Mitigation of Angle Tracking Errors due to Color Dependent Centroid Shifts in SIM-Lite

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ABSTRACT

The SIM-Lite astrometric interferometer will search for Earth-size planets in the habitable zones of nearby stars. In this search the interferometer will monitor the astrometric position of candidate stars relative to nearby reference stars over the course of a 5 year mission. The elemental measurement is the angle between a target star and a reference star. This is a two-step process, in which the interferometer will each time need to use its controllable optics to align the starlight in the two arms with each other and with the metrology beams. The sensor for this alignment is an angle tracking CCD camera. Various constraints in the design of the camera subject it to systematic alignment errors when observing a star of one spectrum compared with a star of a different spectrum. This effect is called a Color Dependent Centroid Shift (CDCS) and has been studied extensively with SIM-Lite’s SCDU testbed. Here we describe results from the simulation and testing of this error in the SCDU testbed, as well as effective ways that it can be reduced to acceptable levels.

Keywords: SIM, Interferometry, Astrometry, Spectral, Angle Tracker, SCDU

1. INTRODUCTION

SIM-Lite is a space-borne astrometric interferometer which, over its five years of operation, will reach sub micro-arcsecond astrometric accuracy for selected narrow angle targets. It is a multi-purpose instrument which will make significant and possibly revolutionary measurements affecting many branches of astrophysics. It is perhaps best known for its unique ability to fully characterize the orbits and masses of any Earth-size planets that may exist in the habitable zones of nearby stars. SIM-Lite is shown in Figure 1 and is described in detail elsewhere.1

Figure 1: SIM-Lite flight system (left) and the astrometric beam combiner optical layout (right). The incoming beams from the two arms are shown by arrows. The ATC optics are highlighted by box (the bottom right).

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At the heart of the instrument’s science interferometer is its astrometric beam combiner (ABC), described in another paper presented at this conference. The ABC receives light from the two arms of the interferometer and combines them to produce spectrally dispersed white light fringes on its fringe tracker camera. Additionally, the ABC splits off part of the light from each arm and images it using its angle tracker camera in order to provide alignment information and angle tracking (Figure 1).

SCDU is a breadboard version of the ABC along with additional optics to create a self-contained testbed interferometer operating in a vacuum chamber. Figure 2 shows both the layout of the testbed and a simplified view. The testbed has a white light source with a tungsten halogen light outside the chamber. The light is sent to the testbed via a single-mode fiber. This simulated star light is fed in from the ‘back end’ of the interferometer, where it is collimated, split into two beams, and travels down each of the ‘left’ and ‘right’ arms of the testbed to two siderostats. The siderostats retro-reflect the beams back to the beam splitter where they are recombined. Before the light is recombined, a separate pair of beam splitters send a small part of each arm’s light to the interferometer’s angle tracker camera (ATC), where they are focused as two separate spots on the ATC CCD. Meanwhile, the combined light is sent through a polarizing beam splitter, then a dispersing prism, and finally imaged on the face of the fringe tracker camera CCD as a spectrally dispersed fringe.

Since the testbed’s purpose is expressly to study the impact of spectral effects on measurements of the optical pathlength difference, the realization of a narrow angle observation consists simply of inserting two different filters into the white light source and measuring the optical pathlength difference (OPD) simultaneously using white light and metrology, once with one filter (designated target) and then with the other (designated reference). In the absence of errors, the OPD should measure out to be zero. In practice, phase dispersion will cause many nanometers of OPD and will need to be calibrated out in order to achieve the SIM-Lite goal of 1 pm OPD error. The interferometer’s performance is measured using the metric:

\[ \varepsilon_{SCDU} = d_{target} - d_{reference} \]  

where \( d \) is the calibrated white light OPD corrected for internal path differences. For SIM this quantity contains signal and is necessarily non-zero. For SCDU, operating as it does in retro condition, there is no signal but only the error that SIM would experience. The expression for the calibrated delay \( d \) is:

\[ d = (\delta \phi - \delta \phi_{\text{cal}}) - \delta M \]  

