Radio Science Measurements with Suppressed Carrier

Sami Asmar
Jet Propulsion Laboratory
California Institute of Technology
4800 Oak Grove Drive
Pasadena, CA 91109
818-354-6288
Sami.Asmar@jpl.nasa.gov

Dariush Divsalar
Jet Propulsion Laboratory
California Institute of Technology
4800 Oak Grove Drive
Pasadena, CA 91109
818-393-5138
Dariush.Divsalar@jpl.nasa.gov

Kamal Oudrhiri
Jet Propulsion Laboratory
California Institute of Technology
4800 Oak Grove Drive
Pasadena, CA 91109
818-393-1143
Kamal.Oudrhiri@jpl.nasa.gov

***Abstract*—Radio Science started when it became apparent with early Solar missions that occultations by planetary atmospheres would affect the quality of radio communications. Since then the atmospheric properties and other aspects of planetary science, solar science, and fundamental physics were studied by scientists. Radio Science data was always extracted from a received pure residual carrier (without data modulation). For some missions, it is very desirable to obtain Radio Science data from a suppressed carrier modulation. In this paper we propose a method to extract Radio Science data when a coded suppressed carrier modulation is used in deep space communications. Type of modulation can be BPSK, QPSK, OQPSK, MPSK or even GMSK. However we concentrate mostly on BPSK modulation. The proposed method for suppressed carrier simply tries to wipe out data that acts as an interference for Radio Science measurements. In order to measure the estimation errors in amplitude and phase of the Radio Science data we use Cramer-Rao bound (CRB). The CRB for the suppressed carrier modulation with non-ideal data wiping is then compared with residual carrier modulation under the same noise condition. The method of derivation of CRB for non-ideal data wiping is an innovative method that presented here. Some numerical results are provided for coded system.**

TABLE OF CONTENTS

1 INTRODUCTION 1
2 CHANNEL AND SIGNAL MODELS 2
3 THEORETICAL ANALYSIS BASED ON THE CRAMER-RAO BOUND 3
4 PROPOSED SYSTEM 7
5 EXAMPLE 7
6 SUMMARY 7
APPENDICES 7
A MODIFIED CRAMER RAO BOUND 7
ACKNOWLEDGMENTS 9
REFERENCES 9
BIOGRAPHY 9

1. INTRODUCTION

Radio Science investigations utilize the telecommunication links between spacecraft and Earth to examine changes in the phase/frequency, amplitude, and polarization of radio signals to investigate: planetary atmospheres, planetary rings, planetary surfaces, planetary interiors, solar corona and wind, comet mass flux, fundamental Physics. The measurements are conventionally made at the earth station. In Deep Space communications in addition to receiving the information data reliably, it is always desirable to receive Radio Science observations accurately to estimate a planet atmosphere structure. Currently the Radio Science observations use only the residual carrier (no data on carrier). For deep space communication systems, the decision of whether or not to suppress the transmitted carrier has always been an issue. In general, for a given total available power, a fraction of the power would be allocated to the carrier signal and the remainder of the power would be allocated to the signal with data. It is well known that the more power allocated to the carrier, the better the synchronization and Radio Science observation accuracy. At the same time the more power allocated to the data-bearing signal the better the error probability of the end-to-end communication link. The question is how to trade off between these two conflicting power limited requirements so as to minimize the average error probability of the system and at the same time enabling accurate Radio Science observations. Obviously there always exists an optimum split of the total power into the two components mentioned above if the cost is a function only of error probability. When the cost is a function of probability of error and goodness of Radio Science data, still there could exist an optimum phase modulation index corresponding to required average error probability performance, while satisfying the required Radio Science data accuracy. For certain missions that use high data rates, the available bandwidth might be a limitation factor. In such case, it is preferred to use a completely suppressed carried system that is more bandwidth efficient. Currently Radio Science measurements are performed by using only a residual carrier scheme. When a carrier signal is passing through the atmosphere of a planet, Radio Science information is extracted from the amplitude, and phase variations of the received carrier. For certain critical missions, pure carrier transmission without data might be used for some duration of time to enhance the quality of received Radio Science observations. In this paper we consider a pure suppressed carrier system and we restrict the modulation to be binary Phase Shift Keying (BPSK). However extension of results to Quadrature Phase-Shift Keying (QPSK), and M-ary Phase-Shift Keying (MPSK) is possible. We present a method to remove the data from the received suppressed carrier and effectively generate a residual carrier for Radio Science from the received suppressed carrier data for estimating the amplitude, and phase variations. For an un-coded system a simple way to remove data is to square the received baseband samples or to multiply the incoming samples with the tangent hyperbolic function of the received samples. Unfortunately using this method, only twice of the actual phase can be estimated. Thus, this method creates phase ambiguity that requires unwrapping of the estimated phase. With such scheme there is also a performance degradation due to the squaring of noisy samples. For coded systems, fortunately this is not the case. The existing coding schemes for space communications include convolutional codes, concatenated Reed-Solomon and convolutional codes, turbo codes, and Low density Parity Check (LDPC) codes. The proposed

978-1-4577-0557-1/12/$26.00 ©2012 IEEE.
[1] IEEEAC Paper #2419, Version 3, Updated 1/2/2013.

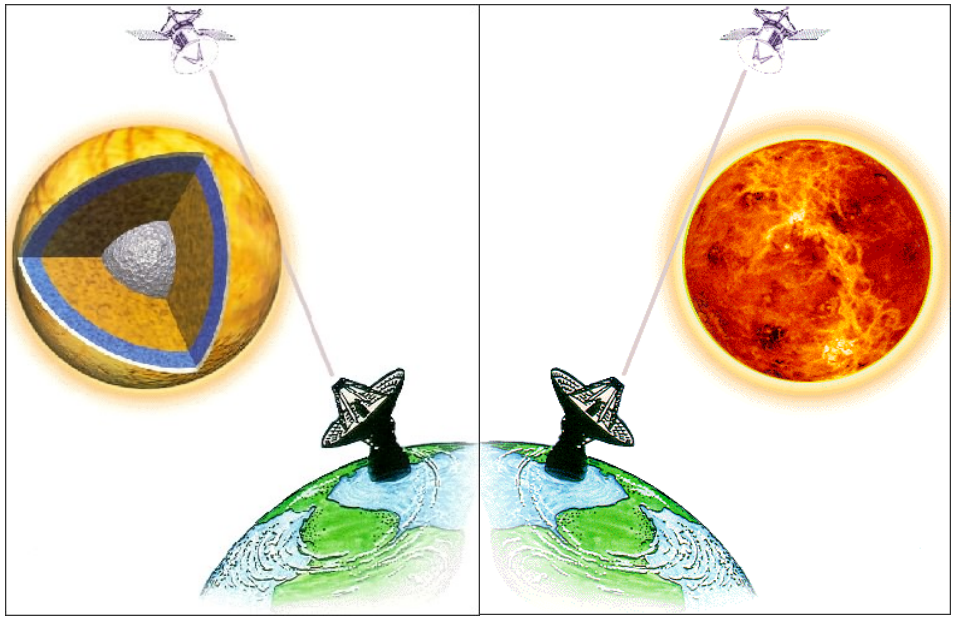


Figure 1. Concept of Radio Science Measurements using RF.

