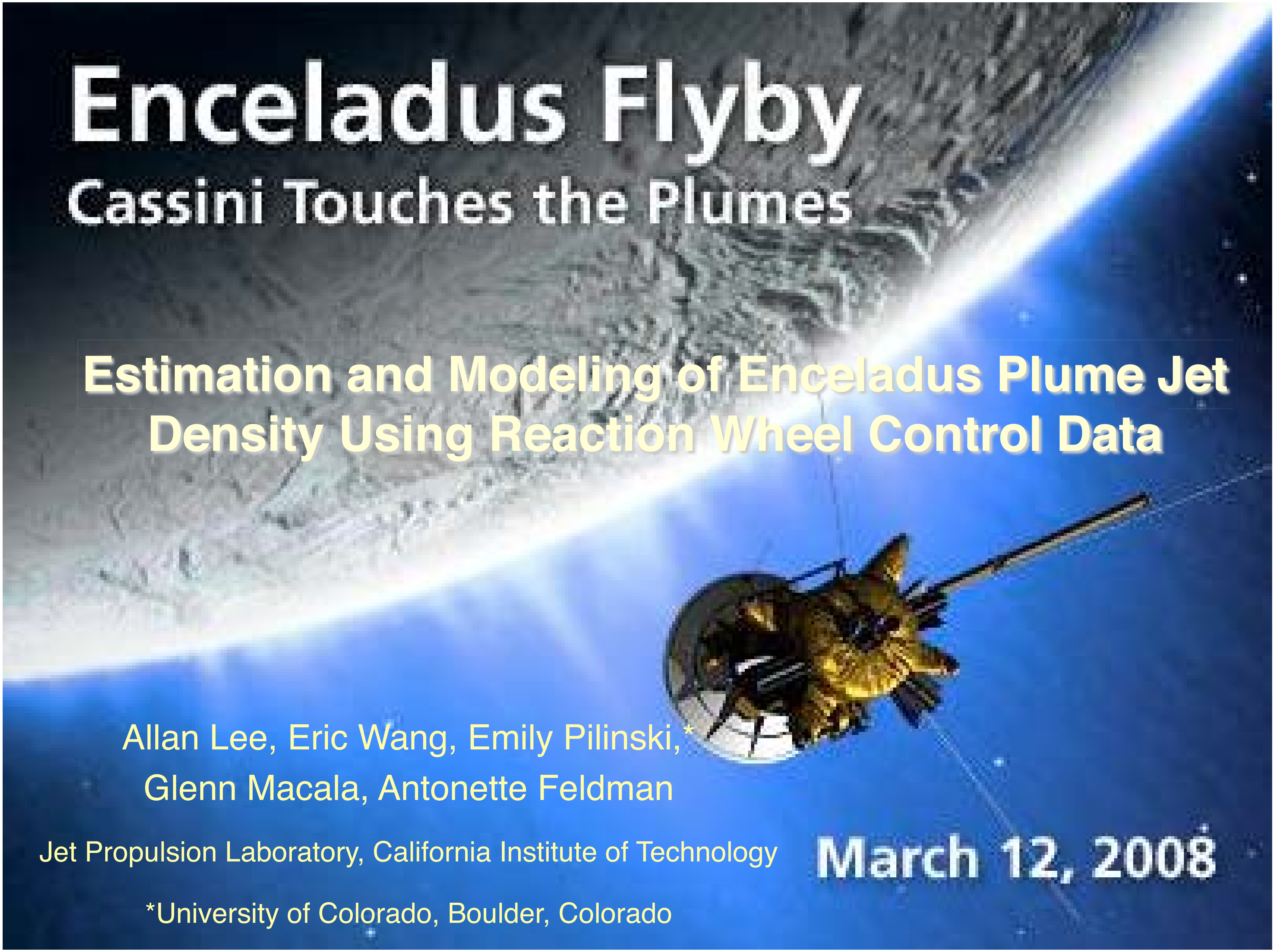

Estimation and Modeling of Enceladus Plume Jet Density Using Reaction Wheel Control Data

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Topics to Cover

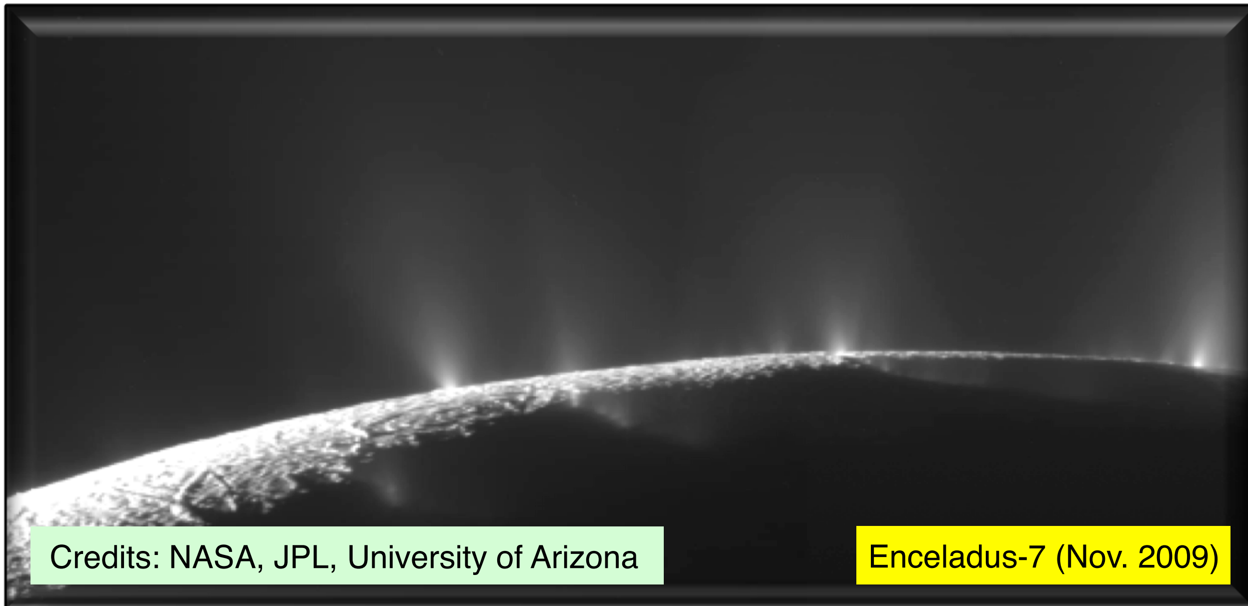
- The Cassini Spacecraft
- Enceladus Plume
- Reaction Wheel Control System Design
- Enceladus Plume Density Reconstruction
- Enceladus Plume Density Modeling
- Conclusions

