Precise Orbit Determination for LEO Spacecraft Using GNSS Tracking Data from Multiple Antennas

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BIOGRAPH

Da Kuang received his Ph.D. in Aerospace Engineering from The University of Texas at Austin in 1995. He joined the Orbiter and Radio Metric Systems Group at JPL in 1996. His work has been focused on analyzing GNSS tracking data for precise orbit determination and precise relative positioning.

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ABSTRACT

To support various applications, certain Earth-orbiting spacecrafts (e.g., SRTM, COSMIC) use multiple GNSS antennas to provide tracking data for precise orbit determination (POD). POD using GNSS tracking data from multiple antennas poses some special technical issues compared to the typical single-antenna approach. In this paper, we investigate some of these issues using both real and simulated data. Recommendations are provided for POD with multiple GNSS antennas and for antenna configuration design. The observability of satellite position with multiple antennas data is compared against single antenna case. The impact of differential clock (line biases) and line-of-sight (up, along-track, and cross-track) on kinematic and reduced-dynamic POD is evaluated. The accuracy of monitoring the stability of the spacecraft structure by simultaneously performing POD of the spacecraft and relative positioning of the multiple antennas is also investigated.

INTRODUCTION

Multiple GNSS antennas have been used on certain Earth-orbiting spacecrafts, for various reasons, to provide tracking data for precise orbit determination. Some missions may require precise positioning of different sensor locations on board the spacecraft, in addition to the center of mass of the spacecraft (e.g. for SRTM mission, Bertiger, et al., 2000). Some spacecraft design may require the GNSS antenna be placed to point to certain direction, away from multipath sources. In such cases multiple antennas may be used to compensate the limited sky view (e.g., Kuang, et al., 2008). For multipurpose scientific missions, spacecrafts with complex structure are expected to carry multiple instruments with different functions and the complexity of the spacecraft structure will limit the options for GNSS antenna placement. Multiple GNSS antenna tracking may provide larger total view of the sky when the field of view for single antenna deployment is limited. Whenever one antenna is added, however, a differential clock bias parameter for that antenna should be added to the estimates in the position solution because the line bias is not usually calibrated for the deployment. Further more, the continues carrier phase data arcs are usually cut short because the same GNSS tracking pass for single antenna would be divided into multiple passes with multiple antenna tracking. For precise orbit determination that depends much on the carrier phase measurements, this is a severe shortcoming. The objective of this paper is to study how these factors would affect the positioning of a lower Earth orbiter using phase measurement and range measurement, to provide a guideline for the optimum design of GNSS tracking plan and estimation strategy for precise LEO orbit determination.

We study these issues through simulation and real data analysis. A truth orbit is created using the orbital elements as defined in Table 1 for a sun-synchronous orbiter at 760-kilometer altitude. Simulated measurements are generated for various antenna location and orientation plans, over 24 hours period. Measurement errors with statistical characteristics gained from real mission tracking data analysis are added to the simulated measurements. Orbital positions over the 24-hour period solved from the simulated measurements in each case can be compared with the truth orbit to evaluate the orbit errors.

Table 1. Orbital elements of the simulated LEO.

Semi-major axis	7148.425104 km
Eccentricity	0.0012411
Inclination	98.45912 degree
Ascending Node	269.999921 degree
Arg. of Perigee	67.65481 degree
Mean anomaly	292.482168 degree
Time of epoch	2008-09-23 08:20:00 UTC

The simulated measurement types in our study are ionosphere-free combinations of dual-frequency pseudo-range (PC) and carrier phase (LC) measurements, with white noise measurement errors of 1 meter and 1 centimeter (1 σ), respectively.

We use the single up-looking antenna tracking plan as a baseline case, to evaluate the performance of different multiple antenna tracking plans. The precise orbit determination of LEO using a single up-looking antenna tracking data has been well studied and widely tested for various missions [Bertiger et al., 1994, 2002; Haines et al., 2003]. It has been long proved that in the presence of dynamic model errors, reduced-dynamic orbit determination technique provide better orbit solution than either the pure kinematic solution or the pure dynamic orbit determination results. A properly tuned reduced-dynamic strategy optimally estimates a time series of stochastic accelerations to compensate the errors in the dynamic models [Yunck et al., 1990; Wu et al., 1991]. Figure 1 shows one example of the RMS of orbit error from the orbit solutions using the simulated single up-looking antenna tracking data, in the presence of simulated dynamic model errors. We use the reduced-dynamic solution with one up-looking antenna tracking as the standard for POD performance in this study. Since the dynamic effect of adding an antenna or orientating an antenna differently is ignorable, all the differences are almost due to the tracking geometry effect alone.