method takes the hard output of the decoder to remove the data from the incoming signal after re-encoding and re-modulation. Since delay is not an issue in Radio Science measurements the process of data removal can also be done in several iterations. We analyze the received data using open loop estimation techniques for amplitude, and phase. Results will be parametrically represented so that it can represent all the different cases of interest to Radio Science observations. In section 2 we provide the received signal models, in section 3 a theoretical analysis based on Cramer-Rao bound is performed. The goal is to obtain a lower bound on minimum variance of unbiased estimation of parameters related to Radio Science data for the suppressed carrier. Then the theoretical results are compared to the residual carrier case. In section 4 a system for suppressed carrier to extract the Radio Science data is proposed. Section 5 provides an example and numerical results. Section 6 provides a summary.

2. Channel and signal Models

For suppressed carrier BPSK modulation the complex envelope of the received signal in absence of planet atmosphere can be modeled as

$$y(t) = Ad(t)e^{j(2\pi f_d t+\phi_o)} + n(t) \tag{1}$$

where $y(t)$ is the complex received signal, with amplitude A , Doppler frequency shift f_d, and the carrier phase ϕ_o. In (1), $n(t)$ is additive zero mean white Gaussian noise and $d(t)$ is transmitted data that can be represented as

$$d(t) = \sum_k d_k p(t - kT_s) \tag{2}$$

where $p(t)$ represent modulation pulse shaping, $d_k \in \{+1, -1\}$ for BPSK, and T_s is symbol duration. In deep space communications usually $p(t)$ is a rectangular pulse shape such that

$$p(t) = 1; 0 < t \leq T_s \tag{3}$$

and zero otherwise. Other pulse shaping such as Nyquist pulse shaping can also be considered if there are strict bandwidth limitation.

When the suppressed carrier BPSK modulation is passed through a planet atmosphere the received signal can be modeled as

$$y(t) = Aa(t)d(t)e^{j(2\pi f_d t+\psi(t)+\phi_o)} + n(t) \tag{4}$$

where $a(t)$ and $\psi(t)$ are the amplitude and phase of Radio Science signal to be measured. For residual carrier the complex envelope of the received signal can simply be modeled as

$$y(t) = Aa(t)e^{j(2\pi f_d t+\psi(t)+\phi_o)} + n(t) \tag{5}$$

Comparing (4), and (5) we see that for pure residual carrier the multiplicative term $d(t)$ is missing. The received observation using (5) provides all information required for Radio Science measurements. The only nuisance interfering signal is the noise $n(t)$. However in (4) the data $d(t)$ represent an additional multiplicative nuisance interfering signal that should be removed. In addition to Doppler frequency, we are interested in estimating $a(t)$ and $\psi(t)$. In order to simplify the signal model, either prior to estimating $\psi(t)$ we can calibrate out the Doppler, and ϕ_o or simply represent the Doppler and ϕ_o as part of $\psi(t)$ phase. Also we assume the received signal

can be normalized (calibrated) by A. Thus a simpler model for the received signal can be written as

$$r(t) = a(t)d(t)e^{j\psi(t)} + n(t) \tag{6}$$

The discrete time model for signal in (6) at a symbol sample rate can be represented as

$$\mathbf{r} = \mathbf{s}(\mathbf{a}; \boldsymbol{\psi}; \mathbf{d}) + \mathbf{n} \tag{7}$$

where the kth component of $\mathbf{r}$ can be represented as

$$r_k = a_k d_k e^{\psi_k} + n_k \tag{8}$$

Suppose the estimation of time varying amplitude and phase is done over windows of N samples. Since these parameters are slow varying, in practice we can make the following approximations for amplitude and phase process. Over every N samples we assume the amplitude a is almost constant and phase can have linear approximation. The estimated amplitude and phase (with possible phase unwarpping) over consecutive windows will represent the time varying amplitude and phase if the number of samples per window frame is chosen properly. Without loss of generality consider the time index for a given window be $k = 0, 1, 2, \ldots (N-1)$. Over this window then $a_k = a$, and $\psi_k = 2\pi\psi_1 k\Delta + \psi_0$ where ψ_0 is the initial phase at the beginning of the window i.e at $k = 0$, ψ_1 represent the slope of the phase, and Δ is the sampling period (here it is equal to T_s). In fact we used piecewise line fitting for the phase over every N samples. In (8), the n_ks are independent and identically distributed zero mean noise samples with variance $\sigma^2 = \frac{N_o}{2E_s}$ per dimension. The symbol energy is given by $E_s = PT_s$ where P is the averaged received power. Note that the variance of noise after normalization is correctly done since if we set $a = 1$, and $\psi_1 = \psi_0 = 0$, by passing the received samples r_k through a hard limiter we get exactly the performance of uncoded coherent BPSK modulation. With this setup we have a simpler signal model to be used in our theoretical analysis in the next section. The received vector is modeled as

$$\mathbf{r} = \mathbf{s}(\boldsymbol{\theta}; \mathbf{d}) + \mathbf{n} \tag{9}$$

where the vector parameter $\boldsymbol{\theta} = (\theta_1, \theta_2, \theta_3) = (a, \psi_0, \psi_1)$ to be estimated given the received complex observation samples

$$r_k = a d_k e^{j(2\pi\psi_1 k\Delta + \psi_0)} + n_k \tag{10}$$

For wiping out the data, we use a genie argument. Assume for every received sample a genie tells us the correct value of data d_k with probability of $1-p$ and incorrect value with probability of p. This is equivalent to have $p(d_k = 1) = 1-p$, and $p(d_k = -1) = p$. Using genie argument simplifies our theoretical analysis. In real implementation of the system we replace the genie with the channel decoder output the provides the same information for non-ideally wiping out the data.

3. Theoretical Analysis based on the Cramer-Rao bound

The Cramer-Rao bound (CRB) [1], [2], [3] in estimation theory, provides the tightest achievable lower bound on the mean square error of an unbiased estimator. The CRB for vector parameter estimation in additive white Gaussian noise in absence of other nuisance random vectors has already been done and there is nothing new. However a closed form solution of the CRB for vector parameter or even for a single parameter estimation when there is a nuisance random vector such as a non-uniform (not equally likely) random data is extremely hard and to the best of our knowledge no closed form solutions have been provided in the past. A modified Cramer-Rao bound (MCRB) was proposed in [4], [5], [6] in order to obtain a closed form solutions to the lower bound of the error variance in estimating the synchronization parameters in digital communications. In most cases equally likely data were assumed. The MCRB represent a looser lower bound. Here we use the CRB rather than MCRB. In appendix B we briefly review MCRB bound. In fact we will show that MCRB is useless for our problem when the data values have unequal probabilities.

