Repetitive Interrogation of 2-level Quantum Systems

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***Abstract*—Trapped ion clocks derive information from a reference atomic transition by repetitive interrogations of the same quantum system, either a single ion or ionized gas of many millions of ions. Atomic beam frequency standards, by contrast, measure reference atomic transitions in a continuously replenished "flow through" configuration where initial ensemble atomic coherence is zero. We will describe some issues and problems that can arise when atomic state selection and preparation of the quantum atomic system is not completed, that is, optical pumping has not fully relaxed the coherence and also not fully transferred atoms to the initial state. We present a simple two-level density matrix analysis showing how frequency shifts during the state-selection process can cause frequency shifts of the measured clock transition. Such considerations are very important when a low intensity lamp light source is used for state selection, where there is relatively weak relaxation and re-pumping of ions to an initial state and much weaker 'environmental' relaxation of the atomic coherence set-up in the atomic sample.**

We shall use the density matrix formalism to investigate this problem since it explicitly describes quantum coherence between the two states of a clock transition. An ensemble of trapped ions where repeated interrogations are carried out on the same atomic system is shown in Figure 1. The process nominally employs a Rabi microwave pulse [1,2] followed by a optical pumping period where de-tunings from the atomic resonance frequency are determined via fluorescence measurements. In trapped mercury microwave standards, where discharge light sources are used for optical pumping or state selection, atomic coherence in the atomic ensemble can be quite long, usually several measurement cycles if no deliberate relaxation of the ensemble coherence is carried out. The light source must restore a population imbalance between the 2 clock levels and also destroy atomic coherence set up in the ion ensemble by the microwave pulse. If the coherence persists through the optical pumping process then the atomic line shape is distorted and the center frequency is pulled by the resulting 'Ramsey effect' of adjacent Rabi pulses. The Ramsey interrogation [1,2] is a double microwave pulse where each pulse is one half the amplitude-duration product as compared to a Rabi microwave pulse. The Ramsey interrogation is sensitive to atomic frequency during the dark period between pulses and this can lead to light shift of the state selection process when the atomic coherence persists between the nominally independent Rabi pulses shown in Fig. 1.

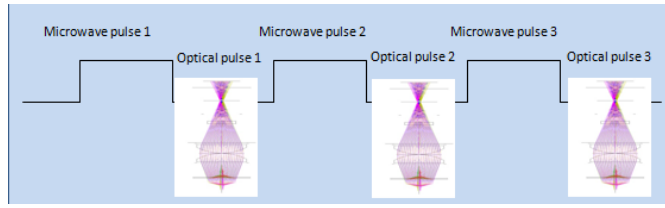

Figure 1. The repetitive interrogation of the ensemble of trapped ions alternates between a microwave pulse and optical pumping state-selection pulse. The weak relaxation of the atomic coherence can lead to a shift of the measured clock reference frequency through frequency pulling during the state selection process.

The density matrix formalism is described in references 3 and 4, but we start with the original probability amplitude notation of Ramsey [1,2]. In matrix notation, the amplitudes evolve from t_0 to t as follows:

$$\begin{pmatrix} P \\ Q \end{pmatrix} = U(t,t_0)\begin{pmatrix} P_0 \\ Q_0 \end{pmatrix} \tag{1}$$

We form a density matrix evolution [3,4] from the above amplitude evolution equation:

$$\rho = \begin{pmatrix} P \\ Q \end{pmatrix}\begin{pmatrix} P^* & Q^* \end{pmatrix} = U(t,t_0)\begin{pmatrix} P_0 \\ Q_0 \end{pmatrix}\begin{pmatrix} P_0^* & Q_0^* \end{pmatrix}U^{\uparrow}(t,t_0) = U(t,t_0)\rho_0 U^{\uparrow}(t,t_0) \tag{2}$$

We begin with an initial condition on the density matrix,

$$\rho_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{3}$$

that is, all the population is in one state and the coherence is between states is zero. Following a single microwave pulse of duration t,

$$\rho(\tau) = \begin{pmatrix} \left(1-\sin^2\theta\sin^2\frac{a\tau}{2}\right) & -\frac{i}{2}e^{i\Delta\tau}\left(\sin\theta\sin a\tau + i\sin 2\theta\sin^2\frac{a\tau}{2}\right) \\ \frac{i}{2}e^{-i\Delta\tau}\left(\sin\theta\sin a\tau - i\sin 2\theta\sin^2\frac{a\tau}{2}\right) & \sin^2\theta\sin^2\frac{a\tau}{2} \end{pmatrix} \tag{4}$$

