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THE ECCENTRIC BEHAVIOR OF NEARLY FROZEN ORBITS

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Frozen orbits are orbits which have only short-period changes in their mean eccentricity and argument of periapse, so that they basically keep a fixed orientation within their plane of motion. Nearly frozen orbits are those whose eccentricity and argument of periapse have values close to those of a frozen orbit. We call them "nearly" frozen because their eccentricity vector (a vector whose polar coordinates are eccentricity and argument of periapse) will stay within a bounded distance from the frozen orbit eccentricity vector, circulating around it over time. For highly inclined orbits around the Earth, this distance is effectively constant over time. Furthermore, frozen orbit eccentricity values are low enough that these orbits are essentially eccentric (i.e., off center) circles, so that nearly frozen orbits around Earth are bounded above and below by frozen orbits.

INTRODUCTION

Frozen orbits for this discussion are orbits which have only short-period changes in their mean eccentricity and argument of periapse, so that they basically keep a fixed shape and orientation within their plane of motion. These orbits are useful for planetary observations because they are easily predictable, in the sense that the radius at any particular latitude will remain close to constant. And when an orbit is set to repeat its ground track, the altitude profile will also repeat (even more exactly) from one cycle to the next.[1]

Just before a spacecraft gets into a frozen orbit, or just after it leaves one, it is in a nearly frozen orbit which is almost tangent to the frozen one. This nearly frozen orbit is a concern to operators of spacecraft still in the frozen orbit because the nearly frozen orbit will change with time. If the nearly frozen orbit could change so that it becomes actually tangent to or crosses the frozen orbit, then this would create a repeating risk of collision with spacecraft in the frozen orbit.

Fortunately, the behavior of the nearly frozen orbit is very regular and easy to describe, and we will show below that the evolution of the orbit is such that the risk of collision does not increase with time.

CLOUDSAT'S EXIT-THE-A-TRAIN PROBLEM

The CloudSat mission[2] uses a cloud-penetrating radar from orbit to generate vertical radar profiles of clouds and measure their water content. The CloudSat spacecraft was launched together with

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the CALIPSO spacecraft in 2006, when they independently joined the A-Train (Afternoon Constellation), one of two International Earth Observing Constellations. The spacecrafts of the A-Train fly in frozen orbits with a reference altitude of 705 km and an ascending node with a mean local time of about 1:30 PM.

In 2011 a malfunction developed in one of the cells of CloudSat's battery which effectively reduced the battery power available to a small fraction of initial capacity. With this reduced battery power CloudSat was unable to operate as planned in Earth's shadow. The spacecraft transitioned to a planned safe mode from which it was unable to recover itself—the battery problem caused a negative feedback situation in which the spacecraft gradually discharged the battery and grew colder and colder. Extraordinary efforts[3] by the CloudSat team at Ball Aerospace, Jet Propulsion Laboratory, and Kirtland Air Force Base (including Aerospace Corporation and other contractors) over the course of the following year led to a remarkable recovery of the CloudSat spacecraft and restoration of science operations (though only on the day side of the orbit). This recovery involved regaining control of the spacecraft, maneuvering it out of the A-Train, developing and implementing entirely new operations modes, and returning to the A-Train in 2012.

The new operations modes restrict CloudSat's maneuvers to the day side of the orbit, which required a new end-of-mission plan to be written to describe how CloudSat would be brought out of orbit to meet NASA's debris limitation requirements. In the process of developing the new plan, special consideration was given to the possibility that the end of mission might be precipitated by imminent failure of the spacecraft, in which case the spacecraft might be able to execute only one maneuver. Since a single maneuver necessarily leaves the spacecraft in a non-frozen orbit near the orbits of the A-Train, concern was raised over how that orbit would evolve over time—would it change in a way that would bring CloudSat back into interaction with the A-Train and disrupt operations there, even present a risk of collision? The question became what would be the best orbital state to put CloudSat in with the first maneuver when it begins to leave the A-Train. The answer to this depends on how nearly-frozen orbits behave relative to frozen orbits and, more importantly, relative to other nearly-frozen orbits nearby (since no spacecraft flies a perfectly frozen orbit).

FROZEN ORBITS

An orbit is called frozen in the most general sense if any of its characteristics stays fixed or exhibits only short-period oscillations. In the particular sense which we are considering, and in the most common use of the term, an orbit is frozen if its mean eccentricity vector stays basically constant with time. The eccentricity vector, $\vec{e}$, is the vector whose polar coordinates are the eccentricity e and the argument of periapse ω. The cartesian coordinates of this vector are usually called the semi-equinoctial elements k' and h', so we have

$$\begin{aligned} k' &= e\cos\omega \\ h' &= e\sin\omega \end{aligned} \tag{1}$$

(For orbits very close to the X-Y plane, the longitude of the ascending node is added to the argument of periapse to avoid singularities when the inclination is 0 deg or 180 deg; in this case, the cartesian coordinates of $\vec{e}$ are the equinoctial orbit elements k and h.)

