High-precision Narrow Angle Astrometry with a Space-borne Interferometer

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Abstract

This paper develops an observing and processing scheme for narrow angle astrometry using a single baseline interferometer without the aid of "grid" stars to characterize the interferometer baseline vector in inertial space. The basic concept derives from the recognition that over a narrow field the set of fundamental unknown instrument parameters that arise because the interferometer baseline vector has large uncertainties (since there are no grid star measurements) is indistinguishable from a particular set of unobservable errors in the determination of star positions within the field. Reference stars within the narrow field of regard are used to circumvent the unobservable modes. Feasibility of the approach is demonstrated through analysis and example simulations.

Keywords: astrometry, interferometry

Introduction. One of the main science objectives of the SIM PlanetQuest mission is to survey nearby stars for earth analogue planets and determine their orbital parameters. The astrometric requirement for such detection and characterization is on the order of 1uas single measurement differential position accuracy. This accuracy is achieved in the narrow angle observing mode in which the field of regard is restricted to about 1°. The nominal narrow angle observing scenario for the SIM PlanetQuest mission uses accurate grid stars over a 15° field obtained via the global astrometry mode to first determine the interferometer baseline vector in inertial space. And then armed with this baseline vector knowledge the narrow angle observations then use a set of astrometrically benign reference stars over the narrow angle field that serves as a local reference frame to measure the motion of the target star. A fundamental question regarding this paradigm is whether it is possible to avoid the grid star measurements altogether and still satisfy the SIM PlanetQuest narrow angle astrometry requirements. (A second, somewhat related question which is not addressed in this paper is what is the minimal grid star accuracy required to achieve these requirements?) Affirmative answers to either of these questions open up the option space for alternative mission concepts.

This paper develops an observing and processing scheme for narrow angle astrometry using a single baseline interferometer without the aid of "grid" stars to characterize the interferometer baseline vector in inertial space. The gridless scheme presented here is an evolution and refinement of the ideas originally presented in [4]. The basic concept derives from the recognition that over a narrow field the set of fundamental unknown instrument parameters that arise because the interferometer baseline vector has large uncertainties (since there are no grid star measurements) is indistinguishable from a particular set of unobservable errors in the determination of star positions within the field. When three baseline orientations are used to derive In classical astrometry utilizing overlapping plate models the star positions and instrument parameters are solved simultaneously [2]. This technique has been successfully applied with the Hubble Space Telescope (e.g. 1 The interferometric analogue of this technique is used in SIM PlanetQuest global astrometry [3]. But because of operational constraints of the instrument, this type of approach is proscribed when observations are restricted to the narrow field. At the conceptual level this difficulty is circumvented by the use of the reference stars to help determine the *changes* in the unobservable instrument modes as the particular narrow angle field is revisited over the course of the mission. A principal drawback in the original approach developed in [4] was the requirement of using three distinct baseline orientations of observations of the target and reference stars at each epoch. While this requirement is not particularly onerous as a limited use observation constraint within the PlanetQuest mission, it is very restrictive if it were the principal mode of observation, as might be envisaged for other missions. This paper extends the methods in [4] to allow for a two baseline orientation observing scenario. There are a number of complications that arise because of this, and these are discussed.

1. Basic concepts. Let n denote a unit vector in R^3 and define

$$D = \{x : \langle x, n\rangle = 0\}. \tag{1}$$

The vector n is near the *a priori* position of the target star (tens of arcseconds is O.K.), and D represents the plane containing a sequence of nearly contemporaneous science baseline orientations. This plane will remain fixed through the lifetime of the mssion and small perturbations from this plane are allowed, and we will quantify their magnitude.

Introduce two additional vectors m and p so that $\{m,\ p,\ n\}$ form a right hand frame. Let S denote the hemisphere with center at the origin with $n \in S$. Define the projection π from R^3 onto D by

$$\pi(s) = s - \langle n, s\rangle n, \tag{2}$$

for any $s \in R^3$. Let D_0 denote the image $\pi(S)$ and note that π is an invertible map from S onto D_0 with $\pi^{-1} : D_0 \to S$ is given by

$$\pi^{-1}(x) = x + \sqrt{1 - |x|^2}n. \tag{3}$$

Hence if $s \in S$ with

$$s = \langle s, m\rangle m + \langle s, p\rangle p + \langle s, n\rangle n, \tag{4}$$