The Cassini Spacecraft

- Cassini, a Saturn orbiter, is one of the largest and most sophisticated interplanetary spacecraft humans have ever built and launched
- Major science objectives of the Cassini mission include investigations of
  - The configuration and dynamics of Saturn's magnetosphere
  - The structure and composition of the rings
  - The characterization of several of Saturn's icy satellites (e.g., Enceladus)
  - Titan's atmosphere constituent abundance

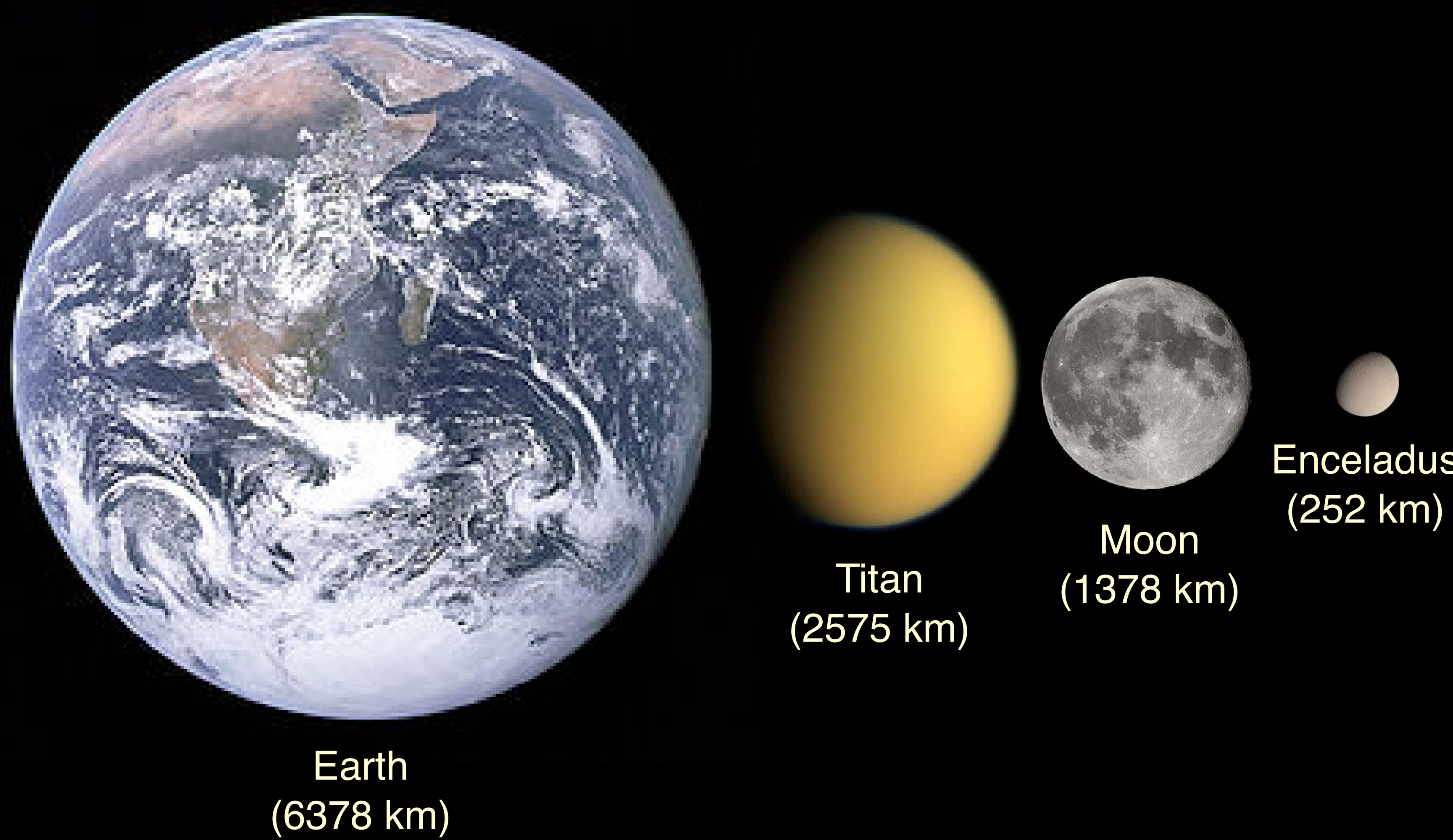
Credits: NASA, JPL

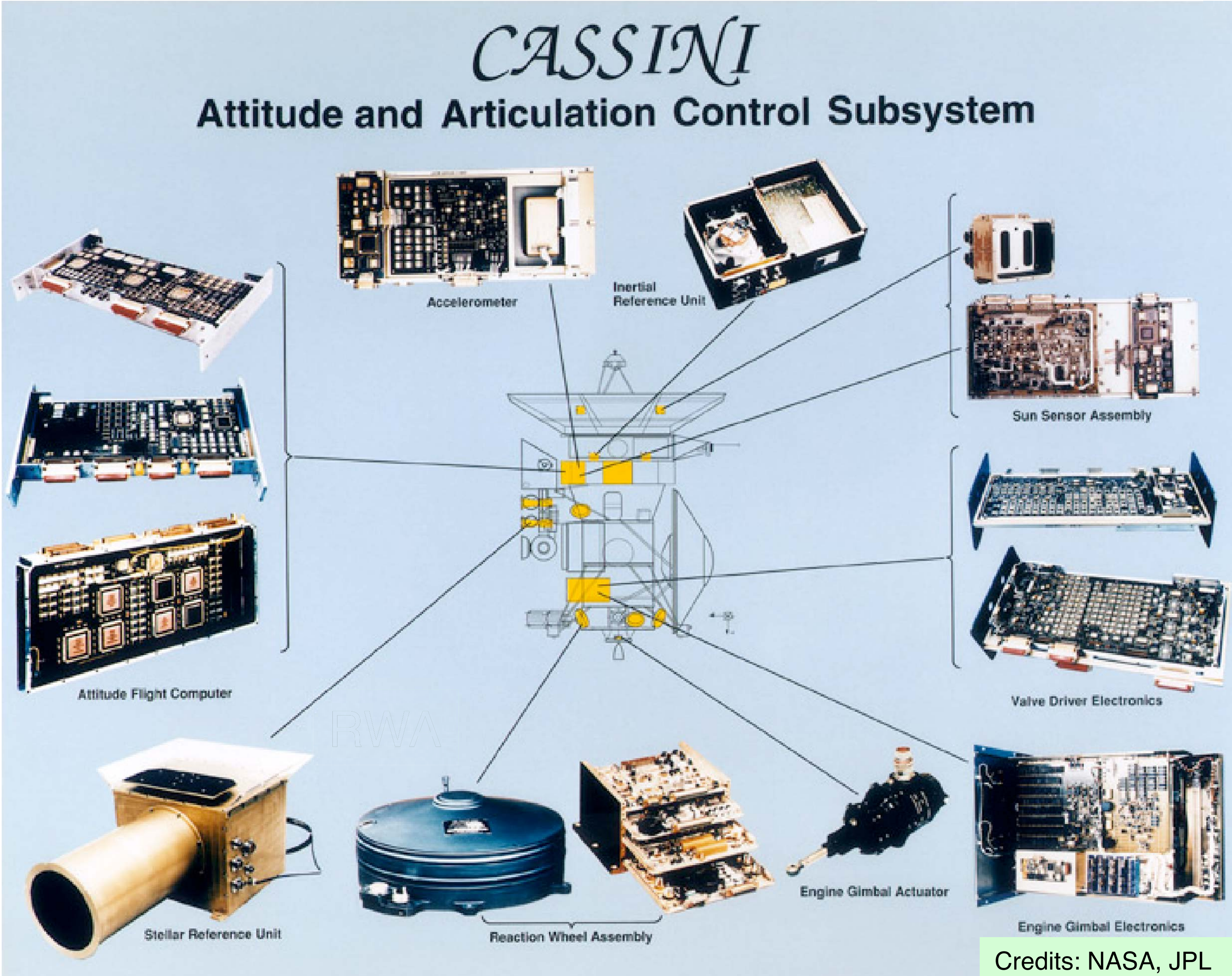
The Discovery of Enceladus Plume in 2005



