MULTIPLE SCATTERING OF LASER PULSES IN SNOW OVER ICE: MODELING THE POTENTIAL BIAS IN ICESAT ALTIMETRY

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ABSTRACT

The primary goal of NASA's current ICESat and future ICESat2 missions is to map the altitude of the Earth's land ice with high accuracy using laser altimetry technology, and to measure sea ice freeboard. Ice however is a highly transparent optical medium with variable scattering and absorption properties. Moreover, it is often covered by a layer of snow with varying depth and optical properties largely dependent on its age. We describe a modeling framework for estimating the potential altimetry bias caused by multiple scattering in the layered medium. We use both a Monte Carlo technique and an analytical diffusion model valid for optically thick media. Our preliminary numerical results are consistent with estimates of the multiple scattering delay from laboratory measurements using snow harvested in Greenland, namely, a few cm. Planned refinements of the models are described.

1. INTRODUCTION, CONTEXT & OUTLINE

In view of the potentially dire consequences of partial loss of Greenland and Antarctica ice sheets, accurate and well-sampled altimetry in the cryosphere is critically important to climate science. So well-coordinated international programs have been established for land-based, airborne, and satellite measurements. They each have their pros and cons but together guarantee success.

In this paper, we address a particular problem faced by space-based laser altimetry as implemented in NASA's current ICESat mission [1] and as planned for its future ICESat2 mission, which is a "Tier 1" element of the Earth observation prioritization for NASA laid out in the National Academy of Sciences' Decadal Survey. Space-based laser altimetry is inherently over a relatively wide footprint determined by laser beam divergence and orbit height. As in any form of remote sensing, it is important to have a conceptual model for how the return signal is formed, starting with the shape of the transmitted pulse and following it through the atmosphere and the terrain.

Each interaction with atmospheric and surface materials can impact the shape of the received pulse. For the most part, we expect in altimetry a simple reflection that does not change the shape of the pulse, only its energy. Consequently, tracing peak energy in the pulse is a direct measurement of the two-way path, hence the targeted altitude. Digitization just needs to be fine enough to detect the maximum with high precision.

If the footprint contains variable terrain, either a gentle slope or a degree of roughness, then the width of the received pulse is wider, but the timing of the peak still reflects the average altitude in the footprint … as long as each detected photon has undergone a single reflection. Multiple reflections in the terrain add path to the light and will move the peak, leading to somewhat negatively biased altitudes. That effect will be studied in detail elsewhere. Here, we examine a physically analogous effect that occurs when the target surface is somewhat transparent. Specifically, we will devise a modeling framework for estimating the *extra* path length cumulated by laser light as it penetrates snow covering the ice.

In Sect. 2, we set up the time-dependent radiative transfer (RT) problem at hand, and we demonstrate on a specific case its numerical solution using a Monte Carlo scheme. Results are compared with preliminary analyses of recent laboratory measurements of polarized laser pulse penetration into snow sampled in Greenland. In Sect. 3, a useful approximation to the RT is set up that uses time-dependent diffusion theory; this model has the huge advantage of being analytically tractable. Both Monte Carlo and diffusion models were adapted from versions used successfully to probe optically thick stratiform clouds with pulsed lasers and innovative receivers from ground [2] or aircraft [3]; the latter configuration was subsequently considered to probe sea ice thickness [4]. The simpler diffusion model, with closed-form expressions, may prove valuable later on if corrections are required to remove the extra path length from the received signal. However, both models still need some refinements.

2. UNDERPINNING RADIATIVE TRANSFER

2.1 Time-dependent azimuthally-invariant 1D RT

Let $I(t,z,\mathbf{\Omega}) \equiv I(t,z,\cos\theta)$ be *diffuse* radiance at time $t \geq 0$, depth z and direction $\mathbf{\Omega}$, determined by polar angles (θ,ϕ), in an optical medium $0<z<H$ that is, along with sources, both (x,y)- and ϕ-invariant. This field obeys:

$$\left[c^{-1}\partial_t + \mu\partial_z + \sigma(z)\right]I = \sigma_s(z)\int_{4\pi} p(z,\Omega\cdot\Omega')I(t,z,\Omega')\mathrm{d}\Omega' + q(t,z,\mu), \quad (1)$$