We use mean orbital elements instead of osculating elements because the osculating values vary widely as the spacecraft goes around its orbit. The extent of this variation is shown in Figure 1,

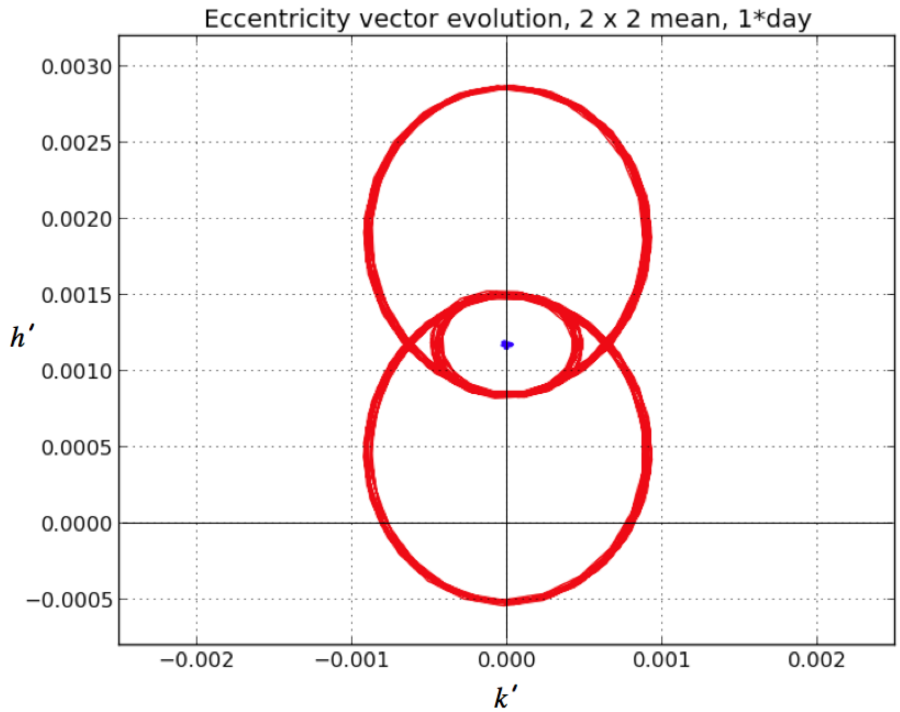

Figure 1. The A-Train osculating eccentricity vector variation (red) through one day's worth of orbits; the mean eccentricity vector (blue) shows much less variation.

which traces out the eccentricity vector for all of the orbits of one day, using the frozen A-Train orbit as an example. The curve of the osculating eccentricity vector starts at the ascending node of the orbit, which corresponds to a point at the left-hand tip of the horizontal inner ellipse. The eccentricity vector goes around counter-clockwise from there to the top of the outer top vertical ellipse (corresponding to the northernmost point of the orbit), comes down and loops around the right half of the inner, horizontal ellipse (corresponding to the descending node), and then out to the bottom of the outer bottom vertical ellipse (corresponding to the southernmost point of the orbit) before returning to the position for the ascending node. The curve looks thick because it is actually traced over 14 times, the number of orbits in a day, with a small change from orbit to orbit due to the sectoral and tesseral components of the Earth's gravity field.

Even the mean $\vec{e}$ for the same orbit changes somewhat in the medium term as the orbit goes through different parts of the gravity field while the central body rotates, as shown in Figure 2, which is a zoom in on Figure 1. But if the orbit has been designed to have a repeating ground track, then $\vec{e}$ will be the same from repeat cycle to repeat cycle for a frozen orbit. The mean $\vec{e}$ shown here was calculated assuming a 2×2 gravity field, even though the orbit itself was integrated in a 36×36 gravity field, because that seemed to minimize the total variation. Some of the variation shown may itself be due to the difference in the gravity models, but the orbit-to-orbit variations really are on the order of hundreds of meters as indicated here. These show up as altitude variations from orbit to orbit, which are shown at weekly intervals in Figure 3. Also shown are the relative altitudes of a nearby smaller coplanar orbit. These medium period variations do not affect the relative altitudes between coplanar orbits, because they are (nearly) the same on nearby orbits. This