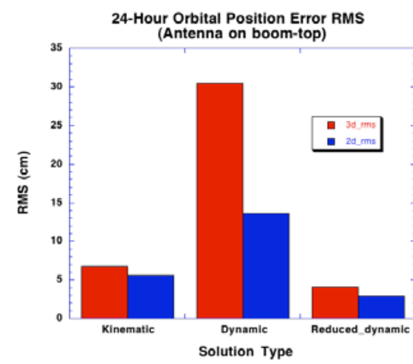

Figure 1. Orbital error of different solution types, where 2d is in horizontal space.

OBSERVABILITY

For positioning purpose the GNSS tracking data can be regarded as range measurement between transmitter and receiver plus the clock bias, with the transmitter position and clock bias known either from the broadcast ephemerides or after-fact precise orbit solution. For a single-antenna positioning problem, the impact of tracking geometry on positioning accuracy is well reflected in the geometric Dilution of Precision (DOP). Given the range measurement noise level as 1, DOP is the variance of the point position solution from the range measurements (e.g., Wells, et al., 1986). For a well-determined solution the DOP value is small, and a poor determined solution has big DOP value. As a special occasion, when all GPS satellites being tracked fall on the surface of a cone, the positioning problem becomes singular, and the position component along the central axis of the cone cannot be separated from the receiver clock bias. Mathematically the axis of the cone is an exact eigenvector of the normal matrix of the point position estimation problem with the eigenvalue zero. This can be easily verified by multiplying the unit vector of this axis by the normal matrix (Kuang, et al., 1996).

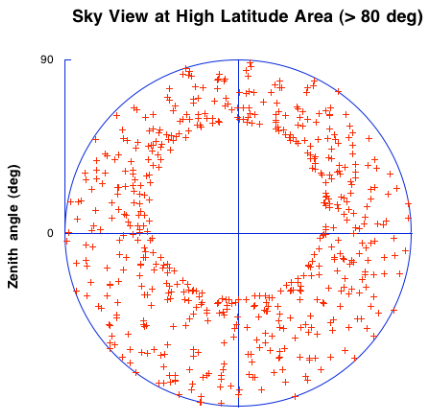

Figure 2. View of all GPS satellites by the LEO at high latitude area.

At high latitude area, the GPS satellites in the sky view are limited inside a narrow ring due to the 55-degree inclination angle of the GPS satellite orbits. Figure 2 shows the sky view of all GPS satellites by the simulated LEO satellite, with one up-looking antenna, at all locations with latitude greater than 80 degrees. There is no GPS satellite can be viewed around the zenith area. Apparently this is not a favorable geometry for positioning. The positioning accuracy (for height component in particular) is affected in the high latitude area and this is reflected in the DOP values as functions of latitude.

DOP of Multiple Antennas vs. Single Antenna

Figure 3 shows the one up-looking antenna tracking plan. An extended boom to holds the GNSS receiver antenna above any possible multipath sources of the spacecraft body, a design actually used in the Topex/Poseidon mission. Values of DOP for each individual component of the point positioning solution corresponding to this tracking plan are plotted, each point being one position solution. Of the three components, cross-track shows most stable accuracy. Both along-track and height components show DOP values varying depending on latitude. In particular the height component shows larger DOP values in the high latitude areas, meaning the height component is worst determined in high latitude areas.

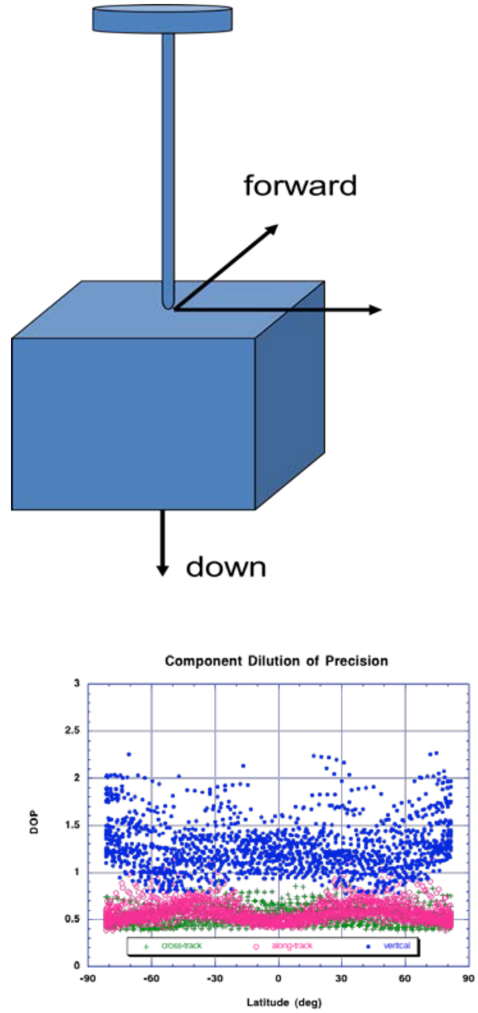

Figure 3. Up-looking antenna tracking and corresponding DOP values.