Where we follow the notation of Ramsey [1,2] where Δ is the angular frequency detuning from the atomic transition line center, and b is the Rabi flopping frequency. In addition, $\sin\theta$ and $\cos\theta$ are given by

$$\sin\theta = -\frac{2b}{a}, \quad \cos\theta = -\frac{\Delta}{a}, \; and \;\; a = \sqrt{\Delta^2 + (2b)^2}. \tag{5}$$

During the period of optical pumping/state selection the density matrix exponentially approaches

$$\rho_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}. \tag{6}$$

That is, after an optical pumping period T,

$$\rho(\tau+T) = \begin{pmatrix} \left(1-\sin^2\theta\sin^2\frac{a\tau}{2}e^{-\gamma T}\right) & -\frac{i}{2}e^{i(\Delta\tau+\delta T)}\left(\sin\theta\sin a\tau + i\sin 2\theta\sin^2\frac{a\tau}{2}\right)e^{-\gamma T/2} \\ \frac{i}{2}e^{-i(\Delta\tau+\delta T)}\left(\sin\theta\sin a\tau - i\sin 2\theta\sin^2\frac{a\tau}{2}\right)e^{-\gamma T/2} & \sin^2\theta\sin^2\frac{a\tau}{2}e^{-\gamma T} \end{pmatrix} \tag{7}$$

where the pumping rate is γ and the atomic clock frequency is shifted by δ from the resonant value when microwaves are applied. When ions are located in the same trap for both optical state selection and microwave interrogation, this is primarily light shift. If ions are shuttled from one trap to another [5], second order Doppler shift and magnetic field shifts would also contribute to δ. Following a second microwave interrogation pulse the density matrix is given by

$$\rho(2\tau+T) = U(T+2\tau, T+\tau)\rho(\tau+T)U^{\uparrow}(T+2\tau, T+\tau), \tag{8}$$

resulting in an expression too long to write out here explicitly. We use the r_{22} element from this expression to examine the probability of transfer to the empty clock state. Notice that in case γ=0, no light has been applied and an overpowered Ramsey line shape for r_{22} vs. frequency detuning results, but with a frequency residual shift δ experienced during the microwave off period.

The frequency shift vs. optical pumping time T is shown below in figure 2. In this data, two parameters were adjusted to give the best fit to the measured shifts shown in the dotted trace. The best fit optical pumping rate $1/\gamma$ was found to be 0.85 seconds with the offset δ= -0.12 Hz. In this measurement the optical pump time T was adjusted between 1.75 and 5 seconds in nine measurement values. The light level was assumed to produce the same intensity and hence the same shift even though the duration varied and this can sometimes lead to variations in light intensity. The shift δ= -0.12 Hz is consistent with direct measurements of line center light shift measured by comparing switched lamp operation and continuous lamp operation.

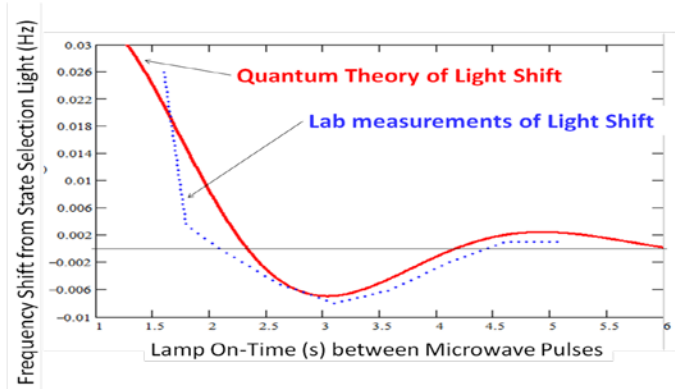

Figure 2. Light shift of the clock transition results when atomic coherence of the atomic ensemble persists through the optical pumping process. This is eliminated by randomizing the phase of the microwave pulse between subsequent microwave pulses.

Such light shifts were investigated in operation of Rb vapor cell clocks by Alekseev and Bazarov [6]. This is the first observation of such shifts in the $^{199}Hg^{+}$ microwave clock system. Weakly relaxing quantum systems with lamp based state selection can thus experience large frequency shifts.

To eliminate such shifts it is necessary to randomize the phase of the microwave pulse between subsequent microwave pulses [7]. In this way, on average, atomic coherence is eliminated and the shifts average to zero although measurement noise can substantially increase degrading short-term clock performance. Longer state selection periods are then necessary to reduce this noise source.

ACKNOWLEDGMENT

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