Cramer-Rao bound

Consider estimating a vector parameter $\boldsymbol{\theta} = (\theta_1, \theta_2, \ldots, \theta_q)^T$. Assume that the estimator $\hat{\boldsymbol{\theta}}$ is unbiased. Using the Cramer-Rao bound we have [3]

$$var(\hat{\theta}_i) \geq [I^{-1}(\boldsymbol{\theta})]_{i,i} \tag{11}$$

where $I(\boldsymbol{\theta})$ is the $q \times q$ Fisher information matrix where the (ij) component of $I(\boldsymbol{\theta})$ is defined as

$$[I(\boldsymbol{\theta})]_{i,j} = -E_{\mathbf{r}}\{\frac{\partial^2 \ln p(\mathbf{r}|\boldsymbol{\theta})}{\partial\theta_i \partial\theta_j}\} \tag{12}$$

for $i, j = 1, 2, \ldots, q$ and it is evaluated at the true value of $\boldsymbol{\theta}$. The vector $\mathbf{r}$ represents the N received observations samples for index set $k = 0, 1, \ldots, (N-1)$. The model of received observation vector $\mathbf{r}$ depends on the vector data vector $\mathbf{d}$, which acts as a unwanted nuisance vector in estimation of $\boldsymbol{\theta}$. Thus the probability density function $p(\mathbf{r}|\boldsymbol{\theta})$ in CRB can be obtained as

$$p(\mathbf{r}|\boldsymbol{\theta}) = E_{\mathbf{d}}\{p(\mathbf{r}|\boldsymbol{\theta}, \mathbf{d})\} \tag{13}$$

Based on the observation model for suppressed carrier BPSK modulation over AWGN channel and model of amplitude and phase we have

$$p(\mathbf{r}|\boldsymbol{\theta}, \mathbf{d}) = \prod_{k=0}^{N-1} p(r_k|\boldsymbol{\theta}, d_k) \tag{14}$$

where

$$p(r_k|\boldsymbol{\theta}, d_k) = \frac{1}{2\pi\sigma^2} e^{-1/2\sigma^2 |r_k - a d_k e^{j(2\pi\psi_1 k\Delta + \psi_0)}|^2} \tag{15}$$

For BPSK modulation data d_k is taking values of $+1, -1$. Using genie argument where genie is not reliable as discussed above we have $p(d_k = 1) = 1-p$ and $p(d_k = -1) = p$ where genie lies independently with probability p, then using (13) we have

$$\begin{aligned} E_{d_k}\{p(r_k|\boldsymbol{\theta}, d_k)\} &= p(r_k|\boldsymbol{\theta}, d_k = 1)(1-p) \\ &\quad + p(r_k|\boldsymbol{\theta}, d_k = -1)p \end{aligned} \tag{16}$$

Using (13) and (14) we have

$$p(\mathbf{r}|\boldsymbol{\theta}) = \prod_{k=0}^{N-1} E_{d_k}\{p(r_k|\boldsymbol{\theta}, d_k)\} \tag{17}$$

Thus

$$\ln p(\mathbf{r}|\boldsymbol{\theta}) = \sum_{k=0}^{N-1} \ln E_{d_k}\{p(r_k|\boldsymbol{\theta}, d_k)\} \tag{18}$$

Using (15) and (16) we obtain

$$\begin{aligned}\ln E_{d_k}\{p(r_k|\boldsymbol{\theta}, d_k)\} &= \ln \frac{\sqrt{p(1-p)}}{2\pi\sigma^2} \\ &\quad - \frac{|r_k|^2 + a^2}{2\sigma^2} \\ &\quad + \ln\cosh(z_k(r_k;\boldsymbol{\theta};p))\end{aligned} \tag{19}$$

where

$$z_k(r_k;\boldsymbol{\theta};p) = \frac{a}{\sigma^2}\Re\{r_k\beta_k^*\} + \ln\sqrt{\frac{1-p}{p}} \tag{20}$$

and

$$\beta_k = e^{j(2\pi\psi_1 k\Delta + \psi_0)} \tag{21}$$

$$\begin{aligned}\frac{\partial}{\partial\theta_i}\ln E_{d_k}\{p(r_k|\boldsymbol{\theta}, d_k)\} &= -\frac{\partial}{\partial\theta_i}\frac{a^2}{2\sigma^2} \\ &\quad + \frac{\partial z_k(r_k;\boldsymbol{\theta};p)}{\partial\theta_i} \\ &\quad \times \tanh(z_k(r_k;\boldsymbol{\theta};p))\end{aligned} \tag{22}$$

$$\begin{aligned}\frac{\partial^2}{\partial\theta_i\partial\theta_j} \quad & \ln E_{d_k}\{p(r_k|\boldsymbol{\theta}, d_k)\} = -\frac{\partial^2}{\partial\theta_i\partial\theta_j}\frac{a^2}{2\sigma^2} \\ &+ \frac{\partial^2 z_k(r_k;\boldsymbol{\theta};p)}{\partial\theta_i\partial\theta_j} \\ &\times \tanh(z_k(r_k;\boldsymbol{\theta};p)) \\ &+ \frac{\partial z_k(r_k;\boldsymbol{\theta};p)}{\partial\theta_i}\frac{\partial z_k(r_k;\boldsymbol{\theta};p)}{\partial\theta_j} \\ &\times(1-\tanh^2(z_k(r_k;\boldsymbol{\theta};p)))\end{aligned} \tag{23}$$

$$[I(\boldsymbol{\theta})]_{i,j} = -\sum_{k=0}^{N-1} E_{r_k}\{\frac{\partial^2}{\partial\theta_i\partial\theta_j}\ln E_{d_k}\{p(r_k|\boldsymbol{\theta}, d_k)\}\} \tag{24}$$

Note that $-\frac{\partial^2}{\partial\theta_i\partial\theta_j}\frac{a^2}{2\sigma^2} = -\frac{1}{\sigma^2}$ when $i = j = 1$, and zero otherwise.

