Telescope Multi-Field Wavefront Control with a Kalman Filter

John Z. Lou, David Redding, Norbert Sigrist and Scott Basinger
Jet Propulsion Laboratory, California Institute of Technology

Abstract

An effective multi-field wavefront control (WFC) approach is demonstrated for an actuated, segmented space telescope using wavefront measurements at the exit pupil, and the optical and computational implications of this approach are discussed. The integration of a Kalman Filter as an optical state estimator into the wavefront control process to further improve the robustness of the optical alignment of the telescope will also be discussed. Through a comparison of WFC performances between on-orbit and ground-test optical system configurations, the connection (and a possible disconnection) between WFC and optical system alignment under these circumstances are analyzed. Our MACOS-based [2] computer simulation results will be presented and discussed.

Keywords: space telescope, multi-field wavefront control, optical state estimation and correction

1. Introduction

The current and future trend in the design of large space telescopes is to use segmentation of the primary mirror to allow the telescope to fit in an existing rocket shroud. Other benefits of segmentation include reduced primary mirror mass and reduced manufacturing cost. Ground-based telescopes, such as the Keck Observatory [8] and the planned Thirty Meter Telescope [9], use 36 and 421 hexagonal mirrors, respectively, to stitch together a large primary mirror. The James Webb Space Telescope will use 18 hexagonal segments for its primary mirror, and a current NASA concept under development, ATLAST, is also considering a large number of hexagonal segments.

These telescopes require some form of wavefront sensing and control (WFS&C) to align the segmented primary mirrors. One such technique is to point the telescope toward an unresolved point source, take out-of-focus data at several foci, and then use a Gerchberg-Saxton type iterative phase retrieval algorithm [3] to estimate the wavefront of the telescope at the exit pupil. Actuators on the optics can then be controlled to correct for any aberrations in the system. This technique works well to align the telescope for a single field point, but most space telescopes are required to operate over a significant field of view (a few degrees) for which a multi-field WFS&C procedure would be desirable to minimize system bias toward any particular field of view.

One common design for telescopes that can operate over a large field of view is a three mirror anastigmat (TMA). One major challenge of aligning a TMA is that certain alignment degrees of freedom are degenerate with respect to wavefront quality at single field points. For example, at a single field point, coma in the system could be compensated for by either lateral translation the secondary mirror, or by changing the shape of the primary mirror using the segments. From the system control point of view, such correlation of optical elements and compensation ambiguity could fail the wavefront control (WFC) process due to the singularity in the control gain matrix. With multi-field WFC, however, such control instability can be significantly mitigated.

In addition to exploring an on-orbit alignment, this paper will also discuss ramifications of ground testing before launch and correlating ground data to on-orbit performance. Ground testing of space telescopes is critical to their success, but problematic due to the sheer size a collimated beam needed to test the full aperture of the telescope. Two approaches are typically considered: A collimated beam created by an off-axis parabola (OAP) to test the telescope in single-pass, or an internal light source and an auto-collimating flat (ACF) to test the telescope in double-pass. In both cases, full-aperture testing requires a mirror (OAP or ACF) that has the same diameter as the primary mirror of the telescope. However, a monolithic mirror of this size is prohibitively expensive and sub-aperture tests must be considered.

We have built a MACOS model of a segmented TMA telescope to simulate the WFS&C technique described in this paper. This model consists of an 18-segment primary mirror (PM), a secondary mirror (SM), a tertiary mirror (TM)
and a fine-steering mirror (FSM), as sketched in Fig. 1. (In section 4 of this paper, we discuss a theoretical sub-aperture test setup and explore performance issues therein.)

Figure 1. Front-end optics for a segmented space telescope

In this study, the controllable optics is assumed to be the PM segments and the SM. Each PM segment can be actuated in seven degrees of freedom (DOF), including tip, tilt, clock, x and y translations, piston and the adjustment in radius of curvature (ROC), and the SM can be controlled in five DOF of tip, tilt, x and y translations and piston. After deployment in space, the telescope can be aligned in several steps; the alignment process can be roughly divided into two parts: a coarse alignment/phasing stage and a fine-phasing stage. The goal of coarse alignment and phasing is to align the telescope into a state for which a wavefront sensing (WFS) operation can be performed to estimate the current system wavefront with an accuracy good enough for the subsequent fine-phasing WFC operations. The algorithms used in the first stage generally do not depend on an accurate knowledge of system wavefront. The goal of the second stage, fine-phasing the telescope, is to reduce the system wavefront error (WFE) to within the mission requirements, typically at nanometer level. The success of the fine-phasing WFC operation, however, requires a reasonably accurate knowledge of system wavefront error. A poor estimate of wavefront error will often lead to instability and failure of the fine-phasing WFC operation. In this paper, we will only discuss issues related to WFC at the fine-phasing stage, assuming the capability of obtaining reasonably good wavefront estimates through WFS.