c being the speed of light in the medium; this describes the conservation of radiant energy in a small volume in phase space (t,z,μ) where $\mu = \Omega_z = \cos\theta$. Coefficients $\sigma(z)$ and $\sigma_s(z)$ are respectively probabilities per unit length of extinction and scattering events, $p(\cdots)$ is the phase function, assumed to depend only on scattering angle $\theta_s = \cos^{-1}\mathbf{\Omega}\cdot\mathbf{\Omega}'$. For an infinitesimal laser pulse impinging normally (direction +**z**, $\mu = +1$) on the upper boundary ($z = 0$) with unit energy, the volume source is

$$q(t,z,\mu) = \exp[-\int_0^z \sigma(z')\mathrm{d}z']\sigma_s(z)p(z,\mu)\delta(t - z/c) \cdot \quad (2)$$

Boundary conditions are $I(t,0,\mu) = 0$, $\mu > 0$, at the upper boundary and $I(t,H,\mu) = 0$, $\mu < 0$, at the lower boundary ($z = H$). The quantity of prime interest here is $I(t,0,-1)$, the time-dependent back-reflected radiance, in essence, it is the "lidar equation" for all orders of scattering.

2.2 Monte Carlo solution for a typical scenario

The easiest way to obtain $I(t,0,-1)$, by far, is implement a numerical solution of (1)–(2) using forward Monte Carlo simulation. We did this for a representative case: the doubled Nd:YAG wavelength (532 nm) irradiating 15 cm of snow above 2 m of ice. Ice optics were described for propagation by an *effective* extinction coefficient of 4.5 m^{-1} followed by isotropic scattering, hence $p \equiv 1/4\pi$, and the absorption coefficient is 0.07 m^{-1}. Snow optics were described as purely scattering with $\sigma = \sigma_s = 1634$ m^{-1} and $p(\cos\theta_s)$ computed from ray tracing (geometric optics) in a monodisperse collection of ice spheres of diameter 0.4 mm embedded in air, corresponding to somewhat aged snow. This phase function is plotted in Fig. 1 where we note the very strong forward (diffraction) peak; as expected, it contains ≈50% of the overall scattering cross section.

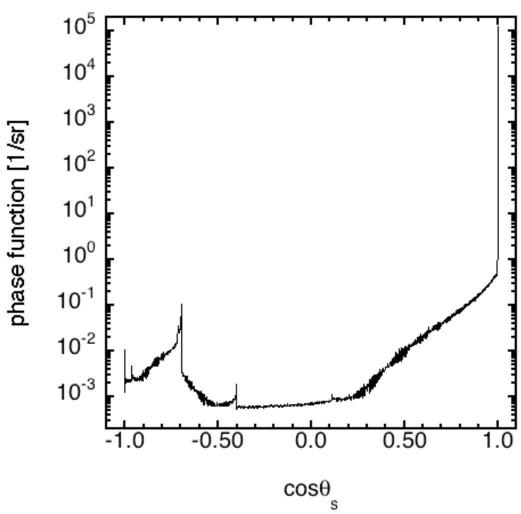

Figure 1. Adopted phase function for snow generated by a ray-tracing code [8,9].

In Fig. 2, we plot $I(t,0,-1)\delta t$ normalized by its integral from 0 to ∞. Here, t is equated with $2z/c$, z being the usual range inside the medium, with $\delta z = 0.1$ m. With this relatively coarse temporal binning, 84% of the probability is in the first 10-cm bin. If that probability is assigned to range $z = ct/2 = 0$, then the mean delay $<ct>/2$ for all orders of scattering is 3.7 cm; if it is assigned to $\delta ct/2 = 5$ cm, then $<ct>/2 \approx 8.7$ cm. Although this seems like a long delay, recall that it is <u>only for the fraction of the laser light that actually penetrates the medium</u>. Light returned upon specular reflection, with $ct/2 \equiv 0$, is not presently incorporated but the low-biased estimation of $<ct>/2$ mimics this contribution. A physics-based model for reflection by the micro-roughened snow-air interface is required to improve the model in this important respect.