$$\frac{\partial z_k(r_k;\boldsymbol{\theta};p)}{\partial\theta_1} = \frac{1}{\sigma^2}\Re\{r_k\beta_k^*\} \tag{25}$$

$$\frac{\partial z_k(r_k;\boldsymbol{\theta};p)}{\partial\theta_2} = \frac{a}{\sigma^2}\Im\{r_k\beta_k^*\} \tag{26}$$

$$\frac{\partial z_k(r_k;\boldsymbol{\theta};p)}{\partial\theta_3} = \frac{2\pi ak\Delta}{\sigma^2}\Im\{r_k\beta_k^*\} \tag{27}$$

$$\frac{\partial^2 z_k(r_k;\boldsymbol{\theta};p)}{\partial\theta_1\partial\theta_1} = 0 \tag{28}$$

$$\frac{\partial^2 z_k(r_k;\boldsymbol{\theta};p)}{\partial\theta_2\partial\theta_1} = \frac{1}{\sigma^2}\Im\{r_k\beta_k^*\} = \frac{\partial^2 z_k(r_k;\boldsymbol{\theta};p)}{\partial\theta_1\partial\theta_2} \tag{29}$$

$$\frac{\partial^2 z_k(r_k;\boldsymbol{\theta};p)}{\partial\theta_3\partial\theta_1} = \frac{2\pi k\Delta}{\sigma^2}\Im\{r_k\beta_k^*\} = \frac{\partial^2 z_k(r_k;\boldsymbol{\theta};p)}{\partial\theta_1\partial\theta_3} \tag{30}$$

$$\frac{\partial^2 z_k(r_k;\boldsymbol{\theta};p)}{\partial\theta_2\partial\theta_2} = -\frac{a}{\sigma^2}\Re\{r_k\beta_k^*\} \tag{31}$$

$$\frac{\partial^2 z_k(r_k;\boldsymbol{\theta};p)}{\partial\theta_3\partial\theta_2} = -\frac{2\pi ak\Delta}{\sigma^2}\Re\{r_k\beta_k^*\} = \frac{\partial^2 z_k(r_k;\boldsymbol{\theta};p)}{\partial\theta_2\partial\theta_3} \tag{32}$$

$$\frac{\partial^2 z_k(r_k;\boldsymbol{\theta};p)}{\partial\theta_3\partial\theta_3} = -\frac{a(2\pi k\Delta)^2}{\sigma^2}\Re\{r_k\beta_k^*\} \tag{33}$$

We note that $\frac{1}{\sigma}\Re\{r_k\beta_k^*\} = \frac{a}{\sigma}d_k + \nu_{I,k}$ and $\frac{1}{\sigma}\Im\{r_k\beta_k^*\} = \nu_{Q,k}$, where $\nu_{I,k} = \Re\{\frac{n_k\beta_k^*}{\sigma}\}$, and $\nu_{Q,k} = \Im\{\frac{n_k\beta_k^*}{\sigma}\}$. Both $\nu_{I,k}$ and $\nu_{Q,k}$ are zero mean, unit variance, independent Gaussian random variables.

Using equations (23) to (33) it is easy to show that the zero components of the Fisher information matrix are

$$[I(\boldsymbol{\theta})]_{1,2} = [I(\boldsymbol{\theta})]_{2,1} = 0$$

and

$$[I(\boldsymbol{\theta})]_{1,3} = [I(\boldsymbol{\theta})]_{3,1} = 0$$

.

Further using the above results we notice that

$$\begin{aligned}[I(\boldsymbol{\theta})]_{2,2} &= \sum_{k=0}^{N-1} E_{r_k}\{\frac{a}{\sigma^2}\Re\{r_k\beta_k^*\}\tanh(z_k(r_k;\boldsymbol{\theta};p)) \\ &\quad - \frac{a^2}{\sigma^4}\Im^2\{r_k\beta_k^*\}(1-\tanh^2(z_k(r_k;\boldsymbol{\theta};p)))\}\end{aligned} \tag{34}$$

It is easy to show that $E_{r_k}\{.\}$ in (34) is independent of index k. Further using the above results we notice that

$$\begin{aligned}[I(\boldsymbol{\theta})]_{2,3} &= \textstyle\sum_{k=0}^{N-1}(2\pi k\Delta) \\ &\times E_{r_k}\{\tfrac{a}{\sigma^2}\Re\{r_k\beta_k^*\}\tanh(z_k(r_k;\boldsymbol{\theta};p)) \\ &- \tfrac{a^2}{\sigma^4}\Im^2\{r_k\beta_k^*\}(1-\tanh^2(z_k(r_k;\boldsymbol{\theta};p)))\}\end{aligned} \tag{35}$$

This implies that

$$[I(\boldsymbol{\theta})]_{2,3} = [I(\boldsymbol{\theta})]_{3,2} = [\frac{\sum_{k=0}^{N-1}(2\pi k\Delta)}{N}][I(\boldsymbol{\theta})]_{2,2} \quad (36)$$

Also

$$[I(\boldsymbol{\theta})]_{3,3} = [\frac{\sum_{k=0}^{N-1}(2\pi k\Delta)^2}{N}][I(\boldsymbol{\theta})]_{2,2} \quad (37)$$

Now we need only to compute $[I(\boldsymbol{\theta})]_{1,1}$ and $[I(\boldsymbol{\theta})]_{2,2}$. It is not possible to obtain closed form solutions for these non-zero components of the Fisher information matrix using (23) and (24) due to the presence of $\tanh$ function. At this point there are few options for approximating $\tanh$ function. For small signal-to-noise ratio (SNR) and when the absolute value of argument of $\tanh$ is very small we have the following approximation

$$\tanh(z_k(r_k;\boldsymbol{\theta};p)) \approx (1-2p) + \frac{4p(1-p)a}{\sigma^2}\Re\{r_k\beta_k^*\} \quad (38)$$

and

$$\tanh^2(z_k(r_k;\boldsymbol{\theta};p)) \approx (1-2p)^2 + \frac{8p(1-p)a}{\sigma^2}\Re\{r_k\beta_k^*\} \quad (39)$$

For large signal-to-noise ratio (SNR) and when the absolute value of the argument of $\tanh$ is very large we have the following approximation

$$\tanh(z_k(r_k;\boldsymbol{\theta};p)) \approx \mathrm{sgn}(z_k(r_k;\boldsymbol{\theta};p)) \quad (40)$$

and

$$\tanh^2(z_k(r_k;\boldsymbol{\theta};p)) \approx 1 \quad (41)$$

where $\mathrm{sgn}(x) = 1$ for $x \geq 1$ and it is equal to -1 otherwise.

Unfortunately both approximations couldn't produce accurate results. Here we propose the following approximation to be used.

$$\tanh(z_k(r_k;\boldsymbol{\theta};p)) \approx \mathrm{sgn}(z_k(r_k;\boldsymbol{\theta};p))(1-e^{-h|z_k(r_k;\boldsymbol{\theta};p)|}) \quad (42)$$

The parameter h can be optimized such that the integrated square error

$$\int_{-x_0}^{x_0} |\tanh(x) - \mathrm{sgn}(x)(1-e^{-h|x|})|^2 dx \quad (43)$$

is minimized over an interval $(-x_0, x_0)$ where x_0 can be greater than 3. The optimum h when the bias a=0 is 1.4. However for a¿1 h is very close to $\frac{3}{2}$. In Figure 2 we compared the proposed approximation with the true $\tanh$ function for $h = \frac{3}{2}$ and $a = 1$. For the example in section 5, $h = \frac{3}{2}$ will be used.