In this expression, \( \delta \phi \) is the measured white light fringe delay, \( \delta \phi_{\text{cal}} \) is the correction for spectral effects as obtained from calibration, and \( \delta M \) is the metrology measurement. Because the light is spectrally dispersed there are in fact six such measurements at a time, from the six spectral channels on the FTC CCD. SIM will have a variable number of spectral channels depending on the target spectrum and brightness.
2. ANGLE TRACKING AND CDCS

The angle tracker’s function is to provide two overlapping and stable beams that produce fringes on the fringe tracker. There are a number of competing constraints on the angle tracker. On the one hand it must have sufficient field of view to allow efficient star acquisition. On the other hand, during tracking where image frames must be obtained at high rate (500 fps), only a sub-window within the CCD frame around the spot can be grabbed during the allowable time. Thus, for SIM-Lite, the angle tracker CCD has a size of 80x80 pixels, and each of the two stars are imaged at the centers of two 40x40 quadrants within the CCD. During tracking, only the central 6x6 region-of-interest (ROI) within each quadrant is grabbed. An additional requirement is that the star image on the camera should be sampled as finely as possible by the CCD. Finally, to allow fast processing of the image data, a simple centroiding algorithm is used. The angle tracker’s error signal $e$ is the computed brightness centroid $C$ for the spot, minus any desired tracking offset, $O$. In the $x$ direction this can be written as:

$$e_x = C_x - O_x$$

$$C_x = \frac{\sum_{i=1}^{6} \sum_{j=1}^{6} I_{i,j} x_{i,j}}{\sum_{i=1}^{6} \sum_{j=1}^{6} I_{i,j}}$$  \hspace{1cm} (3)$$

In the above equation, $I_{i,j}$ is the measured (and dark-subtracted) CCD intensity for pixel $(i, j)$, and $x_{i,j}$ is the x-coordinate of that pixel, in pixel units, such that $x_{4,4} = +0.5$. A similar equation exists for the $y$ direction. The centroid formula of Equation (3) has a well-known non-linearity, with a periodicity of 1 pixel.\(^7\) The formula is used despite this shortcoming because it lends itself to fast computation of the spot position needed for angle tracking.

In an astrometric measurement with the interferometer, the instrument acquires one star and measures the OPD for that star. It then switches to the next star and repeats the process. The limited ROI, the coarse pixelation, and the non-linearity in Equation (3) together conspire with wavefront errors in the measured beams to produce a systematic pointing error that depends on the difference in the spectra of the two stars being astrometrically compared. This occurs because wavefront errors distort the point-spread function, and certain wavefront errors can cause asymmetric distortions. In addition, a point spread function’s size scales with the (mean) wavelength of the imaged light. When the ROI is too small, part of the spot intensity pattern is lost beyond the edges of the ROI, and not available for a centroid computation. When the instrument looks at a different star with a different spectrum, the size of the error is different. Thus, from one star to another, the centroid measurement has an error. This Color-Dependent Centroid Shift (CDCS) can then couple with other errors and create astrometric errors.

Figure 3: Simple model of interferometer siderostat alignment errors. White light and metrology beams are shown before and after an angle error in one of the siderostats. The detector is simplified to a plane at the bottom of the figure. The boxes on the right show zoomed-in portions of the diagram on the left.

