Determining Wheel-Soil Interaction Loads using a Meshfree Finite Element Approach Assisting Future Missions with Rover Wheel Design

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A wheel experiencing sinkage and slippage events poses a high risk to rover missions as evidenced by recent mobility challenges on the Mars Exploration Rover (MER) project. Because several factors contribute to wheel sinkage and slippage conditions such as soil composition, large deformation soil behavior, wheel geometry, nonlinear contact forces, terrain irregularity, etc., there are significant benefits to modeling these events to a sufficient degree of complexity. For the purposes of modeling wheel sinkage and slippage at an engineering scale, meshfree finite element approaches enable simulations that capture sufficient detail of wheel-soil interaction while remaining computationally feasible. This study demonstrates some of the large deformation modeling capability of meshfree methods and the realistic solutions obtained by accounting for the soil material properties. A benchmark wheel-soil interaction problem is developed and analyzed using a specific class of meshfree methods called Reproducing Kernel Particle Method (RKPM). The benchmark problem is also analyzed using a commercially available finite element approach with Lagrangian meshing for comparison. RKPM results are comparable to classical pressure-sinkage terramechanics relationships proposed by Bekker-Wong. Pending experimental calibration by future work, the meshfree modeling technique will be a viable simulation tool for trade studies assisting rover wheel design.

Nomenclature

Φ	kernel function
C	correction function
b	coefficients of n-th order monomial
x	spatial coordinate
t	time
NP	number of particles
$\mathbf{M}$	moment matrix
$\mathbf{H}$	n-th order polynomial basis vector
Ψ	meshfree shape function

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Ω approximation domain
$\widetilde{\epsilon}$ smoothed nodal strain
$\overline{A}$ area of smoothing domain
V smoothing domain
S boundary of smoothing domain
N normal to smoothing domain
Γ boundary of representative nodal domain

Subscript
i, j, k non-negative indices
a measure of compact support
I arbitrary point
L refers to nodal integration

Superscript
i, j, k non-negative indices
R reproduced function
I, J, K arbitrary points
T transpose
h indicates deviatoric

I. Introduction

In planetary exploration missions, it is essential for a mobile robotic vehicle to operate robustly in a variety of soil conditions. Ensuring mobility of the rover enables one-of-a-kind science and data observations to be made. If mobility is compromised, the scope and capability of science exploration is severely limited. Often, it is difficult to replicate experimentally the soil and gravity conditions that planetary rovers experience. Therefore, it is critical to model and simulate the soil-wheel interaction event. The objective of this study is to develop a practical soil-wheel interaction benchmark problem capable of simulating the interaction phenomena of sinkage and slippage and verify that it can be analyzed using existing commercial codes.

I.A. Relevant Missions

Since the late 90s, the Jet Propulsion Laboratory (JPL) has enjoyed highly profiled success in the arena of robotic, rover exploration missions. Beginning with the Mars Pathfinder project and associated *Sojourner* rover, followed by the Mar Exploration Rover(s) (MER) *Spirit* and *Opportunity*, and presently the Mar Science Laboratory (MSL) *Curiosity*, there has been a gradual progression in science mission scope and mechanical capability of the rovers. As the scope and capability increase, there have been several well documented mobility challenges, particularly on the MER missions.[1]

All the rovers engage a heritage mobility system consisting of a rocker-bogie suspension configuration with six wheel drive capability enabling the self equilibration of wheel loads during drives.[2] The MER *Spirit* rover's mobility system became compromised starting in April 23, 2009 (sol 1886) due to the right-front wheel becoming undrivable. After embedding further into ferric sulphate sand, *Spirit* was rendered immobile at a site called "Troy.[3]" However, prior to its ultimate demise, JPL mission operations commissioned a ground-based testing program to devise an extrication plan. In order to accurately test possible extrication sequences, a Mars-like "sand box" was constructed with simulant of Martian regolith and varying obstacles/slope conditions.[4]

While *Opportunity* is presently exploring the surface of Mars following its inception in 2004, it too has encountered instances of high slippage. Although visual odometry data accumulated from all drives shows little overall slippage ($\approx 5\%$), there were several isolated instances of high slippage and wheel sinkage despite several path correction manuevers.[5] These instances included traverses over loose sand indicated by low thermal inertia and the ascension of a slope ($\approx 10^{\circ}$) to arrive at the Santa Maria crater rim.[8]