then

$$\pi(s) = \langle s, m\rangle m + \langle s, p\rangle p \in D_0. \tag{5}$$

Observe that the differential maps π_* and π_*^{-1} between the corresponding tangent spaces to S and D_0 will also be needed. Since π is linear the differential π_* restricted to a tangent vector h to S at $s \in S$ is simply

$$\pi_*(s) : h \to \pi(h), \tag{6}$$

where $\pi(h)$ is a tangent vector to D_0 at $\pi(s)$. And for any tangent vector k to $x \in D_0$,

$$\pi_*^{-1}(x) : k \to k - \frac{\langle x, k\rangle}{\sqrt{1 - |x|^2}}n. \tag{7}$$

Next consider a set of observations of the science target with direction vector s_T and reference stars with direction vectors s_i. The nominal *a priori* positions of these objects will be denoted with a superscript 0, and as remarked before, $s_T^0 = n$. Introduce the perturbation vectors δs_i so that the relationship between the true and nominal direction vectors is given as

$$s_i = s_i^0 + \delta s_i, \quad i = T, 1, ..., N. \tag{8}$$

In a similar manner we take the nominal baseline vectors as B_0^j and write the relationship between the true and nominal baseline vectors as

$$B^j = B_0^j + \delta B^j. \tag{9}$$

The standard regularized delay equation (without noise or systematic error terms) has the form

$$d_i^j - \langle s_i^0, B_0^j \rangle = \langle B_0^j, \delta s_i \rangle + \langle s_i^0, \delta B^j \rangle + c_j + \langle \delta B^j, \delta s_i \rangle, \quad i = T, 1, ..., N, \; j = 1, ..., M. \tag{10}$$

Without loss of generality we impose the B_0^j to be constrained to lie in the $m-p$ plane.

So far there are no approximations made in (10) above. Now we will introduce some simplifications that will enable the formulation of the GNAA approach. First we restrict the δs_i to be tangent vectors. The error in the right side of this equation introduced by this assumption is on the order of $|B_0||\delta s_i|^2$. This is sub–picometer for *a priori* star position errors of 100mas. When analyzing the sensitivity of the resulting equations we will see that this does not propagate to astrometric errors of any consequence (small fraction of a uas). We would also like to dismiss the quadratic term in (10). The assumption so far is that δB^j is an unrestricted 3–vector. We many assume that the components in the plane D are small as they represent knowledge errors. However, the component δB_z^j which is out of the plane must be controlled. Let's see how large this component can be and still be disregarded. Here we will take advantage of the small narrow angle field and write

$$\delta s_i = k - \frac{\langle x, k \rangle}{\sqrt{1-|x|^2}} n, \tag{11}$$

for a tangent vector k at $x = \pi(s_i^0)$. Now, $|\delta s_i| \geq |k|$ by (11) above. Thus

$$|\langle \delta B_z n, \delta s_i \rangle| \leq |\delta B_z| \frac{|x||\delta s_i|}{\sqrt{1-|x|^2}}. \tag{12}$$

But since $|x| \approx .02$ and $|\delta s_i| \approx 10^{-7}$, we need $|\delta B_z| \approx 10^{-3}$, which corresponds to about a picometer error, even with the conservative estimates of reference star position error and out of plane baseline error. However, another restriction on δB_z will appear later.

Note that

$$\langle B_0^j, \delta s_i \rangle = \langle B_0^j, h_i \rangle, \quad h_i = \pi(\delta s_i), \tag{13}$$

since $\langle B_j^0, n \rangle = 0$ for all j. Parameterize δB^j as

$$\delta B^j = \epsilon B_0^j + \epsilon_T^j E(B_0^j) + \delta B_z^j n, \tag{14}$$

where E is the rotation in $m-p$ plane

$$E(m) = p, \quad E(p) = -m. \tag{15}$$

Note that δB^j has a component out of the $m-p$ plane. We allow c_j, ϵ_T^j, and δB_z^j to vary with each baseline vector, but ϵ is fixed over the entire set of observations. External metrology is used to keep track of the change in baseline length. But there is a second order effect in the parameterization of δB^j when the components orthogonal to the nominal baseline are large. Since the model uses the projection of the baseline onto the plane D, a second order length change in the baseline vector is introduced by the out of plane contribution via

$$B = B_0 + \epsilon B_0 + \omega \times (B_0 + \epsilon B_0) + \frac{1}{2} \omega \times (\omega \times B_0). \tag{16}$$

Observe that the last term above has a component in the B_0 direction with magnitude

$$\langle \omega \times (\omega \times B_0), \frac{B_0}{|B_0|} \rangle = \frac{|\omega \times B_0|^2}{|B_0|}. \tag{17}$$

This term contradicts the model assumption that ϵ is a fixed parameter in the model over multiple baseline orientatons. To bound this error without conservatism requires tracking its contribution to the final astrometric error. It will be shown that this error is significantly less than the bound in (17).