- In 2005, Cassini completed several flybys of Enceladus, a small, icy satellite of Saturn
- Observations made during these flybys confirmed the existence of a water vapor plume in the south polar region of Enceladus
- The discovery of watery geysers from Enceladus is an important and unexpected discovery made by Cassini

Enceladus – A Small Saturnian Moon

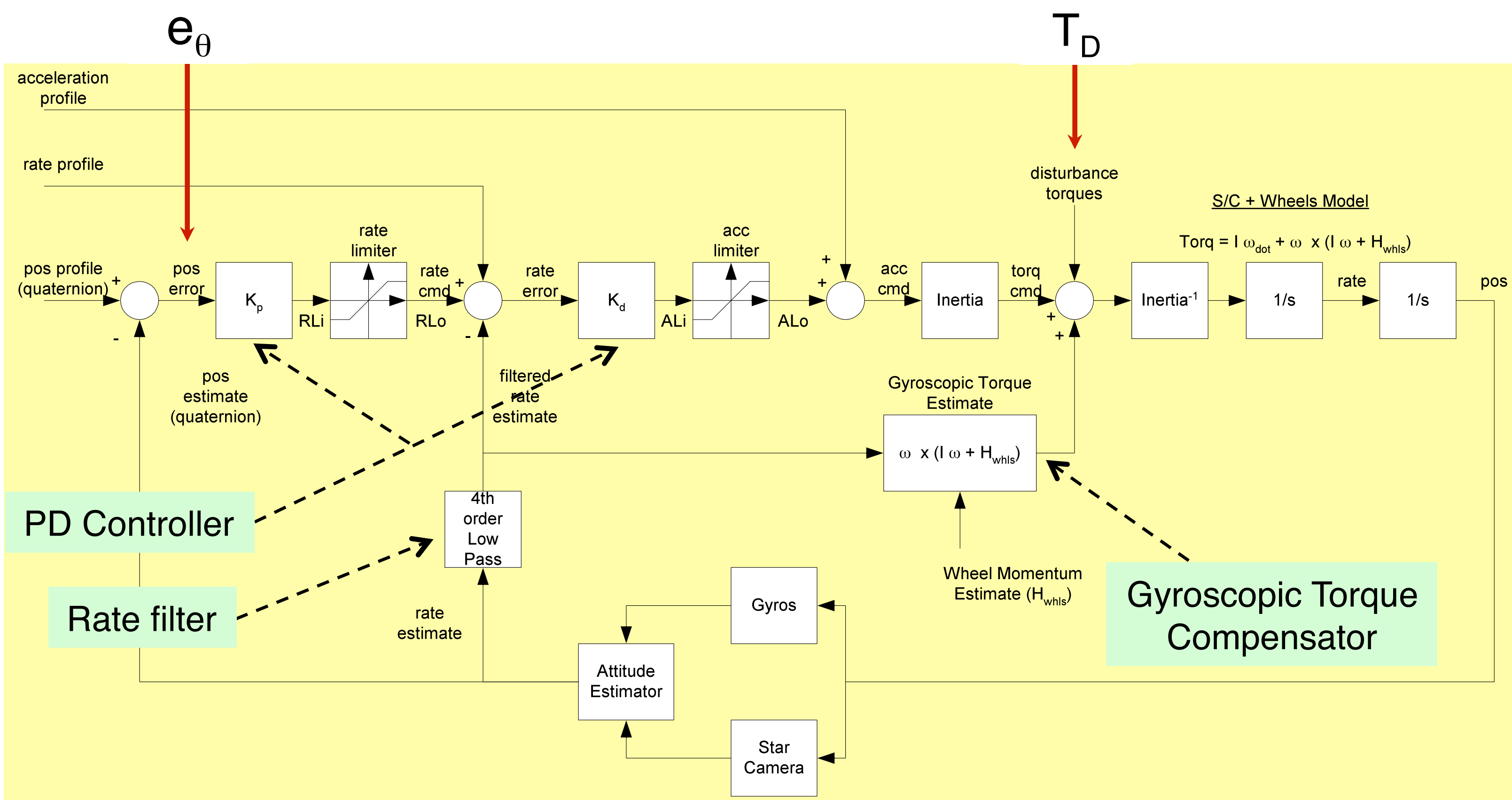




Credits: NASA, JPL

Reaction Wheel Assembly Controller (RWAC) Design

- The basic structure of the RWAC is a decoupled, three-axis, Proportional and Derivative (PD) controller
  - With rate and acceleration feed-forward commands
  - With compensation for gyroscopic torque



Enceladus Density Reconstruction

- The transfer function between the disturbance torque $T_D(s)$ and the attitude control error $e_\theta(s)$ is given by:

$$\frac{e_\theta(s)}{T_D(s)} = -\frac{(s^2 + 2\xi\omega s + \omega^2)^2 / I_{SC}}{Den(s)} \quad (1)$$

- The denominator is given by the following expression:

$$\begin{aligned} Den(s) = s^6 + 4\xi\omega s^5 + (4\omega^2\xi^2 + 2\omega^2 + 4K_P K_D)s^4 + \\ (4\omega^3\xi + 4K_P K_D \omega\xi)s^3 + (\omega^4 + 4K_P K_D \omega^2\xi^2 + 2K_P K_D \omega^2)s^2 + \\ (K_D\omega^4 + 4K_P K_D \omega^3\xi)s + K_P K_D \omega^4 \end{aligned} \quad (2)$$