A space telescope typically serves light to a science instrument (SI) module at the backend within its Field of View (FOV). An example of a FOV window with five SIs is depicted in Fig. 2. The camera of the center SI (SI 1) will be used for wavefront sensing and control in our modeling.

Figure 2. Assumed science instruments in the sky field of view

The objective of telescope on-orbit fine-phasing WFC is to align the telescope using the wavefront information measured in the central camera pupil by minimizing the differences of RMS WFE between the current wavefront and predetermined nominal wavefront at a set of selected control fields in the central camera FOV. A successful WFC operation will drive an initially misaligned telescope towards its design optical state, and when that happens, the WFE in the FOV of all the science instruments will also be approaching their nominal values – the values obtainable from a perfectly aligned telescope. As will be discussed in more detail in Section 4, the WFC operation does not always converge to an optical state that is close to the design state, and a consequence of this could be that the residual RMS WFE across science instruments FOV could be rather poor even though the WFE at selected control fields have been quite effectively minimized – something we refer to as a disconnection between WFC and telescope alignment.

When the initial optical system misalignments are not very small, the WFC problem is mathematically a nonlinear optimization problem. In practice, however, a linear WFC algorithm is preferred because a nonlinear WFC procedure would be too expensive to be practical in a space mission. We found the combination of a Kalman Filter optical state
estimator and a linear WFC optimizer to be quite effective for space telescope on-orbit alignment when using the linear
optimizer alone would cause computational instability for certain relatively large initial misalignments.

This paper is organized as follows: Section 2 discusses telescope on-orbit WFC with multi-fields and a comparison of
WFC performances between single and multi-fields. Section 3 discusses the incorporation of a Kalman optical state
estimator into the WFC process. Section 4 makes a comparison of WFC performances between on-orbit and ground-test
configurations, and discusses the factors that influence the connection between WFC and optical alignment. The last
section provides a brief summary and some possible future work.

2. The Principle of Multi-Field WFC

It seems reasonable to expect that by using multiple FOV in WFC, improved residual wavefront errors (WFE)
can be achieved in both the controlled region (central camera FOV window in this case) and in FOV regions assigned to
other science instruments. The benefits of using multiple FOV in WFC can be justified from at least two aspects: when
wavefront fields from multiple FOV points are used in aligning the telescope, field bias can be reduced in the optics
alignment process, and the optical state thus produced will be more balanced in generating uniformly good WFE across
the entire SI module FOV field; on the numerical computing side, multiple FOV WFC tends to reduce the computational
 singularity associated with the solution of the least-square problem in the telescope alignment process, and the improved
computational stability will also contribute positively to the quality of the telescope alignment.

We extended the single FOV feedback control law [1] to the case of multiple FOV by applying the least-square
optimization of WFE to multiple field points simultaneously [4]. Let \( w^i \) be the aberrated wavefront obtained from \( i \)th
FOV location in the central camera window and \( w_0^i \) be the nominal wavefront from design residual at the same location,
let \( dw^i/du \) be the sensitivity matrix about its nominal state at \( i \)th FOV location, and let \( u \) be the perturbation vector
applied to the system state, the linear model of the control law for multiple FOV can be expressed in vector form as

\[
W = \begin{bmatrix} w^1 \\ w^2 \\ \vdots \\ w^n \end{bmatrix} = \begin{bmatrix} \frac{dw^1}{du} \\ \frac{dw^2}{du} \\ \vdots \\ \frac{dw^n}{du} \end{bmatrix} u + \begin{bmatrix} w_0^1 \\ w_0^2 \\ \vdots \\ w_0^n \end{bmatrix} = \frac{dW}{du} u + W_0
\]  

(2.1)

The goal of the WFC operation is to minimize the following objective function \( J(u) \) in the least-square sense

\[
\min J(u) = \min \{ (dw - Cu)^{\top} A (dw - Cu) \}
\]  

