of these results apply to a single LL_2 halo orbit.
  - Mapping technique tested successfully on a range of LL_1 and LL_2 halo orbits.
  - Mapping technique also tested successfully on a range of unstable distant prograde orbits about the Moon.

- The results of these surveys provide mission managers with more information about the available options of low-energy transfers.
  - Assist in the development of a rapid design tool to construct low-energy transfers.

Acknowledgments

The research described in this paper was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration.

Copyright 2010 California Institute of Technology. Government sponsorship acknowledged.

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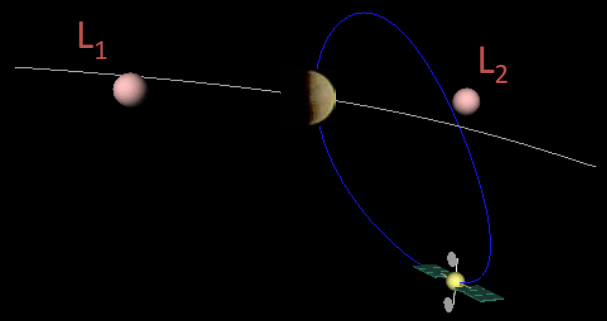
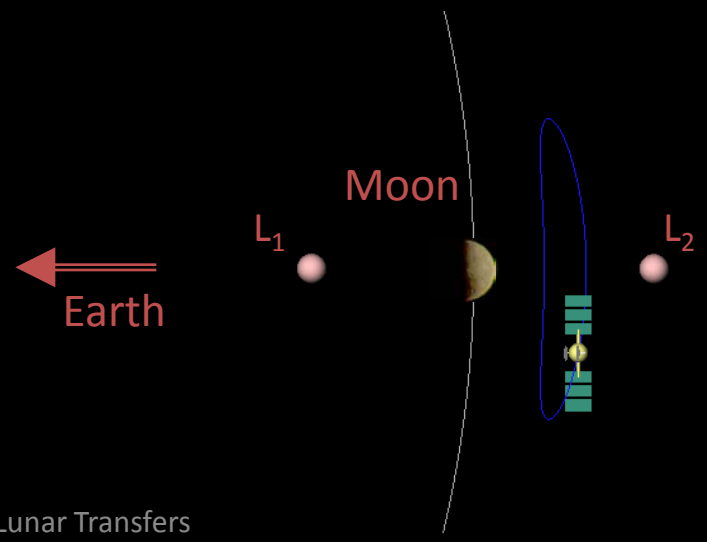
Appendix Slides

Background: Halo Orbits

- Well-known three-dimensional simple periodic orbit
- Mission Heritage
  - ISEE3 – SOHO
  - ACE – Genesis
  - WMAP
- Potential Missions
  - James Webb Space Telescope
  - Terrestrial Planet Finder
  - Space Station (L_1)