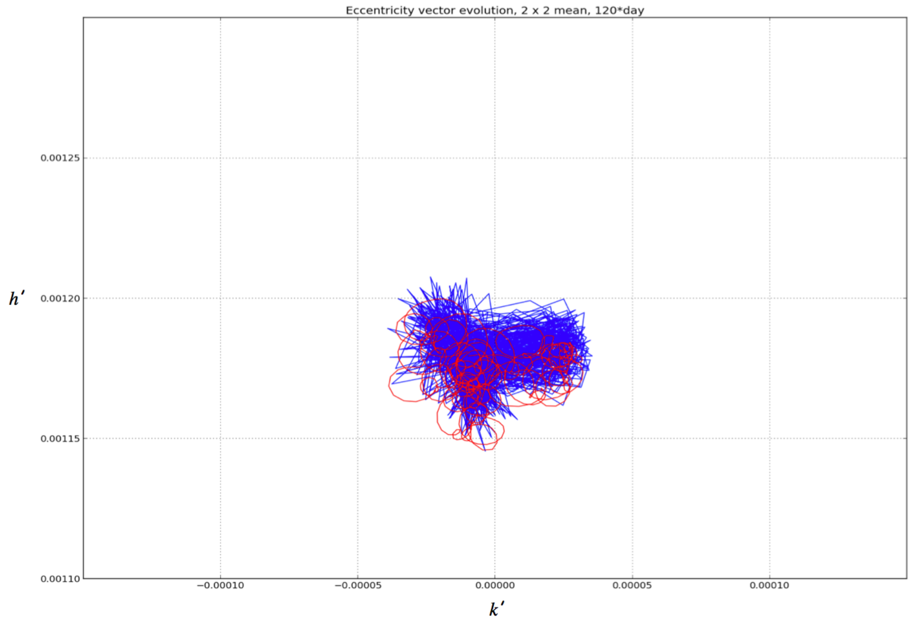

Figure 2. The A-Train mean eccentricity vector variation (red) through one day's worth of orbits; the mean eccentricity vector for one entire 120-day evolution cycle (blue) shows about the same variation. (The jaggedness of the longer evolution is an artifact of plotting points that are spaced farther apart in time.)

is seen in Figure 4, where the altitude comparison is made as contemporaneous as possible. Those weeks when the altitudes of the nearby orbit through the day are more spread out are weeks when the nearby orbit is most out of phase with the frozen orbit, so that the ground tracks are farthest apart and the nearby and frozen orbits are seeing the most difference in the gravity field. When the orbits are in phase the altitudes are much more repeatable during the day shown.

In a gravity field with only J_2 and J_3 terms in the spherical harmonics of the field, the formula for the frozen $\vec{\mathbf{e}}$ is well known:

$$e = -\frac{1}{2}(\frac{R_m}{a})\frac{J_3}{J_2}\sin i \sin\omega, \text{ for } \omega = 90 \text{ deg or } 270 \text{ deg.} \tag{2}$$

In this equation R_m is the mean equatorial radius corresponding to the gravity field model, a is the semi-major axis of the orbit, and i is the inclination, and the value of ω is chosen to be 90 deg or 270 deg to make the value of e positive. For Earth, $\omega = 90$ deg because J_2 and J_3 have opposite signs.

This formula for e has been extended to include the effects of other zonals by Cook[4–6] (note that Reference 6 has typographical errors in its Equations (6)-(8)—we have used the corresponding Equations (21), (29), and (30) in Reference 5, which are correct). The extended formula for the frozen eccentricity is given by

$$e = \frac{\rho}{(\epsilon - \eta)}\sin\omega, \text{ for } \omega = 90 \text{ deg or } 270 \text{ deg} \tag{3}$$

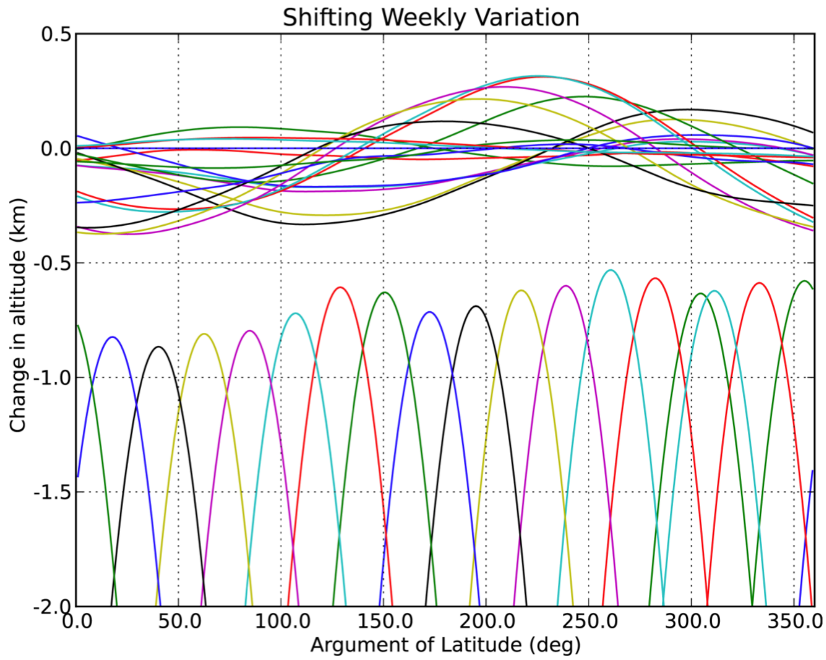

Figure 3. Altitudes of the A-Train frozen orbit (top wavy lines) and a nearby smaller orbit (lower peaks), shown for the first orbits of each week relative to the first A-Train orbit. This propagation is for a full 114 day evolution cycle of the eccentricity vector of the smaller orbit, as will be discussed later.