- $\omega_n = 2\pi \times 0.0299$ rad/s (RWAC bandwidth) and $\xi_n = 0.4138$

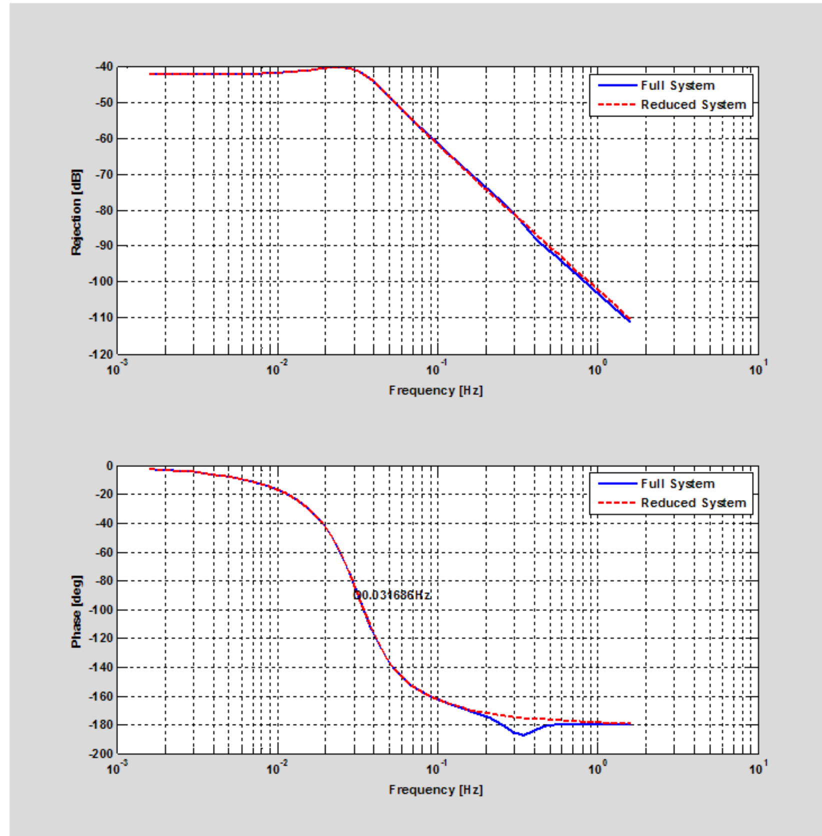
- The natural frequency (ω) and damping coefficient (ξ) of the 4th order low-pass filter are 2.34048 rad/s and 0.4000, respectively

Enceladus Density Reconstruction

- Since the BW of the RWA controller is more than an order of magnitude lower than the center frequency of the low-pass filter, a low-order approximation of Eq. (1) could be derived:

$$\frac{e_\theta(s)}{T_D(s)} = -\frac{1/I_{SC}}{s^2+K_D s+K_P K_D} = -\frac{1/I_{SC}}{s^2+2\xi_n \omega_n s+\omega_n^2} \qquad (3)$$
$$= -\frac{0.0002747}{s^2+0.15548s+0.03529}$$

- The second-order transfer function Eq. (3) approximates the full-order transfer function very well; thus, it is used in the study



A comparison of the frequency responses of transfer functions

States with small Hankel singular values are eliminated

Estimate Plume Torque from Attitude Control Data

- Plume torque imparted on S/C could be estimated using attitude control error data

$$T_D(t) \approx -3640.4\{\ddot{e}_\theta(t) + 0.15548\dot{e}_\theta(t) + 0.03529 e_\theta(t)\}\ \mathrm{Nm}$$

  - Per-axis attitude error control errors (e_θ) and per-axis attitude rate control errors ($\dot{e}_\theta$) are available via telemetry
  - The time rate of change of $\dot{e}_\theta$, $\ddot{e}_\theta$, could be estimated numerically

Targeted Low-altitude Enceladus Flybys on RWA Control

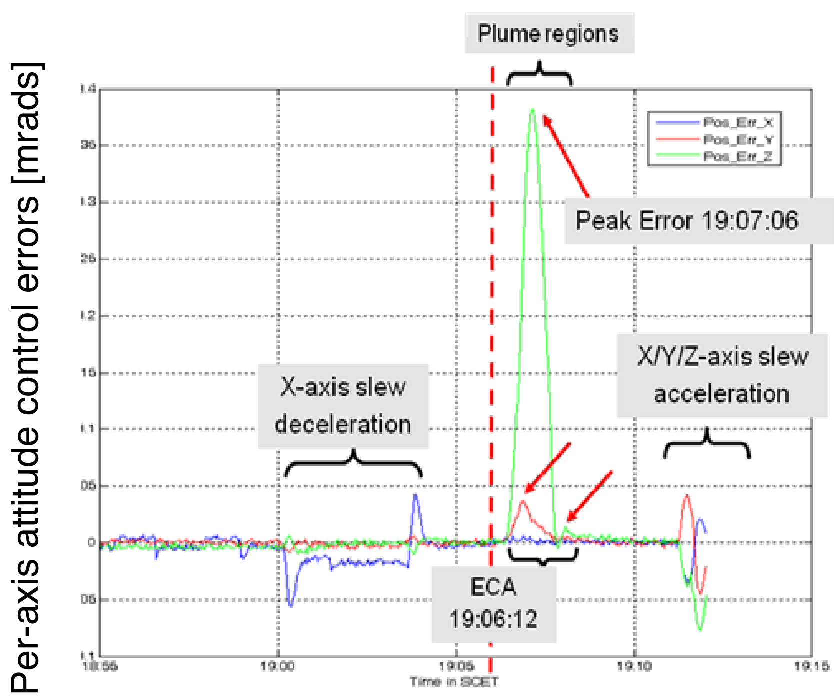
- During most Enceladus flybys, Cassini will experience significant plume torque, and only thrusters have the control authority to guarantee spacecraft safety