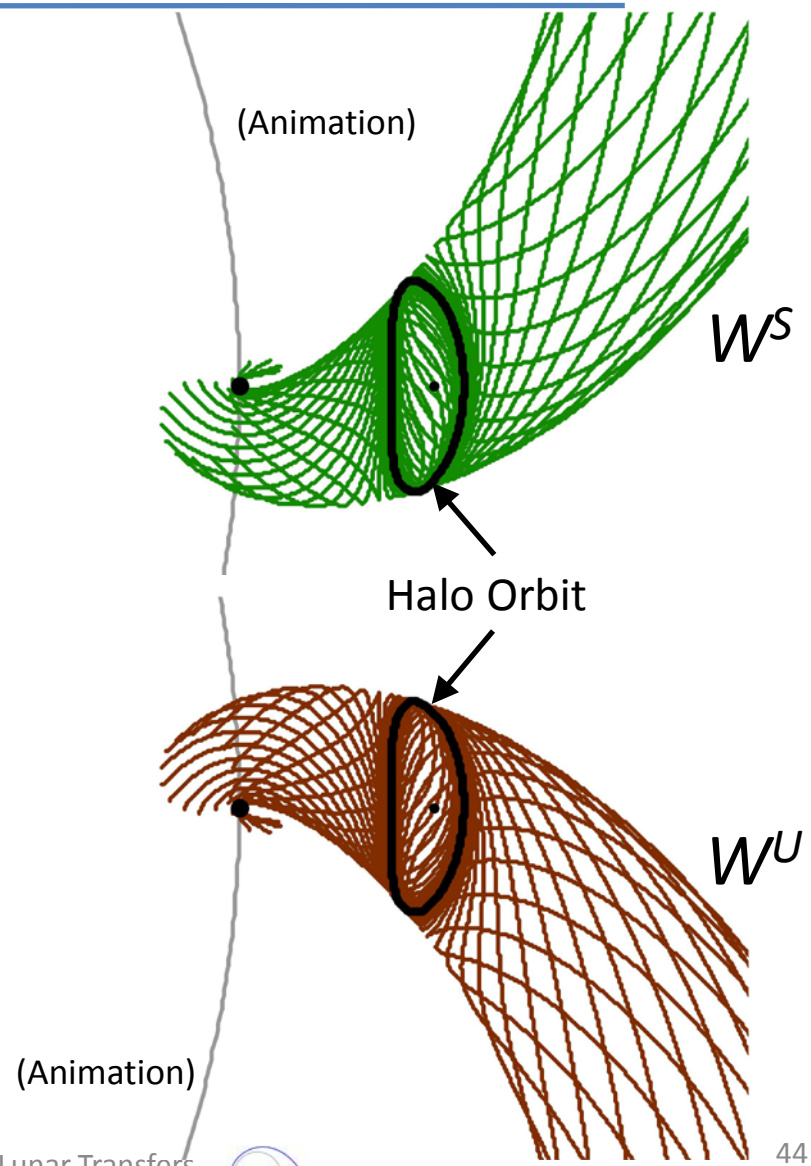




Background: Halo Orbit Manifolds

Halo Orbits: Manifolds

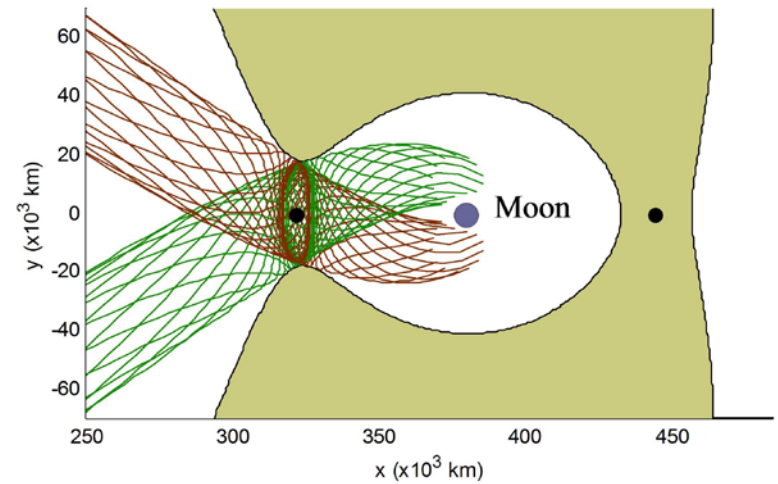
- Unstable Periodic Orbits (UPOs) have invariant manifolds.
- Stable Invariant Manifold (W^S)
  - *The set of all trajectories a particle could use to arrive onto the UPO.*
- Unstable Invariant Manifold (W^U)
  - *The set of all trajectories a particle could take after a small perturbation from the UPO.*



Background: Low-Energy Lunar Transfers

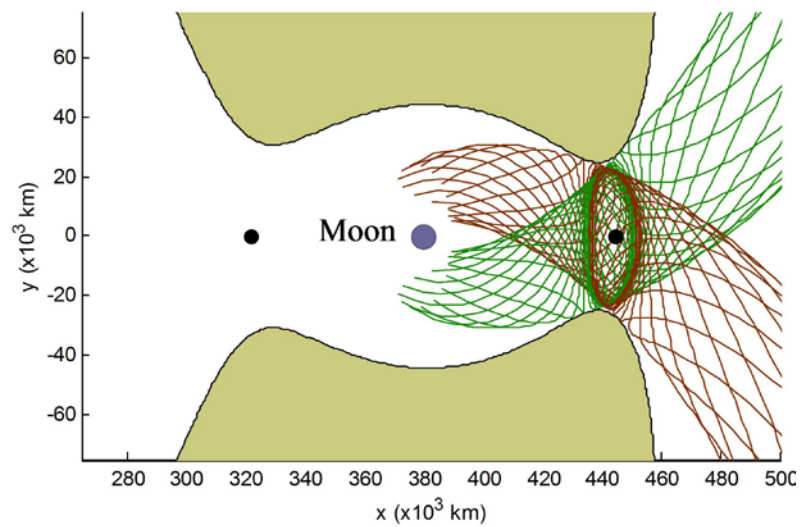
Conley (1968); Koon et al. (2000)

