HETEROCLINIC, HOMOCLINIC CONNECTIONS BETWEEN THE
SUN-EARTH TRIANGULAR POINTS AND QUASI-SATELLITE
ORBITS FOR SOLAR OBSERVATIONS

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Investigation of new orbit geometries exhibits a very attractive behavior for a
spacecraft to monitor space weather coming from the Sun. Several orbit transfer
mechanisms are analyzed as potential alternatives to monitor solar activity such
as a sub-solar orbit or quasi-satellite orbit and short and long heteroclinic and
homoclinic connections between the triangular points \( L_4 \) and \( L_5 \) and the collinear
point \( L_3 \) of the Circular Restricted Three-Body Problem (CRTBP) in the Sun-
Earth system.

INTRODUCTION

Heteroclinic and homoclinic connections are of special interest for space mission applications,
such as low energy orbits to the Moon\(^{11}\) and the Petit Grand Tour to the moons of Jupiter.\(^3\) We
know that in the Earth-Moon system\(^{10}\) a spacecraft in orbit near the triangular points \( L_4 \) and \( L_5 \) can
migrate back and forth between these points through the collinear point \( L_3 \) without encountering the
Moon. Similarly, a spacecraft in the Sun-Earth system can exhibit this motion, which has recently
been observed in asteroids. The first Earth Trojan asteroid\(^2\) (2010 TK7), was discovered in 2010 by
the Wide Infrared Survey Spacecraft (WISE) spacecraft. This Earth co-orbital asteroid (ECA) is in
a 1:1 mean motion resonance with the Earth, that is, it goes around the Sun in the same amount of
time as the Earth. Asteroid 2010 TK7 has an approximately 390-year cycle. Currently, this asteroid
orbits in tadpole-shaped loops around \( L_4 \). These loops are large, reaching as close as 20 million
km from Earth (about 50 times the distance from the Earth to the Moon) and nearly as far as the
opposite side of the Sun from the Earth. Eventually, the motion of 2010 TK7 will reverse direction
and come back to its current position. This irregular motion of this type of orbit will be analyzed in
some examples in this paper. But there are many more asteroids following a horseshoe orbit motion,
such as asteroids 2003 YN107 and 2010 SO16. These asteroids are in 1:1 resonance with the Earth
because they share Earth’s orbit. So, when they are in front of the Earth, they are slowed down and
they are accelerated when they are behind the Earth.

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MOTIVATION

In previous papers, we investigated transfer trajectories and orbits around the triangular points (Trojan orbits) that are above or below the ecliptic plane. We also analyzed other orbits that are displaced from \( L_5 \), called "sub-\( L_5 \) orbits" that are more attractive from a science perspective because a spacecraft in any of these orbits could anticipate space weather up to 7 days earlier than its arrival at Earth. Other nominal Trojan orbits provide only a 3-5 day advance warning at the Earth. Some of these orbits are illustrated in Fig 1. Although some of these orbits are very promising in studying solar events, there are other orbits where the spacecraft does not need to be strictly off the ecliptic plane, requiring less \( \Delta V \) and therefore yielding a lower mission cost. This paper will address some of these orbits in the ecliptic plane as shown in Figure 2(a), potential heteroclinic connections between the triangular points, and heteroclinic connections between the triangular points and the collinear point \( L_3 \) in the Sun-Earth system. Besides the tadpole Trojan and horseshoe motion, we could place a satellite into a retrograde quasi-satellite (QS) motion as shown by some of the orbits displayed in Figure 2(a). The QS orbit (green) is a simulated trajectory depicted in Figure 2(a) with an amplitude of \( A_y = 2 \cdot A_x = 0.85 \) AU as seen in the rotating frame from the Earth. In Figure 2(b), we exhibit a schematic of a quasi-satellite orbit recommended by the scientists as an attractive option for both an operational space weather perspective and a research perspective. Similar to the
sub-$L_5$ orbits, which can anticipate space weather up to a week in advance of the Earth, which is sooner than reception in a Trojan orbit around $L_5$, these quasi-satellite orbits (sub-$L_1$ orbits) can also anticipate space weather earlier than other orbits around $L_1$.

Some of these orbits are placed closer to the Sun than other orbits located around the collinear point $L_1$. These closer orbits will provide advance warning of solar events and CMEs coming toward the Earth. In the rotating coordinate system, a spacecraft located in a quasi-satellite orbit appears to orbit (oval shape) the Earth (see Figure 2(a)), but it is actually orbiting in a heliocentric orbit around the Sun as seen in inertial coordinates. This quasi-satellite orbit (sometimes called a retrograde orbit) is relatively close to the Earth-Sun line. A constellation of satellites can be arranged so that at least one spacecraft will be closer to the Sun than the Earth while providing continuous transmission of solar conditions. This orbit geometry will help to anticipate space weather, which generates geomagnetic storms and other interplanetary disturbances that can disrupt communications in our infrastructures on Earth and endanger future human space flights.