(2.2)

where \( dW = W - W_0 \), \( C = dw/du \) and \( A \) is a weighting matrix that allows us to set priorities for different FOV
locations in the wavefront error reduction process (\( A = I \) if a uniform weighting is used for all FOV locations). The
solution to the minimization problem (2.2) can be obtained by a singular value decomposition (SVD) procedure. When \( C \)
has full column rank, the SVD computation is numerically stable and the solution to equation (2.2) is unique [6]. For
most telescope WFC, however, the system sensitivity matrix \( C \) is often nearly rank deficient, which could make the
stability of the control process sensitive to the magnitude and distribution of initial system aberrations. The use of
multiple FOV in WFC tends to make the combined sensitivity matrix \( C \) less singular than the one from using a single
FOV, so the WFC with multiple FOV is in general computationally more stable.

In our closed-loop WFS&C simulation, the set of control fields used in the central camera FOV are shown in Figure 3.
Figure 3. Five control fields are distributed in right half region of the central camera FOV, as shown with blue dots.

As a preprocessing step of the closed-loop WFS&C simulation, sensitivity matrices with respect to the nominal state first are computed for all of the control points. Nominal wavefront fields for all control points are also pre-computed. A randomly perturbed telescope optical system state is then generated, and perturbed wavefront fields for all FOV locations under that state are generated using the MACOS program. A correction to the current optical state is then generated to minimize the difference between the perturbed wavefront fields and their nominal ones. The correction is applied in the form of actuator controls to generate a new optical state. The newly generated optical state is then used as the starting point in the next iteration of the control loop. To reflect the reality, random wavefront measurement noises and actuator errors are included in the control operations. A nearly optimal system state can be typically obtained in a few control iterations. A schematic for the feedback control is shown in Fig. 4.

Table 1. Initial error allocations for telescope optics (one-sigma standard deviations)

<table>
<thead>
<tr>
<th>Tip (urad)</th>
<th>Tilt (urad)</th>
<th>Clock (urad)</th>
<th>dX (um)</th>
<th>dY (um)</th>
<th>Piston (um)</th>
<th>RoC (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PM</td>
<td>100</td>
<td>100</td>
<td>1000</td>
<td>70</td>
<td>70</td>
<td>100</td>
</tr>
<tr>
<td>SM</td>
<td>350</td>
<td>350</td>
<td>0</td>
<td>700</td>
<td>700</td>
<td>85</td>
</tr>
<tr>
<td>TM</td>
<td>35</td>
<td>35</td>
<td>225</td>
<td>150</td>
<td>150</td>
<td>225</td>
</tr>
<tr>
<td>FSM</td>
<td>7</td>
<td>7</td>
<td>0</td>
<td>1600</td>
<td>1600</td>
<td>140</td>
</tr>
</tbody>
</table>

PM surface Zernike error | 10 nm/mode
Wavefront sensing noise | 10 nm

Initial misalignments are assumed on all telescope optical elements, including the primary mirror segments, secondary mirror, tertiary mirror and fine steering mirror. It is assumed that the controllable optics are primary and secondary mirrors. Each optical element, including each PM segment, is perturbed independently in each of the six degrees of freedom as in a rigid body motion, and the PM segments have an additional DOF of radius of curvature. The actual amount of initial telescope misalignment for an optical element is assumed to be a random realization of Gaussian distribution with its one-sigma standard deviation specified by an assumed on-orbit error budget. Table 1 shows the one-sigma values used in our WFC simulations. The values for PM ROC and SM piston values are smaller than might be expected and this was to ensure WFC computational stability (more on that in the next section).

It has been found that wavefront errors, especially in the directions off the boresight, are very sensitive to the SM misalignment. And when PM and SM are both misaligned, PM and SM can not be controlled entirely separately to
achieve good global alignment. To maintain the computational stability of the control algorithm, it is also important to perform the control in steps that combine PM and SM while avoiding any coupling among PM and SM DOF. An effective control sequence for the five-point control is as follows:

1) PM tip, tilt, piston, ROC, and SM tip, tilt, ~ 3 iterations
2) PM clock, x and y translations, piston, and SM dx, dy, ~ 3 iterations
3) Repeat 1)
4) PM tip, tilt and piston to clean up actuator errors ~ 2 iterations