where again the value of ω is chosen to be 90 deg or 270 deg to make the value of e positive, and

$$\rho = n^* \sum_{\substack{n=3\\ odd}}^{\infty} \left(\frac{R_m}{a}\right)^n J_n \left[\sum_{\beta=0}^{(n-1)/2} (n-1) D_\beta^n \sin^{n-2\beta} i\right] \tag{4}$$

$$\eta = n^* \sum_{\substack{n=4\\ even}}^{\infty} \left(\frac{R_m}{a}\right)^n J_n \left[\sum_{\beta=0}^{(n-2)/2} \frac{1}{2}(n-1)(n-2) E_\beta^n \sin^{n-2\beta} i\right] \tag{5}$$

$$\begin{aligned} \epsilon = & -n^* \sum_{\substack{n=2\\ even}}^{\infty} \left(\frac{R_m}{a}\right)^n J_n \left[\sum_{\beta=0}^{n/2} \frac{1}{2} n(n+1) A_\beta^n \sin^{n-2\beta} i\right] \\ & + n^* \sum_{\substack{n=2\\ even}}^{\infty} \left(\frac{R_m}{a}\right)^n J_n \cos^2 i \left[\sum_{\beta=0}^{n/2-1} (n-2\beta) A_\beta^n \sin^{n-2\beta-2} i\right] \end{aligned} \tag{6}$$

where n^* is the mean motion in the orbit, R_m is the mean equatorial radius used in the gravity field expansion, a is the semi-major axis of the orbit, J_n is the nth zonal coefficient in the spherical

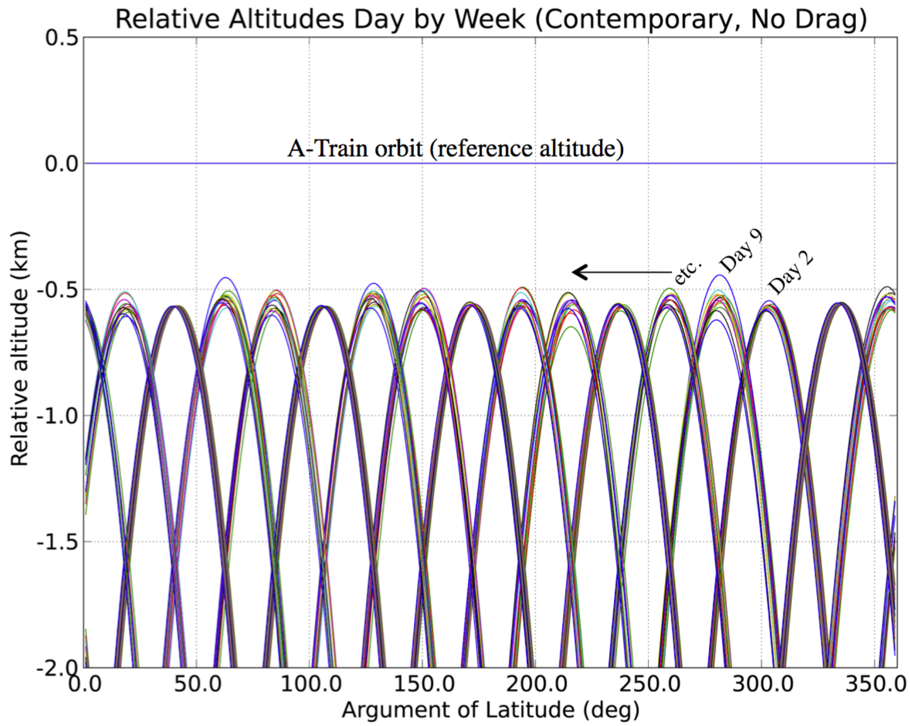

Figure 4. Altitudes of the same nearby smaller orbit, shown for all orbits during the first day of each week relative to the A-Train orbit altitude at the same latitude which is nearest in time.

harmonic model of the gravity field, and i is the inclination, and with

$$A^n_\beta = \frac{(2n-2\beta)!\,(-1)^\beta}{\beta!\,(n-\beta)!\,(\frac{1}{2}n-\beta)!\,(\frac{1}{2}n-\beta)!\;2^{2n-2\beta}} \tag{7}$$

$$D^n_\beta = \frac{(2n-2\beta)!\,(-1)^\beta}{\beta!\,(n-\beta)!\,(\frac{1}{2}(n-1)-\beta)!\,(\frac{1}{2}(n+1)-\beta)!\;2^{2n-2\beta}} \tag{8}$$

$$E^n_\beta = \frac{(2n-2\beta)!\,(-1)^\beta}{\beta!\,(n-\beta)!\,(\frac{1}{2}(n-2)-\beta)!\,(\frac{1}{2}(n+2)-\beta)!\;2^{2n-2\beta}} \tag{9}$$

Note that if we apply Equation (3) to a gravity field model that only includes J_2 and J_3 then it reduces exactly to Equation (2), which is reassuring.