Figure 2. a: Several shapes of quasi-satellite orbits as seen in the corotating frame. The green orbit is a simulated quasi-satellite orbit. b: Artist’s conception of a quasi-satellite orbit (QSO) around the Earth.

MODEL AND METHODOLOGY

We used the circular restricted three-body problem (CRTBP) in the trajectory analysis for this study. The Sun is the primary body, the Earth is the secondary body and the spacecraft is the third body or infinitesimal mass in this system. To simplify the analysis, we used the normalized and non-dimensionalized convention so that the mass of the secondary body is $0 < \mu < 1$ and the mass of the primary body is $1-\mu$. The distance between the primary and secondary bodies is normalized to one with the primary body located on the x-axis at $-\mu$ and the secondary body at $1-\mu$. The x-axis is directed from the primary body to the secondary body. The y-axis is $90^\circ$ from the x-axis in the primary plane of motion. Finally, the x-axis completes the right-handed system, defining the out-of-plane direction. For this work, $\mu = 3.040423389123456E-6$ for the Earth-Moon barycenter model based on the combined mass of the Earth and Moon, $M_{EM baryc}$, using JPL DE405 constants. Finally, time corresponds to the angle between the x-axis of the rotating frame and the x-axis of the inertial frame so that the period of the rotating frame becomes $2\pi$. Using this convention, the
motion of the infinitesimal mass in the rotating frame can be described by the governing equations of motion:

\[
\ddot{x} - 2\dot{y} = x - (1 - \mu) \frac{x + \mu}{r_1^3} - \mu \frac{x - 1 + \mu}{r_2^3}
\]

\[
\ddot{y} + 2\dot{x} = \left(1 - \frac{1 - \mu}{r_1^3} - \frac{\mu}{r_2^3}\right) y
\]

\[
\ddot{z} = - \left(\frac{1 - \mu}{r_1^3} + \frac{\mu}{r_2^3}\right) z
\]

where \( r_1 \) and \( r_2 \) are the distances from the spacecraft to the Sun and Earth, respectively.

\[
r_1 = \sqrt{(x + \mu)^2 + y^2 + z^2}
\]

\[
r_2 = \sqrt{(x - 1 + \mu)^2 + y^2 + z^2}
\]

and

\[
\mu = \frac{M_{EMbaryc}}{M_{EMbaryc} + M_s}
\]

where \( M_s \) denotes the mass of the Sun. The CRTBP is nondimensionalized so that the sum of the primary and secondary masses, the mean motion of the rotating frame, the distance between the primary and secondary, and the gravitational constant are all unity.

In the last section of this work, we will analyze other orbit geometries using low thrust. For this, we use similar two-body equations of motion in polar coordinates\(^\text{12}\) described below:

\[
\ddot{r} = r \dot{\theta}^2 - \frac{\mu_{Earth}}{r^2}
\]

\[
\ddot{\theta} = \frac{1}{r} \left( - 2r \dot{\theta} + \frac{T}{m} \right)
\]

where \( r \) is the spacecraft’s distance from the central body (in this case Earth), \( \theta \) is the true anomaly of the spacecraft’s position along the orbit, \( \mu_{Earth} \) is the mass parameter of the Earth, \( m \) is the varying mass of the spacecraft due to the thrust (T), which is assumed to be in the tangential direction. The mass variation is governed by:

\[
\dot{m} = - \frac{T}{g_0 I_{sp}}
\]

RESULTS

\( L_5-L_4 \) AND \( L_4-L_5 \) HETEROCLINIC CONNECTIONS

Figure 3 exhibits a long heteroclinic connection between \( L_4-L_5 \) and Figure 4(a) shows a \( L_5-L_4 \) heteroclinic connection. The long \( L_4-L_5 \) transfer takes 1685.6 days and requires an injection deltaV of 897.4 m/s to depart from the Trojan orbit (amplitude of 0.73 AU) around \( L_4 \) and a deltaV of 861.3 m/s to insert into the Trojan orbit (0.73 AU amplitude) around \( L_5 \). The tick marks in Figures 3 and 4(a) are separated by 30 days and 60 days, respectively.
Figure 3. Long heteroclinic connections between the triangular points $L_4$ and $L_5$ in the Sun-Earth system. Every loop or bounce is one Earth year long. Long heteroclinic connection with a time of flight of 1685.59 days. The spacecraft requires 897.38 m/s to depart from a Trojan orbit (0.72 AU) around $L_4$ and arrives at a Trojan orbit (0.72 AU) around $L_5$, requiring 861.25 m/s at insertion.