DOP values for multiple antenna cases are not any better. As a matter of fact, the observability is worse for multiple antenna cases because additional differential clock parameter has to be solved for, holding some of the positioning data strength. In particular, the positioning accuracy is reduced in the direction of antennas' pointing. To study the effect of the multiple antenna on position observability, we simulated two cases to compare with the single antenna case. In the first case the LEO has two receiver antennas, one pointing horizontally towards the front direction, the other pointing to the rear direction. In the second case the two antennas point horizontally in orbital cross-track direction, one towards left and the other towards right. With both designs the combined sky view of the two antennas together is the same as the single up-looking antenna, limited by the local horizon only, so that all the three cases have the same numbers of observed GPS satellites and total number of tracking data points.

Figure 4 shows the front-rear looking tracking plan and the corresponding DOP values. With the same total number of observed GPS satellites, the sky is divided by the two antennas into the front-looking half and the rear-looking half. The DOP value for the along-track component is significantly raised compared with the single antenna case while the other two components remain largely the same. The along-track component is the one that correlates with the differential clock bias most thus has positioning accuracy reduced most.

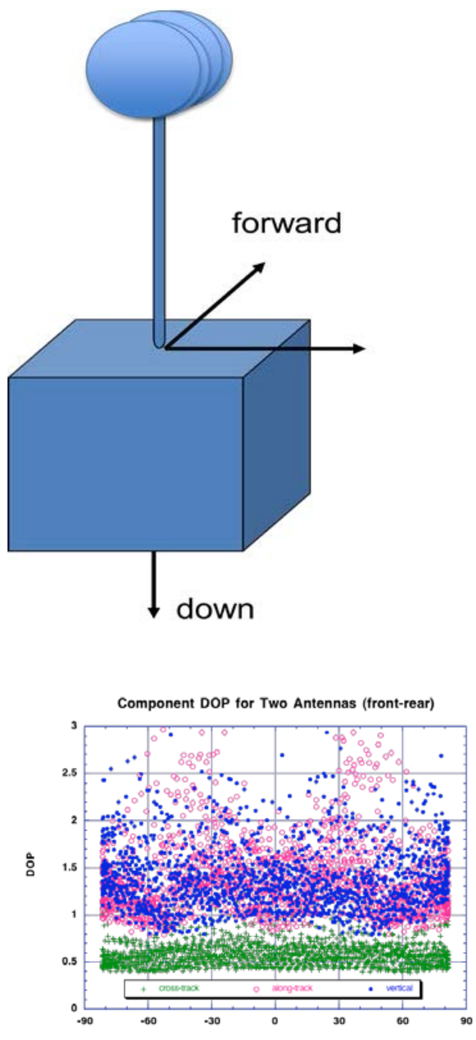

Figure 4. Front-rear looking antenna tracking and corresponding DOP values.

Similarly the positioning accuracy is reduced in the cross-track direction for side-looking antennas tracking plan, as shown in Figure 5. In this case the sky is divided into left-looking and right-looking halves by the two antennas, and the added differential clock bias estimation correlates with the cross-track component position estimate. The accuracy in along-track and height components remains about the same as those in the single antenna tracking case.