Preceding the landing of MSL, the project has conducted several sloped drive tests of the *Curiosity* rover similar to those conducted on the MER program. One such test is shown in Fig. 1 where the *Curiosity* rover is traversing soft sand on a sloped surface. While extensive drive tests were conducted on all programs, robust wheel-soil interaction analysis and simulation capability is still in development. Experimentation coupled with less costly simulations can reduce overall risk during the planning and operational phases of rover missions. Also, simulation capability can assist wheel design trade studies and guide experimentalists in developing a mission-like test program.

Figure 1. Mars Science Laboratory (MSL) engineering model tested on dry, loose sand

I.B. Existing Rover Wheel-Soil Analysis Approaches

Modeling and simulation of wheel-soil interaction loads for rover mobility applications is a growing body of research and has gained significant momentum following the 2011 Keck Institute for Space Studies workshop at Caltech addressing the topic. While foundational concepts stem from the field of terramechanics dating back to the work of Bekker in the 1950's,[6] a variety of analysis approaches have been developed recently which are primarily driven by a diverse list of objectives including path planning, controller design and verification, capturing effects of granular behavior, and experimental validation.

In the area of path planning for planetary exploration rovers, Krenn and Hirzinger of the German Aerospace Center (DLR) have successfully integrated updated formulations of Bekker's equations with a contact detection algorithm to produce an in-house multi-body-dynamics software.[7] Also toward the objective of rover path planning, Trease *et al.* have developed a software tool, ARTEMIS - Adams-based Rover Terramechanics and Mobility Interaction Simulator, that combines state of the art multi-body-dynamics commercial software with classical terramechanics for simulation of rover drives over high resolution digital elevation maps of the Martian surface.[8] Aside from path planning, rover controller design is heavily influenced by wheel-soil interaction loads. For this reason, Jain *et al.* have developed ROAMS - Rover Analysis, Modeling and Simulation for development and testing of on-board control systems[9] and demonstrated its capability to perform operator in-the-loop simulations.[11] In 2008, ROAMS was validated on a set of rover mobility experiments on sloped terrain.[10] While the previous software modeling approaches implement classical interpretation of the wheel-soil contact forces, there have been efforts to utilize Discrete Element Modeling (DEM) to capture the multi-scale effects of granular soil behavior on the contact forces. Hopkins *et al.* and Knuth *et al.* have simulated a flight model of the Mars Exploration Rover (MER) wheel in a soil bed of 400,000 computational particles and compared the model to experimental results[12].[13] Finally, experimental studies have been performed to gain an in depth understanding of the soil behavior just beneath the rover wheel contact surface. Moreland *et al.* have developed the Shear Interface Imaging Analysis Tool that used optical flow software to capture detailed images of shear interfaces and sub-surface soil displacement.[14] In an additional investigation, Moreland *et al.* validate experimentally an inching locomotion strategy to overcome the loss in tractive capability due to wheel sinkage in conventional rolling mobility.[15]

II. Meshfree Motivation

Conventional finite element methods exhibit a number of shortcomings in analyzing problems involving large deformation, high gradient, material separation, and multiple-scale phenomena. These difficulties are partially due to the regularity requirement of the finite element mesh. Due to the inability to effectively model large material distortion and separation, finite element methods with traditional meshing have not been successfully applied to the analysis of wheel-soil interaction models where large strains are present. There is also a fundamental difficulty associated with the numerical solution of strain localization that often exists in unstable soil motion. Grid-based numerical methods introduce a length scale (i.e. the mesh size) creating a bifurcation problem which results in the numerical solution being very sensitive to mesh size. The multiple-scale nature of shear band formation inherent in soil materials also adds considerable complication to conventional finite element approaches.