This model is assume to hold on a single tile measurement in which M different baseline orientations are made somewhat contemporaneously. Over the mission lifetime the following model will be assumed to hold for the reference model tangent vectors:

$$h_i(t) = h_i(0) + tv_i + \pi q_i(t), \tag{18}$$

where v_i is the proper motion of reference star i and $q_i(t)$ is observed motion due to parallax. For simplicity we take

$$q_i(t) = \pi[\langle s_i^0, \phi(t)\rangle s_i^0 - \phi(t)], \tag{19}$$

where $\phi(t)$ is the position of the instrument in solar system barycenter coordinates and π_i is the parallax of the star. Note that this model assumes that the reference stars are accurately modeled by just proper motion and parallax astrometric parameters. The model for the target star is similar but allows for (non–parallax) nonlinear motion:

$$h_T(t) = h_T(0) + tv_T + \pi q_T(t) + r(t), \tag{20}$$

where $r(t)$ is the motion we wish to ultimately characterize.

An important point here is that all of these tangent vectors are in D; thus at various epochs in the mission the underlying plane remains the same. It seems plausible that we may be able implement the scenario inwhich only a pair of baseline orientations are used in a more flexible way. Clearly each pair of baseline orientations defines its own plane. We can conceivably map the tangent vectors from any epoch back to anyother epoch with a posteriori knowledge of the plane (i.e. reconstructed attitudes of the baseline vector at each epoch). We will see that the pointing requirement while restricting the same plane assumption is not very onerous, but it may still be relaxed some more.

The matrix model. The single tile parameterization above generates a linear model for the observations and unknown parameters. We will let h denote the star position differential (h is a $2N$ vector with $h(2i) = h_i(1),\ h(2i+1) = h_i(2)$). The $3M+1$ vector r shall denote the vector of baseline and constant term parameters. And the MN vector y is the set of regularized delay measurements. Thus we obtain

$$y = Ax + \eta, \quad x = \begin{bmatrix} h \\ r \end{bmatrix}, \tag{21}$$

where η denotes measurement error.

The narrow angle observing scenario "chops" between the target and reference star observations to reduce the effect of time–varying errors in the instrument. So instead of a single observation of the target star as modeled, the observation sequence goes more like this: $T \to R_1 \to T \to R_2 ...$, where T denotes an observation on the target star, R_i is an observation on the reference target, and the arrow denotes the transition between the observations (they are not made simultaneously). The small modifications in the measurement model above required to take advantage of the true observing scenario will be discussed shortly, but first we make some very important observations about the basic linear model (21).

The overall objective is to detect and determine the motion $r(t)$ from the measurements y made over the mission lifetime. We first show that it is impossible to determine the basic astrometric and instrument parameters, h and r, at the tile level as shown by the argument below.

Proposition 1. For $M \geq 3$ the matrix A has a null space of dimension greater than or equal to six.

Proof. Using the parameterization in (13)–(15)

$$d_i^j - \langle s_i^0, B_0^j \rangle = \langle B_0^j, h_i \rangle + \epsilon \langle s_i^0, B_0^j \rangle + \epsilon_T^j \langle s_i^0, E(B_0^j) \rangle + \delta B_z^j \langle s_i^0, n \rangle + c_j. \tag{22}$$