When initial telescope misalignments are large, especially with PM tip and tilt, it could be impossible to perform ray-tracing properly through the telescope and central camera due to existing apertures in those optics which can cause a large number of rays lost in the process. In practice, very large initial telescope misalignments will be first reduced at a coarse alignment stage by, for example, a dispersed fringe sensing procedure to make sure the telescope can produce reasonably measurable wavefront, which is then followed by the fine-phase control procedure described here. To emulate a coarse alignment operation to make it possible to perform our fine-phase control simulation in the presence of large initial misalignments, we added a “prefiltering” step which removes most of the initial tip and tilt misalignments from the PM segments. The operation of the prefilter can be described with the following equation:

\[ p_{rb}^{\text{filtered}} = (I - A\left(\frac{dw}{dq}\right)^{\dagger})_{p_{rb}} \]

where \( p_{rb} \) and \( p_{rb}^{\text{filtered}} \) are state misalignment perturbation vectors before and the filter is applied, \( dw/du \) is the overall system sensitivity, \( (dw/dq)^{\dagger} \) is the sensitivity for those PM DOF from which we would like to remove the initial misalignments, and \( A \) is an elementary selection matrix that maps the perturbation from the subspace of those DOF that we wish to remove from PM initial misalignments to the entire misalignment perturbation space.

Figure 5. Nominal (first row) and initially aberrated (second row, after initial prefiltering) wavefront maps at the center and four corners of central camera FOV window.

We now present some results of the WFC with five FOV control points in the central camera window. For the results shown, twenty Monte Carlo runs were performed with Gaussian distributed initial misalignments applied to the telescope, using the error standard deviations given in Table 1. Root mean square (RMS) WFE values are first computed at five FOV positions within the central camera for the nominal states, which are the design residuals from the telescope optical prescription, and then the mean and standard deviations of RMS WFE for the perturbed states after prefiltering.
PM tip and tilt misalignments are computed, as shown in Fig. 5. The statistical errors of residual RMS WFE at the five points in the central camera after WFC are shown in Fig. 6.

**Figure 7.** Exit pupil OPD images and residual RMS WFE across science instrument fields after five-point WFC. The pair of values shown below each OPD map is the mean and standard deviation of RMS WFE in nanometers over 20 Monte Carlo runs.

To evaluate the robustness of the telescope alignment through our WFC procedure, we also computed the residual wavefront error footprints in various instrument windows as shown in the instruments FOV map depicted in Fig. 2. The WFE evaluation for all science instruments is performed in an approximation sense, as described below. When evaluating RMS WFE in instrument FOV windows, we first perform a WFC with the central camera in the system to align the telescope; the central camera is then removed from the system, and RMS WFE are computed at various locations corresponding to the FOV windows of various SIs. The result is shown in Fig. 7. As a comparison, Fig. 8 shows the RMS WFE in instrument FOV windows after a single-field (at the center of central camera) WFC. Compared to the residual WFE from WFC with a single point at the center of the central camera window, the results in Figures 6 and 7 show significant improvements.

**Figure 8.** Exit pupil OPD images and residual RMS WFE across science instrument fields after center-field WFC. The pair of values shown below each OPD is the mean and standard deviation of RMS WFE in nanometers over 20 Monte Carlo runs.
3. Kalman Filter as an Optical State Estimator

A Kalman Filter (KF) has been successfully applied to diverse applications as a robust and optimal state estimator [7]. In the context of optical system alignment based on wavefront measurements, a KF state estimator can combine all available wavefront measurements, past and present, as well as measurement and actuation error statistics to generate a Maximum-Likelihood optimal state estimator [5]. The flexibility and robustness of the KF algorithm make it attractive to be used in real-time optical system alignment when WFC alone can not effectively align the system. For example, if a KF state estimator can reasonably estimate the state of a misaligned optical system based on wavefront information, an effective correction to the misalignment can then made since we do have the knowledge of the system’s design or nominal state. The hope is that the KF state estimator is more robust and computationally more stable than a wavefront optimizer when system aberrations are large. A wavefront optimizer could also have difficulty separating misalignment effects at different components of the system when presented a total WFE map measured at the system exit pupil, and that confusion could either lead to control instability, when, in WFC, the system is further misaligned to the point a reasonable measurement of wavefront becomes impossible, or result in an optical state that minimizes WFE at controlled fields but with large WFE outside the control region.