Meshfree methods offer several advantages for simulating the types of responses that are critical to wheel-soil interaction. These methods remove the necessity of having to maintain a well formed, single domain mesh. For the last fifteen years or so, a family of methods, collectively called meshfree methods or meshless methods, has attracted much interest in the community of computational mechanics. This family of numerical methods incorporates the main advantages of the finite element method such as compact supports of shape functions and good approximation properties, while overcoming the main disadvantages caused by the mesh-dependence. All meshfree methods share a common feature in that no mesh is needed in the approximation. The shape functions are constructed from sets of points with overlapping domains, thus eliminating the difficulties associated with mesh distortion in large deformation problems such as wheel-soil interaction.

Soil materials usually undergo large deformation, shear band formation, damage evolution, and material separation when subjected to wheel maneuvers. These involved soil characteristics are critical to rover mobility in a variety of mission contexts. A realistic wheel-soil interaction model must account for the aforementioned soil behavior while capturing the wheel interface behavior in the form of soil sinkage, vertical force, drag force, and torque on the wheel axle. Therefore, a thorough investigation of soil behavior and properties is a vital task prior to the development of complex wheel-soil interaction model and is a topic for future work.

III. Benchmark Problem Formulation

A semi-Lagrangian meshfree formulation has been developed to effectively simulate material distortion, damage, separation, and free surface formation and closure during the soil-wheel interaction. A stabilized nodal integration[16] has been introduced and incorporated into the semi-Lagrangian meshfree formulation to achieve stability and efficiency of meshfree computation. The developed meshfree methods have been applied to the simulation of soil-wheel interaction and a characterization of torque response. The meshfree soil-wheel interaction problem is verified by comparison to a semi-empirical solution.

III.A. Reducing Kernel Particle Method Overview

Reproducing Kernel Particle Method (RKPM)[17,18] belongs to this class of meshfree methods and it has been successfully applied to large deformation and contact/impact problems.[18−21] The foundation of RKPM is a reproducing kernel approximation where the approximation of a function $u_i(x)$ is

$$u_i^R(\mathbf{x}) = \sum_I C(\mathbf{x}; \mathbf{x} - \mathbf{x}_I)\Phi_a(\mathbf{x} - \mathbf{x}_I)u_{iI} \tag{1}$$

Here Φ_a is the kernel function that defines the smoothness of the approximation function where subscript, a, is the measure of compact support, $C(\mathbf{x}; \mathbf{x} - \mathbf{x}_I)$ is called the correction function that is to be constructed to fulfill consistency conditions, and $u_i^R(\mathbf{x})$ is the "reproduced" function of $u_i(\mathbf{x})$. The correction function,

$C(\mathbf{x};\mathbf{x}-\mathbf{x}_I)$, is a linear combination of monomial basis functions,

$$C(\mathbf{x};\mathbf{x}-\mathbf{x}_I) = \sum_{i+j+k=0}^{n} b_{ijk}(\mathbf{x})(x_1 - x_{1I})^i (x_2 - x_{2I})^j (x_3 - x_{3I})^k, 0 \leq i+j+k \leq n \tag{2}$$

where i, j, k are non-negative integers, and $b_{ijk}(\mathbf{x})$ are the coefficients solved by requiring exact representation of the n-th order monomial in Eq. 1; often referred to as the n-th order consistency. The final reproducing kernel approximation can be expressed as

$$u_i^R(\mathbf{x}) = \sum_I \Psi_I(\mathbf{x}) u_{iI} \tag{3}$$

where

$$\Psi_I(\mathbf{x}) = C(\mathbf{x};\mathbf{x}-\mathbf{x}_I)\Phi_a(\mathbf{x}-\mathbf{x}_I)u_{iI} \tag{4}$$

$\Psi_(x)$ corresponds to the meshfree shape functions constructed without the need for a mesh, and u_{iI} being the coefficients of the approximation. An example of RKPM discretization of problem domain and the corresponding reproducing kernel shape functions $\Psi_(x)$ are shown in Fig 2.