- Demonstrated the use of invariant manifold theory
  - Invariant manifolds of Lyapunov orbits separated the energy space
  - Target the interior of W^S_{LL2}
- Does not scale to three-dimensions



Belbruno et al. (1991+), Ivashkin (2002+)

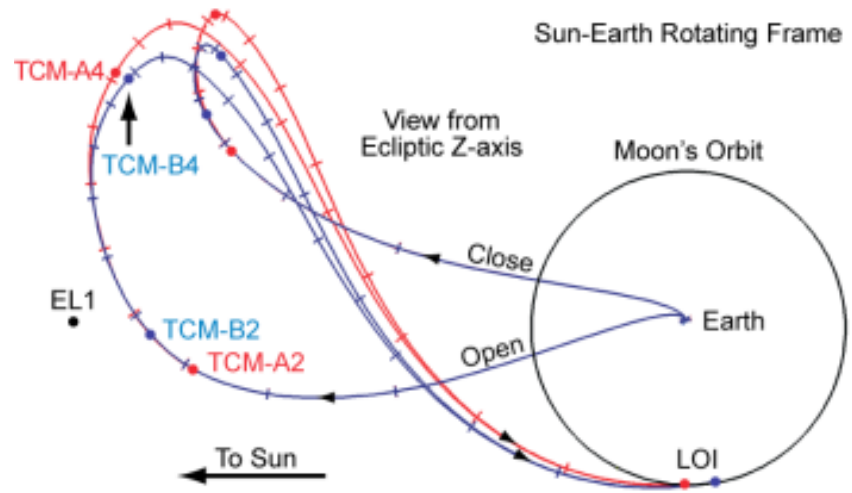
- Targeting methods
  - Use *a priori* knowledge of the dynamics of the system
  - Target "Weak Stability Boundary"
  - Methods applied to save the Japanese mission "Hiten"



Background: Low-Energy Lunar Transfers

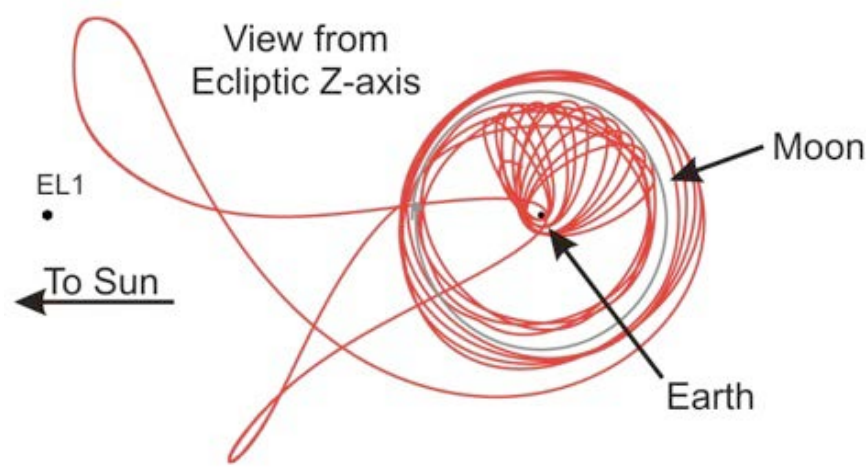
GRAIL

- Targeting scheme to design low-energy transfer to low lunar orbit
  - Designed for a single type of transfer
  - Targeting scheme assumes a single perigee passage