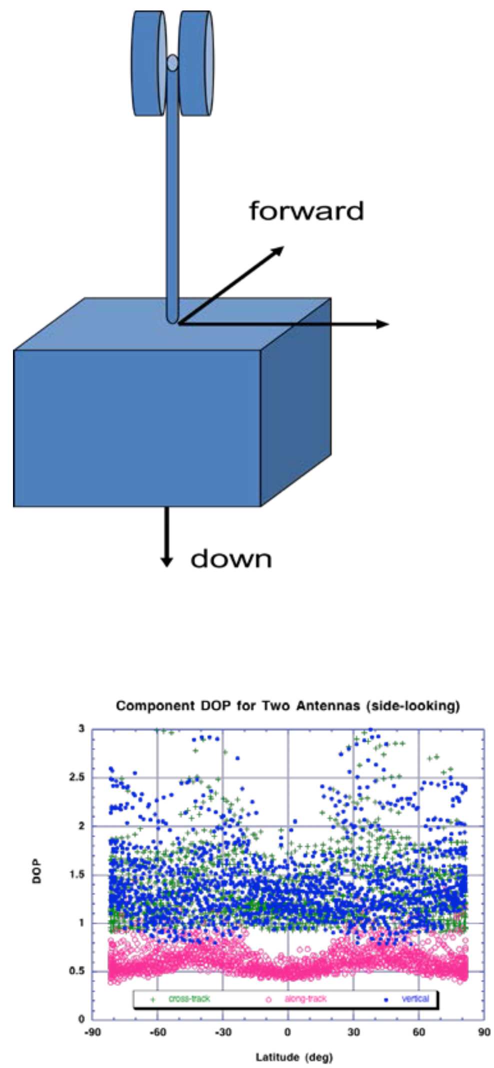

Figure 5. Side-looking antenna tracking and corresponding DOP values.

This epoch-by-epoch feature of DOP reflects the observability with range data only, and by estimating the differential clock at each data epoch without constraint it is same as using two independent receivers with one antenna for each receiver. For precise orbit determination, we always rely on carrier phase measurements for accurate positioning solution. In addition, in the real tracking data the differential clock bias is expected to be relatively stable, compared with the receiver clock oscillation itself. As a result, we can usually apply certain type of constraint on it instead of estimating it as a free white noise process.

Formal Errors in Kinematic Solution

To study further the position observability for more realistic orbit determination practice, we investigate the formal error of the orbit position solution. 1 meter of pseudo-range measurement noise and 1 cm of phase data nose are used to compute the formal error in the position solution. First we focus on the tracking geometry effect, and constrain the differential clock bias as a constant over the whole 24-hour arc. The effect of differential clock bias variation will be investigated later separately.

Figure 6 shows the formal error of the kinematic positioning solution with one up-looking antenna. With carrier phase measurements included, the cross-track component is still the best-determined one in the kinematic position solution. Height component accuracy shows less latitude dependency now because of the time smoothing effect of the carrier phase measurements.

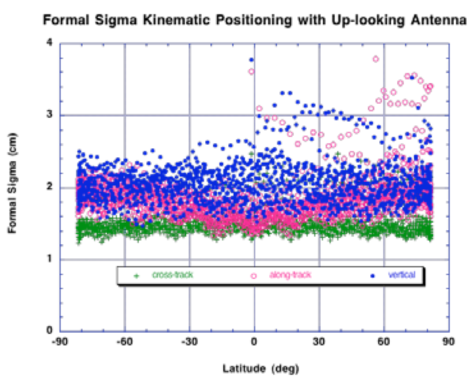

Figure 6. Formal sigma values as function of latitude (with one up-looking antenna).

In comparison with the single antenna tracking case, the formal errors of the kinematic positioning solutions with two-antenna tracking are all higher, regardless the ways the antennas are oriented. Figure 7 shows the increase of formal error from the single antenna case if only pseudo-range measurements are used. For the front-rear looking tracking, the error in along-track direction is increased, while for the side-looking tracking the cross-tracking direction is increased. This is consistent with the results from the DOP value comparisons. The only real difference here is that the differential clock bias is estimated as one constant over 24 hours instead of white noise time series updated at each measurement time. The increase of formal error is very small in this case, smaller than 3%. This means that estimating the differential clock bias as an additional parameter is the only reason for the weakened observability for multiple antennas tracking with range measurements, and this weakening can be limited by constraining the parameter with proper knowledge of the feature of the differential clock bias.

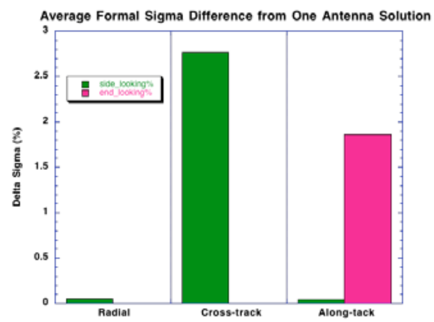

Figure 7. Formal sigma increase from the single-antenna case (using range data only).

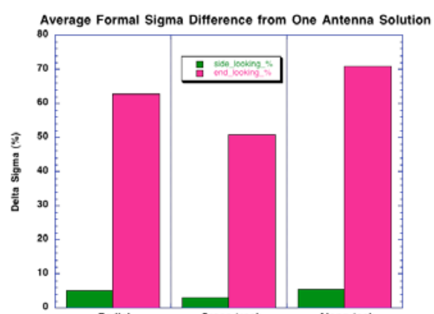

Figure 8. Formal sigma increase from single-antenna case (using both range and phase data).