One important instance of CDCS-induced error is when there is a shear error between white light and metrology at a siderostat. The shear $S_{WM}$ is defined as the distance between the white light beam’s chief ray and the metrology corner.
cube’s vertex. On SCDU, $S_{WM}$ can be brought down to below 100 $\mu$m. If the siderostat pointing changes by an amount $\theta_{sid}$ due to CDCS, it can be shown that the white light-metrology path difference on that arm changes by:

$$\delta d = 2S_{WM}\theta_{sid}$$  \hspace{1cm} (4)

In 3D $S_{WM}$ and $\theta_{sid}$ are vectors and their product is a vector dot product. We can motivate this expression in a simple 2D case by the highly simplified interferometer shown in Figure 3. In this illustration, the angle tracker is in fact not shown for simplicity. We assume the metrology beams are incident on the vertices of both siderostat corner cubes. White light is introduced from point F, with an alignment error $\theta_W$ (typically 10 $\mu$rad) relative to metrology. $L$ is the (one-way) length of one arm of the interferometer (about 5 m in length for SCDU). $r_m$ is the lever arm (typically under 100 $\mu$m) between the actual pivot point of the siderostat and the vertex of metrology corner cube. In the initial state, the (unseen) angle tracking system has so pointed the siderostats that the star light is correctly retro-reflected. At a later time, one siderostat undergoes a pointing error of amount $\theta_{sid}$ (if due to CDCS, typically under 0.3 $\mu$rad). Assuming the other arm has no such change, the resulting white light minus metrology path difference change is given by:

$$\delta d = \left(\frac{r_m + S_{WM}}{ED}\theta_{sid} + (L + (r_m + S_{WM})\theta_{sid})\left(1 + \frac{1}{2} (\theta_W + 2\theta_{sid})^2\right) - \frac{1}{2} L \cdot 2\theta_{sid} (\theta_W + 2\theta_{sid})\right) \cdot \frac{\delta M}{ED}$$  \hspace{1cm} (5)

where, wanting an estimate good to about 1 pm, we have dropped terms second order in $\theta_{sid}$ and third order in any combination of $\theta_{sid}$ and $\theta_W$.

Using the typical values given above, we can see from Equation (4) that the CDCS-induced error can easily be tens of picometers. Since CDCS is systematic, producing the same pointing error each time the instrument switches from a given target to a given reference (or vice versa), no amount of averaging can reduce it. Since the allocation from the SIM-Lite instrument error budget to this error source is under 1 pm, the only options are to 1) to measure CDCS and compensate for it, or 2) minimize either or both of the shear and the CDCS. In SCDU it was easier to do the former, but for SIM-Lite, it is only possible to do the latter, as we shall discuss later.

### 3. MODELING CDCS

We can simulate the CDCS effect in a simple diffraction model to assess which factors are most important in producing this error. The Matlab™ simulation, outlined in Figure 4, includes a phase screen at the pupil with a SCDU-like annular mask (but no spiders) and imaging (via FFT) onto the ATC focal plane. For the simulation, the pixelation at the focal plane is 4096 pixels on a side, scaled such that each pixel is 1 $\mu$m in width. The actual CCD pixel width is 24 $\mu$m. In the Fourier transform, the DC term (corresponding to the image center) falls on pixel 2049, corresponding to $(N/2)+1$, where $N$ is the number of dimension of the image (4096). The nominal spot position on SCDU is the crosshairs of 4 pixels so that the sensitivity of the sensor to tilt changes is maximized. Matlab’s interp2 function is used to move the diffraction pattern to a desired location near the center of the image plane.

![Figure 4: CDCS simulation elements. The counts in a single 24 um pixel are simulated by adding the 24x24, 1 um pixels contained within it.](image-url)
The spectra associated with SCDU’s two filters are simulated as 6 wave numbers with relative amplitudes matching those of the kg03 and bg38 filters. For each wavenumber the intensity is computed at the image plane, and the intensities for the whole set of wavenumbers are added in order to simulate a polychromatic spot on the image plane. Figure 5 shows the spectra used and the resulting asymmetric point spread function (PSF) profiles for the two spectra. The AT parabola focal length is 0.6 m, and the outer diameter of the starlight beam is 42 mm, giving an F# of 14.3 for the focusing optic. The mean wavelengths for the KG03 and BG38 filters are approximately 720 nm and 660 nm, respectively. From the figure we see that the full width of the PSF at these wavelengths is about 16 um so that a pixel size of 8 um would have Nyquist-sampled the spot. The ATC pixel size, on the other hand, is 24 um.