Three elements of the null space corresponding to common rotations are immediately identified by setting $c_j = 0$, $(j = 1, ..., M)$ and recalling (10):

$$\delta s_i = m \times s_i, \quad \delta B^j = m \times B_0^j, \tag{23}$$

$$\delta s_i = p \times s_i, \quad \delta B^j = p \times B_0^j, \tag{24}$$

$$\delta s_i = n \times s_i, \quad \delta B^j = n \times B_0^j. \tag{25}$$

Recalling (13), this corresponds to

$$h_i = \sqrt{1 - |\pi(s_i^0)|^2} m, \quad \delta B_z^j = -\langle m, B_0^j \rangle, \tag{26}$$

$$h_i = \sqrt{1 - |\pi(s_i^0)|^2} p, \quad \delta B_z^j = -\langle p, B_0^j \rangle, \tag{27}$$

and

$$h_i = E(\pi(s_i^0)), \quad \epsilon_T^j = -1. \tag{28}$$

From (22) it is very easy to identify three other elements of the null space. Namely we set $\epsilon = \epsilon_T^j = \delta B_z^j = 0$, together with $h_i = m$ and $c_j = -\langle B_0^j, m \rangle$ to get the first. We do the same thing for the second, but with $h_i = p$ and $c_j = -\langle B_0^j, p \rangle$. And the last one is the dilation of the field obtained by setting $c_j = \epsilon_T^j = \delta B_z^j = 0$ together with $h_i = \pi(s_i^0)$ and $\epsilon = -1$. Using (7) we can the map these null vectors back to the sphere if we like./

Let these six identified null vectors be denoted as $u_1, ..., u_6$. Thus we have $Au_i = 0$. Corresponding to the partition of x in (31) we may partition A as

$$A = [A_h \; A_r], \tag{29}$$

so that

$$Ax = A_h h + A_r r, \; x = \begin{bmatrix} h \\ r \end{bmatrix}. \tag{30}$$

We will also have use for the partition of each of the u_i as

$$u_i = \begin{bmatrix} u_i^+ \\ u_i^- \end{bmatrix}, \tag{31}$$

so that $Au_i = A_h u_i^+ + A_r u_i^-$. Observe that four of these vectors $(u_3^+, ..., u_6^+)$ represent the "standard" affine transformation on a photographic plate. (See early paper and Hubble paper on astrometry.)Namely, u_3^+ is an expansion of the field, u_4^+ is a rotation of the field, and u_5 and u_6 are the translations of the field. However, the first two vectors are nearly translations modulo a small nonlinear function of the field position. We will see later that these contributions cannot in general be neglected, except in the case that the distance between the target and reference stars are (nearly) the same since then these vectors collapse to full translations. An important point here is that the u_i^+ only depend on the reference and target star nominal position vectors. We will see later that when $M = 2$, a seventh null vector appears such that its "+" component *is* a function of the baseline orientation vectors. This produces a significant difference in the ultimate narrow angle processing scheme for the case of just two baseline orientations.

The chopped matrix is obtained by subtracting the row for each baseline orientation corresponding to the target star from each of the rows containing the reference star parameters. This is tantamount to multiplying A on the left by an elementary matrix. Since an elementary matrix is invertible, the chopped matrix, which is the product, has the same null space as A.

As a corollary to the above proposition we have the following.

Proposition 2. Let h_i, $i = T, 1, ...N$, denote the true tangent vector corrections to $s_T, ...s_N$, and let $\hat{h}_T, ...\hat{h}_N$ denote the component of the estimate obtained from a least squares solution. Then there exist constants $\alpha_1, ..., \alpha_6$ such that

$$h_i - \hat{h}_i = \sum_{j=1}^{6} \alpha_j u_j^+, \quad i = T, 1, ..., N, \tag{32}$$

where the u_i^+ are given in (31).

Proof. By Proposition 1, if x denotes the true parameter vector (astrometric and instrument components) and $\hat{x}$ is any other solution, then $x - \hat{x}$ lies in the null space of A. This says that there exist α_j, $j = 1, ..., 6$ such that

$$x - \hat{x} = \sum_{j=1}^{6} \alpha_j u_j. \tag{33}$$

The result now follows from the partitioning of the vectors.///

Obtaining the target star motion estimate: three baseline case. In this section we will show how (32) can be used to generate the trajectory of the target star. There are two fundamental issues to resolve in the course of producing this solution. The first is a matter of sensitivity. That is, how stable is the solution $\hat{h}_i$ with respect to the measurement noise vector η in (21). The second issue is precisely how the unobservable modes in (32) are handled to obtain the trajectory. This section will deal with these obstacles for the case of 3 or more baseline orientations. The case of just 2 baseline orientations will be dealt with in the following section.