Let \( \mathbf{X}_i \) be the optical state vector at time step \( i \), which usually includes all the DOF of the optical system elements that we wish to estimate their values, and let \( \mathbf{u}_i \) be the control actuation vector at time \( i \). The general form of an optical state transition equation and a measurement equation can be written as

\[
\begin{align*}
\mathbf{X}_{i+1} &= \mathbf{A}_i \mathbf{X}_i + \mathbf{B}_i \mathbf{u}_i + \mathbf{\omega}_i \\
\mathbf{W}_i &= \mathbf{H}_i \mathbf{X}_i + \mathbf{\gamma}_i
\end{align*}
\]

where \( \mathbf{A}_i \) and \( \mathbf{B}_i \) are matrices relating state and control vectors at time \( i \) to the state vector at time \( i+1 \), \( \mathbf{H}_i \) is a sensitivity matrix relating state \( i \) to wavefront \( \mathbf{W}_i \), \( \mathbf{\omega}_i \) is typically a Gaussian random vector of process noise, and \( \mathbf{\gamma}_i \) is Gaussian random vector of measurement errors. The KF algorithm can be viewed as a recursive predictor-corrector procedure, where the time update equation (4.1) makes an a priori estimate of the state \( \mathbf{X}^{-}_{i+1} \) at time \( i+1 \) and a measurement update, with a computed Kalman gain \( \mathbf{K}_i \), corrects the a priori estimate \( \mathbf{X}^+_{i+1} \) to produce a posteriori estimate of the state \( \mathbf{X}^\ast_{i+1} \) at time \( i+1 \). In our model, the state change is caused by control and noise only, and we separate the control from the KF estimation process, so that \( \mathbf{A}_i \) is identity and \( \mathbf{B}_i \) is zero. We also use the sensitivity computed at the nominal state for all \( \mathbf{H}_i \), so \( \mathbf{H}_i =\mathbf{H} \). Under these assumptions and without going into further details of the KF equation derivations (see ref [7]), we present the equations that are used for computing the Kalman gain in (4.3) and the a priori and a posteriori state update equations in (4.4)

\[
\begin{align*}
P_{i+1} &= (\mathbf{H}^T \mathbf{R}^{-1} \mathbf{H} + \mathbf{M}_i)^{-1} \\
\mathbf{K}_{i+1} &= P_{i+1} \mathbf{H}^T \mathbf{R}^{-1} \\
\mathbf{M}_{i+1} &= \mathbf{P}_i \\
\mathbf{X}^-_{i+1} &= \mathbf{X}^+_{i+1} + \mathbf{\omega}_i \\
\mathbf{X}^\ast_{i+1} &= \mathbf{X}^-_{i+1} + \mathbf{K}_{i+1} (\mathbf{W}_i - \mathbf{H} \mathbf{X}^-_{i+1} - \mathbf{W}_{\text{nom}})
\end{align*}
\]

where \( \mathbf{R} = \mathbf{E}(\mathbf{\gamma}\mathbf{\gamma}^T) \) is the error covariance of wavefront measurement assumed here to be independent of time, and \( \mathbf{M}_i \) is the covariances of initial state errors (also used as the initial a priori state estimate). Since the equations in (4.3) are independent of wavefront measurement, all the Kalman gains can be computed before the actual state estimation process starts. Since most of the KF computations are for updates in equations (4.3), the ability to pre-compute all Kalman gains makes the KF state estimation process very efficient and especially attractive for real-time operations. The a posteriori state update equation for \( \mathbf{X}^\ast_{i+1} \) is a balance between a priori state estimate and a state correction with Kalman gain. As the wavefront measure error approaches zero (\( \mathbf{R} \rightarrow 0 \)), it can be seen from equations (4.3) that \( \mathbf{K}_i \rightarrow \mathbf{H}^{-1} \), and
therefore the a posterior state update equation approaches the least-square WFC equation. On the other hand, as $P_i$, which is the a priori state estimate error covariance, approaches zero, $K_i$ approaches zero and the a posterior state update weighs less on the Kalman gain correction. Therefore the KF state estimate algorithm can be adjusted depending on our degree of confidence in the wavefront measurement and state estimate errors.