Figure 4. a: Heteroclinic connection from $L_5$ to $L_4$ of 1583.14 days. The transfer orbit is clockwise and outside the path of the Earth around the Sun. b: Short heteroclinic connection between the triangular points $L_4$ and $L_5$ in the Sun-Earth system. The satellite loops around the Earth before heading towards the Trojan orbit around $L_5$. The time of flight is 596.62 days. The spacecraft needs 2.426 km/s when departing $L_4$ and 2.366 km/s at insertion into the Trojan orbit around $L_5$.

Figure 4(b) displays a short $L_4$-$L_5$ heteroclinic connection with a time of flight of 596.6 days. The spacecraft departs from the same Trojan orbit around $L_4$ as in the previous case, but it requires a larger injection burn of 2426.7 m/s and it arrives at a Trojan orbit around $L_5$, where it needs an
Heteroclinic and Homoclinic Connections

In Figure 6(a), we show a homoclinic connection where the satellite departs from an asymmetric orbit with a 342 m/s injection burn and a time of flight of about 14 years. Then it stays for about 2 years before returning counterclockwise towards the same vicinity of the asymmetric orbit. The time of flight of the return trajectory is 8 years.

For the heteroclinic connections displayed in Figure 6(b), the spacecraft can orbit the vicinity of $L_3$ for about 34 years and it does not need any deterministic maneuvers to be captured around $L_3$. However, the satellite requires an injection burn at the $L_5$-orbit of about 730 m/s for the red trajectory and 536 m/s for the purple trajectory. After this time, the spacecraft moves away from the vicinity of $L_3$ clockwise towards $L_4$ for the trajectory in red and counterclockwise for the trajectory in purple as shown in Figure 7 (amplified section of Figures 6(b) and 6(c)).

Similarly, the heteroclinic connection displayed in Figure 6(c) illustrates a satellite leaving $L_5$ with a required burn of only 299 m/s, arriving 11 years later in the vicinity of $L_3$ with a stay time of about 17 years. After this time, the satellite follows a counterclockwise motion around the Sun towards $L_4$.
Figure 6. a: Long homoclinic connection between $L_5$ and $L_3$. b: Short heteroclinic connections between the triangular points $L_5$ and $L_3$ in the Sun-Earth system of 6 years (red) and 7 years (purple). c: Long heteroclinic connection from $L_5$ to $L_3$ of 11 years.

Other quasi-periodic orbits were also analyzed around $L_3$ with amplitudes large enough (0.1 AU or larger as seen in Figure 6) so that a spacecraft will be able to communicate directly with Earth and without being occulted by the Sun. An amplified section of the orbits around $L_3$ is illustrated in Figure 7. Although these orbits do not require an insertion burn into the $L_3$ orbits in the CRTBP, the spacecraft may need minor station-keeping maneuvers when using a high fidelity ephemeris model. A spacecraft in one of these orbits will obtain observations of CMEs coming from behind the Sun (as seen from the Earth) that cannot yet be seen by a spacecraft at $L_1$ and therefore will have the advantage of earlier warnings when the CME or any other solar event travels along the Parker spiral within the corotating zone.
Figure 7. Large amplitude orbits around $L_3$ in the Sun-Earth system. a: Maximum amplitude excursions of about 0.1-0.18 AU. b: Maximum amplitude excursions of about 0.15 AU.

QUASI-SATELLITE ORBITS

Perhaps one of the most attractive orbits for solar observations is the quasi-satellite orbit depicted in Figure 8(a). The satellite departs from a 200-km parking orbit around the Earth with an injection burn of 3.613 km/s. The transfer time to the insertion into the quasi-satellite orbit ($A_x=0.425$ canonical units or 63,833,411 km and $A_y=0.825$ canonical units or 127,666,828 km) is about 245.8 days, requiring an insertion $\Delta V$ of 3.07 km/s. Among all the trajectories that were explored by varying slightly the velocity at injection up to 1%, we found that our lowest $\Delta V$ solution (not optimized) of 3.07 km/s corresponds to $X = 1.132251$ AU and $Y = -0.008929$ AU as we illustrate in Figure 8(b). Other trajectories were also obtained for shorter times of flight but at the expense of very high insertion $\Delta V$s which may not be suitable for mission design purposes for current propulsion systems. The trajectories of the probe intersect the quasi-satellite orbit at different points so each trajectory will also have a different time of flight. In Figure 8(c), we display $\Delta V$-insertion into the QSO for a given transfer time.