We can use the A-Train frozen orbit to compare Equations (2) and (3). The defining parameters of the A-Train frozen orbit as determined empirically using a 36×36 gravity field model are

$$\begin{aligned} a &= 7077.732\,\text{km} \\ e &= 0.00118 \\ i &= 98.20\,\text{deg} \\ \omega &= 90\,\text{deg} \end{aligned} \tag{10}$$

The following gravity field parameters are from GGM02C,[7] a gravity field for Earth that is a product of the GRACE mission.

$$\begin{aligned} R_m &= 6378.1363\,\mathrm{km} \\ gm &= 398600.4356\,\mathrm{km}^3/\mathrm{s}^2 \\ J_2 &= 1.082635666551 \times 10^{-3} \\ J_3 &= -2.532473691333 \times 10^{-6} \\ J_4 &= -1.619974305782 \times 10^{-6} \\ J_5 &= -2.279051260821 \times 10^{-7} \\ J_6 &= 5.406167899402 \times 10^{-7} \end{aligned} \tag{11}$$

Using this data we calculate from Equation (2) a value of 0.00104 for the frozen eccentricity, 12% lower than the 0.00118 actual value needed. If we use Equation (3) up to $n = 6$, then we get a value of 0.00112, only 5% low. For your convenience, here is the formula for the frozen eccentricity for n up to 6:

$$e = (\frac{R_m}{a}) \left[\frac{J_3(\frac{15}{8}\sin^2 i - \frac{3}{2}) + (\frac{R_m}{a})^2 J_5(\frac{315}{32}\sin^4 i - \frac{105}{8}\sin^2 i + \frac{15}{4})}{J_2(-\frac{15}{4}\sin^2 i + 3) + (\frac{R_m}{a})^2 J_4(-\frac{105}{4}\sin^4 i + \frac{975}{32}\sin^2 i - \frac{15}{2}) + \tilde{J}_6} \right] \sin i \sin\omega \tag{12}$$

where

$$\tilde{J}_6 = \Big(\frac{R_m}{a}\Big)^4 J_6\Big(-\frac{79695}{512}\sin^6 i + \frac{16695}{64}\sin^4 i - \frac{3885}{32}\sin^2 i + \frac{105}{8}\Big) \tag{13}$$

and where again the value of ω is chosen to be 90 deg or 270 deg to make the value of e positive. Note that, as mentioned before, if we include only the J_2 and J_3 terms then Equation (12) reduces to Equation (2).

One interesting consequence of Equation (2) is that in a J_2/J_3 gravity field all frozen orbits at a given inclination are concentric. We can see this because the vector to the center, $\vec{\mathbf{c}}$, of the ellipse corresponding to a mean orbit is given by

$$\vec{\mathbf{c}} = -a\vec{\mathbf{e}} \tag{14}$$

and Eq. (2) can be rewritten as

$$ea = -\frac{1}{2}R_m\frac{J_3}{J_2}\sin i \sin\omega, \text{ for } \omega = 90 \text{ deg or } 270 \text{ deg.} \tag{15}$$

Since ω is fixed, this means that $\vec{\mathbf{c}}$ depends only on i in this low-order model.

In higher order models there is some dependence on the semi-major axis. We have found empirically that this is small. The center of the A-Train frozen ellipse according to Equations (12) and (14) (i.e., in a gravity field with zonals up to order 6) is 7.961 km south of the Earth's center (in the orbit plane, of course), and this center moves 18 m farther south for every 100 km reduction in the semi-major axis. The stability of the orbit center will be significant in understanding how nearly frozen orbits can interact as they evolve.

NEARLY FROZEN ORBITS AND HOW THEY EVOLVE

In his analysis of orbit evolution in a zonal gravity field, Cook[5,6] found more than just a more general formula for the frozen eccentricity; he deduced a differential equation for the motion of $\vec{\mathbf{e}}$ in eccentricity space, the space of eccentricity vectors, and he found closed-form analytic solutions to this differential equation. Which closed-form solution applies depends on the value of a parameter Λ^2, which is defined by $\Lambda^2 = \eta^2 - \epsilon^2$. When $\epsilon^2 > \eta^2$ then $\Lambda = \sqrt{\epsilon^2 - \eta^2}$ and the closed-form analytic solution is the equation for harmonic motion given in Equation (16), i.e., $\vec{\mathbf{e}}$ moves in an ellipse in eccentricity space which is centered on the eccentricity vector of the frozen orbit with the same semi-major axis.

$$
\begin{aligned}
k'(t) &= -\frac{\Lambda}{\epsilon+\eta}(h'_0 - \frac{\rho}{\epsilon-\eta}) \sin \Lambda t + k'_0 \cos \Lambda t \\
h'(t) &= (\frac{\rho}{\epsilon-\eta}) + (h'_0 - \frac{\rho}{\epsilon-\eta}) \cos \Lambda t + k'_0 \frac{\Lambda}{\epsilon-\eta} \sin \Lambda t
\end{aligned}
\tag{16}
$$

where k'_0 and h'_0 are the initial values of k' and h'.