Laboratory measurements were recently performed on *polarized* laser pulses (~1 ns FWHM, equiv. $ct/2$ ~ 15 cm) returned by real Greenland snow collected in 2006. Interestingly, preliminary analyses based on increase in FWHM yield ≈3 cm for delay of *parallel* light [David Harding, pers. comm.]. However, parallel returns emphasize very low orders of scattering, i.e., shallow penetration. Moreover, it is not known what part of these returns is from the surface per se and what part comes from the volume. Here, we only model the later.

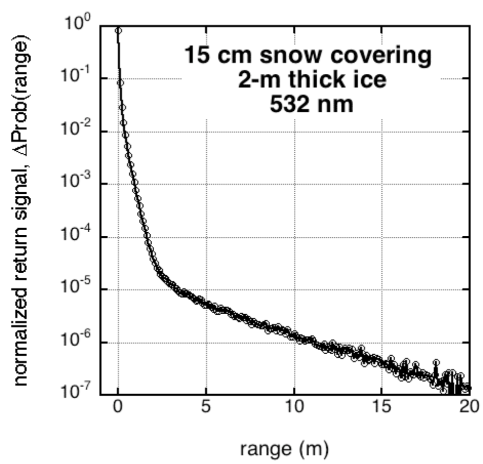

Figure 2. Returned waveform computed by Monte Carlo simulation for an infinitesimally narrow incoming light pulse. This impulse response must be convolved with the actual pulse shape to obtain the measured return.

3. "1+1D" RT IN THE DIFFUSION LIMIT

3.1 Diffusive transport problem definition

Assume *total* radiance $I(t,z,\mu) \approx$

$$[J(t,z) + 3\mu F(t,z) + f_{\mathrm{col}}{}^{\times}2\delta(1-\mu)\,\delta(t-z/c)\mathrm{e}^{-\sigma z}] / 4\pi, \quad (3a)$$

and, accordingly, $p(\mu_s) \approx (1+3g\mu_s) / 4\pi$. (3b)

We used here

$$J(t,z) = 2\pi\int_{-1}^{+1} I(t,z,\mu)\mathrm{d}\mu\,, \quad (4a)$$

$$F(t,z) = 2\pi\int_{-1}^{+1} \mu I(t,z,\mu)\mathrm{d}\mu\,, \quad (4b)$$

and the asymmetry factor $g = \langle\cos\theta_s\rangle = \int\mu_s p(\mu_s)d\mu_s$. The snow phase function in Fig. 1 yields $g \approx 0.9$. The implicit model for *diffuse* (once or more scattered) radiance in (3a) is a two-term expansion of the angular dependence in spherical harmonics. This bland picture is reasonable deep inside optically thick media. So we require H, the outer scale of the problem, to be much larger than the inner scale, mean-free-path (MFP) $1/\sigma$.

The transport problem in (1)–(2), for uniform media, now reduces to

$$\begin{aligned} c^{-1}\partial_t J + \partial_z F &= -\sigma_a J + f_{col} \times \sigma_s \exp(-\sigma z)\delta(t - z/c) \\ c^{-1}\partial_t F + \partial_z J/3 &\approx -\sigma_t F + f_{col} \times \sigma_s g \exp(-\sigma z)\delta(t - z/c) \end{aligned}, \quad (5)$$

with $\sigma_a = \sigma - \sigma_s$ (= 0, here) and $\sigma_t = \sigma - g\sigma_s = (1-g)\sigma_s + \sigma_a$ being the coefficients for absorption and "transport" (or "scaled") extinction, respectively. Boundary conditions for the pair $\{J(z),F(z)\}$ are

$$\left(J + 3\chi F\right)\big|_{z=0} = 4(1 - f_{col})\delta(t) \text{ and } \left(J - 3\chi F\right)\big|_{z=H} = 0, \quad (6)$$

with $\chi = 2/3$ when f_{col}, the fraction of collimated illumination, is unity. In standard diffusion theory, we furthermore assume that $\partial_t F \equiv 0$.