The sensitivity of CDCS to various forms of wavefront errors is studied by generating pure Zernike functions starting from tip and tilt and moving on to higher radial orders. A comparison of the first few orders shows that the strongest effect (after tilt) comes from Coma. With coma as the most important wavefront error for CDCS, we study the sensitivity of CDCS first to the size of the ROI window, and then to the granularity with which the PSF is sampled. To see why these should be important, consider the energy lost outside the ROI. When the spectrum changes, the shape and size of the PSF changes. This means a different fraction and pattern of light is missed outside the ROI, changing the centroid estimate and the CDCS magnitude.

Figure 6 shows the simulation results on the sensitivity of CDCS to ROI size for a hypothetical case with 100 nm of Coma assumed, for focal length of 0.6 m (left) and 1.2 m (right).
remains. The sharp annular nature of the pupil means that the PSF contains pronounced rings (Figure 4), thus it is not surprising to see oscillatory behavior in the ROI or position dependence of CDCS.

With pixelation, we have to consider the non-linearity of the centroid formula, Equation (3). In the limit where the spot is entirely contained in one pixel, the centroid estimate is insensitive to the movement of the spot. In reality the spot is never fully contained so there is always some sensitivity to spot movement, but the sensitivity is not linear. As we move away from this extreme to the other extreme of infinitesimal pixels, the centroid estimation become perfect and linear. So we expect CDCS to become smaller with smaller pixels.

Figure 7 shows simulation results on the dependence of CDCS on the AT parabola focal length. The nominal SCDU focal length is 0.6 m. As the focal length is increased the PSF becomes larger. This in turn means that the sampling of the PSF improves. At the same time, more of the energy falls outside the ROI. In the study shown, the ROI is to be significantly larger than the actual SCDU value (4096 um rather than 960 um on a side), so the ROI effect is small and the pixelation effect dominates. We see that the dependence on sampling is approximately quadratic, both in pixel-size dependence of each individual focal length and in comparing the two focal lengths at a common pixel size. As we saw earlier, at \( f = 0.6 \text{ m} \) the spot is Nyquist sampled by a pixel size of approximately 8 um for the geometry and spectra of SCDU.

![Figure 7: CDCS dependence on PSF sampling. As the pixel size increases, the sampling becomes coarser, leading to larger CDCS. Alternatively, increasing the focal length improves the sampling and decreases CDCS.](image)

4. **CDCS AS MEASURED BY SCDU**

In SCDU it is possible to measure the CDCS effect. To do this, the angle tracker controller is stopped and the centroid is measured as the spectrum is alternated (using filter switches) between that of the target and reference ‘stars.’ The chopping reduces the error due to pointing drifts during the test. This is repeated over a range of star spot positions on the face of the angle tracker CCD. The results of one such scan is shown in Figure 8 for the left arm beam. Each of the two plots shows two traces, one using a 4x4 pixel ROI and the other using a 6x6 pixel ROI. Widening the window obviously helps. The CDCS error when tracking at the center with a 6x6 pixel ROI is seen to be about .0025 pixels, or about 0.1 µrad.

The source of wavefront error that leads to CDCS in SCDU is likely multi-mode effects occurring in the white light source’s single mode fibers which are designed for wavelengths around 600 nm and above, while SCDU has broadband light that goes below that range. For SIM-Lite this will not be a source of wavefront error because the stellar wavefront is essentially perfect.

Once the CDCS is known, it can be applied as a correction to the pointing. Note that the CDCS is the change in pointing in going from the target star to the reference star. Thus to apply the correction, the measured CDCS in each direction of each arm is inserted into the error signal as the offset \( O_x \) in Equation (3). The test of whether the measured CDCS accurately reflects the true pointing shift during operations is to check, using some other means, whether the siderostats move immediately after a filter change. This is provided by inspecting the applied voltages to the three PZT actuators on
the siderostats during closed loop operation. If the CDCS correction is accurate, then the voltages should be essentially continuous across the time boundaries between filter switches. Changes in the PZT voltages are thus a proxy for the actual pointing of the beam. For SCDU the conversion factor is 2 µrad per volt. As seen in Figure 9, CDCS goes from 0.2 µrad down to about 10 nrad, a 20-fold suppression. This level of CDCS is tolerable for achieving SIM goals.