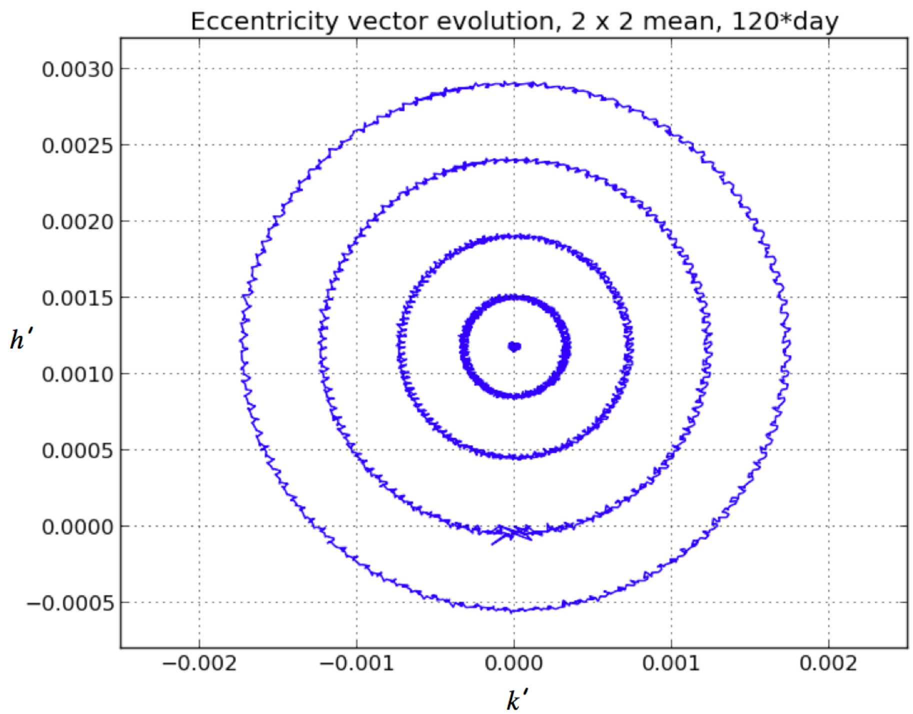

Figure 5. Eccentricity vector evolution for a variety of orbits.

For Earth orbits with the same semi-major axis and inclination as the A-Train orbit but with varying eccentricities, the evolution of the eccentricity vector is circular to the limits of our measurement ability (about 1%) and is centered around the frozen eccentricity vector. This is shown in Figure 5 for orbits that start with a variety of eccentricities; the Figure shows the time history or evolution of the mean eccentricity vector in eccentricity space, coordinatized by h', k'. In Figure 6, we've zoomed in and show the evolution of $\vec{\mathbf{e}}$ for orbits whose eccentricity is closer to the frozen value. All of these orbits were integrated using a full 36×36 subset of the GGM02C gravity field for Earth. The fuzziness of the curves is from medium period (i.e., orbit to orbit) perturbations on the orbit by the sectoral and tesseral terms of the gravity field; this fuzziness is what limits how well we can measure the true circularity of the evolution of the eccentricity vector.

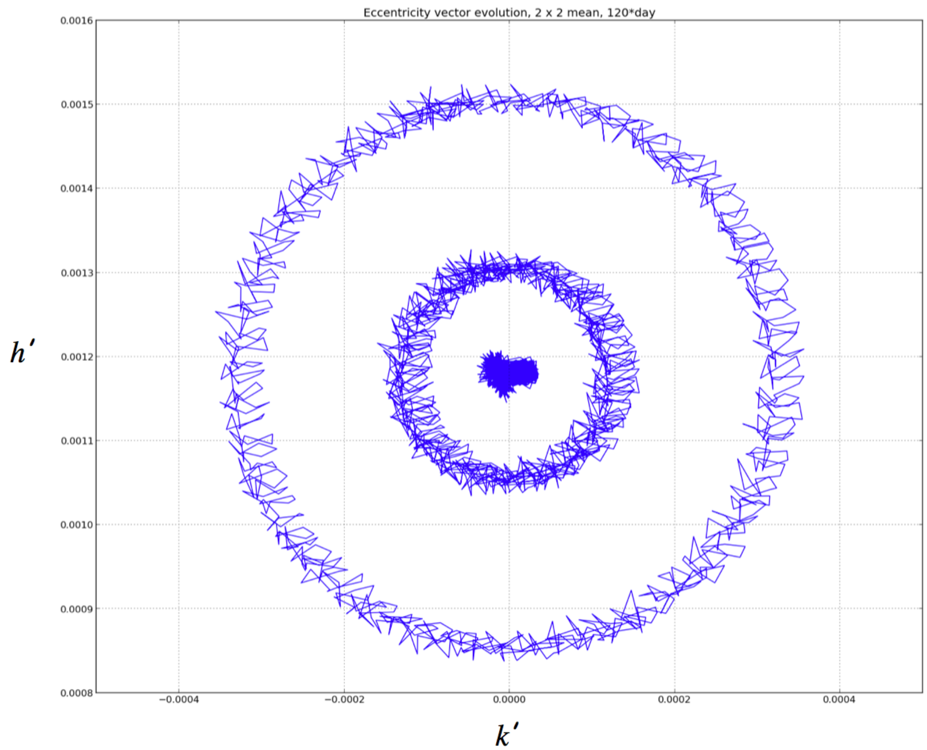

Figure 6. Eccentricity vector evolution for orbits closer to the frozen orbit.