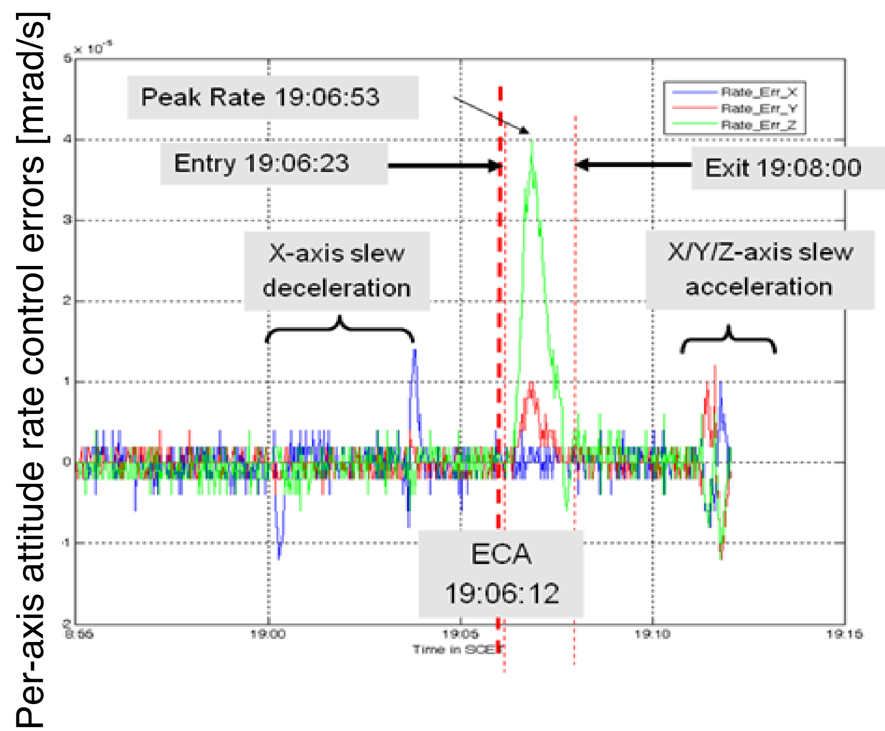
- Because the predicted magnitudes of plume torque imparted on the spacecraft during selected Enceladus flybys are within the control authority of the wheels, some Enceladus flybys were executed using reaction wheels

Flyby	Distance at Closest Approach [km]	Location of Closest Approach	Date of Flyby
Enceladus-3	50	20° S, 135° W	March 12, 08
Enceladus-4	50	28° S, 98° W	August 11, 08
Enceladus-9	99	89° S, 147° W	April 28, 10

E3 Flyby's Attitude and Attitude Rate Control Errors

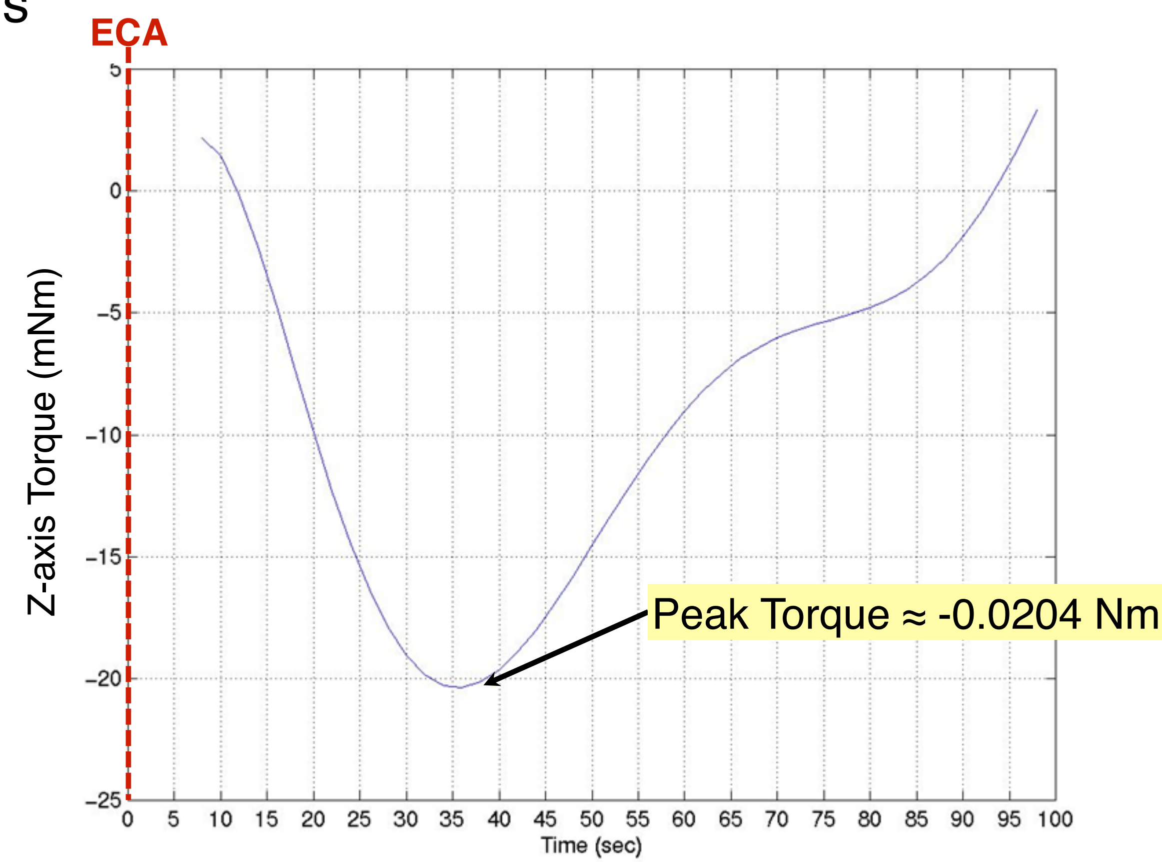
- During the flyby, multiple plume jets imparted disturbance torque on the spacecraft resulting in small but visible attitude control errors
  - Attitude control errors occurred even with zero commanded attitude rate
  - Most of the disturbance torque is about the spacecraft's Z-axis





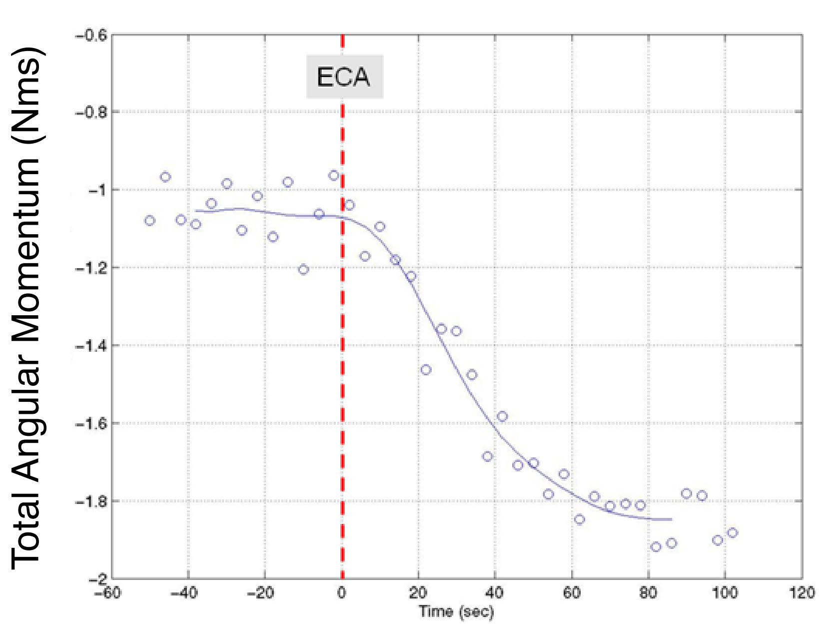
Reconstructed E3 Flyby's Disturbance Torque