Now using (42) we can obtain closed form solutions to our problem. First $[I(\boldsymbol{\theta})]_{1,1}$ is computed

$$\begin{aligned}[I(\boldsymbol{\theta})]_{1,1} &= \sum_{k=0}^{N-1} E_{r_k}\{\frac{1}{\sigma^2} - \frac{1}{\sigma^4}\Re^2\{r_k\beta_k^*\} \\ &\times(1-\tanh^2(z_k(r_k;\boldsymbol{\theta};p)))\} \end{aligned} \quad (44)$$

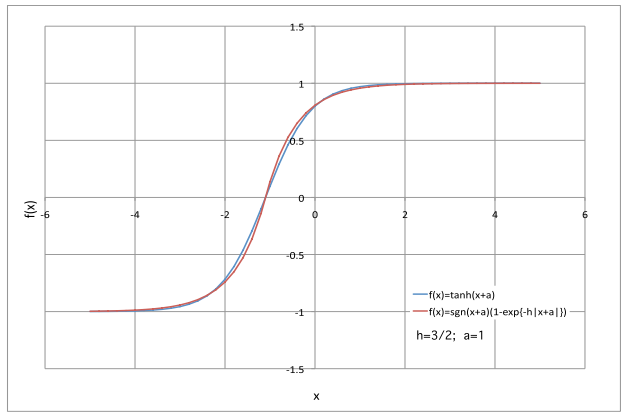


Figure 2. Comparing the proposed approximation with $\tanh$ function.

In order to compute the Fisher information matrix component $[I(\boldsymbol{\theta})]_{1,1}$ we need the following results. For a Gaussian random variable ν with zero mean and unit variance, and constants h^* and ν^* we have

$$E\{e^{-h^*|\nu+\nu^*|}\} = g(\nu^*, h^*) + g(-\nu^*, h^*) \quad (45)$$

$$E\{\nu e^{-h^*|\nu+\nu^*|}\} = h^*[g(\nu^*, h^*) - g(-\nu^*, h^*)] \quad (46)$$

$$\begin{aligned} E\{\nu^2 e^{-h^*|\nu+\nu^*|}\} &= (1+h^{*2})[g(\nu^*, h^*) + g(-\nu^*, h^*)] \\ &- \frac{2h^*}{\sqrt{2\pi}}e^{-\frac{\nu^{*2}}{2}} \end{aligned} \quad (47)$$

where

$$g(\nu^*, h) \triangleq e^{\frac{1}{2}h^{*2}+h^*\nu^*}Q(\nu^*+h^*) \quad (48)$$

and

$$Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}}e^{-\frac{t^2}{2}}dt = \frac{1}{2}\,\mathrm{erfc}(\frac{x}{\sqrt{2}}) \quad (49)$$

Define

$$W(h^*) = E_{r_k}\{\frac{1}{\sigma^2}\Re^2\{r_k\beta_k^*\}e^{-h^*|\frac{1}{\sigma}\Re\{r_k\beta_k^*\}+\frac{\sigma}{a}\ln\sqrt{\frac{1-p}{p}}|}\} \quad (50)$$

Using equations (45), (46), and (47), we get

$$\begin{aligned} W(h^*) &= (1-p)\{[1+(\frac{a}{\sigma}+h^*)^2]g(\nu_1^*, h^*) \\ &+ [1+(\frac{a}{\sigma}-h^*)^2]g(-\nu_1^*, h^*) \\ &- \frac{2h^*}{\sqrt{2\pi}}e^{-\frac{\nu_1^{*2}}{2}}\} \\ &+ p\{[1+(\frac{a}{\sigma}-h^*)^2]g(\nu_2^*, h^*) \\ &+ [1+(\frac{a}{\sigma}+h^*)^2]g(-\nu_2^*, h^*) \\ &- \frac{2h^*}{\sqrt{2\pi}}e^{-\frac{\nu_2^{*2}}{2}}\} \end{aligned} \quad (51)$$

where

$$h^* = h\frac{a}{\sigma} \tag{52}$$

$$\nu_1^* = \frac{a}{\sigma} + \frac{\sigma}{a}\ln\frac{\sqrt{1-p}}{p} \tag{53}$$

$$\nu_2^* = -\frac{a}{\sigma} + \frac{\sigma}{a}\ln\frac{\sqrt{1-p}}{p} \tag{54}$$

Then

$$[I(\boldsymbol{\theta})]_{1,1} = \frac{N}{\sigma^2}[1 - 2W(h^*) + W(2h^*)] \tag{55}$$

In order to compute $[I(\boldsymbol{\theta})]_{2,2}$, in addition to equations (45) to (47) we need the following results. Again for a Gaussian random variable ν with zero mean and unit variance, and constants h^* and ν^* we have

$$E\{\mathrm{sgn}(\nu+\nu^*)\} = 1 - 2Q(\nu^*) \tag{56}$$

$$E\{\nu\,\mathrm{sgn}(\nu+\nu^*)\} = \frac{2}{\sqrt{2\pi}}e^{-\frac{\nu^{*2}}{2}} \tag{57}$$

$$E\{\mathrm{sgn}(\nu+\nu^*)e^{-h^*|\nu+\nu^*|}\} = g(-\nu^*,h^*) - g(\nu^*,h^*) \tag{58}$$

$$\begin{aligned} &E\{\mathrm{sgn}(\nu+\nu^*)\nu e^{-h^*|\nu+\nu^*|}\} = \\ &-h^*[g(\nu^*,h^*) + g(-\nu^*,h^*)] \\ &+\tfrac{2}{\sqrt{2\pi}}e^{-\frac{\nu^{*2}}{2}} \end{aligned} \tag{59}$$

Using equations (56) to (59), and (42) in (34) we get

$$[I(\boldsymbol{\theta})]_{2,2} = \frac{Na^2}{\sigma^2}Z(h^*) \tag{60}$$

where

$$\begin{aligned} Z(h^*) &= (1-p)\{1 - 2Q(\nu_1^*) + (h-1)g(\nu_1^*,h^*) \\ &+ (h-3)g(-\nu_1^*,h^*) \\ &+ g(\nu_1^*,2h^*) + g(-\nu_1^*,2h^*)\} \\ &+ p\{-1 + 2Q(\nu_2^*) + (h-3)g(\nu_2^*,h^*) \\ &+ (h-1)g(-\nu_2^*,h^*) \\ &+ g(\nu_2^*,2h^*) + g(-\nu_2^*,2h^*)\} \end{aligned} \tag{61}$$