The formal errors in kinematic orbit solution with multiple antennas tracking increase much more from the single antenna tracking case when both pseudo-range and carrier phase measurements are used, because of the phase breaks. In particular the front-rear looking antennas cut almost every phase data pass into two, and significantly decrease the data strength in the position solution. As shown in Figure 8, formal error increases more than 50% in all three components for the front-rear looking antennas tracking plan, while for the side-looking tracking plan the formal errors increase by about 5% only.

Orbit Error in Reduced-dynamic Solution

In reduced-dynamic orbit solution, stochastic accelerations are estimated from tracking data fit to compensate the orbit dynamic model error. The dynamic model errors are primarily independent of the receiver antenna location and orientation, but the data strength change due to tracking geometry still affects the solution. Figure 9 shows the true orbit error RMS of the reduced-dynamic orbit solution with multiple antennas tracking increased from the one antenna tracking solution. Differential clock bias is estimated as white noise time series at each measurement time. Again, the front-rear looking antennas tracking shows significantly bigger orbit errors, because the phase data passes are cut short. The side-looking antennas tracking shows relatively mild increase of orbit error, mainly in cross-track direction.

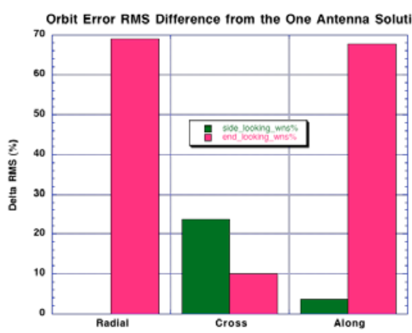

Figure 9. True orbit error increase due to multiple antennas tracking geometry.

All the simulation studies show that with the same number of tracked GPS satellites and the total number of data points, quality of position solution with multiple antennas is always lower than that with single antenna. For orbit accuracy purpose, using single up-looking GNSS receiver antenna without multipath is always the best choice. When that option is not available and multiple antennas have to be used, the optimal way of placing the multiple antennas is to keep longer data pass for phase measurements. In that sense, side-looking tracking plan is much more desirable than the front-rear looking plan.

DIFFERENTIAL CLOCK BIAS

Each antenna has a differential clock bias due to the extra delay in the cable line and channel circuit. The differential clock bias for each antenna and the main receiver clock bias cannot be all estimated together and simultaneously. To avoid the singularity of the estimation problem, a reference antenna has to be chosen, similar to the situation that a reference clock has to be defined for the network of receivers and transmitters. Nevertheless, the choice of reference antenna has no effect on positioning result. The time tag difference due to the line-bias at the spacecraft dimension size (~10m) is on the level of 30-nanosecond, equivalent to 0.2 mm of a LEO motion, and negligible to our concern for now. For the simulation cases in this study, we tested using either one of the two antennas as the reference antenna, they all yield the same orbit position solution.

Differential Clock Bias from Simultaneous Tracking

Estimating the differential clock biases reduces the data strength by increase the number of estimated parameters. The simulation studies in the previous session have shown that this decrease of the accuracy is in the general direction of the bore sight of the additional antenna. This decrease, however, is limited if the differential clock bias can be constrained.

So far our simulation does not include simultaneous tracking of the same GNSS transmitter. That means, at any epoch, if one GNSS transmitter is tracked by one receiver antenna then it is not tracked by another antenna at the same time. In theory the simultaneous tracking of the same GNSS transmitter by multiple antennas does not provide any new information about the LEO position, except reducing the measurement noise through averaging. It does, however, provide the information about the differential clock bias at each moment of measurement, similar to what two receivers provide on a zero baseline.

To study how the simultaneous tracking would improve the data strength for differential clock bias estimation and thus improve the orbit solution, we slightly modify the antenna orientation in the simulation plans shown in Figure 4 and Figure 5. We let each antenna tilted up by 15 degree now so that there is 30 degree of overlap in the combined sky view. Each GNSS satellite within the overlapping area is tracked by both antennas simultaneously.

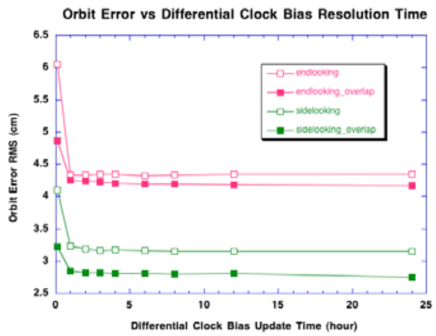

Figure 10. 3D orbit position error with and without overlapping sky.