The first predicate of diffusion theory in (3a) is not a good approximation near (within a few MFPs of) strongly anisotropic sources and absorbing boundaries. Similarly, (3b) is a poor model for strongly forward-peaked phase function such as displayed for snow in Fig. 1. These liabilities can be largely mitigated by using δ-Eddington rescaling [6]: replace (3b) with $p(\mu_s) \approx [f \times 2\delta(1-\mu_s) + (1-f) \times (1+3g'\mu_s)] / 4\pi$, yielding

$$\sigma' = (1-\varpi_0 f)\sigma, \text{ with}$$
$$\sigma_a' = \sigma_a \text{ and } \sigma_t' = \sigma_t. \quad (7)$$

The new parameter f is the fraction of the single-scattered light that is reassigned to directly transmitted radiance, and $\varpi_0 = \sigma_s/\sigma \approx 1$ is the single-scattering albedo. The first and last relations in (7) lead to $g' = (g-f)/(1-\varpi_0 f)$. A natural choice here is $f = ½$, to account for the diffraction peak. The snow phase function in Fig. 1, with $g = 0.9$, will now use $g' = 0.8$.

Outgoing hemispheric flux at the upper boundary is then $R_{col}(t) = ¼\,[J-3\chi F]_{z=0} = J(t,0)/2$. Therefore, the new lidar equation for multiple scattering is

$$I(t,0,-1) \approx R_{col}(t) / \pi, \quad (8)$$

making the assumption of isotropic emittance from the illuminated side of the optically thick target medium.

3.2 Solutions in Laplace space

The above problem in coupled PDEs is then Laplace-transformed in t, $\int_{[0,\infty)}(\cdots)e^{-st}dt$, yielding a classic ODE problem in z, indeed a close analog to the well-known "δ-Eddington 2-stream" model used in climate models for solar radiation parameterization, with s/c being the equivalent of a gaseous absorption coefficient. We find

$$\begin{aligned} dF^*/dz &= -(s/c + \sigma_a)J^* + f_{col} \times \sigma_s \exp[-(s/c + \sigma)z] \\ dJ^*/dz &\approx -3\sigma_t F^* + f_{col} \times 3\sigma_s g \exp[-(s/c + \sigma)z] \end{aligned}, \quad (9)$$

with

$$\left(J^* + 3\chi F^*\right)\big|_{z=0} = 4(1 - f_{col}) \text{ and } \left(J^* - 3\chi F^*\right)\big|_{z=H} = 0 \cdot \quad (10)$$

The resulting Laplace transform $R^*_{col}(s) = L[R_{col}(t)]$ has a closed-form expression provided by Davis et al. [5]; similarly for transmitted flux, $T^*_{col}(s) = L[T_{col}(t)]$.

Another problem of interest here is when $f_{col} = 0$ in (9)–(10), with no exponential term in (3a) for $I(t,z,\mu)$, corresponding to a pulsed isotropic boundary source. The quantities of interest are again the time-dependent reflectance $R_{iso}(t) = J(t,0)/2-1$ and transmittance $T_{iso}(t) = J(t,H)/2$ or, equivalently, their Laplace transforms $F^*_{iso}(s) = L[F_{iso}(t)]$, for $F = R,T$.

Thinking about a simple model for semi-transparent ice, this last problem of diffuse pulsed illumination is of interest in the limit $H \to \infty$. This corresponds to an isotropic source on the upper boundary of a semi-infinite medium, which we must now take as weakly absorbing ($0 < \sigma_a \ll \sigma_s \approx \sigma$). This yields a time-resolved albedo for the ice denoted $\alpha_{iso}(t)$, and the associated $\alpha^*_{iso}(s) = L[\alpha_{iso}(t)]$. Specifically, we find

$$\alpha^*_{iso}(s) = \frac{\sqrt{\sigma_t/3} - \chi\sqrt{\sigma_a + s/nc}}{\sqrt{\sigma_t/3} + \chi\sqrt{\sigma_a + s/nc}}, \quad (11)$$

where $n \approx 1.31$ is the refractive index of ice; c/n is the group velocity of the transported energy, we account for it by scaling s, the Laplace conjugate of t, by $n \geq 1$. We note that this expression is invariant under the δ-scaling in (7). The so-called diffusion length scale of an optical medium is $L_d = (3\,\sigma_a\sigma_t)^{-½}$ and it should be smaller than H to justify the semi-infinite assumption.