According to Cook's analytic solution applied to our A-Train model (with zonals J_2 through J_6), these evolutions of the eccentricity vector are actually ellipses which are about 0.3% smaller in the k' axis than in the h' axis; the actual ratio is $\sqrt{\frac{\epsilon-\eta}{\epsilon+\eta}}$, where $\epsilon \approx (-5.9 \times 10^{-4})n^*$ and $\eta \approx (-1.7 \times 10^{-6})n^*$ for orbits near the A-Train frozen orbit. This gives a variation of 0.3% in the distance between the eccentricity vectors of the A-Train frozen orbit and a nearly frozen orbit, which corresponds to a shift of the latter orbit's center of some tens of meters. Also, Cook's solution for this case gives a period of 116 days for the evolution of the nearly frozen eccentricity vector around the A-Train frozen eccentricity vector, in good agreement with the integrated propagation in the larger 36×36 gravity field.

A PTOLEMAIC ASIDE

We are still faced with the question of whether two elliptical orbits intersect each other, or whether they come close to doing so, when they have different semi-major axes, eccentricities, and arguments of periapse. Precisely because this question does not have a simple answer, A-Train operators in the past have regarded any smaller non-frozen ellipse whose apoapse was higher than the A-Train periapse as a potential risk to the A-Train, and there was some residual uneasiness even when the apoapse was lower because the apoapse altitude of a non-frozen orbit changes. Fortunately we can use an approximation to restate the question so that it does have a simple answer.

Recall that the ratio of the semi-minor axis of an ellipse to its semi-major axis is $\sqrt{(1-e^2)}$. This means that the distance between an ellipse and its circumscribing circle is never more than about $a(e^2/2)$ (the error in this approximation is of order e^4). The eccentricity of a frozen A-Train orbit is just under 0.0012. If we consider orbits with eccentricities up to 0.003 (two and a half times as large), then the maximum departure from circularity is less than 32 m. Hence we may model orbits in a rather large neighborhood of the A-Train orbit by adopting Ptolemy's model,[8] which takes them simply to be eccentric (i.e., off center) circles with radii equal to their corresponding semi-major axes; we just need to keep in mind that actual orbital points might be some meters in from the modeled points.

It is easy to see if two coplanar circles intersect when they both go around some common interior point—they intersect if and only if the positive difference in their radii is less than the distance between their centers. If they don't intersect then one circle is inside the other and it is easy to calculate how close the inside one gets to the outside one—the minimum distance between the circles equals the positive difference in their radii minus the distance between their centers. And if the distance between the circles is more than 32 m, then ellipses near the A-Train orbit that are approximated by those circles (i.e., with eccentricity less than 0.003) cannot intersect. (Note that we only need to consider the outer orbit's departure from circularity, since the inner orbit's departure is away from the outer orbit.) This is true in the non-coplanar case even more strongly—in the non-coplanar case the two orbits can come as close to each other as in the coplanar case only if the line of nodes between the orbit planes is the line through the centers of the orbits.

HOW FROZEN ORBITS BOUND NEARLY FROZEN ONES

This research began when one of us (Mark) noticed in trajectory plots that when a smaller coplanar orbit is tangent at one point to the A-Train frozen orbit, the smaller orbit would remain tangent and the point of tangency would circulate clockwise around the frozen orbit. We now have all the pieces in place to explain why this should be so:

- Frozen orbits in any given orbit plane are (nearly) concentric.
- The mean center of an orbit is located at $-a\vec{\mathbf{e}}$ in the orbit plane.
- The relative positions of the eccentricity vectors of nearby coplanar orbits are (nearly) not affected by short period (over one orbit) and medium period (over one day) variations in the gravity field.
- The mean center of a nearly frozen orbit moves (nearly) in a circle around the mean center of the frozen orbit of the same size in the same orbit plane.
- Keplerian orbits with low eccentricity are (nearly) eccentric, i.e., off-center, circles.
- Two nested circles in a plane are tangent if and only if the distance between their centers equals the positive difference in their radii.

In each case, the qualification "(nearly)" means within a few tens of meters.

Thus if a smaller coplanar orbit is tangent to the A-Train frozen orbit, then it is starting with its mean center some distance from the center of the A-Train orbit, and that distance equals the amount by which its semi-major axis is smaller. Over time, the smaller orbit's center moves in a clockwise circle around the A-Train orbit's center, keeping their distance fixed. Since the mean semi-major axes of the orbits, and hence their difference, are also fixed, the smaller orbit remains tangent to the A-Train orbit and the point of tangency moves clockwise around the A-Train orbit.