Figure 10 compares the orbit position error from the two different tracking strategies, with and without overlapping sky. The result shows that the overlapping data improve the orbit solution mostly if we solve the differential clock bias as white noise at each measurement time. The improvement is very limited if the differential clock bias can be constrained as constant over certain period of time, especially for the front-rear looking configuration where carrier phase data passes are short. For the cross-track looking case, the overlapping sky added some longer carrier phase data passes and made a little more improvement. Still, the improvement is most significant only if we estimate the differential clock bias at each measurement time.

Variation of Differential Clock Bias

The next question is whether a real differential clock bias should be constrained as a constant over certain period of time. Real tracking data from missions using multiple GPS receiver antennas such as COSMIC show that the differential clock biases are indeed relatively stable, compared with the receiver clock oscillation itself.

In the analysis of GPS data from COSMIC satellites (Kuang, et al., 2008), we choose antenna-1 as the reference antenna and estimate only one differential clock bias for antenna-2 over the daily orbit arc, while estimating the main receiver clock bias as unconstrained white noise process at each measurement time. Figure 11 shows the variation of the estimated daily differential clock bias for COSMIC4 over 40 days' period. The standard deviation of the time series is 20 cm.

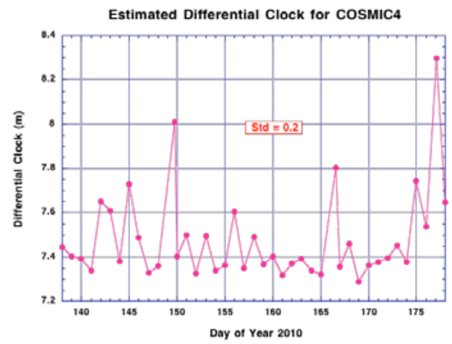

Figure 11. Estimated differential clock bias for COSMIC4 satellite.

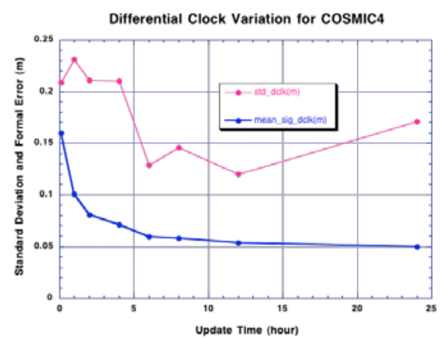

Figure 12. Variation of estimated differential clock bias for COSMIC4 satellite at different update time.

We can also examine the variation of the differential clock bias at higher rate by estimating it more frequently. Figure 12 shows the standard deviation of the same differential clock bias estimated at every 5 minutes to every 24 hours, over one week's period. Also included in Figure 12 is the average formal sigma of the estimated differential clock bias for the corresponding update time. Both Figure 11 and Figure 12 show the variation is at 20 cm level or lower. Although the standard deviation is relatively higher at the higher frequency regime, there is a possibility that part of the variation is due to the estimation error because the formal error increases rapidly as the update time is reduced. This possibility is also suggested by

Figure 13, which shows the RMS of orbit position overlap error for different differential clock bias update time. Again, the formal sigma is included for comparison. As we can see, the orbit solution quality does not improve, as indicated by the overlap error RMS, as we try to solve for the differential clock bias more frequently. Instead, the orbit error increase significantly as we push to the limit to solve the differential clock bias at every measurement time.

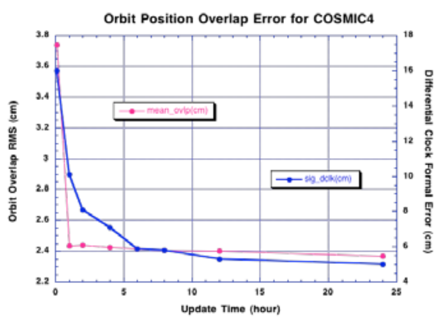

Figure 13. 3D Orbit overlap error for COSMIC4 satellite for different differential clock bias update time.

Whether it is because the differential clock bias does not change much in time, or the tracking data does not support the resolution of the differential clock bias at higher rate, the experience from COSMIC data analysis shows that constraining the differential clock bias as a constant over the daily orbit arc is a good practice for better orbit precision. This limits the unfavorable data strength of the multiple antennas tracking to minimum as discussed in previous sections.