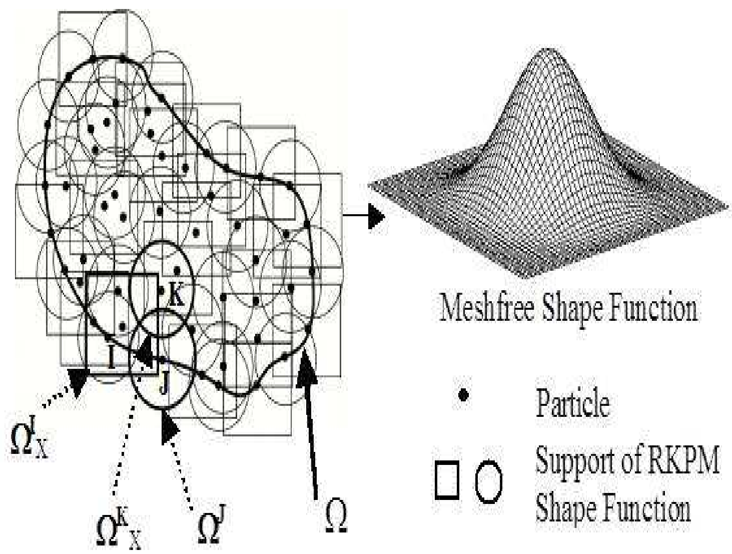


Figure 2. RKPM discretization: The union of the supports of all particles (such as Ω^I, Ω^J, Ω^K) should cover the problem domain Ω

To effectively model large degrees of material deformation and damage, a semi-Lagrangian form of RKPM has been developed. The Lagrangian meshfree discretization that considers evaluation of kernels based on particle distance measured in the undeformed configuration breaks down in modeling penetration and fragmentation processes. In the semi-Lagrangian approach, all variables are expressed as functions of spatial coordinate $\mathbf{x}$ of deformed configuration and time, t, and the discrete meshfree particles follow the material motion, $\mathbf{x}_I = \mathbf{x}(\mathbf{X}_I, t)$, where $\mathbf{x}_I$ and X_I are the spatial and material coordinates of point I, respectively. In this approach, the compact support of the kernel function involved in the meshfree shape function is defined in the deformed configuration. In the semi-Lagrangian formulation, since the approximation for displacement and velocity is a function of spatial coordinate x and time t, the material time derivative of the kernel function in RKPM has been considered to account for the non-conservative particle interaction in the semi-Lagrangian kernel. A semi-Lagrangian formulation based on Eulerian kernel originally developed for an earth-moving simulation has been extended to soil-wheel interaction. The semi-Lagrangian formulation employs a distance measure in the kernel function which is defined in the deformed configuration. Also, the semi-Lagrangian approach allows the neighbors to be redefined in the deformation process, and it avoids the need for inverse mapping from the deformed configuration to the undeformed configuration. By imposing reproducing conditions in the deformed configuration, the semi-Lagrangian shape function is obtained as

$$\Psi_I(\mathbf{x}) = C(\mathbf{x};\mathbf{x}-\mathbf{x}(\mathbf{X}_I,t))\Phi_a(\mathbf{x}-\mathbf{x}(\mathbf{X}_I,t)) \tag{5}$$

where

$$C(\mathbf{x};\mathbf{x}-\mathbf{x}(\mathbf{X}_I,t)) = \mathbf{H}^T(\mathbf{0})\mathbf{M}^{-1}(\mathbf{x})\mathbf{H}(\mathbf{x}-\mathbf{x}(\mathbf{X}_I,t)) \tag{6}$$

$$\mathbf{M}(\mathbf{x}) = \sum_{I=1}^{NP}\mathbf{H}(\mathbf{x}-\mathbf{x}(\mathbf{X}_I,t))\mathbf{H}^T(\mathbf{x}-\mathbf{x}(\mathbf{X}_I,t))\Phi_a(\mathbf{x}-\mathbf{x}(\mathbf{X}_I,t)) \tag{7}$$

$$\mathbf{H}^T(\mathbf{x}-\mathbf{x}(\mathbf{X}_I,t)) = [1 \;\; x_1-x_1(\mathbf{X}_I,t \;\; x_2-x_2(\mathbf{X}_I,t \;\; (x_1-x_1(\mathbf{X}_I,t))^2\ldots(x_2-x_2(\mathbf{X}_I,t))^n] \tag{8}$$