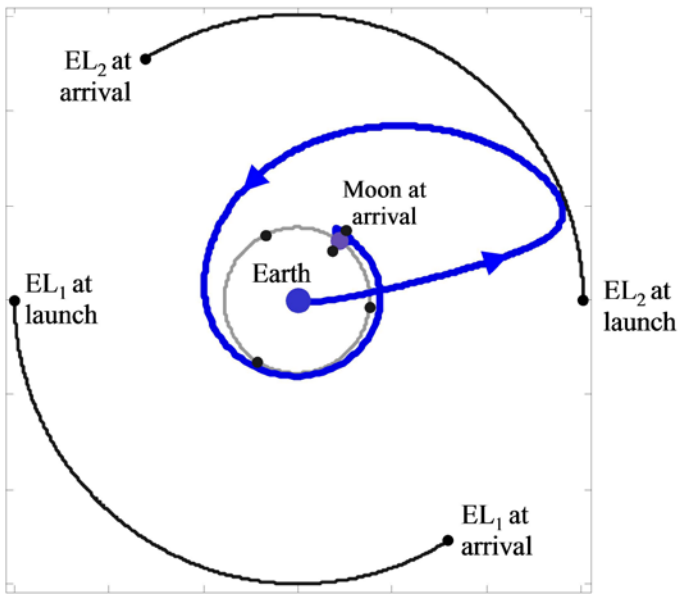
Artemis

- Targeting scheme to target lunar orbit from Earth
  - Use *a priori* knowledge of the dynamics of the system
  - Very computationally intensive scheme

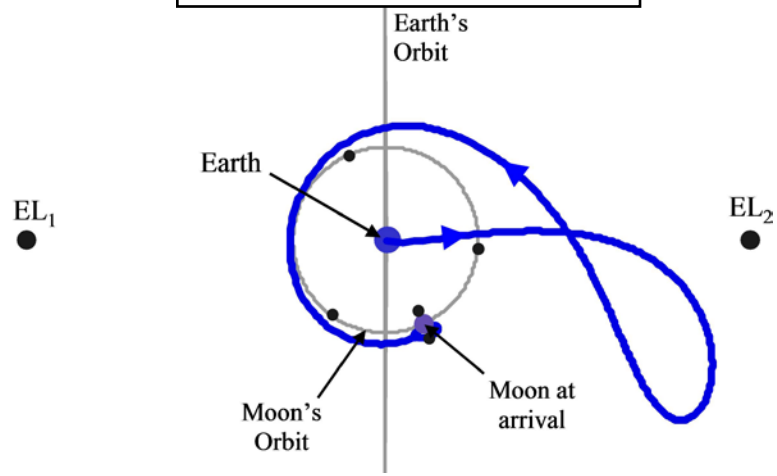


Low-Energy Ballistic Lunar Transfers (BLTs)

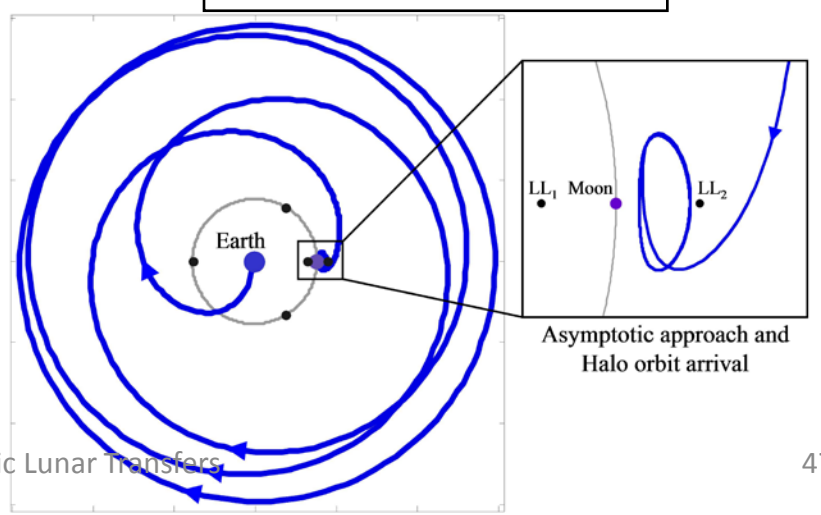
Inertial Frame



Sun-Earth Synodic



Earth-Moon Synodic



(Click for Animations)