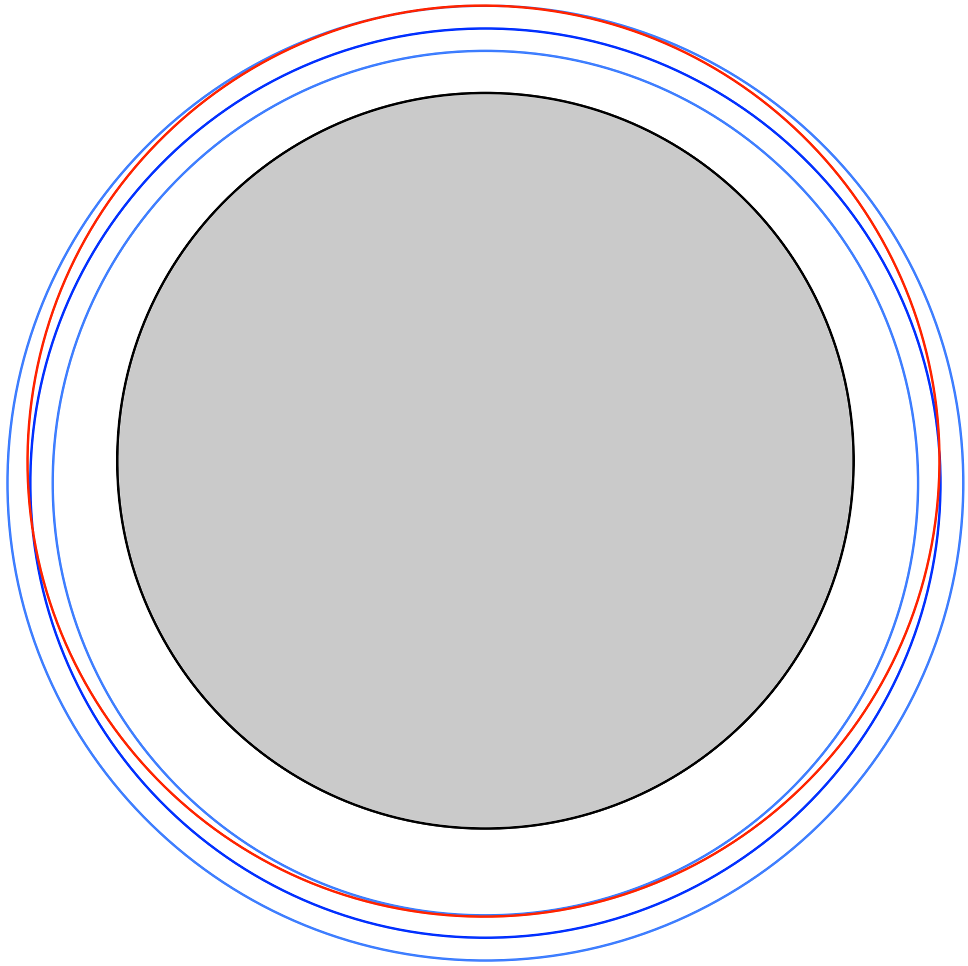

Figure 7. A nearly frozen orbit (red) bounded by two tangent concentric frozen orbits (light blue). The dark blue frozen orbit is the same size as the nearly frozen orbit. As the nearly frozen orbit evolves with time, it will stay tangent to the same frozen orbits (within a few tens of meters) and the points of tangency will move clockwise around the inner and outer frozen orbits.

More generally, all the frozen orbits in any given orbit plane form a nested set of concentric circles. Any nearly frozen orbit in that plane will be tangent on the outside to a larger frozen orbit and on the inside to a smaller frozen orbit, as shown in Figure 7. As the nearly frozen orbit evolves with time it will stay tangent to the same frozen orbits and the points of tangency will move clockwise around the orbits. If the A-Train frozen orbit is even larger than the outside tangent frozen orbit, then it stays the same distance from the outside tangent frozen orbit all the way around, namely by how much its semi-major axis exceeds that of the outside tangent frozen orbit. This means that the nearly frozen orbit starts out that same distance from the A-Train frozen orbit and stays that same distance as it evolves, as we had observed in Figure 4.

Since the actual spacecraft in the A-Train have orbits that are not exactly the A-Train frozen orbit, they themselves are nearly frozen orbits with their own tangent bounding frozen orbits. They may be tangent to their bounding orbits at different latitudes from other nearly frozen orbits, but their bounding frozen orbits will stay the same distance apart. For example, some particular A-

Train spacecraft may be on an orbit whose inner tangent bounding frozen orbit is $100\,\mathrm{m}$ below the A-Train frozen orbit. If a smaller nearby orbit has a circumscribed bounding frozen orbit which is $600\,\mathrm{m}$ below the A-Train frozen orbit, then there is $500\,\mathrm{m}$ between the circumscribed bound of the smaller nearby orbit and the inscribed bound of the A-Train spacecraft orbit, so the two orbits will never be closer together than $500\,\mathrm{m}$. Note that all of these statements are approximate, but the approximations are all within tens of meters of the truth, so gaps between orbits are maintained to within a few tens of meters.

OPPORTUNITIES FOR FUTURE WORK

There's a lot more we wanted to do in this area, if only there were money enough and time. This is an opportunity for others, students in particular, to extend this research. We would like to see more examples worked out, with different inclinations and a larger spread of orbit sizes. It would be very interesting to see the same analysis applied to Mars, the Moon, or other planets. In a more theoretical area, a convergence analysis should be done for the series defined by Cook.[5]

CONCLUSION

Although nearly frozen orbits have varying eccentricity, their behavior is not nearly as eccentric as has been assumed in the past. Their well-behaved evolution means that we can design maneuver plans using orbits which are more closely nested than has been the practice, which means that we now have more space to work in.

ACKNOWLEDGMENT

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