To provide stabilization of nodal integration in the Galerkin meshfree formulation, a stabilized nodal integration method[16,22,23] has been employed in the simulation of soil-wheel interaction. In this approach, a modified smoothed nodal strain $\tilde{\epsilon}^h_{ij}(\mathbf{x}_L)$ has been introduced:

$$\tilde{\epsilon}^h_{ij}(\mathbf{x}_L) = \frac{1}{2\overline{A}_L}\int_{V_L}(u^h_{i,j}+u^h_{j,i})d\Omega = \frac{1}{2\overline{A}_L}\int_{S_L}(u^h_i N_j + u^h_j N_i)d\Gamma \tag{9}$$

where $\tilde{\epsilon}^h_{ij}(\mathbf{x}_L)$ is the smoothed strain at $\mathbf{x}_L$, $\overline{A}_L$ is the area of smoothing domain V_L bounded by boundary S_L of particle $\mathbf{x}_L$ as shown in Fig. 3. In the same figure, the nodal representative domain Ω_L has boundary Γ_L and normal component, N, for a particle $\mathbf{x}_L$ creating a Voronoi diagram. Although the previous nodal integration method satisfies integration constraints in the Galerkin approximation, the conforming condition and integration constraints are no longer imposable in the modeling of large material separation and is therefore relaxed in the proposed approach. Although, this modified stabilized nodal integration is "nonconforming," the method is consistent with the semi-Lagrangian RKPM.

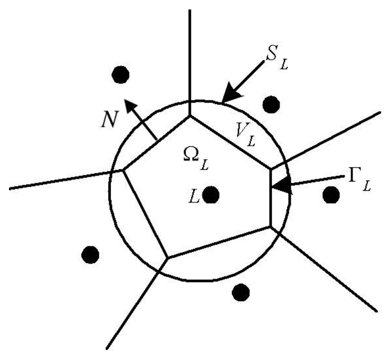


Figure 3. Nodal smoothing zone in stabilized nodal integration

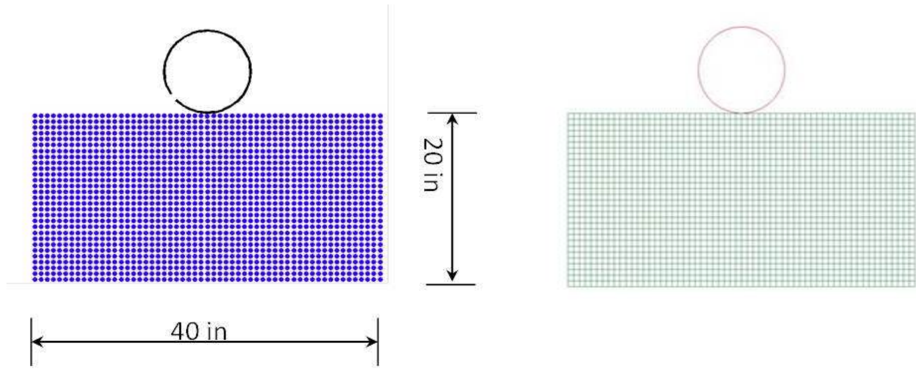
III.B. Benchmark Problem Details

A simulation model consisting of a rigid wheel and a meshfree soil domain is shown in Fig. 4(a). The rotating wheel is compressed vertically to the soil with prescribed angular rotation. The wheel geometry, wheel motion, and simulation parameters used in the RKPM simulation are as follows: wheel diameter, 10 *in*; vertical compression, 0.6 *in/sec*; angular velocity, 1.0 *rad/sec*; simulation time, 8 *sec*. The wheel is assumed to be rigid, and frictional contact between wheel and soil is considered.

The soil properties are characterized by the following parameters: elastic modulus, 2880.8 *psi*; Poisson's ratio, 0.28; cohesion, 6.6 *psi*; friction angle, 31.68°; density, 129.86 lb/ft^3; initiating damage strain, 2.5%; coefficient of friction between wheel and soil, 0.4. The soil properties and nonlinear behavior are implemented through a soil plasticity constitutive model proposed by Dimaggio *et al.*[27] which is based on a Drucker-Prager yield condition.