![Figure 9. State transition for Kalman Filter based optical alignment as shown along the green path](image)

In Fig. 9, we illustrate how a Kalman optical state estimator can be incorporated into a space telescope on-orbit alignment to improve the robustness of the telescope alignment process. Suppose we start from an initially misaligned optical system (state 1). A least-square based WFC operation tries to go from state 1 to state 3, which would be fine if the WFC procedure is computationally stable and the resulting optimal WFE state 3 is reasonably close to the nominal state 4. When initial system misalignment is not small and/or when there is ambiguity in waterfront as a function of different DOF in the system, a linear (or even nonlinear) wavefront controller often encounters difficulty in generating a reasonably aligned optical state. In those situations, an iterative WFC procedure could either produce states diverging further and further away from the nominal state or converge to a state that minimizes the WFE at controlled fields but leaves the optical system still badly misaligned. We tested an optical alignment procedure where the Kalman state estimator is first applied to make state estimate followed by a state collection, which is the path from state 1 to state 2 in Fig. 9. The test case we used in our simulation increased PM ROC error $\sigma$ from 50 nm to 20 um and SM piston error $\sigma$ from 84 um to 3.2 mm, as shown in Table 2. With the initial errors in Table 2, the pure WFC procedure is not computationally stable in a Monte Carlo simulation – that is for many of the initial error distributions, the WFC procedure does not converge to an optimal WFE state. To apply KF algorithms of Eqns (4.3) and (4.4) for optical state estimation, initial conditions for WF measurement error covariance $R$ and initial state error covariance $M_i$ must be specified. Table 3 shows the one-$\sigma$ statistics for the DOF of optical elements whose states are estimated and for the
wavefront measurement error, which are used in constructing \( R \) and \( M \). For simplicity, it is assumed that \( R \) and \( M \) are diagonal.

Table 2. One-Sigma Initial Errors Used in the Simulation

<table>
<thead>
<tr>
<th></th>
<th>Tip (urad)</th>
<th>Tilt (urad)</th>
<th>Clock (urad)</th>
<th>dX (um)</th>
<th>dY (um)</th>
<th>Piston (um)</th>
<th>RoC (um)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PM</td>
<td>500</td>
<td>500</td>
<td>1000</td>
<td>71</td>
<td>71</td>
<td>100</td>
<td>20</td>
</tr>
<tr>
<td>SM</td>
<td>350</td>
<td>350</td>
<td>0</td>
<td>707</td>
<td>707</td>
<td>3200</td>
<td>0</td>
</tr>
<tr>
<td>TM</td>
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<td>35</td>
<td>224</td>
<td>142</td>
<td>142</td>
<td>224</td>
<td>0</td>
</tr>
<tr>
<td>FSM</td>
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<td>7</td>
<td>0</td>
<td>1600</td>
<td>1600</td>
<td>141</td>
<td>0</td>
</tr>
<tr>
<td>PM surface Zernike error</td>
<td>10 nm mode</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Wavefront measurement noise</td>
<td>10 nm</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>

Table 3. One-Sigma Statistics for Initial State Error Covariance and Wavefront Measurement Error Covariance, Used in Computing Kalman Gains

<table>
<thead>
<tr>
<th></th>
<th>Tip (urad)</th>
<th>Tilt (urad)</th>
<th>Clock (urad)</th>
<th>dX (um)</th>
<th>dY (um)</th>
<th>Piston (um)</th>
<th>RoC (um)</th>
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<td>PM</td>
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<td>500</td>
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<td>70</td>
<td>10</td>
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<tr>
<td>SM</td>
<td>200</td>
<td>200</td>
<td>N/A</td>
<td>500</td>
<td>500</td>
<td>2500</td>
<td>N/A</td>
</tr>
</tbody>
</table>

One-\( \sigma \) for WF measurement error = 10 nm

Fig. 11. Results of a telescope alignment with KF state estimate, correction, followed by wavefront control. WFE values are mean and standard deviation of 20 Monte Carlo runs.