Returning to the two-layer medium with snow over ice, we can obtain its overall albedo in Laplace space, $R^*(s)$, using the classic decomposition into path radiance (reflectance with a black surface) and successive surface reflections, assumed isotropic [e.g., 5]:

$$R^*(s) = R^*_{col}(s) + T^*_{col}(s)\alpha^*_{iso}(s)\frac{1}{1 - \alpha^*_{iso}(s)R^*_{iso}(s)}T^*_{iso}(s) \cdot \quad (12)$$

What can we do with a diffusion-theoretical expression for the Laplace transform $R^*(s)$ of $\pi I(t,0,-1)$? First, we can readily compute the total albedo

$$R = ½\int J(t,0)dt = R^*(0). \quad (13)$$

Then we can compute low-order temporal moments, starting with

$$\langle t\rangle = -\partial \ln R^*/\partial s|_{s=0}; \quad (14)$$

from there, the multiple-scattering path delay is $½\langle ct\rangle$.

3.3 Results and analysis

As in Sect. 2, we follow [4, and references therein] and consider ice as a weakly-absorbing ($\sigma_a \approx 0.07$ 1/m, due to impurities) isotropically-scattering (g = g' = 0) medium using an "effective" (i.e., transport) extinction coefficient σ_t that can range from 3 to 6 m^{-1}. This puts L_d in the range 0.9 to 1.3 m, significantly less than the 2-m thickness used previously, thus rationalizing our use of the semi-infinite medium assumption.

From the same source [4], we take snow to be a densely-packed purely-scattering layer of thickness H between 0.1 and 0.5 m with spherical mono-disperse ice particles of diameter 0.1, 0.4, and 2 mm; these choices lead respectively to $\sigma = \sigma_s$ = 6521, 1634, and 326 1/m (no absorption at 532 nm). This gives us a range of snow optical depths $\tau = \sigma H$ going from 32.6 to 3260. Scaled optical depth $\tau_t = (1-g)\,\tau = (1-g')\,\tau'$ is then in the range 3.3 to 330, which is plenty to establish a diffusive transport regime throughout the layer.

In Fig. 3, we plot overall albedo R from (13) for $\log\tau$ between –1.5 ($\tau \approx 0.03$) to +3.5, with the region of interest being +1.5 to +3.5, for the extreme ice opacities (σ_t = 3 and 6 m^{-1}). This demonstrates how the gradual addition of snow brightens the scene, and we recall that this does not even account for specularly reflected light, neither by snow nor by ice. Ice's albedo is approached on the left-hand side, saturation on the right.

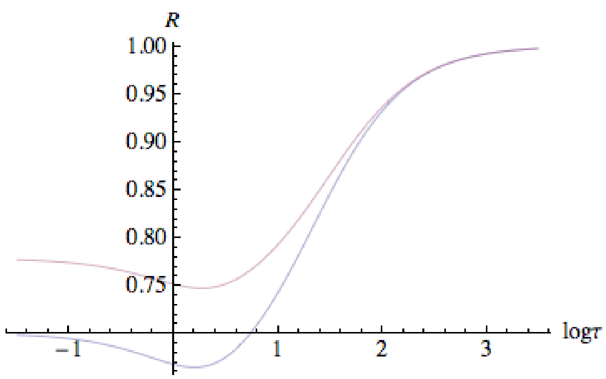

Figure 3. Overall reflectance R of the layered snow/ice medium for increasing snow optical depth $\tau = \sigma H$ and two extreme assumptions about σ_t, the effective (a.k.a. transport) extinction coefficient of ice: top, 6 m^{-1}; bottom, 3 m^{-1}.