III.C. Commercial Code Implementation

As a precursor to meshfree implementation, the benchmark problem was modeled and simulated in LS-Dyna using conventional Lagrangian meshing for the soil domain. Dimensions of the wheel and soil domain were identical to the benchmark problem. In the LS-Dyna environment, the contact surface was defined using CONTACT_AUTOMATIC_SURFACE_TO_SURFACE assumptions which allows for the input of the benchmark coefficient of friction between the wheel and soil interface. The wheel was assumed to be rigid



(a) Benchmark Problem with RKPM Soil Domain (b) Benchmark Problem with Lagrangian Soil Domain

Figure 4. Benchmark Problem Implementation

with prescribed angular velocity and vertical translation listed in the benchmark problem details. Nonlinear soil behavior based on a polynomial approximation of the Drucker-Prager yield surface was modeled using a MAT_005 soil constitutive model with experimentally obtained parameters captured in a study by Bojanowski and Kulak.[24] Due to the numerical accuracy and stability of the MAT_005 material model in soil structure interaction problems, it is widely used in research simulation applications such as earth landing[26] and soil penetration.[25] A picture of the benchmark problem modeled by LS-Dyna is shown in Fig. 4(b).

IV. Simulation Results

Visual simulation results for the RKPM technique are depicted in a sequence of snapshots (left to right, top to bottom) in Fig. 5. As the simulation progresses through 8 sec, the wheel begins to dig into the soil, effectively transporting RKPM nodes to larges deformations from their original location. The same event simulated in LS-Dyna using a Lagrangian soil mesh is displayed in Fig. 6. It can be observed that the Lagrangian mesh elements penetrate the wheel surface in several instances due to large contact penalties. The Lagrangian soil also has difficulties capturing the soil transport effect. The pressure-sinkage relations

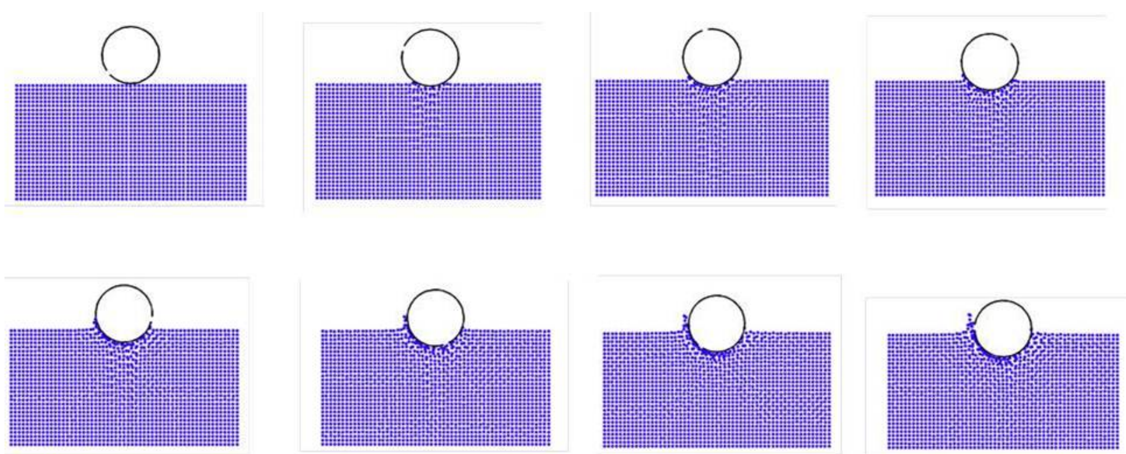


Figure 5. Simulation of soil-wheel interaction using RKPM

obtained from RKPM simulations are then fitted into Bekker's semi-empirical equation:[28]