Fig. 10 shows the results of KF state estimates in two cases, the group of plots on the left used initial errors shown in Table 2. The one on the right changed PM ROC error from 20 um to 50 nm. It can be seen from the plots on the left that reasonably good state estimates were obtained for PM tip, tilt, piston, ROC and SM tip, tilt, but the estimate on SM piston was quite off. The plots on the right show, when PM ROC error is reduced to a smaller value, the estimate on SM piston is significantly improved. This comparison suggests it would be effective to perform several iterations of state estimate followed by state correction for telescope alignment. In the test case above, the first iteration of state estimate-correction would correct most of the PM ROC errors, and the second iteration would then be able to estimate and correct SM piston as well. As can also be seen from the results in Fig. 10, KF gives better estimates on the DOFs of the system that have a significant impact on system WFE, those so-called strong DOF components, than on weak DOF components. This makes KF estimator an effective tool for reducing initially large telescope misalignments, which paves the way for a computationally stable fine-tuning of WFE to mission WFE requirements using a least-square based WFE controller. Fig. 11 shows such a closed-loop alignment simulation. Shown are nominal, perturbed, KF state estimated/corrected and wavefront controlled exit-pupil OPD maps, corresponding to each of the four rows, at five central camera fields as shown in Fig. 3. As before, WFE values shown below each OPD map are the mean and standard deviation of 20 Monte Carlo runs.
Carlo simulations. With initial OPD errors at about 183 um, the two KF state estimate-correction iterations reduced the errors down to about 2.7 um, and the final WFC fine-tuning cut the errors further down to about 64 nm.

Values shown in Table 3 are the error statistics for the initial state and wavefront estimates, which reflects our confidence in the estimate and measurement accuracies of those quantities. These values are used in computing the Kalman gains and obviously will affect the performance of the state estimation process. Values in Table 3 are of the same order as the initial misalignment errors shown in Table 2. From KF algorithm point of view, however, we do have the flexibility of adjusting the values in Table 3. For example, we can reduce the error statistics in Table 3 by an order of magnitude, which implies a higher confidence in the accuracy of the estimation and measurement process. The result of doing that is shown in Fig. 12. In comparison to Figure 11, the residual errors after WFC in Fig. 11 exhibit noticeably smaller deviations in the Monte Carlo simulation. The performance of this alignment is also evaluated by checking residual wavefront errors at FOV windows of all science instruments using the optical state produced by the closed-loop alignment process. With the resulting optical state corresponding to the case shown in Fig. 12, the distribution of WFE across the entire SI field is shown in Fig. 13.

Figure 12. Result of reducing initial state estimate and wavefront measurement error statistics by an order of magnitude.

Figure 13. WFE distribution across all science instruments fields after a closed-loop telescope alignment with a Kalman Filter.

4. WFC versus Optical System Alignment

Figure 14. A simplified view of a double-pass configuration with a sub-aperture ACF.
We now briefly discuss the connection between *wavefront control* and *optical system alignment*. To some people, the two phrases may mean virtually the same thing. For example, when wavefront errors are minimized at control fields, the optical system is expected to be reasonably aligned or close to its design state. In reality, however, whether that connection exists depends on the nature of an optical system. In this section, we compare two telescope system configurations, one for on-orbit operation and one for ground-test, to explore the factors that may influence the relation between WFC and optical system alignment. The purpose of the ground-test configuration is to provide a laboratory means for verifying the telescope on-orbit wavefront sensing and control approach. Fig. 14 shows a sketch of a simplified ground-test double-pass configuration, where TM and FSM are not drawn. In double-pass configuration, the light emitted from the (laser) source first goes through the telescope backwards (first pass), and upon reflecting from the auto-collimating flat (ACF), the light then goes through the telescope in the forward path as in on-orbit mode. The ACF contains six sub-aperture triads, each reflecting light beam onto part of three distinct PM segments. The ground-test configuration differs from the on-orbit configuration in two main aspects, one is double-pass and the other is sub-aperture at ACF. In our simulation study, we used both full-aperture ACF and sub-aperture ACF to try to understand what kind of impact the sub-aperture would have on WFC performance.

Using the WFC procedure discussed in Section 2 for on-orbit configuration, we tried to minimize the wavefront errors in double-pass at the five fields in the central camera FOV region as shown in Fig. 3. The results obtained are quite different from those for on-orbit WFC. The first difference is the WFC procedure is computationally less stable in the double-pass configuration for both full and sub-aperture ACF, so we had to reduce the initial misalignment errors shown in Table 1 by 50% to stabilize the WFS&C simulation process. The computational stability of WFC can be affected by the internal coupling of system components sensitivity. A strong coupling of PM tilts and SM translations, for example, can result in a near-singular system sensitivity matrix that contributes negatively to the WFC stability. Another major difference is the post-WFC optical state in double-pass; we found PM and SM are badly misaligned even when residual wavefront errors at all controlled fields have been reduced to reasonable values. A consequence of that misalignment is that the telescope optical performance will be poor outside the controlled FOV region, which negatively impacts the performance of science instruments. In contrast, WFC in telescope on-orbit configuration always produces optical state of controllable elements (PM and SM) that is close to the aligned (design) state. The breakdown of connection between WFC and optical alignment in the double-pass configuration is also related to the stronger coupling of PM and SM, where we have seen the wavefront controller move SM by quite a few millimeters from its nominal position and in the meantime tilt PM segments to offset that SM move to minimize WFE. Therefore in the double-pass configuration we face a greater challenge aligning the controllable optics through the WFC procedure because of the ambiguity of PM and SM to the controller. Without any additional constraints, the wavefront control often finds a misaligned optical state with optimal WFE at controlled fields.