Having components of the Fisher information matrix, next we obtain the inverse of this matrix to compute $[I^{-1}(\boldsymbol{\theta})]_{i,i}$, for $i = 1,2,3$ to find the CRB for the parameters as

$$var(\hat{\theta}_1) \geq \frac{1}{[I(\boldsymbol{\theta})]_{1,1}} \tag{62}$$

$$var(\hat{\theta}_2) \geq \frac{[I(\boldsymbol{\theta})]_{3,3}}{[I(\boldsymbol{\theta})]_{2,2}[I(\boldsymbol{\theta})]_{3,3} - [I(\boldsymbol{\theta})]_{2,3}^2} \tag{63}$$

$$var(\hat{\theta}_3) \geq \frac{[I(\boldsymbol{\theta})]_{2,2}}{[I(\boldsymbol{\theta})]_{2,2}[I(\boldsymbol{\theta})]_{3,3} - [I(\boldsymbol{\theta})]_{2,3}^2} \tag{64}$$

For large N, we have $N-1 \approx N$, and $2N-1 \approx 2N$ then using (63), (64) we get

$$var(\hat{\theta}_2) \geq \frac{4}{[I(\boldsymbol{\theta})]_{2,2}} \tag{65}$$

$$var(\hat{\theta}_3) \geq \frac{12}{(2\pi\Delta N)^2}\frac{1}{[I(\boldsymbol{\theta})]_{2,2}} \tag{66}$$

For the residual carrier case $p = 0$, the CRB reduces to

$$var(\hat{\theta}_1) \geq \frac{\sigma^2}{N} \triangleq (\mathrm{CRB})_{\mathrm{res},1} \tag{67}$$

$$var(\hat{\theta}_2) \geq \frac{4\sigma^2}{Na^2} \triangleq (\mathrm{CRB})_{\mathrm{res},2} \tag{68}$$

$$var(\hat{\theta}_3) \geq \frac{12\sigma^2}{Na^2(2\pi\Delta N)^2} \triangleq (\mathrm{CRB})_{\mathrm{res},3} \tag{69}$$

For suppressed carrier then

$$var(\hat{\theta}_1) \geq \gamma_1(p,a^2,\sigma^2)(\mathrm{CRB})_{\mathrm{res},1} \triangleq (\mathrm{CRB})_{\mathrm{sup},1} \tag{70}$$

$$var(\hat{\theta}_2) \geq \gamma_2(p,a^2,\sigma^2)(\mathrm{CRB})_{\mathrm{res},2} \triangleq (\mathrm{CRB})_{\mathrm{sup},2} \tag{71}$$

$$var(\hat{\theta}_3) \geq \gamma_3(p,a^2,\sigma^2)(\mathrm{CRB})_{\mathrm{res},3} \triangleq (\mathrm{CRB})_{\mathrm{sup},3} \tag{72}$$

where $\gamma_i(p,a^2,\sigma^2)$, for $i = 1,2,3$ represent the amount of degradation for the estimation of parameters for the suppressed carrier with respect to the residual carrier $(\mathrm{CRB})_{\mathrm{res,i}}$.

$$\gamma_1(p,a^2,\sigma^2) = \frac{1}{1 - 2W(h^*) + W(2h^*)} \tag{73}$$

$$\gamma_2(p,a^2,\sigma^2) = \frac{1}{Z(h^*)} \tag{74}$$

$$\gamma_3(p,a^2,\sigma^2) = \frac{1}{Z(h^*)} \tag{75}$$

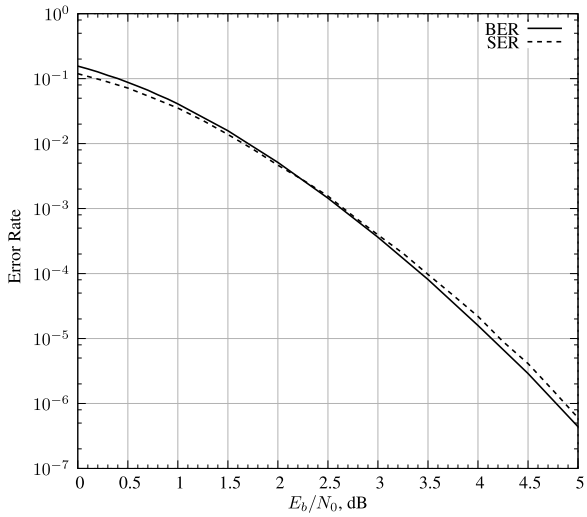


Figure 4. SER performance of Viterbi decoder with re-encoder with perfect synchronization

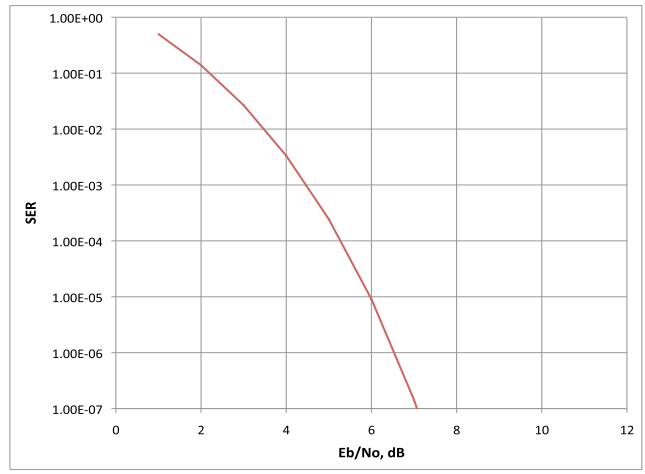


Figure 5. SER performance of Viterbi decoder with re-encoder with quantization, synchronization, and amplitude variation losses

4. Proposed System

The proposed system for suppressed carrier is shown in Fig. 3. The channel codes may include convolutional codes, concatenated Reed-Solomon with convolutional codes, turbo codes, and Low density Parity Check (LDPC) codes. Type of modulations can be BPSK, QPSK, OQPSK, MPSK or even GMSK. However we concentrate mostly on BPSK modulation. At the receiver after frequency down conversion and analog to digital conversion (ADC), the digital signal is demodulated. The receiver is equipped with timing, carrier phase and frequency synchronization circuits that are not shown in the figure.The channel decoder decodes the transmitted sequence. The decoded data is re-encoded, and re-modulated to be used for wiping out the data in the receiver part for Radio Science. FIR type filter is used at the output of ADC. Since the exact delay between the ADC and the output of re-modulator can be obtained, the delay element in the figure can be adjusted accurately prior to the data wiping process. The output of data wiping circuit provides the samples to the Doppler, phase, and amplitude estimation circuits for Radio Science measurements. The output of decoder with re-encoding and re-modulation in fact replaces the genie that was used to perform the theoretical analysis.

5. Example

Consider a rate 1/2 $K = 7$ standard convolutional code (generator polynomials in octal are 171, 133) with BPSK modulation. The output of Viterbi decoder is re-encoded by the same convolutional encoder. The coded symbol error rate (SER) of re-encoded bits at the output of floating point Viterbi decoder using 60-bit traceback under perfect synchronization is shown in Fig. 4 and it is compared with bit error rate at the output of Viterbi decoder. The SER performance of Viterbi decoder including quantization, synchronization and amplitude variations losses is shown in Fig. 5. Compared to the ideal SER performance of Viterbi decoder in Fig. 4 this loss is approximately 1.75 dB.

Based on the performance of Viterbi decoder in Fig.5, and assuming $a^2 = 0.8$ we computed the degradation functions $\gamma_i(p, a^2, \sigma^2)$ in dB for $i = 1, 2, 3$ given by equations (73), (74), and (75) in section 3 and are shown in Fig. 6 (note that $\gamma_3(p, a^2, \sigma^2) = \gamma_2(p, a^2, \sigma^2)$). The degradation functions $\gamma_i(\frac{1}{2}, a^2, \sigma^2)$ in dB for $i = 1, 2, 3$ when the decoder output is not used are also shown in Fig. 6 (note that $\gamma_3(\frac{1}{2}, a^2, \sigma^2) = \gamma_2(\frac{1}{2}, a^2, \sigma^2)$).

6. Summary

In this paper we theoretically analyzed the achievable performance of amplitude and phase estimation for Radio Science data using Cramer Rao bound. We derived closed form CRB both for suppressed carrier and residual carrier and compared them. For practical implementation we proposed a system the removes the nuisance data from the received signal to be used for Radio Science measurements.

Appendices

A. Modified Cramer Rao bound

In this appendix we review the modified Cramer Rao bound. Consider estimating a vector parameter $\boldsymbol{\theta} = (\theta_1, \theta_2, \ldots, \theta_q)^T$. Assume that the estimator $\hat{\boldsymbol{\theta}}$ is unbiased. Define the Modified Cramer-Rao bound as [4]

$$var(\hat{\theta}_i) \geq [I_M^{-1}(\boldsymbol{\theta})]_{i,i} \tag{76}$$

where $I_M(\boldsymbol{\theta})$ is the $q \times q$ modified Fisher information matrix. The (i, j) component of $I_M(\boldsymbol{\theta})$ is defined as

$$[I_M(\boldsymbol{\theta})]_{i,j} = -E_{\mathbf{d}} E_{\mathbf{r}|\mathbf{d}} \{ \frac{\partial^2 \ln p(\mathbf{r}|\boldsymbol{\theta}, \mathbf{d})}{\partial \theta_i \partial \theta_j} \} \tag{77}$$

for $i, j = 1, 2, \ldots, q$ and it is evaluated at the true value of $\boldsymbol{\theta}$. The vector $\mathbf{r}$ represents the N received observations samples for index set $k = 0, 1, \ldots, (N-1)$. The model of received observation vector $\mathbf{r}$ depends on the vector data vector $\mathbf{d}$, which acts as a unwanted nuisance vector in estimation of $\boldsymbol{\theta}$. Based on the observation model for suppressed carrier BPSK modulation over AWGN channel and the model of amplitude and phase we have

$$p(\mathbf{r}|\boldsymbol{\theta}, \mathbf{d}) = \prod_{k=0}^{N-1} p(r_k|\boldsymbol{\theta}, d_k) \tag{78}$$

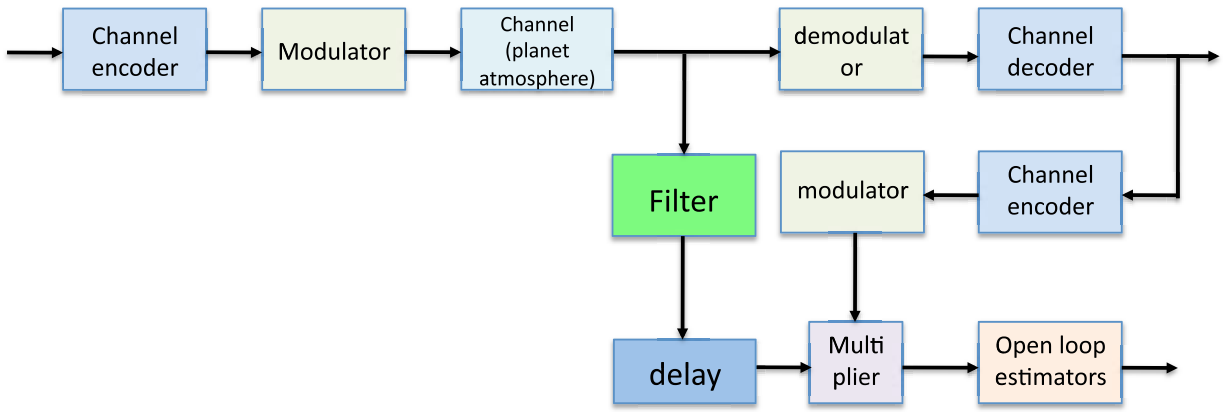


Figure 3. Proposed system for Radio Science Measurements using suppressed carrier.