$$p = \left(\frac{k_c}{b} + k_\phi\right) z^m = k_{eq} z^m \tag{10}$$

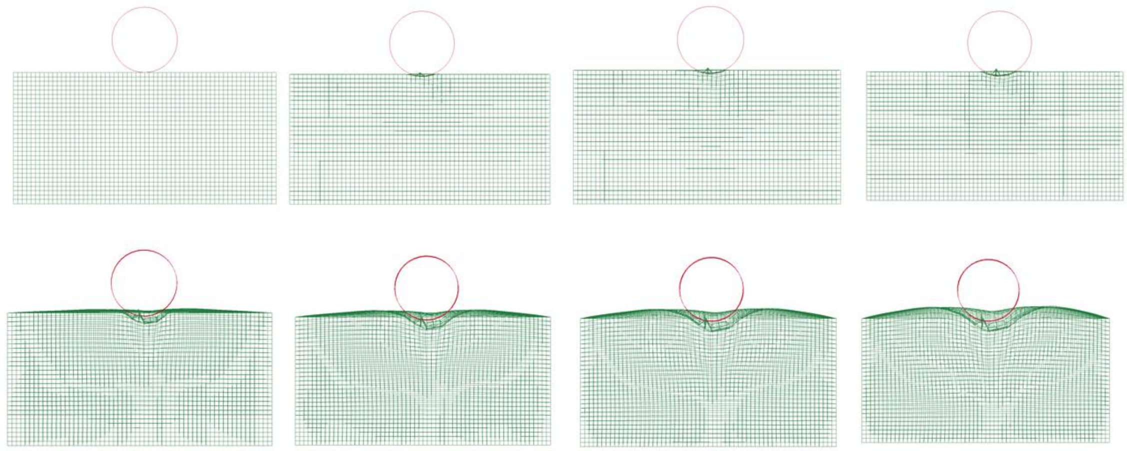


Figure 6. Simulation of soil-wheel interaction using Lagrangian soil mesh

where m is the exponent of sinkage, b is the width of wheel, k_c is the cohesive coefficient, and k_ϕ is the friction parameter. Fitting of the numerical results was performed for two reasons. First, given that the Bekker relationships were derived to 'reasonably' mimic soil interaction experiments, the meshfree solutions demonstrating 'Bekker-like' behavior concludes that they nominally agree with classical theory. Secondly, having a simplified expression that describes complex wheel-soil interaction behavior is advantageous for real time simulations that do not possess the necessary computational capability. In this fashion, these adequately constructed Bekker relationships are suitable for online applications, such as feedback control. The fitted Bekker's pressure-sinkage parameters using RKPM solution with plasticity soil model are $m = 0.32$ and $k_{eq} = 159$, and the fitted curve for pressure versus sinkage is shown compared to all simulation results in Fig. 7. Fig. 7 demonstrates that the Bekker relationship is capable of fitting the nominal pressure-sinkage

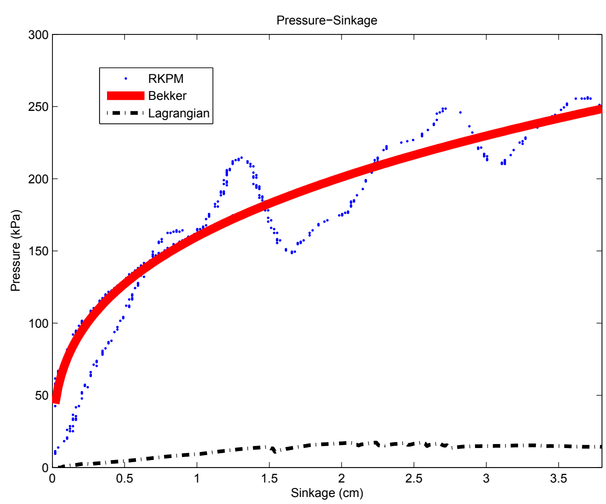


Figure 7. Fitted Bekker's pressure-sinkage curve using plasticity soil constitutive law

behavior of the meshfree solution; however, depending on whether or not meshfree techniques are used in the simulation, accuracy can be affected greatly affected. This is illustrated by the curve of pressure-sinkage data points produced from the Lagrangian soil mesh simulation. From these data points, it can be inferred that the Lagrangian simulation results do not fit the Bekker relationship as well as those produced from the RKPM solution.

V. Conclusion

A straightforward, benchmark problem was developed that accurately captures wheel-soil sinkage and slippage. A general form of meshfree methods, RKPM, was employed in its analysis. The benchmark problem was formulated and analyzed in LS-Dyna using a Lagrangian mesh for the soil domain. The simulation results demonstrated that the RKPM meshfree technique better simulated wheel sinkage and slippage in soil when compared to empirical relationships proposed by Bekker. The RKPM solution combined with a finite element approach is a viable analysis method for the wheel-soil interaction event. Future work will involve verification of the general form of RKPM using the built-in meshfree capability of LS-Dyna and validations of the technique using flight-like model geometry.

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