RELATIVE POSITIONING BETWEEN ANTENNAS

As previously mentioned, simulation and real data analysis show that up-looking antenna provide the best POD result. However, some times we do not have that choice for various reasons, and multiple antennas have to be used as alternative. Further more, even if we place an up-looking GNSS antenna on an extended boom, as it was done on Topex, we may still have to use multiple antennas to monitor the relative position variation of the POD antenna on top the boom with respect to the spacecraft body and other onboard instruments. Figure 14 shows the conceptual structure of a spacecraft, as an example. A big radar reflector and other scientific instruments limit the options for the GNSS antenna deployment. The best way for precise POD tracking is to put one antenna on top of a boom, above all multipath sources. However the location of the antenna with respect to the center of mass of the spacecraft cannot be precisely determined before launch. Actual location of the antenna after deployment has to be precisely measured in space, because without precisely knowing the antenna location, the location of the center of mass of the spacecraft cannot be precisely determined. Additional GNSS antennas are proposed be placed on the spacecraft body and the tracking data be use to measure the location of the top antenna in relative positioning.

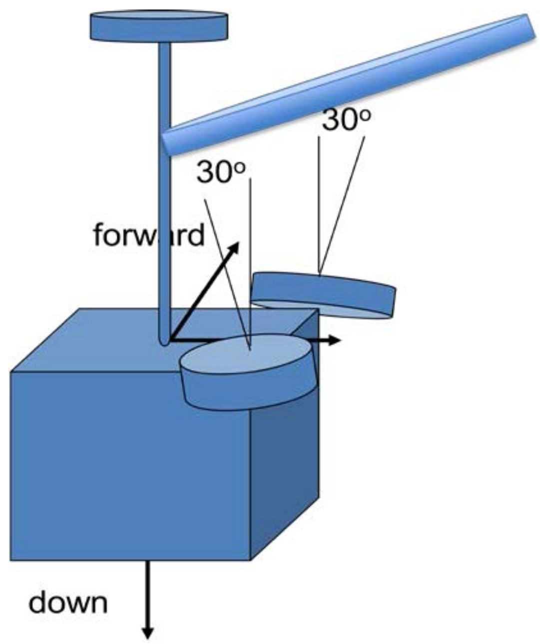

Figure 14. The concept of using multiple GNSS antennas on a radar mission.

Simulated data are generated, based on orbit elements in Table 1, to study how well this tracking plan can support the precise orbit determination. Figure 15 shows all the data passes over 24 hours period, on the top antenna. Figure 16 shows all the data passes, of the same GPS satellites over the same period as those in Figure 15, tracked by the two antennas on the spacecraft body. This tracking geometry, limited by the radar reflector and the local horizon, is definitely not a favorable one, worse than the one in Figure 4. If we use the data from Figure 16 for orbit determination, the orbit error would be worse than those shown in Figure 9, as compared to using data from the top antenna. Using data from the top antenna we have the best tracking geometry for the position solution of the antenna as we can have, the question now is whether the antenna position solution can be accurately linked to the spacecraft center of mass position and thus locations of other scientific instruments. Since the POD antenna location error in the spacecraft body-fixed frame maps one to one into the center of mass location error, we need to determine the antenna location to centimeter level accuracy to support centimeter level precise orbit determination.

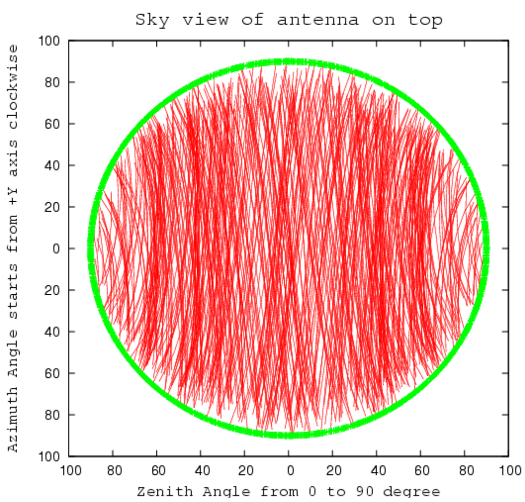

Figure 15. The sky view of the antenna on the top of the boom.

Figure 17 shows the true error of the relative position of the top antenna, solved from the simulated data from all 3 antennas in relative positioning. Location of the two antennas on the spacecraft body are held fixed, receiver clock bias is estimated as white noise at each measurement time, differential clock biases are estimated as constant over the daily orbit arc. The location of the top antenna is estimated as white noise process at different time interval. Average of the 3D true errors for each update time rate is plotted. Similar to the result in the differential clock bias variation study, Figure 17 shows that the tracking data strength does not support precise monitoring of the antenna location at high frequency. But if the top antenna location is stable over 8 hours, the tracking plan is well suited to determine the antenna location at accuracy of better than 1 cm.

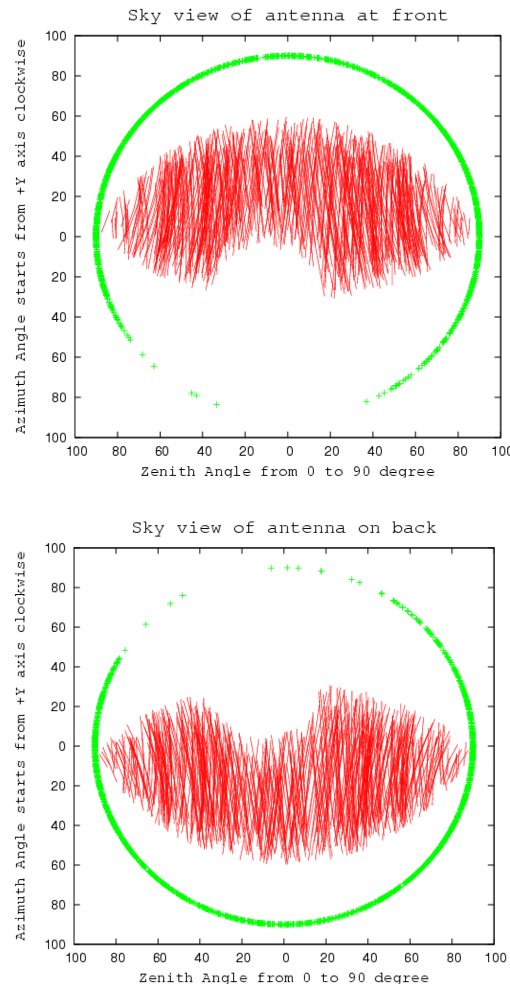

Figure 16. The sky views of the two antennas on the spacecraft body.

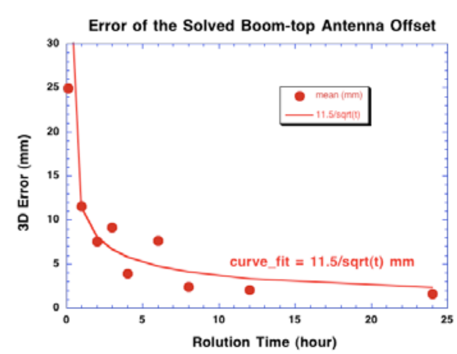

Figure 17. Mean RSS error of estimated location of the boom-top antenna.

The location of the top antenna in the spacecraft body-fixed frame can be solved simultaneously with the spacecraft center of mass position in the precise orbit determination process. Figure 18 shows the true error of the top antenna offset solved simultaneously with the orbit position in the precise orbit determination. All differential clocks and antenna-offset parameters are estimated same as in the relative positioning case. The antenna location is determined slightly better when solved together with the orbit solution. For comparison the orbit error is also plotted in Figure 18. The error of the antenna location estimation is well below the orbital position error for update time longer than a few hours, thus good for monitoring the antenna variation to support precise orbit determination.

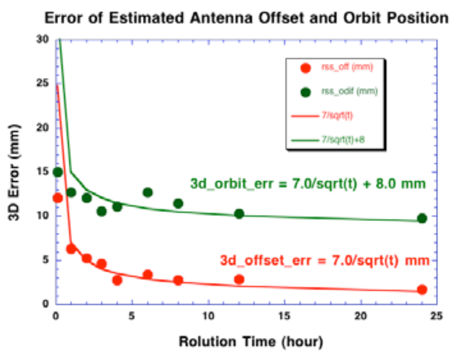

Figure 18. Mean RSS error of estimated location of the boom-top antenna and orbital position.

SUMMARY

Using GNSS tracking data from one up-looking antenna usually provides best tracking geometry and most precise orbit determination solution for LEO spacecraft. When the sky view of a single antenna is limited, multiple antennas can be used to make some compensation. Multiple antenna tracking weakens the data strength, as compared with the single up-looking antenna, in two aspects. One is the correlation of the differential clock bias with position information in the direction of antenna bore sight. The other is the breaking of the carrier phase data pass. For the relatively stable differential clock bias, the estimation of the differential clock bias can be constrained as a constant over certain period of time, and the first effect of the data strength weakening by the multiple antennas tracking can be limited to minimum. Overlapping tracking by multiple antennas is not necessary unless the differential clock bias is white noise. When carrier phase data are used, the breaking of the phase data passes is a much more severe damage. When possible the antennas should be placed to retain longer carrier phase data pass. In that sense, side-looking tracking is more desirable than front-rear looking tracking.

Multiple antennas tracking can also be used to monitor a spacecraft's physical structure, through relative positioning the locations of individual antennas. The combination of one up-looking antenna on extended boom with multiple antennas on the spacecraft main body may provide best precise orbit determination solution for spacecraft with complex structure.

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