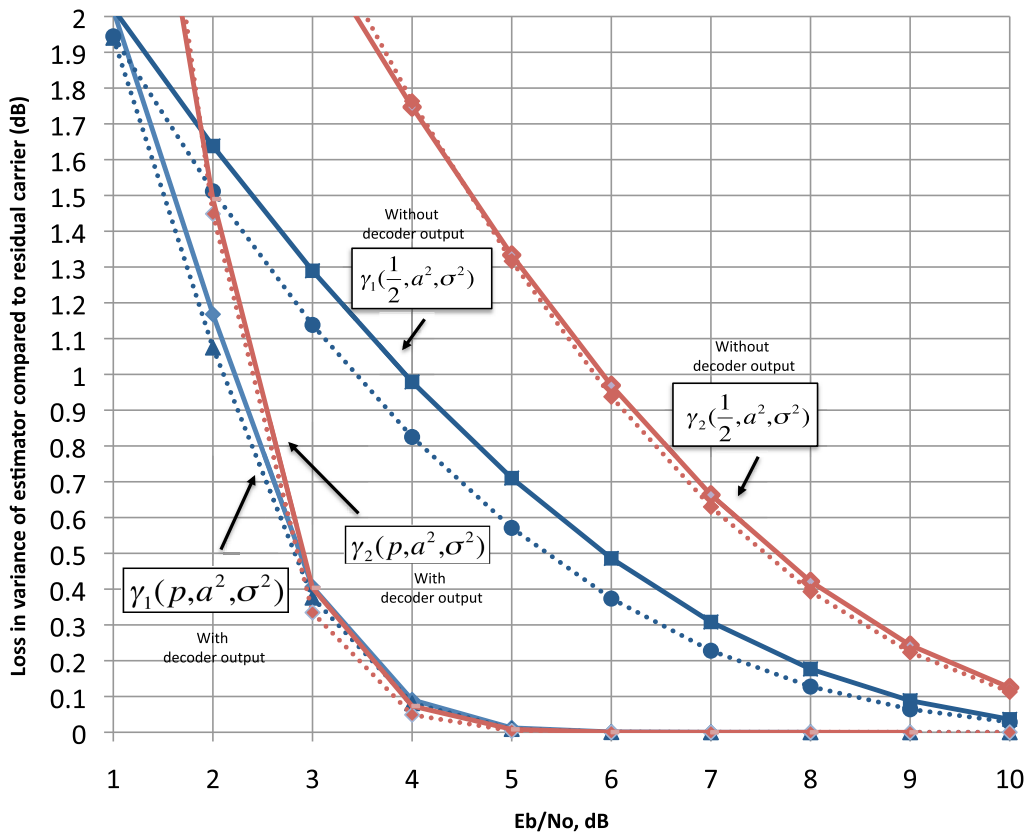


Figure 6. Performance loss in the CRB for suppressed carrier compared to the CRB for residual carrier

where

$$p(r_k|\boldsymbol{\theta}, d_k) = \frac{1}{2\pi\sigma^2} e^{-1/2\sigma^2 |r_k - a d_k e^{j(2\pi\psi_1 k\Delta + \psi_0)}|^2} \tag{79}$$

For BPSK modulation, the data d_k is taking values of $+1, -1$. Again using the genie argument where genie is not reliable as discussed above we have $p(d_k = 1) = 1 - p$ and $p(d_k = -1) = p$ where genie lies independently with probability p, then

$$\ln p(\mathbf{r}|\boldsymbol{\theta}, \mathbf{d}) = \sum_{k=0}^{N-1} \ln p(r_k|\boldsymbol{\theta}, d_k) \tag{80}$$

where

$$\begin{aligned} \ln p(r_k|\boldsymbol{\theta}, d_k) &= -\ln(2\pi\sigma^2) \\ &\quad -\frac{1}{2\sigma^2}|r_k - a d_k e^{j(2\pi\psi_1 k\Delta + \psi_0)}|^2 \end{aligned} \tag{81}$$

However

$$|r_k - ad_k e^{j(2\pi\psi_1 k\Delta + psi_0)}|^2 = |r_k|^2 + a^2 \\ -2ad_k \Re\{r_k e^{-j(2\pi\psi_1 k\Delta + \psi_0)}\} \tag{82}$$

After taking the second partial derivatives all terms in the expressions will be in form of linear function of $d_k \Re\{r_k \beta_k^*\}$ or $d_k \Im\{r_k \beta_k^*\}$. This implies that

$$E_{r_k|d_k=1}\{\frac{\partial^2 \ln p(r_k|\boldsymbol{\theta}, d_k)}{\partial\theta_i \partial\theta_j}\} = E_{r_k|d_k=-1}\{\frac{\partial^2 \ln p(r_k|\boldsymbol{\theta}, d_k)}{\partial\theta_i \partial\theta_j}\} \tag{83}$$

So in the final step that we require to take expectation with respect to data the final results will be independent of p, and $1-p$. Thus the modified Cramer Rao bound is useless for our problem.

ACKNOWLEDGMENTS

We would like to thank Jon Hamkins of Jet Propulsion Laboratory for providing us Figure 4.

This work was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration.

REFERENCES

[1] R. A. Fisher, On the mathematical foundations of theoretical statistics, Phil. Trans. Royal Soc., vol. 222, pp. 309368, 1922.

[2] H. Cramer, Mathematical Methods of Statistics. Princeton, NJ: Princeton Univ. Press, 1946.

[3] H. L. Van Trees, Detection, Estimation, and Modulation Theory - Part I. John Wiley and Sons, 1968.

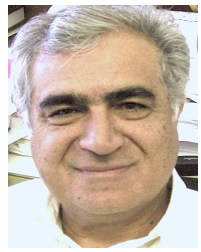
[4] A. DAndrea, U. Mengali, and R. Reggiannini, The modified CramerRao bound and its application to synchronization problems, IEEE Trans. Commun., vol. 42, no. 2/3/4, pp. 13911399, Feb./Mar./Apr. 1994.

[5] F. Gini, R. Reggiannini, and U. Mengali, The modified Cramer-Rao bound in vector parameter estimation, IEEE Trans. Commun., vol. 46, no. 1, pp. 5260, Jan. 1998.

[6] M. Moeneclaey, On the true and the modified Cramer-Rao bounds for the estimation of a scalar parameter in the presence of nuisance parameters, IEEE Trans. Commun., vol. 46, no. 11, pp. 15361544, Nov. 1998.

BIOGRAPHY

Sami Asmar is the supervisor of JPL's Radio Science Systems Group, the Project Scientist for the GRAIL lunar mission, and an active gravity science co-investigator on US and European planetary missions.



Dariush Divsalar received the Ph.D. degree in electrical engineering from the University of California, Los Angeles (UCLA), in 1978. Since then, he has been with the Jet Propulsion Laboratory (JPL), California Institute of Technology (Caltech), Pasadena, where he is a Principal Scientist. At JPL, he has been involved with developing state-of-the-art technology for advanced deep-space communications systems and future NASA space exploration. Since 1986, he has taught graduate courses in communications and coding at UCLA and Caltech. He has published more than 200 papers, coauthored a book entitled An Introduction to Trellis Coded Modulation with Applications, contributed to two other books, and holds 22 U.S. patents in the above areas. Dr. Divsalar was a co recipient of the 1986 paper award of the IEEE Transactions on Vehicular Technology. He was also a co recipient of the joint paper award of the IEEE Information Theory and IEEE Communication Theory societies in 2008. The IEEE Communication Society has selected one of his papers for inclusion in a book entitled The Best of the Best: Fifty Years of Communications and Networking Research, containing the best 56 key research papers ever published in the Society's 50-year history. He has received over 40 NASA Tech Brief awards and a NASA Exceptional Engineering Achievement Medal in 1996. He served as an Area Editor for the IEEE Transactions on communications from 1989 to 1996. Dr. Divsalar is a fellow of IEEE since 1997.

Kamal Oudrhiri is a senior member in the Radio Science Systems Group at NASA's Jet Propulsion Laboratory. As a contract technical manager, Oudrhiri led multi-disciplinary teams through the design, implementation and delivery of flight hardware to the radio science community. Over the last decade, Oudrhiri served in key roles on multiple NASA missions: The Mars Exploration Rovers (MER), the International Cassini mission to Saturn,The GRAIL lunar mission and The Mars Science Laboratory (MSL).