For the on-orbit configuration, we have seen that the five-field WFC significantly improves the telescope alignment over the single-field WFC as indicated in the residual WFE across the entire SI FOV. The question then is why five-field WFC does not show similar effectiveness in the double-pass configuration. Fig. 15 shows the comparison from a simple field-sensitivity test between single-pass and double-pass, where the only inserted misalignment is a tilt of 100 micro radians of the SM. Two field points were selected in the central camera FOV, labeled as Gm and G, and wavefront is
measured at each point. The OPD maps shown in the first two columns in Fig. 15 are the difference of perturbed and nominal OPD maps, and the last column shows the difference between second and first columns. It can be seen in this test that the field-diversity signal in double-pass is about five-times weaker than in single-pass. Generally speaking, the weak signal in double-pass can be shown to be the result of a cancellation of first-order aberrations in the double-pass configuration.

![Figure 16a. WFC results with a double-pass, sub-aperture ACF configuration, five controlled fields in central camera FOV used](image1)

One way to improve the WFC performance in double-pass is to add more fields to the control process. Since the weak field-diversity signal in double-pass makes the sensitivity matrix more numerically singular, which contributes to both a misaligned post-control optical state and control instability, it is hoped that additional controlled fields, e.g. using more than five fields in central camera FOV, would alleviate the singularity problem associated with the sensitivity matrix. Our simulation results, as shown in Fig. 16a and 16b, indicate such an improvement in WFC performance with additional fields is possible. When the number of control fields is increased from five to eight, the residual WFE after WFC is reduced from over 500 nm to around 100 nm. Though not shown here, a similar WFE improvement across the entire science instruments field is also achieved.

5. **Summary and Future Work**

We have demonstrated that for a segmented space telescope on-orbit optical system configuration, a multi-field WFC procedure is quite effective and robust for the telescope fine-phasing, where initial errors are assumed to be present on all telescope optical elements and the control is applied to PM segments and SM simultaneously. Through Monte-Carlo computer simulations, the multi-field WFC procedure demonstrated that it can reduce the initial system WFE, as caused by random initial system misalignments within an assumed telescope fine-phasing error budget, to about 50 nm, and the WFC procedure is also computationally stable. With the incorporation of a Kalman filter as an optical state estimator into the WFC process, we can improve the robustness of the telescope alignment process even further. In the
presence of some large optical misalignments, say at the beginning of the fine-phasing, the Kalman state estimator can provide us with a reasonable estimate of the optical state, especially for those DOFs that have a significant impact on the system WFE. That state estimate allows us to make a few corrections in the optical state to push the system towards its nominal state, and the result is a large part of the WFE can be eliminated in this step. When the WFC procedure is applied after Kalman state estimate and correction, the control stability is much better guaranteed.

We also discussed WFC for a double-pass ground-test configuration and the challenges of aligning the double-pass system through a WFC procedure. Due to the much weaker field-diversity signal present in the double-pass configuration, the WFC operation is much less stable, and the multi-field WFC procedure could leave the system severely misaligned even though the WFEs are minimized at selected control fields. To improve the control stability and better align the telescope, several options have been explored or planned to be explored. For example, our simulation results suggest additional control fields would help. Other options include adding constraints into the WFC operations, so controllable elements can not move beyond a certain range, which of course is always the case in reality. With additional constraints in WFC, the singularity problem associated with the double-pass configuration will also be reduced. The drawback of a constrained WFC is its much larger computational cost since the control algorithm becomes nonlinear and the system sensitivity update will be needed in the WFC process. It is also interesting to investigate whether the use of a Kalman state estimator, linear or nonlinear, would provide some help in this case.

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References:


