Metrology Optical Power Budgeting in SIM Using Statistical Analysis Techniques

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ABSTRACT

The Space Interferometry Mission (SIM) is a space-based stellar interferometry instrument, consisting of up to three interferometers, which will be capable of micro-arc second resolution. Alignment knowledge of the three interferometer baselines requires a three-dimensional, 14-leg truss with each leg being monitored by an external metrology gauge. In addition, each of the three interferometers requires an internal metrology gauge to monitor the optical path length differences between the two sides. Both external and internal metrology gauges are interferometry based, operating at a wavelength of 1319 nanometers. Each gauge has fiber inputs delivering measurement and local oscillator (LO) power, split into probe-LO and reference-LO beam pairs. These beams experience power loss due to a variety of mechanisms including, but not restricted to, design efficiency, material attenuation, element misalignment, diffraction, and coupling efficiency. Since the attenuation due to these sources may degrade over time, an accounting of the range of expected attenuation is needed so an optical power margin can be book kept. A method of statistical optical power analysis and budgeting, based on a technique developed for deep space RF telecommunications, is described in this paper and provides a numerical confidence level for having sufficient optical power relative to mission metrology performance requirements.

Keywords: stellar interferometry, optical metrology, metrology, optical power budget, statistical analysis

Note: *the SIM PlanetQuest (SIM PQ) design referred to in this paper was current as of early 2007 and has since evolved. However, the techniques described below will be applied to all subsequent designs.*

1. INTRODUCTION

SIM Planet Quest is a stellar interferometer instrument capable of micro-arcsecond metrology.[1] The instrument includes three interferometers – one science interferometer and two guide interferometers – and precision metrology gauges to monitor the optical path length difference (OPD) between the arms of each interferometer as well as the orientation of all interferometer baselines. The optical interferometry-based metrology subsystem plays a pivotal role, measuring absolute distances at the micrometer level and relative distances at the picometer level.[2,3] Two types of metrology measurements are deployed: internal metrology (iMet) and external metrology (xMet). The function of iMet is to measure the relative starlight OPD between two arms of an interferometer and to provide feedback to ensure the OPD remains equal during an observation. The function of xMet is to measure the absolute and relative distances between metrology truss fiducials which define the 3-D orientation of the interferometer baselines.

During observation, while both Guide Interferometers lock on to guide stars, the Science Interferometer points at a reference star by slewing the siderostats. The Optical Delay Lines are swept until starlight fringes are detected. Once the interferometer locks onto the reference star the iMet gauge monitors the relative OPD as the siderostats slew to point at a target star. The ODLs are again swept until starlight fringes are detected. The change in the OPD between these two observations, as monitored with picometer-level precision by the iMet system, is used to compute the angular distance of the target star to the reference star. This procedure is repeated with additional reference stars to generate very precise knowledge of the relative position of the target star with respect to a field of reference stars.[4] In addition to the internal metrology measurements, the length of both interferometer baselines and the relative orientation of the Science baseline with respect to

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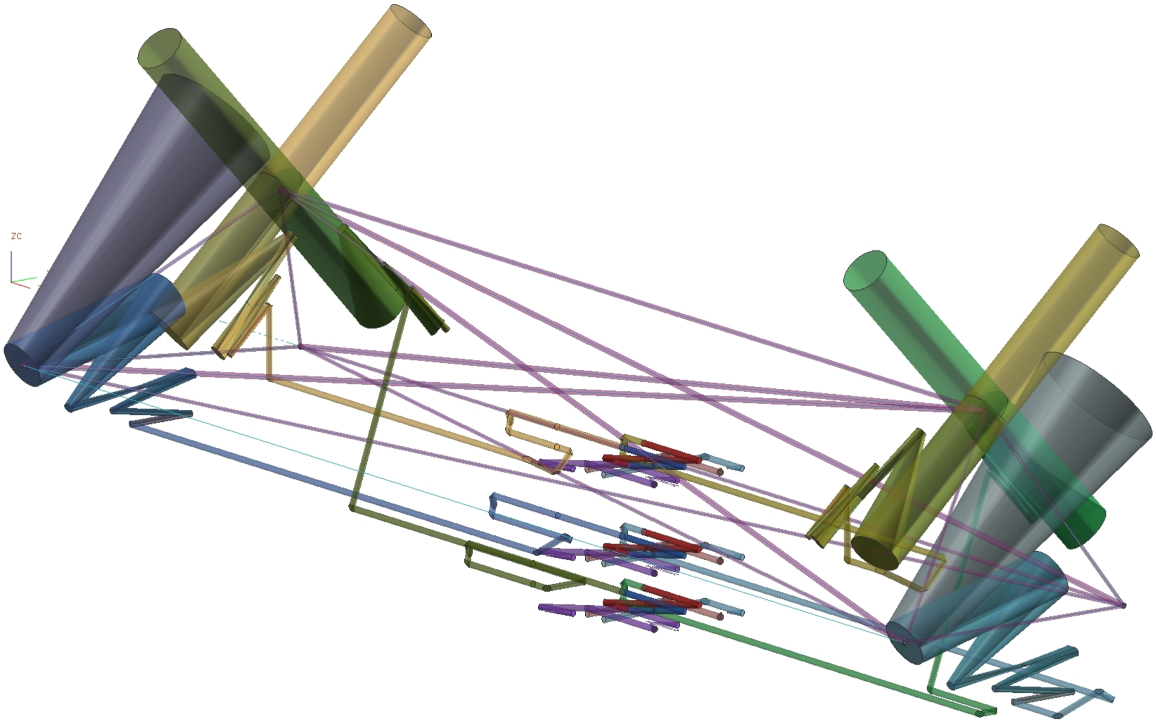


Figure 1. The SIM PlanetQuest 2007 optical design consists of 1 Science Interferometer, 2 Guide Interferometers, 3 Astrometric Beam Combiners, 3 Internal Metrology gauges, 14 External Metrology gauges, and 6 shared-vertex corner cube fiducials

the Guide baseline are needed to compute the angular position of the target star relative to the reference star. This is achieved by precisely monitoring six fiducials that make up the external metrology truss. Each end of an interferometer baseline is defined by the position of a corner cube retroreflector fiducial. Two additional fiducials are placed out of the plane ideally defined by the two co-planar baselines to create a three-dimensional truss. External metrology gauges monitor each of the legs connecting fiducial pairs with the exception of the Science Baseline, for a total of 14 picometer-level precision measurements. These xMet measurements are used to calculate the length of the Guide Interferometer baseline, the length of the Science Interferometer baseline, and the relative orientation of the Science baseline with respect to the Guide baseline.[4,5,6]

Both iMet and xMet gauges have two optical fiber inputs – measurement and local oscillator – each delivering coherent optical power. The measurement power is split between probe and reference beams, while the frequency shifted local oscillator power is split between two local oscillator beams. The probe beam monitors the optical path to be measured and returns to the gauge to interfere with a local oscillator. The xMet reference beam remains inside the gauge and also interferes with a local oscillator. The iMet reference beam actually monitors the alternate arm of the interferometer. Each beam pair produces an optical phase that varies with the path length difference. The optical path difference (OPD) between the measurement and reference beams is computed from the phase difference between the two beam pairs. A more detailed description of these two types of metrology gauges have been described elsewhere.[2,3]

The accuracy and precision of these optical metrology gauges depend on the heterodyne optical power received at the detectors. The power in both the measurement beam (whether probe or reference) and the local oscillator beam contribute to the heterodyne signal power as well as the DC power. The purpose of this manuscript is to present a method of accounting for the many loss mechanisms in the metrology system and recommend a metric by which to evaluate the performance and reliability of the system.

There are a total of seventeen metrology gauges – 14 xMet and 3 iMet – which require coherent optical power delivered continuously over the operational lifetime of the instrument. For SIM PlanetQuest, the baseline mission lifetime is 5.5 years with sufficient resources to not preclude a mission lifetime of 10 years. Given that both Internal and External Metrology Gauges have been proven as brassboards in developmental testbeds, the optical power needed for each type of metrology gauge to meet performance requirements is known. What is not known, however, is the throughput coefficients for each and every optical element in the gauges as well as all of the external optics which the metrology beams intersect. More importantly, how much would these throughput coefficients change over the mission lifetime?

The initial metric applied to the SIM PQ optical power requirement was to maintain 80% margin, where the margin is defined as the fraction of the capability that is in excess of the requirement as shown here:

$$M = \frac{P_C - P_R}{P_C} \tag{1}$$

This definition is commonly used by JPL to monitor mass allocations at different stages of spacecraft development. The 80% value was applicable to SIM PQ at that particular stage of development.

However, an 80% optical power margin (OPM) would require a much larger metrology laser source, thereby increasing mass and electrical power. Furthermore, barring a major design change, the number of metrology gauges needed and the total number of optical elements was already established, and therefore the required optical power should not dramatically increase as the flight design matured further. A more appropriate optical power requirement metric and perhaps an alternate definition of OPM was needed.

1.1 TELECOMMUNICATIONS SYSTEMS DESIGN

For years, JPL has employed a technique for evaluating deep space RF telecommunications system design.[7] This method involves conducting a statistical accounting of various loss mechanisms in an RF communications link.

The throughput of a communications link is the product of the source power and n throughput coefficients as shown here:

$$P_R = P_S T_1 T_2 T_3 \cdots T_n \tag{2}$$

where P_R is the received power, P_S is the source power, and T_n is the n^{th} throughput coefficient.

In log space, Equation 2 becomes a sum of throughput parameters such that

$$\mathbf{P}_R = \mathbf{P}_S + \mathbf{T}_1 + \mathbf{T}_2 + \mathbf{T}_3 + \cdots + \mathbf{T}_n \tag{3}$$

where the bold typeface indicates the logarithmic $\mathbf{X} = 10 \log_{10} X$ of the variable.

In this statistical analysis technique, these throughput parameters, or variables, are assumed to represent random events, each having a probability distribution, or if data exists, a statistical distribution. If each throughput parameter is assumed to be statistically independent, and assuming no single distribution outweighs any other, then, according to the Central Limit Theorem, the received power may be approximated by a Gaussian probability density function (PDF)[8]

$$p(\mathbf{P}_R) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2}\left[\frac{(\mathbf{P}_R - \mu)^2}{\sigma^2}\right]\right) . \tag{4}$$

The mean of this Gaussian distribution is the sum of the means of the n variables

$$\mu = \sum_{i=1,2,3\ldots}^{n} \mu_i \tag{5}$$

while the variance of the Gaussian is the sum of the variances of the n variables

$$\sigma^2 = \sum_{i=1,2,3\ldots}^{n} \sigma_i^2 \ . \tag{6}$$

With the Central Limit Theorem in mind, the first step in link analysis is to identify every source of signal loss and to assign a design value, as well as an adverse and favorable tolerance, to each. In addition, each variable must be assigned a distribution or probability density function (PDF) in log space. From these three values and the distribution type, the mean and variance of the variables may be computed. In some cases, the variable is highly deterministic and may be represented by a single value with a PDF in the form of a Dirac-delta function. A Design Control Table[7] of all loss parameters – with design values, favorable and adverse tolerances, mean, standard deviation, and variance listed – provides a complete overview of the link from which the PDF of the received power can be computed.

From this Gaussian distribution, power requirements can be set by establishing the performance margin needed to ensure mission success by referring to the standard deviation, σ. For a 99% probability of meeting a given received power requirement, the transmitted power must be great enough such that the mean received power is 3-σ greater than the requirement.

Since the SIM PQ Metrology subsystem is essentially made up of many communication links, this technique for designing a telecommunications system seems appropriate for evaluating SIM PQ optical power requirements.

2. SIM PLANETQUEST OPTICAL LAYOUT

In SIM PQ, each starlight interferometer – the Science Interferometer and two Guide Interferometers – has a multitude of optical elements. These include siderostats, TMA compressors, steering and modulation mirrors, optical delay lines, relay mirrors, fold mirrors, and alignment mirrors. The astrometric beam combiner also contains many optical elements including fold mirrors, relay mirrors, alignment mirrors, compressor mirrors, and transmissive optics such as compensators, beamsplitters, prisms, and lenses. There are also six hollow corner cube fiducials that make up the metrology truss and fourteen External Metrology gauges, each with many internal optical elements. The Metrology Source contains acousto-optic modulators and a fiber distribution array (FDA) which distributes the optical power to each metrology gauge.

A graphical representation of the many optical elements involved in each optical path from source to detector is shown in Figure 2. Each optical element imparts loss of optical power by way of absorption, reflection, transmission, diffraction and/or physical clipping of the beam. Misalignments, fabrication tolerances, jitter, and 1-g to 0-g deformations also contribute to optical power throughput. For this exercise, each optical path is considered analogous to a communications link on which link analysis may be conducted. The Design Control Table shown in Figure 3 represents the probe and probe local oscillator optical paths of one arm of the Science Interferometer. These two beams interfere to produce a heterodyne signal. Note that some elements in the probe beam contain optical elements that the beam intersects twice due to the double pass configuration. Although some elements, such as physically identical mirrors, may experience identical fabrication processes they are still separate and distinct components that will be installed and degrade independent of each other. Therefore it is safe to assume that these elements are statistically independent. However, the losses due to identical fabrication processes may be evaluated separately from independent environmental conditions if deemed necessary.

3. ANALYSIS

Once the DCT has been compiled, a metric for evaluating the available optical power is needed. Both Internal and External Metrology gauges have optical power specifications in order to meet metrology performance requirements. One aspect of optical power in an interferometer is the minimum rms heterodyne power.

Given a probe beam optical power, P_P, and a probe local oscillator optical power, P_{PLO}, the total interferometric optical power incident on the detector is

$$P = P_P + P_{PLO} + 2\sqrt{P_P P_{PLO}}\cos(\Delta\theta) \tag{7}$$

where the last term on the right is the heterodyne signal and $\Delta\theta$ is the phase difference between the probe and local oscillator beams.

If P_H is the rms heterodyne power, then from Equation 7

$$\begin{aligned} \sqrt{2}P_H &= 2\sqrt{P_P P_{PLO}} && (8) \\ P_P P_{PLO} &= \frac{1}{2}P_H^2 && (9) \end{aligned}$$

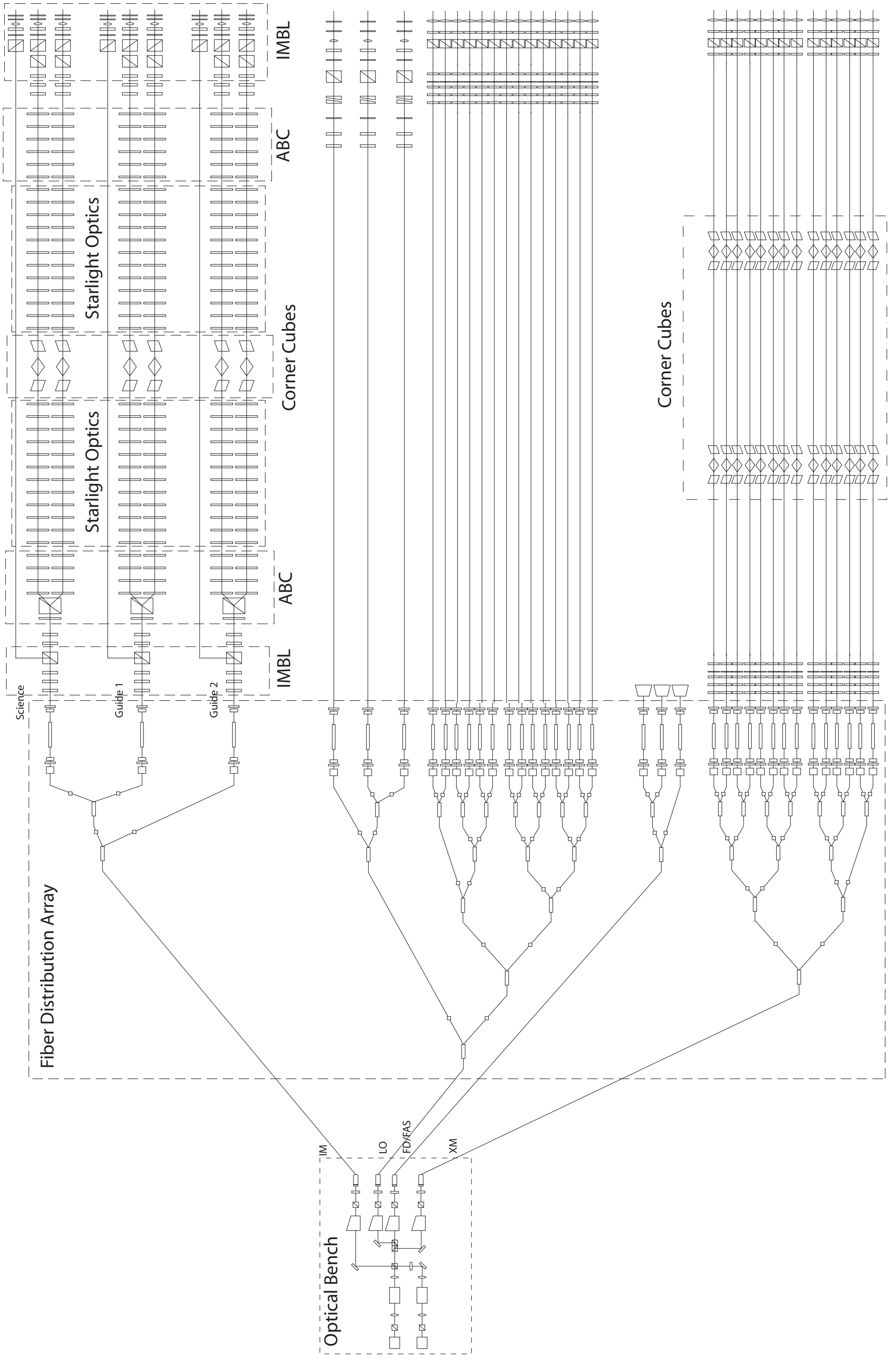
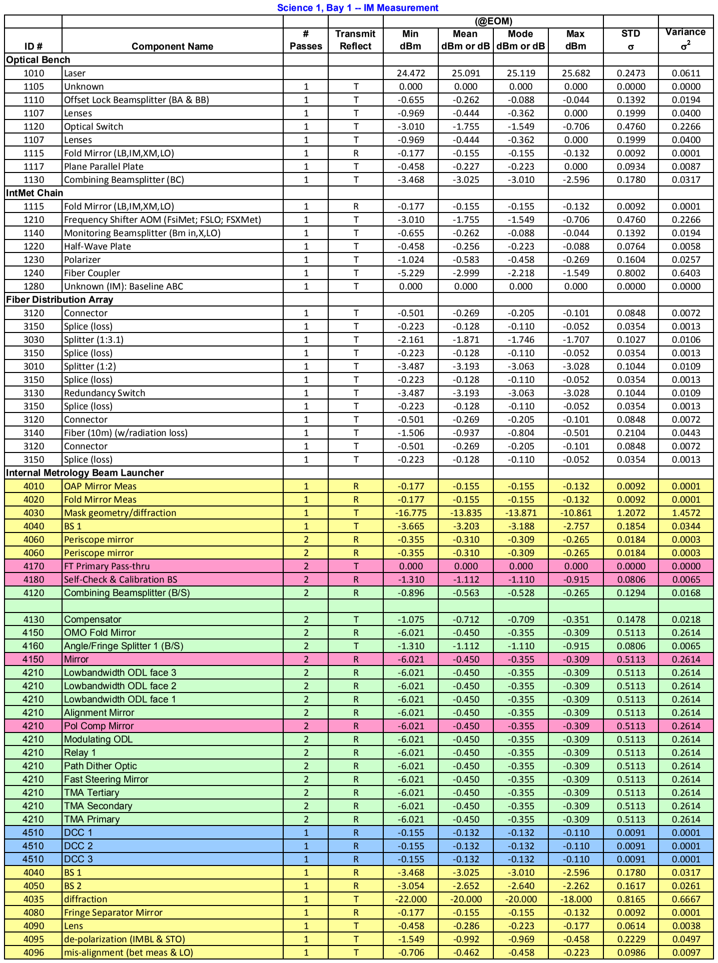
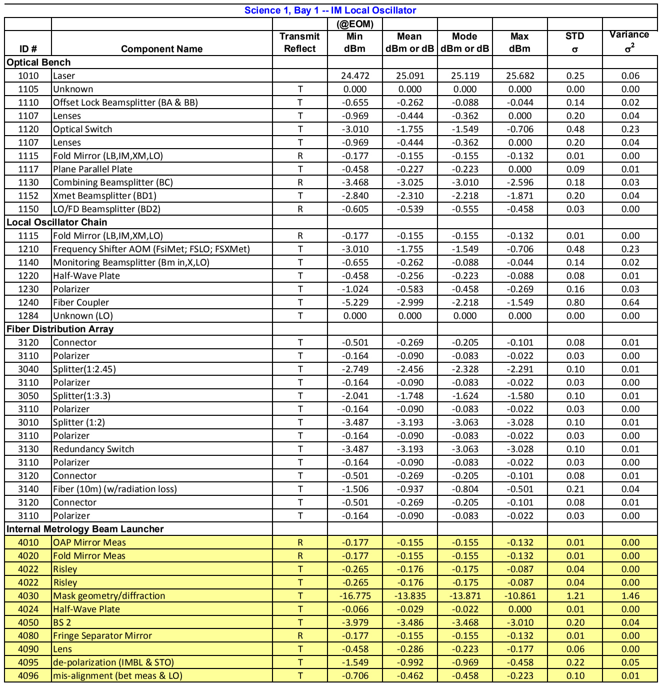