At each observation epoch t_j estimates of $h_i(t_j)$ are generated such that (32) holds. The first thing to know is that the small perturbation, η, introduced into the measurement equation (21) produces a small perturbation in the estimates $\hat{h}_i(t_j)$. That is, if we consider $A^\dagger$ and just partition the part off that produces the astrometric estimate:

$$\hat{h}(t_j) = PA^\dagger y, \quad P \begin{bmatrix} h \\ r \end{bmatrix} = h, \tag{34}$$

then $PA^\dagger$ has small norm. The magnitude is computed numerically and depends on the reference star geometry. It is also be shown numerically that the null space for three baseline orientations has dimension 6. Thus the null vectors are precisely those in Proposition 1.

Let U^+ denote the subspace generated by u_j^+; and let H_0 denote a matrix with 6 columns composed of any choice of six spanning vectors of U^+. Then we may rearrange (32) so that

$$\begin{bmatrix} h_T(t_j) \\ 0_{2N\times 1} \end{bmatrix} + H\alpha = \begin{bmatrix} \hat{h}_T(t_j) \\ \hat{h}_1(t_j) \\ \vdots \\ \hat{h}_N(t_j) \end{bmatrix} - \begin{bmatrix} 0_{2\times 1} \\ \hat{h}_1(t_j) \\ \vdots \\ \hat{h}_N(t_j) \end{bmatrix}, \tag{35}$$

for the proper choice of the six vector α of coefficients of the spanning vector set. In a more compact notation we may write (35) as

$$H \begin{bmatrix} h_T(t_j) \\ \alpha \end{bmatrix} = \hat{h}(t_j) - h^0(t_j), \tag{36}$$

where it is understood that $\hat{h}(t_j)$ is the first term on the right and $h^0(t_j)$ is the second term, and the matrix H is defined from H_0 adn (35) in the obvious manner. Note that $\hat{h}(t_j)$ is known, but $h^0(t_j)$ is not. However $h^0(t_j)$ is the vector of true perturbations of the reference stars and these by assumption obey (18). Thus we have

$$h^0(t_j) = h^0(t_0) + (t_j - t_0)V^0 + Q(t_j), \quad Q(t_j) = \begin{bmatrix} \pi_1 q_1(t_j) \\ \vdots \\ \pi_N q_N(t_j) \end{bmatrix}, \tag{37}$$

and v is the vector of proper motions. We note that $h^0(t_0)$, V^0, and π_i are all unknown.

Now consider $H^\dagger$, and let K denote the submatrix defined by its first two rows. Then K operating on the right side of (36) yields $h_T(t_j)$. The result of this is

$$h_T(t_j) = K\hat{h}(t_j) - Kh^0(t_j). \tag{38}$$

Now $\hat{h}(t_j)$ is the least squares estimate obtained from the regularized delay measurements, and thus is known. However $h^0(t_j)$ is not known. But observe that

$$Kh^0(t_j) = Kh^0(t_0) + (t_j - t_0)Kv^0 + K\pi Q(t_j), \quad j = 1, ..., N_{visits}, \tag{39}$$

where N_{visits} is the total number of visits to the narrow angle field. Now recall that since H is independent of t_j, then K is also. Thus there are two vectors h and v such that

$$Kh^0(t_j) = h, \quad Kv^0 = v, \quad j = 1, ..., N_{visits}. \tag{40}$$

Now

$$\pi q_i(t_j) = \pi_i[\langle s_i, \phi(t_j)\rangle\langle s_i, m\rangle - \langle s_i, \phi(t_j)\rangle]m + \pi_i[\langle s_i, \phi(t_j)\rangle\langle s_i, p\rangle - \langle s_i, \phi(t_j)\rangle]p. \tag{41}$$

And $\phi(t)$ is of the form

$$\phi(t) = a\sin(t)e_1 + b\cos(t)e_2, \tag{42}$$

for orthogonal unit vectors e_1 and e_2 that span the ecliptic plane. Thus

$$\pi q(t_j) = \pi[a_{mi}\sin(t) + b_{mi}\cos(t)]m + \pi[a_{pi}\sin(t) + b_{pi}\cos(t)]p, \tag{43}$$

for constants a_{mi}, b_{mi}. Next write $K = [K_1 \; K_2 \cdots K_N]$, where each K_i is a 2×2 submatrix of K. Then

$$\begin{aligned} K\pi Q(t_j) &= \sum_i K_i \begin{bmatrix} a_{mi}\sin(t) + b_{mi}\cos(t)] \\ a_{pi}\sin(t) + b_{pi}\cos(t) \end{bmatrix} \\ &= \begin{bmatrix} A_m\sin(t) + B_m\cos(t) \\ A_p\sin(t) + B_p\cos(t) \end{bmatrix}, \end{aligned} \tag{44}$$

for some constants A_m, A_p, B_m, B_p. In general these constants are unknown unless the parallaxes are known (and then there would be a small error due to the uncertainty in the positions of the stars). However, what is most significant is that these constants are *independent* of time. So now we have the relationship

$$h_T(t_j) = K\hat{h}(t_j) + h^0 + (t_j - t_0)v + \begin{bmatrix} A_m \sin(t) + B_m \cos(t) \\ A_p \sin(t) + B_p \cos(t) \end{bmatrix}. \tag{45}$$

On the right side of the equation only the first term is known and obtained from the observations. But note that the subsequent terms constitute, respectively, a position offset, a proper motion, and a periodic component with one year period. These terms are indistinguishable from the contributions of the target star proper motion, parallax, and position error at the initial epoch. Recall the model of target star motion defined in (20). Let P denote the projection onto the space of motions spanned by proper motion, one year periods (without harmonics), and constant offset. Then P has the same range as the matrix

$$J = \begin{bmatrix} J_0 & 0 \\ 0 & J_0 \end{bmatrix}, \quad J_0 = \begin{bmatrix} 1 & t_1 - T_0 & \sin(t_1) & \cos(t_1) \\ \vdots & \vdots & \vdots & \vdots \\ 1 & t_{N_{visits}} - T_0 & \sin(t_{N_{visits}}) & \cos(t_{N_{visits}}) \end{bmatrix}. \tag{46}$$

Applying the projection $I - P$ to (20) annihilates the proper motion and *all* motions with pure one year periods (in particular the parallax of the target). Thus applying $I - P$ to (45) results in

$$(I - P)r = (I - P)K\hat{h}. \tag{47}$$

The term on the right is determined from the data. Thus the projection of the "interesting" part of the trajectory of the target onto the orthogonal complement of proper motion and pure one year periods is observable.

Obtaining the target star motion estimate: two baseline case In the two baseline case a 7th vector appears in the null space of A, which necessitates a few changes in the data reduction process. What makes the largest change however, is that this vector is dependent on the orientation, and thus the spanning set of vectors for the null space changes from epoch to epoch. In the event that a fixed pointing can be used, then this is not an issue. But because of solar exclusion angles, etc., this case is only realizable for a very small portion of the sky. Thus in general we may expect this 7th null vector to not be fixed (recall that the linear combinations of the null vectors are in general never the same, regardless of whether the pointing is the same or not). So to define this new null vector let B_1^0 and B_2^0 denote the nominal baseline orientations. We set the astrometric differentials so that they are all orthogonal to baseline B_1^0:

$$\langle B_1^0, h_i \rangle = 0, \ \ i = T, ..., N, \tag{48}$$

together with $c_1 = \epsilon = 0$ and $\delta B_1 = 0$. Next we determine δB_2 to satisfy

$$\langle B_2^0, h_i \rangle + \langle s_i^0, \delta B_2 \rangle = 0, \ \ i = T, ...N, \tag{49}$$

with $c_2 = 0$. It is easily shown that the following choices work:

$$h_i \langle E B_2^0, s_i^0 \rangle w/d, \quad w = E(B_1^0)/|E(B_1^0)|, \ \ d = \langle B_2^0, w \rangle, \tag{50}$$

withe $\delta B_2 = E(B_2^0)$.

Let $w = E(B_1)/|E(B_1)|$, and set $d = \langle B_2^0, w \rangle$. For $j = 1, ..., N$, $g(j) = \langle E(B_2^0), s_j^0 \rangle / d$. Then the associated star astrometric parameter is $[g(1)w \ \cdots \ g(N)w]$. This is the new unobservable motion.

The processing now proceeds just as in the three baseline orientation case, except now the matrix H in (36) has an additional column to accommodate the extra null space vector and since this vector changes from epoch to epoch, the submatrix K comprised by the first two row of $H^{\dagger}$ is no longer constant over the mission. This necessitates a change in the dimension of the subspace of motions corresponding to the proper motion and parallax of the reference stars.

Now we give an example of this method. We implement the SIM PlanetQuest narrow angle scenario which relies on grid stars and compare it with the two baseline orientation gridless solution. Proper motions, parallax, are included in these examples. In addition a single measurement error of 1uas is also present. Both simulations use 100 2–D measurements made uniformly over a 5 year period. The target star has two planets with circular orbits of periods .83 and 1.66 years with amplitudes of 3uas and 2uas, respectively. The first figure below shows the trajectories determined by the two methods. The trajectory on the left uses grid information, while the trajectory on the right is the result of the two baseline orientation gridless processing.

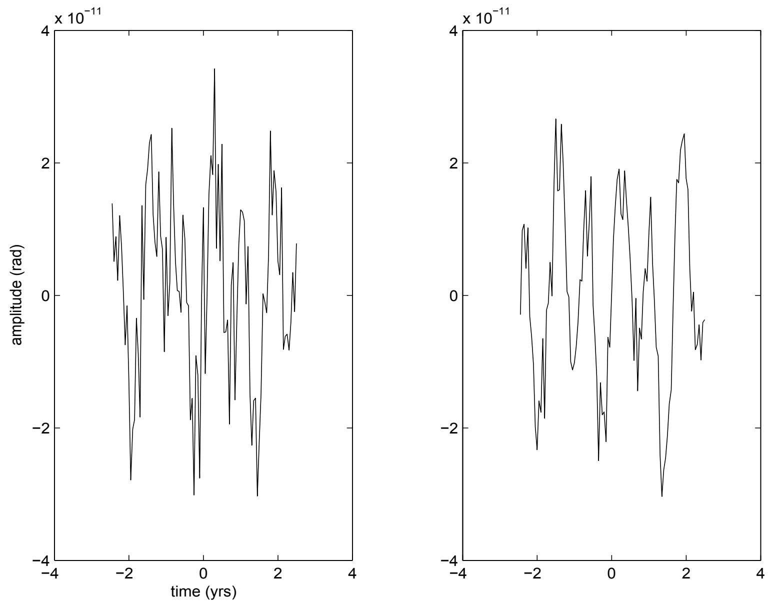

Figure 1. Target star trajectory signals produced with gridless (left) and grid (right) astrometry.

The actual proof is in the pudding however. We applied the super–resolution technique MUSIC to these signals to pull out the two largest harmonics that are present. These peaks are shown in the figure below. Note that the data created with either the solution that uses the grid and the gridless data readily identify these peaks.

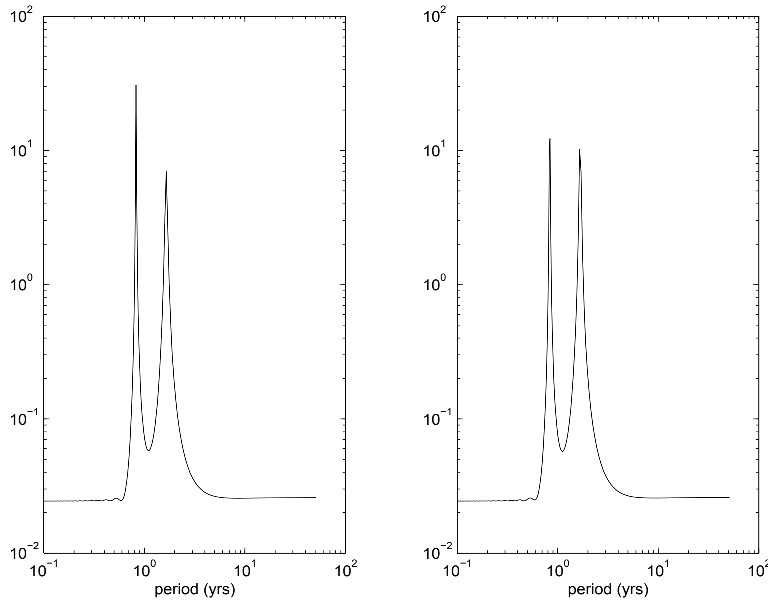

Figure 2. Estimation of periods — gridless (left) and grid (right).

Concluding remarks. Basic feasibility has been established for a gridless two baseline orientation approach to narrow angle astrometry. Future work will concentrate on further comparisons with the full grid approach. Notably these would include actually extracting planetary orbits from the data, the effect of nonuniform sampling, and a more complete characterization of the effects of parallax on the detection and determination of planetary orbits with periods close to one year.

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