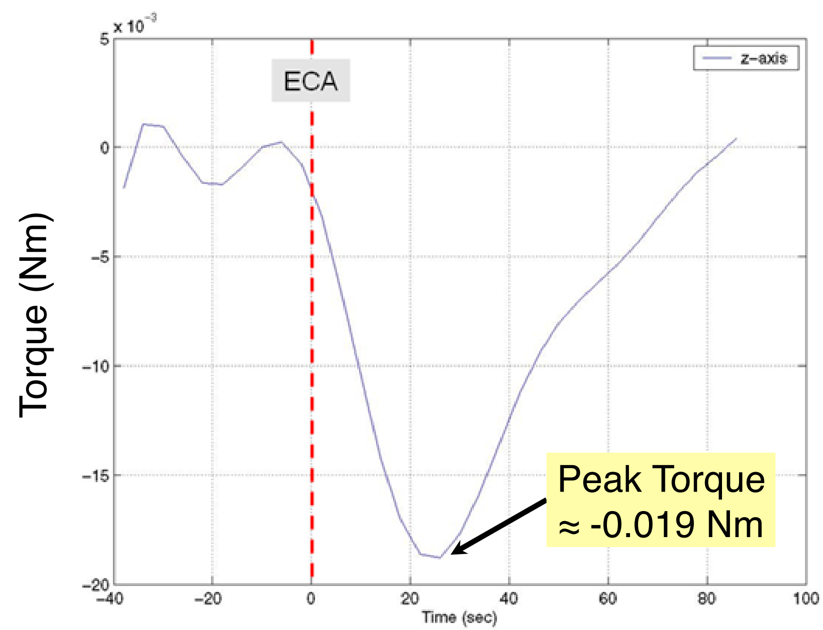
- The Z-axis plume torque imparted on Cassini during E3 could be computed using Eq. (3)
- The peak Z-axis torque is -0.0204 Nm, and it occurred at time ≈ ECA+34 s



Estimate Torque Via Rate of Change of Angular Momentum

- In order to maintain the quiescent inertial attitude of the spacecraft, the three RWAs must absorb the angular momenta imparted on Cassini due to the plume torque
  - As a result, the RWA spin rates changed as the spacecraft passed through the plume cloud
- The total angular momentum of the spacecraft could be computed using knowledge of the RWAs' inertia properties, the S/C inertia properties, and the telemetry data of the S/C's rates and RWA spin rates
  - The rate of change of the total angular momentum is the disturbance torque





Reconstructed Density of Enceladus Plume

- The Enceladus plume density is related to the torque imparted on the spacecraft by the following approximate relation:

$$\vec{T}_{Plume}(t) \approx \frac{1}{2} C_D \rho_{Plume}(t) V(t)^2 A_{Projected}(t) \quad \vec{u}_V(t) \times [\vec{r}_{CP}(t) - \vec{r}_{CM}(t)] \qquad (4)$$

where:

$T_{Plume}(t)$	Torque imparted on the spacecraft [Nm]
$\rho_{Plume}(t)$	Enceladus plume density [kg/m^3]
$V(t)$	S/C velocity relative to Enceladus [m/s]
$A_{projected}$	Pprojected area of the spacecraft [m^2]
C_D	Drag coefficient [-]
r_{cp}-r_{cm}	Moment arm vector [m]

- For E3, at the time of closest approach, V = 14.41 km/s, $A_{projected}$ = 18.401 m^2, and the Z-axis moment arm of the r_{CP}-r_{CM} vector = 0.853 m, Eq. (4) becomes:

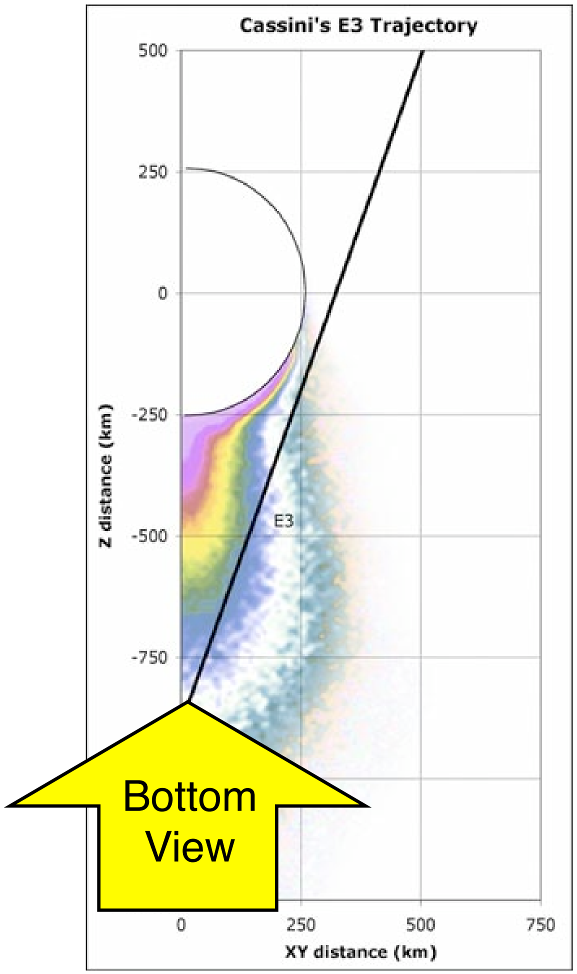
$$\vec{T}_{Plume}(t)\ [Nm] = 3.422 \times 10^{+9} \rho_{Plume}(t) \qquad (5)$$

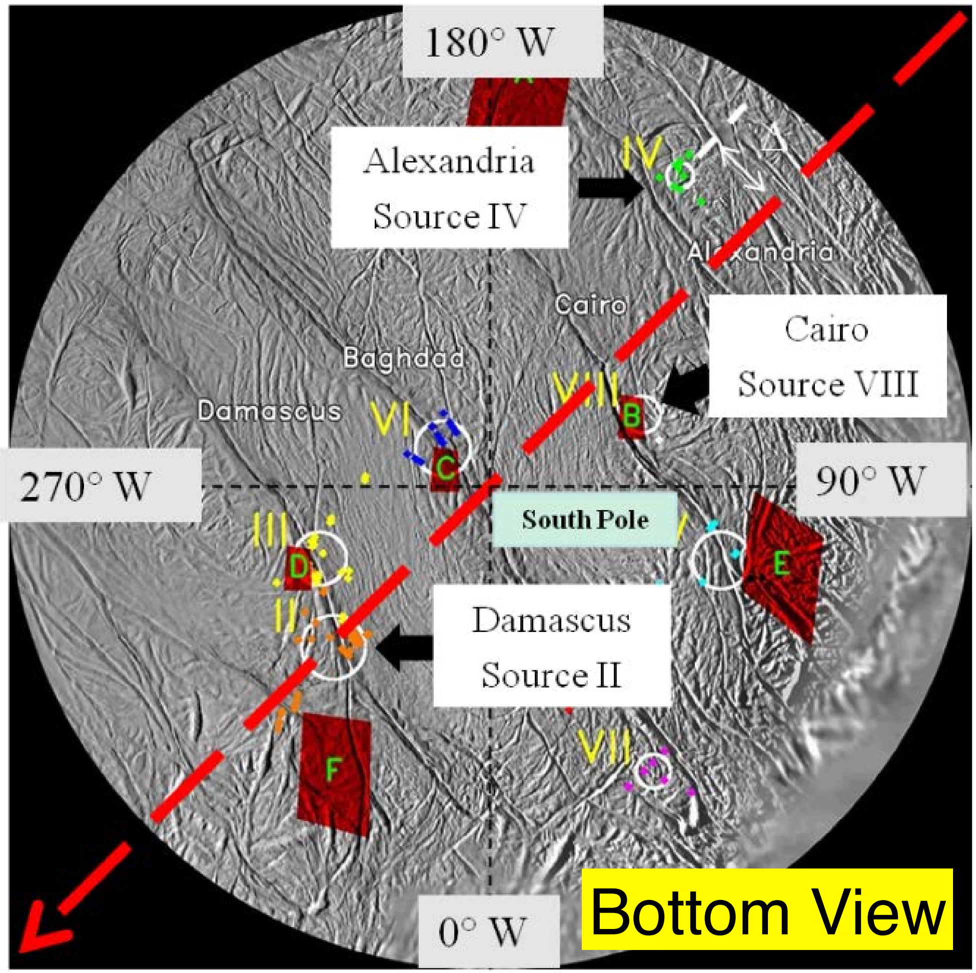
Reconstructed by	Enceladus Flyby	ECA Altitude [km]	Peak density occurred at	Peak density [10^{-12} kg/m^3]
AACS	E3	47.9	ECA+34 s	5.9±0.3
INMS	E3	47.9	ECA+50 s	1.8±0.2

The discrepancy between AACS and INMS-based density estimates drives on-going research

E3 Flyby Trajectory

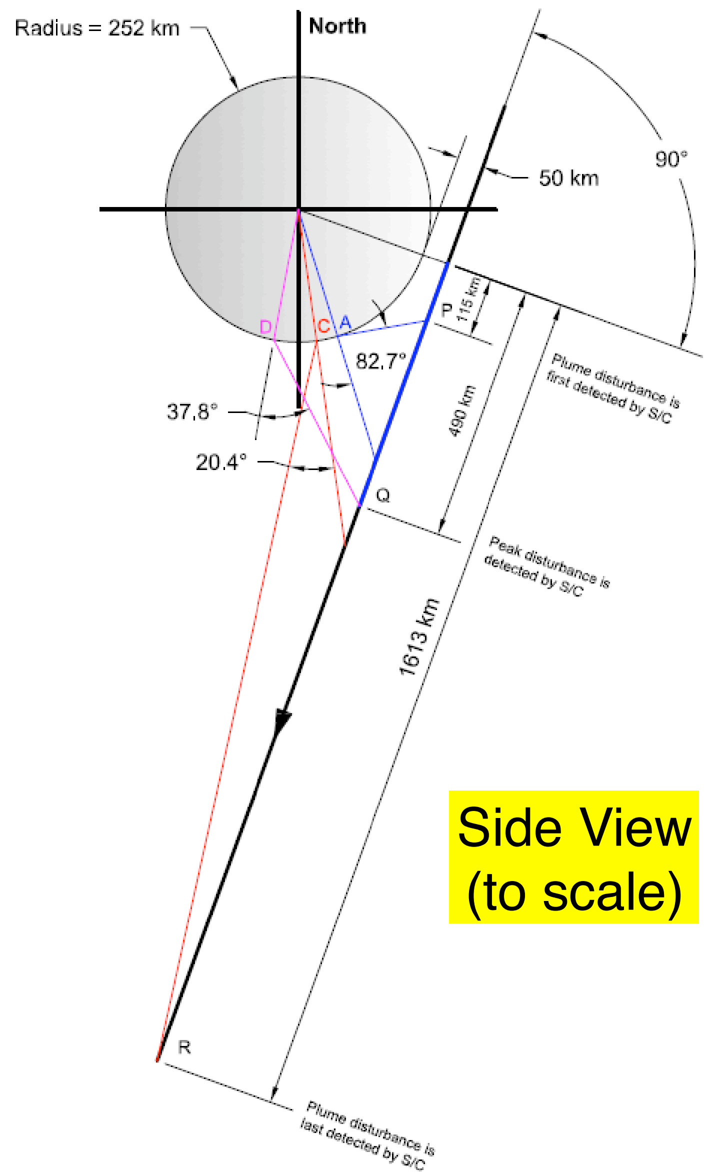
- The thick red line depicts Cassini's flyby trajectory projected onto a plane that is perpendicular to the Enceladus' axis
- The trajectory passed over Alexandria, Cairo, and Damascus





Side View of Enceladus-3 Flyby

- “A”, “C”, and “D” denote the locations of the plume sources, Alexandria, Cairo, and Damascus, respectively
- Three points on the spacecraft trajectory are labeled “P”, “Q”, and “R”
  - Point “P” denotes the time at which the RWA control system first detected and responded to plume torque
  - After point “R”, the RWA control system no longer detected any disturbance torque
  - Point “Q” is the time at which the detected disturbance torque peaked



Enceladus Plume Density Model

- In the literature, the structure of the Enceladus plume density is commonly modeled using the following relation:

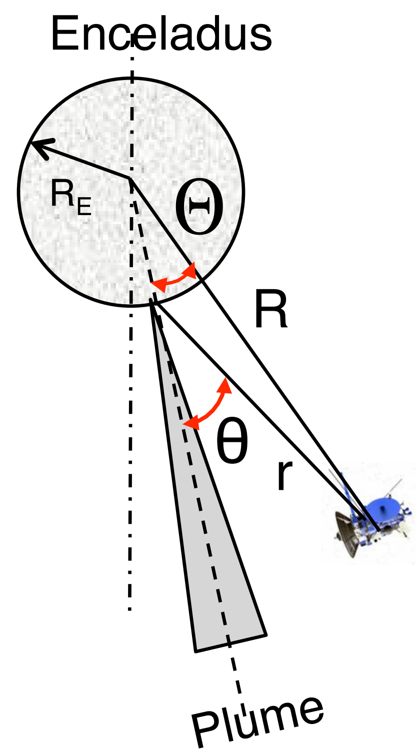
$$\rho(R,\Theta) = \rho_0 [\frac{R_E}{R}]^2 \exp[-(\frac{\Theta}{H_\Theta})^2] \exp[-\frac{R - R_E}{H_d}]$$

where

R is the radial distance of the S/C from Enceladus' c.m.

Θ is the angular distance of the S/C from the plume axis

Other details are in the paper



- In this study, a similar but simplified model is used:

$$\rho(r,\ \theta) = K_\rho [\frac{R_E}{r}]^{\frac{3}{2}} \exp[-\frac{\theta}{K_\theta}]$$

r is the radial distance of the S/C from plume source

θ is the angular distance of the S/C from the plume axis

Other details are in the paper

Two free model parameters per plume: K_ρ and K_θ

Modeling of Enceladus Plume Density

- If the estimated plume density during the Enceladus-3 flyby is given by

$$\rho_{\text{Model}}^{\text{E3}}(\text{t}) = \sum_{\text{i= Alexandria, Cairo}}^{\text{Damascus}} \rho^{\text{i}}(\text{r}_\text{i}(\text{t}),\theta_\text{i}(\text{t}))$$

- The six "free" plume density model parameters, and (i = Alexandria, Cairo, and Damascus), are to be selected to minimize the following modeling error

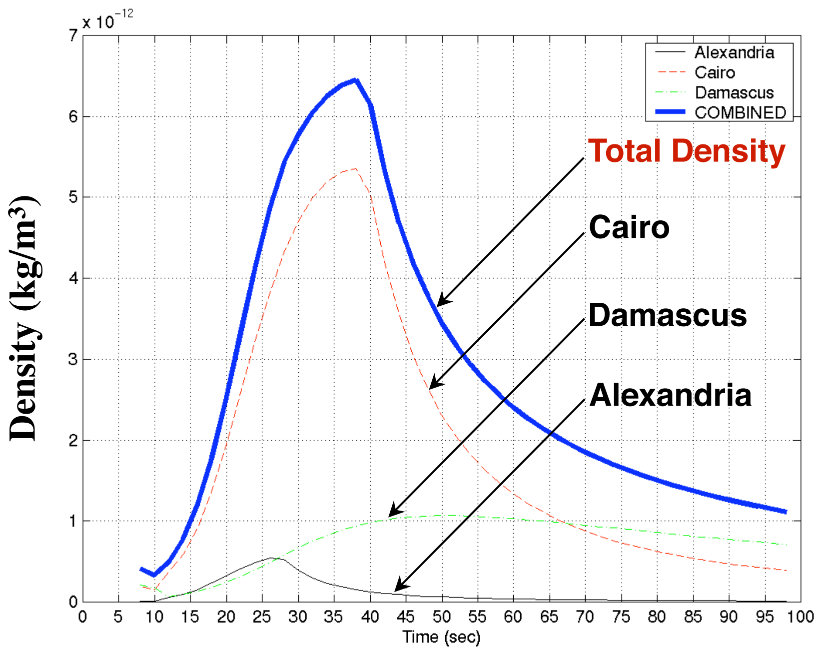
$$\text{Model Error} = \sqrt{\frac{1}{\text{t}_{\text{Exit}} - \text{t}_{\text{Entry}}} \int_{\text{t= Entry}}^{\text{t= Exit}} [\rho_{\text{Model}}^{\text{E3}}(\text{t}) - \rho_{\text{Reconstructed}}^{\text{E3}}(\text{t})]^2 \text{dt}}$$

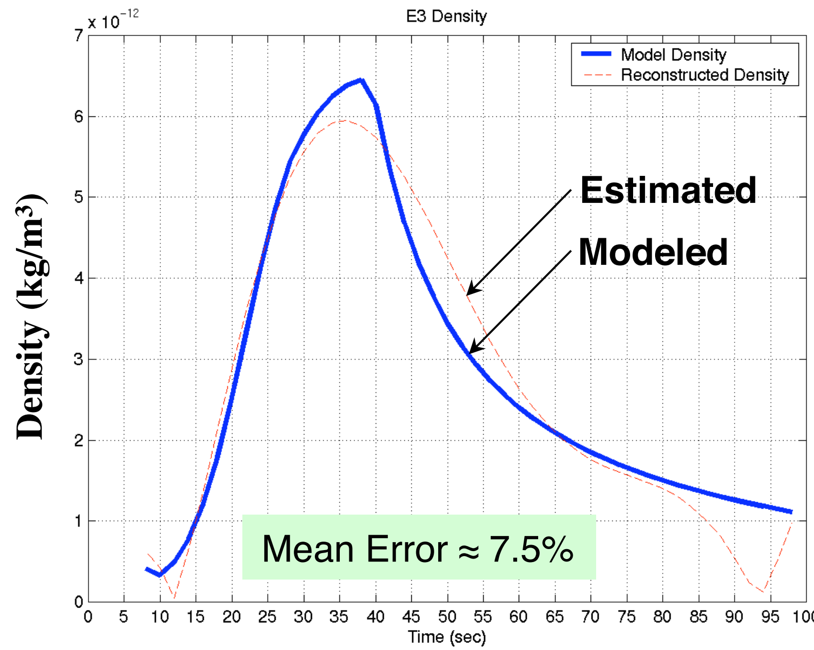
- These parameters could be determined using any optimization techniques or "Monte-Carlo-ly". The E3 results are given below

Plume Sources	Alexandria	Cairo	Damascus
K_ρ (kg/m^3)	0.55×10^{-12}	10.3×10^{-12}	8.5×10^{-12}
K_θ (radians)	0.36 (20.6°)	0.36 (20.6°)	0.36 (20.6°)

- The angular widths of the plumes (i = Alexandria, Cairo, and Damascus) that achieved a good match between the model and reconstructed plume density happened to be almost identical
- The "strengths" of the plumes Cairo and Damascus are about the same

Estimated and Modeled Enceladus Plume Densities





Conclusions

- Using the known and unique transfer function between the disturbance torque and the attitude control error, collected attitude control error telemetry could be used to estimate the very small disturbance torque imparted on Cassini during Enceladus flybys
  - The effectiveness of this methodology is confirmed using the E3 data
- Given good estimates of spacecraft's projected area, c.m. and c.p. locations, and spacecraft velocity, the time history of the Enceladus plume density during a flyby could be reconstructed
  - The 1σ uncertainty of the estimated density is 7.7%
  - Our density estimates deviate from those determined by the INMS science instrument. Disagreement is significant and it is being studied
- Building on Enceladus density models available in the literature, an alternative and simpler Enceladus density model is proposed in this study
  - In spite of its simplicity, it can approximate the reconstructed plume density with a mean modeling error of only 7.5% (for the E3 data)
  - … ***and the research continues*** ….