Figure 2. SIM PlanetQuest 2007 Metrology Subsystem optical paths. Each block represents an optical element. The laser source is at the far right. Met gauge detectors are on the far left. There are four main branches out of the source bench that feeds the Fiber Distribution Array: Internal Metrology (IM) measurement, External Metrology (XM) measurement, Local Oscillator (both IM and XM local oscillators), and a Full-Aperture Stimulus (FAS).



Science 1, Bay 1 -- IM Measurement

				(@EOM)					
ID #	Component Name	# Passes	Transmit Reflect	Min dBm	Mean dBm or dB	Mode dBm or dB	Max dBm	STD σ	Variance σ^2
Optical Bench									
1010	Laser			24.472	25.091	25.119	25.682	0.2473	0.0611
1105	Unknown	1	T	0.000	0.000	0.000	0.000	0.0000	0.0000
1110	Offset Lock Beamsplitter (BA & BB)	1	T	-0.655	-0.262	-0.088	-0.044	0.1392	0.0194
1107	Lenses	1	T	-0.969	-0.444	-0.362	0.000	0.1999	0.0400
1120	Optical Switch	1	T	-3.010	-1.755	-1.549	-0.706	0.4760	0.2266
1107	Lenses	1	T	-0.969	-0.444	-0.362	0.000	0.1999	0.0400
1115	Fold Mirror (LB,IM,XM,LO)	1	R	-0.177	-0.155	-0.155	-0.132	0.0092	0.0001
1117	Plane Parallel Plate	1	T	-0.458	-0.227	-0.223	0.000	0.0934	0.0087
1130	Combining Beamsplitter (BC)	1	T	-3.468	-3.025	-3.010	-2.596	0.1780	0.0317
IntMet Chain									
1115	Fold Mirror (LB,IM,XM,LO)	1	R	-0.177	-0.155	-0.155	-0.132	0.0092	0.0001
1210	Frequency Shifter AOM (FsiMet; FSLO; FSXMet)	1	T	-3.010	-1.755	-1.549	-0.706	0.4760	0.2266
1140	Monitoring Beamsplitter (Bm in,X,LO)	1	T	-0.655	-0.262	-0.088	-0.044	0.1392	0.0194
1220	Half-Wave Plate	1	T	-0.458	-0.256	-0.223	-0.088	0.0764	0.0058
1230	Polarizer	1	T	-1.024	-0.583	-0.458	-0.269	0.1604	0.0257
1240	Fiber Coupler	1	T	-5.229	-2.999	-2.218	-1.549	0.8002	0.6403
1280	Unknown (IM): Baseline ABC	1	T	0.000	0.000	0.000	0.000	0.0000	0.0000
Fiber Distribution Array									
3120	Connector	1	T	-0.501	-0.269	-0.205	-0.101	0.0848	0.0072
3150	Splice (loss)	1	T	-0.223	-0.128	-0.110	-0.052	0.0354	0.0013
3030	Splitter (1:3.1)	1	T	-2.161	-1.871	-1.746	-1.707	0.1027	0.0106
3150	Splice (loss)	1	T	-0.223	-0.128	-0.110	-0.052	0.0354	0.0013
3010	Splitter (1:2)	1	T	-3.487	-3.193	-3.063	-3.028	0.1044	0.0109
3150	Splice (loss)	1	T	-0.223	-0.128	-0.110	-0.052	0.0354	0.0013
3130	Redundancy Switch	1	T	-3.487	-3.193	-3.063	-3.028	0.1044	0.0109
3150	Splice (loss)	1	T	-0.223	-0.128	-0.110	-0.052	0.0354	0.0013
3120	Connector	1	T	-0.501	-0.269	-0.205	-0.101	0.0848	0.0072
3140	Fiber (10m) (w/radiation loss)	1	T	-1.506	-0.937	-0.804	-0.501	0.2104	0.0443
3120	Connector	1	T	-0.501	-0.269	-0.205	-0.101	0.0848	0.0072
3150	Splice (loss)	1	T	-0.223	-0.128	-0.110	-0.052	0.0354	0.0013
Internal Metrology Beam Launcher									
4010	OAP Mirror Meas	1	R	-0.177	-0.155	-0.155	-0.132	0.0092	0.0001
4020	Fold Mirror Meas	1	R	-0.177	-0.155	-0.155	-0.132	0.0092	0.0001
4030	Mask geometry/diffraction	1	T	-16.775	-13.835	-13.871	-10.861	1.2072	1.4572
4040	BS 1	1	T	-3.665	-3.203	-3.188	-2.757	0.1854	0.0344
4060	Periscope mirror	2	R	-0.355	-0.310	-0.309	-0.265	0.0184	0.0003
4060	Periscope mirror	2	R	-0.355	-0.310	-0.309	-0.265	0.0184	0.0003
4170	FT Primary Pass-thru	2	T	0.000	0.000	0.000	0.000	0.0000	0.0000
4180	Self-Check & Calibration BS	2	R	-1.310	-1.112	-1.110	-0.915	0.0806	0.0065
4120	Combining Beamsplitter (B/S)	2	R	-0.896	-0.563	-0.528	-0.265	0.1294	0.0168
4130	Compensator	2	T	-1.075	-0.712	-0.709	-0.351	0.1478	0.0218
4150	OMO Fold Mirror	2	R	-6.021	-0.450	-0.355	-0.309	0.5113	0.2614
4160	Angle/Fringe Splitter 1 (B/S)	2	T	-1.310	-1.112	-1.110	-0.915	0.0806	0.0065
4150	Mirror	2	R	-6.021	-0.450	-0.355	-0.309	0.5113	0.2614
4210	Lowbandwidth ODL face 3	2	R	-6.021	-0.450	-0.355	-0.309	0.5113	0.2614
4210	Lowbandwidth ODL face 2	2	R	-6.021	-0.450	-0.355	-0.309	0.5113	0.2614
4210	Lowbandwidth ODL face 1	2	R	-6.021	-0.450	-0.355	-0.309	0.5113	0.2614
4210	Alignment Mirror	2	R	-6.021	-0.450	-0.355	-0.309	0.5113	0.2614
4210	Pol Comp Mirror	2	R	-6.021	-0.450	-0.355	-0.309	0.5113	0.2614
4210	Modulating ODL	2	R	-6.021	-0.450	-0.355	-0.309	0.5113	0.2614
4210	Relay 1	2	R	-6.021	-0.450	-0.355	-0.309	0.5113	0.2614
4210	Path Dither Optic	2	R	-6.021	-0.450	-0.355	-0.309	0.5113	0.2614
4210	Fast Steering Mirror	2	R	-6.021	-0.450	-0.355	-0.309	0.5113	0.2614
4210	TMA Tertiary	2	R	-6.021	-0.450	-0.355	-0.309	0.5113	0.2614
4210	TMA Secondary	2	R	-6.021	-0.450	-0.355	-0.309	0.5113	0.2614
4210	TMA Primary	2	R	-6.021	-0.450	-0.355	-0.309	0.5113	0.2614
4510	DCC 1	1	R	-0.155	-0.132	-0.132	-0.110	0.0091	0.0001
4510	DCC 2	1	R	-0.155	-0.132	-0.132	-0.110	0.0091	0.0001
4510	DCC 3	1	R	-0.155	-0.132	-0.132	-0.110	0.0091	0.0001
4040	BS 1	1	R	-3.468	-3.025	-3.010	-2.596	0.1780	0.0317
4050	BS 2	1	R	-3.054	-2.652	-2.640	-2.262	0.1617	0.0261
4035	diffraction	1	T	-22.000	-20.000	-20.000	-18.000	0.8165	0.6667
4080	Fringe Separator Mirror	1	R	-0.177	-0.155	-0.155	-0.132	0.0092	0.0001
4090	Lens	1	T	-0.458	-0.286	-0.223	-0.177	0.0614	0.0038
4095	de-polarization (IMBL & STO)	1	T	-1.549	-0.992	-0.969	-0.458	0.2229	0.0497
4096	mis-alignment (bet meas & LO)	1	T	-0.706	-0.462	-0.458	-0.223	0.0986	0.0097



Science 1, Bay 1 -- IM Local Oscillator

			(@EOM)					
ID #	Component Name	Transmit Reflect	Min dBm	Mean dBm or dB	Mode dBm or dB	Max dBm	STD σ	Variance σ^2
Optical Bench								
1010	Laser		24.472	25.091	25.119	25.682	0.25	0.06
1105	Unknown	T	0.000	0.000	0.000	0.000	0.00	0.00
1110	Offset Lock Beamsplitter (BA & BB)	T	-0.655	-0.262	-0.088	-0.044	0.14	0.02
1107	Lenses	T	-0.969	-0.444	-0.362	0.000	0.20	0.04
1120	Optical Switch	T	-3.010	-1.755	-1.549	-0.706	0.48	0.23
1107	Lenses	T	-0.969	-0.444	-0.362	0.000	0.20	0.04
1115	Fold Mirror (LB,IM,XM,LO)	R	-0.177	-0.155	-0.155	-0.132	0.01	0.00
1117	Plane Parallel Plate	T	-0.458	-0.227	-0.223	0.000	0.09	0.01
1130	Combining Beamsplitter (BC)	R	-3.468	-3.025	-3.010	-2.596	0.18	0.03
1152	Xmet Beamsplitter (BD1)	T	-2.840	-2.310	-2.218	-1.871	0.20	0.04
1150	LO/FD Beamsplitter (BD2)	R	-0.605	-0.539	-0.555	-0.458	0.03	0.00
Local Oscillator Chain								
1115	Fold Mirror (LB,IM,XM,LO)	R	-0.177	-0.155	-0.155	-0.132	0.01	0.00
1210	Frequency Shifter AOM (FsiMet; FSLO; FSXMet)	T	-3.010	-1.755	-1.549	-0.706	0.48	0.23
1140	Monitoring Beamsplitter (Bm in,X,LO)	T	-0.655	-0.262	-0.088	-0.044	0.14	0.02
1220	Half-Wave Plate	T	-0.458	-0.256	-0.223	-0.088	0.08	0.01
1230	Polarizer	T	-1.024	-0.583	-0.458	-0.269	0.16	0.03
1240	Fiber Coupler	T	-5.229	-2.999	-2.218	-1.549	0.80	0.64
1284	Unknown (LO)	T	0.000	0.000	0.000	0.000	0.00	0.00
Fiber Distribution Array								
3120	Connector	T	-0.501	-0.269	-0.205	-0.101	0.08	0.01
3110	Polarizer	T	-0.164	-0.090	-0.083	-0.022	0.03	0.00
3040	Splitter(1:2.45)	T	-2.749	-2.456	-2.328	-2.291	0.10	0.01
3110	Polarizer	T	-0.164	-0.090	-0.083	-0.022	0.03	0.00
3050	Splitter(1:3.3)	T	-2.041	-1.748	-1.624	-1.580	0.10	0.01
3110	Polarizer	T	-0.164	-0.090	-0.083	-0.022	0.03	0.00
3010	Splitter (1:2)	T	-3.487	-3.193	-3.063	-3.028	0.10	0.01
3110	Polarizer	T	-0.164	-0.090	-0.083	-0.022	0.03	0.00
3130	Redundancy Switch	T	-3.487	-3.193	-3.063	-3.028	0.10	0.01
3110	Polarizer	T	-0.164	-0.090	-0.083	-0.022	0.03	0.00
3120	Connector	T	-0.501	-0.269	-0.205	-0.101	0.08	0.01
3140	Fiber (10m) (w/radiation loss)	T	-1.506	-0.937	-0.804	-0.501	0.21	0.04
3120	Connector	T	-0.501	-0.269	-0.205	-0.101	0.08	0.01
3110	Polarizer	T	-0.164	-0.090	-0.083	-0.022	0.03	0.00
Internal Metrology Beam Launcher								
4010	OAP Mirror Meas	R	-0.177	-0.155	-0.155	-0.132	0.01	0.00
4020	Fold Mirror Meas	R	-0.177	-0.155	-0.155	-0.132	0.01	0.00
4022	Risley	T	-0.265	-0.176	-0.175	-0.087	0.04	0.00
4022	Risley	T	-0.265	-0.176	-0.175	-0.087	0.04	0.00
4030	Mask geometry/diffraction	T	-16.775	-13.835	-13.871	-10.861	1.21	1.46
4024	Half-Wave Plate	T	-0.066	-0.029	-0.022	0.000	0.01	0.00
4050	BS 2	T	-3.979	-3.486	-3.468	-3.010	0.20	0.04
4080	Fringe Separator Mirror	R	-0.177	-0.155	-0.155	-0.132	0.01	0.00
4090	Lens	T	-0.458	-0.286	-0.223	-0.177	0.06	0.00
4095	de-polarization (IMBL & STO)	T	-1.549	-0.992	-0.969	-0.458	0.22	0.05
4096	mis-alignment (bet meas & LO)	T	-0.706	-0.462	-0.458	-0.223	0.10	0.01

Figure 3. Design Control Table for the Internal Metrology Measurement Beam and Local Oscillator of one arm of the Science Interferometer.

In log space, Equation 9 becomes

$$\mathbf{P}_P + \mathbf{P}_{PLO} = 10\log_{10}\left(\frac{1}{2}\right) + 2\mathbf{P}_H \tag{10}$$

$$= -3.01 + 2\mathbf{P}_H \tag{11}$$

Therefore the sum of the probe and local oscillator power must be greater than or equal to $-3.01 + 2$ times the required rms heterodyne power in log space. If the probe and local oscillator powers are variable over the mission lifetime, each having gaussian statistics, then their sum is also a random variable having gaussian statistics such that

$$p(x) = \frac{1}{\sqrt{2\pi(\sigma_P^2 + \sigma_{PLO}^2)}} \exp\left[-\frac{(x - (\mu_P + \mu_{PLO}))^2}{2(\sigma_P^2 + \sigma_{PLO}^2)}\right] \tag{12}$$

$$= \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right] \tag{13}$$

where $\mu = \mu_P + \mu_{PLO}$ and $\sigma^2 = \sigma_P^2 + \sigma_{PLO}^2$.