Figure 8: Measuring CDCS in the x and y directions for the left arm beam in SCDU. Each trace is a scan that starts at the center, then +1 pixel, then to -1 pixel and finally back to the center. Each plot shows CDCS for two ROI conditions: 4x4 pixels and 6x6 pixels. The vertical axis in each plot is the measured difference in centroid when using the reference (BG38) filter minus the same when using the target (KG03) filter. Each pixel corresponds to 40 µrad.

Figure 9: Checking the efficacy of CDCS corrections by looking at siderostat PZT commanded voltages. A siderostat with its central metrology corner cube and its three PZT’s is also shown. The spikes in the corrected set of plots occur during filter changes and are not part of the observations.
The final measure of the efficacy of the CDCS correction is whether it improves the narrow angle astrometric performance. Figure 10 shows a comparison of narrow angle test results for two sets of runs taken over the same time period. In one set the CDCS corrections was applied, and in the other it was not applied. In each plot, each trace contains six measurements of the error metric (given by Equation (1)) for each of the six spectral channels on the FTC. Ideally, the results from all six channels should be the same and near zero. We see that the range of error in the uncorrected case is significantly greater than that in the corrected case.

![Figure 10: The measured error metric for the six spectral channels of the fringe camera for a number of runs with CDCS corrections applied (left) and runs without CDCS corrections (right).](image)

CDCS characterization of this kind is not feasible for SIM. In SCDU we can assume the target and reference stars are in exactly the same location on the sky, and attribute all of the observed centroid shift, in going from one star to another, to the CDCS effect. For SIM, however, the reference frame of the optical system cannot be placed repeatably in the same orientation with respect to two different stars. The level of repeatability required would be less than 10 nrad divided by the optical compression factor in SIM, currently designed to be 7X. Consequently, SIM would have to either model the effect (requiring additional testbed demonstrations) or “design out” the problem by a) sampling the start spot better (by at least a factor of 3 relative to the SCDU sampling) and b) by using a more sophisticated acquisition technique. Such a process would consist of starting with a large acquisition window, determining the centroid error for the tracking sub-window, and applying the measured correction when changing modes from large window acquisition to small-window tracking. The tracking sub-window should ideally span at least $15 \lambda/D$, where $\lambda$ is the mean star wavelength for NA observations. A simple way to improve the sampling without requiring a smaller-pixel camera is to increase the effective focal length of the angle tracker camera. This modification is in fact being considered for the SIM-Lite brassboard ABC currently under construction.

5. SUMMARY

Color dependent centroid shifts (CDCS) can arise when there are wavefront errors in the beam used for angle tracking and high-speed tracking necessitates simple centroiding over a small region of interest in an angle tracking camera. In an astrometric interferometer like SIM-Lite, switching between different targets opens the possibility of CDCS-induced errors. The SCDU testbed has measured and quantified the error, and has pointed the way to avoid CDCS errors in SIM-Lite. SIM-Lite will avoid CDCS effects by better (approximately Nyquist) sampling of the star spot in the angle tracker and by using a more sophisticated acquisition process that allows the estimation of the tracking-mode, limited-ROI pointing error while still full-ROI acquisition mode. In addition, if the wavefront errors in the SCDU starlight beam are indeed from multimode effects as tests currently suggest, SIM-Lite will avoid these errors simply because it is looking at the nearly perfect wavefronts of real stars.

† For a concrete example, we can define the reference frame of the optical system based on three pixel-corner cross hairs on the FTC CCD. Any deformation of the optical bench on SIM can then be book-kept as an error relative to this defined frame.
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