Figure 4 shows then mean delay caused by multiple scattering $\langle ct\rangle_{dif}/2$ from (14) as a function of H and $\log\tau$ in the regions of interest; the ice optics are set by σ_t = 4.5 m^{-1}, and have very little influence in this regime of very opaque snow anyway. Since $T_{col}^*(s)$ is small, only the first term, $R_{col}^*(s)$, really matters in (12). This term was previously investigated thoroughly [7]. The asymptotic behavior of $\langle ct\rangle_{dif}/2$ for large τ (approached algebraically from above) was shown to be $\approx(5/6)H$. So for the snow modeled in Fig. 2, with H = 0.15 m and $\tau = \sigma H = 245 \approx 10^{2.4}$ ($\tau_t \approx 25$), we predict $\langle ct\rangle_{dif}/2 \approx 13$ cm. This is somewhat more than found for the coarse-binned Monte Carlo simulation in Fig. 2. We attribute this to diffusion theory's poor account of the very lowest orders of scattering, but we propose next a simple refinement.

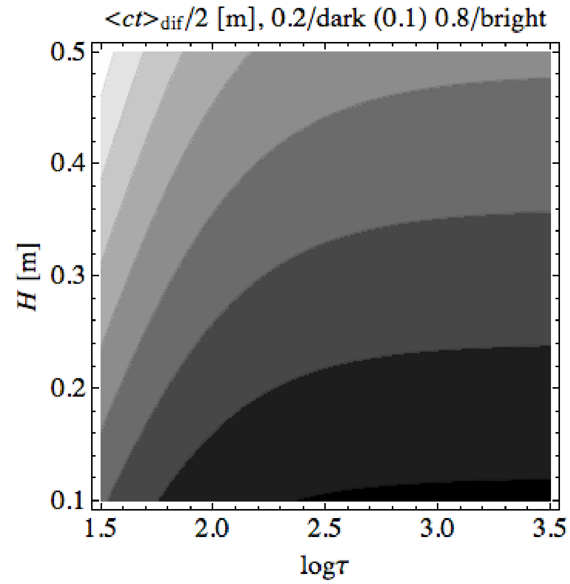

Figure 4. Mean snow-induced multiple-scattering path delay $\langle ct\rangle_{dif}/2$ as function of H and τ, from diffusion theory.

3.4 Refinements, present and future

Diffusion is the asymptotic limit of RT for small MFPs; the opposite limit, large MFPs (small optical depths), takes into account only the uncollided and once-scattered light. Uncollided light does not contribute to $I(t,0,-1)$ but for single scattering the element from ranges $[z,z+dz)$ is $dI_1(t,z,-1) = \delta(t-z/c)e^{-2\sigma z}\sigma p(-1)dz$, cf. (1)–(2), where we can think of σ and $p(-1)$ as "effective" ones. Thus total contribution is $I_1(t,0,-1) = \sigma c p(-1)e^{-2\sigma ct}$. Time-integrated radiance is $I_1(0,-1) = p(-1)/2$ and mean delay $\langle ct\rangle_{1sc}/2 = 1/4\sigma$, a ¼ of a MFP.

So a reasonable hybrid asymptotic estimate would be

$$\langle ct\rangle/2 = ½\,(\,\pi I_1\langle ct\rangle_{1sc}+R\langle ct\rangle_{dif}\,)/(\,\pi I_1+R\,). \qquad (15)$$

For the case in Fig. 2, we can use the scaled MFP, for both the forward peak of $p(\mu_s)$ near $\mu_s \approx 1$ and the residual anisotropic scattering: $1/(1-g')\sigma' \approx 3$ mm, hence $\langle ct\rangle_{1sc}/2 \approx 0.8$ mm with $\pi I_1 \approx 1/8$, as compared to $R \approx 0.95$. So (15) yields $\langle ct\rangle/2 \approx 10$ cm, very close to the more objective estimate from the coarse-binned Monte Carlo simulation. However, still no account has been taken for light with $ct \equiv 0$ in either model.

Turning to polarization effects, the Monte Carlo model will need a major overhaul to incorporate them. However, there is an easy fix for the diffusion model: just to use $R/2$ in (15) for each linear polarization. That will reduce by almost ½ the current estimate, bringing it into close agreement with the experimental counterpart, which was based on a FWHM metric, not an averaging.

Lastly, one can design an adaptive correction scheme for any potential altimetry bias due to snow cover by enabling the estimation of the 2^{nd}-order moment of ct as well as its mean. See [5] for illustrative algorithms to infer both H and τ using only time-domain information.

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