Using Equation 13 we can determine the probability that the heterodyne optical power is greater than the requirement. The probability, $\mathcal{P}$, that a random variable having a Gaussian PDF is greater than a value, R, is found by integrating the PDF from R to infinity

$$\mathcal{P}(R) = \int_R^\infty \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right] dx\ . \tag{14}$$

If the argument of the exponential is parameterized such that

$$t = \frac{x-\mu}{\sqrt{2\sigma^2}} \tag{15}$$

and

$$dt = \frac{1}{\sqrt{2\sigma^2}} dx \tag{16}$$

then Equation 14 can be written in the form

$$\mathcal{P}(R) = \frac{1}{2}\frac{2}{\sqrt{\pi}} \int_{\frac{R-\mu}{\sqrt{2}\sigma}}^\infty \exp\left(-t^2\right) dt \tag{17}$$

$$= \frac{1}{2}\mathrm{erfc}\left(\frac{R-\mu}{\sqrt{2\sigma^2}}\right) \tag{18}$$

where the complementary error function is

$$\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty \exp\left(-t^2\right) dt = 1 - \mathrm{erf}(x) \tag{19}$$

The probability that the received optical heterodyne power, $\mathbf{P}_R$, is greater than the required heterodyne power, $\mathbf{P}_H$, is found by substituting $\mathbf{P}_H$ from Equation 11 into Equation 18 such that

$$\mathcal{P}(\mathbf{P}_H) = \frac{1}{2}\mathrm{erfc}\left(\frac{-3.01 + 2\mathbf{P}_H - \mu}{\sqrt{2\sigma^2}}\right)\ . \tag{20}$$

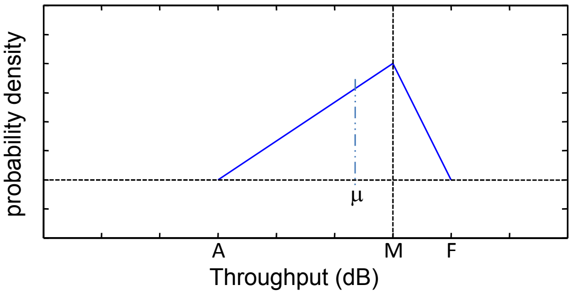


Figure 4. The triangle distribution. M is the mode, or “design-to” parameter, A is the adverse parameter, F is the favorable parameter. The mean, μ, and the variance, σ^2, are computed from these three parameters based on the type of distribution.

4. RESULTS

By summing the means and variances of both probe and probe local oscillator beams, we can compute the mean and variance of $\mathbf{P}_P + \mathbf{P}_{PLO}$. Using Equation 20 and given an rms heterodyne power requirement based on experiment, P_H, we can now determine the probability that the Metrology Source provides sufficient optical power to the metrology gauge such that it meets its performance requirements. This new metric is used to evaluate and express confidence in the available coherent optical power. This metric, of course, does not apply to other issues that affect the gauges such as vibration and thermal drift.

One example of a distribution assigned to a loss mechanism is the triangle (Figure 4). Given “design-to” (or mode of the distribution), adverse (min), and favorable (max) parameters that represent the distribution, the mean is

$$\mu = \frac{D + F + A}{3} \tag{21}$$

while the variance is

$$\sigma^2 = \frac{(F - D)^2 + (A - D)^2 - (A - D)(F - D)}{18} \,. \tag{22}$$

Each distribution type − such as the gaussian distribution, tophat distribution, or delta function − have similar equations to find their mean and variances.

The metrology design presented in the sample DCT in Figure 3 reflects a laser source power of 325mW rms ±45mW. From the table, the total mean of the Probe beam is -53.61 dBm with a variance of 7.44. The total mean of the Local Oscillator is -22.86 dBm with a variance of 3.06. The probability that this beam pair will have greater than a 100nW rms heterodyne signal is 97.8%.

The DCT itself is the optical power budget where an accounting of individual loss mechanisms is kept. The optical system design can also be evaluated and adjusted based on the performance metric. By creating the DCT in a spreadsheet where the equations are coded, we can modify any parameters or add or subtract optical elements and easily evaluate the effects on the probability. The statistical parameters assigned to an optical element can be updated with actual statistical data when the data are available, such as by conducting accelerated lifetime testing on optics or glass fibers or monitoring alignment following thermal cycling or vibration testing.

One notable loss mechanism in SIM PQ is the protected silver coatings applied to the many starlight mirrors. The durability of these coatings over time are highly dependent on the fabrication environment as well as the terrestrial environment prior to launch. Degradation or failure of a 3mm diameter patch of silver coating on which an internal metrology beam is incident could severely compromise the performance of the entire instrument. Given the 16+ mirrors in the path of a single internal metrology beam, the probability of failure or degradation is much higher in SIM PQ than on optical systems having very few optical elements. Future lifetime testing of a large number of witness samples having the protected silver coating will be conducted in order to better model the throughput (reflectivity) parameter. Not only will this provide a better statistical representation for our optical power model and budget, but it may reveal sources of degradation that may be mitigated with proper handling and storage of the optics during pre-launch integration and test phases.

5. SUMMARY

The statistical technique of evaluating the available optical power with respect to the required heterodyne power of metrology gauges, as described above, has become an accepted method for the complex instrument that is SIM Planet Quest and is being carried through to the most current SIM designs. Although JPL has applied the basic technique to deep space RF telecommunications system design of past missions, this approach is still new with respect to optical metrology subsystems.

Despite the "best estimate" nature of the design-to, adverse, and favorable parameters, as well as the assigned probability distributions, with enough variables the overall total statistics should provide a very reasonable metric with which to evaluate the power capability over the mission lifetime. In the case of SIM PQ, the sheer number of optical elements and loss mechanisms allow the gaussian distribution estimation, based on the Central Limit Theorem, to be a good estimate of the statistics of the power throughput. It is hoped that subsequent risk reduction testbeds will provide an experimental forum with which to evaluate the accuracy of this model.

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