The spacecraft is inserted into the quasi-satellite orbit that has been numerically integrated over 20 years, showing very good stability properties for very long periods of time. We also analyzed the invariant manifolds from the quasi-satellite orbit. Given the stability properties of this QSO, the orbits departing (unstable manifolds) from the QSO or arriving (stable manifolds) to the QSO have small orbit drifts of less than 200,000 km when the perturbation from the QSO is 1,000 km
Figure 8. a: Possible transfer trajectories from a 200-km parking orbit around the Earth to a QSO. b: Transfer trajectory from a 200-km Earth parking orbit ($\Delta V_{\text{inj}} = 3.613 \text{ km/s}$) to a quasi-satellite orbit. The transfer time to the QS orbit is 264.43 days. The insertion burn, indicated by a black star, $\Delta V_{QS}$, requires 3.07 km/s. The QS orbit has been integrated over a time span of 20 years. c: $\Delta V$ in km/s of each of the possible transfer trajectories (magenta) depicted in Figure 8(a).

(standard perturbations are of the order of 200 km when computing the invariant manifolds around the collinear points in the Sun-Earth system). When considering very large perturbations of the order of an Earth-Moon distance, these trajectories obviously drift much more (see Figure 9(a) and 9(b)). The integration over 5 years included fifty trajectories for each manifold.

Another possibility for transferring to a quasi-satellite orbit is via the heteroclinic connection between the triangular points in the Sun-Earth system as observed in Figure 10. There are four...
Figure 9. Left: Stable invariant manifolds for large QSO perturbation. Right: Unstable invariant manifolds. The black orbit represents the quasi-satellite orbit (QSO).

Figure 10. Locations (red dots) where possible maneuvers can be performed to transfer the spacecraft between the triangular points heteroclinic connection into a QSO in the Sun-Earth system.

locations (indicated by red dots) where deterministic maneuvers can be performed to transfer the spacecraft from this heteroclinic connection into the QSO. In this particular case, the heteroclinic transfer departs from the Trojan orbit around $L_4$ towards $L_5$ so the insertion into the QSO would be such that the spacecraft would have a direct motion. Another scenario (not shown in figure) would be an heteroclinic connection departing from the Trojan orbit around $L_5$ (and their corresponding transfer orbits$^{6,9}$ from Earth) so that the spacecraft would follow a retrograde motion after insertion into the QSO.
OTHER ORBIT GEOMETRIES

The quasi-satellite orbit (green) analyzed in Figure 11 is symmetric about the Sun-Earth direction. This quasi-satellite orbit (magenta) can also be found displaced (in the direction of $L_4$) from the Sun-Earth direction as illustrated in Figure 11. In this case, the spacecraft has a direct motion around the QSO. The transfer trajectory started from a 200-km orbit with an initial mass of 10,600 kg. The thrust is performed during a time period of about 65 days consuming a total of about 2,054 kg. This leaves a spacecraft wet mass of 8,546 kg on arrival to the QSO, which corresponds to a similar mass of the first Chinese space station, Tiangong 1, of about 8,500 kg.

![Displaced Quasi-Satellite Orbit](image)

**Figure 11.** Displaced QSO in the Sun-Earth system.

Similarly, orbits (not shown) displaced towards $L_5$ can also be found with the spacecraft having a retrograde motion around the QSO.

There seem to be other attractive orbits not only for weather observations but also for searching new undiscovered asteroids (the first Earth Trojan asteroid, 2010 $TK7$, was discovered in 2010). In this example, we discuss a possible scenario for the Hubble Space Telescope (HST) after NASA decides to decommission its use. The HST has an Advanced Camera for Surveys (ACS) that can be used not only to search very distant objects outside the Solar System but also to search new comets and asteroids. The HST can be used for imaging asteroids from a Hight Earth Orbit (HEO) with higher resolution that will provide new data on the dynamics and geophysical properties of these pristine bodies.

In our simulations, we assumed that the HST is in its original orbit of about 559 km with a mass of about 17,000 kg (mass of HST is about 11,110 kg). Figure 12 displays a possible low-thrust trajectory from a Low Earth Orbit (LEO) at 559-km to a High Earth Orbit (HEO) with a semimajor axis of about 183,556 km. Boosting the HST from LEO to this HEO, requires burning approximately 5,497 kg over almost 437 days of thrusting. This burn leaves a final spacecraft mass of 11,503 kg which is nearly the actual mass of HST. The apogee of this HEO is 189,630 km and...
the perigee is about 177,480 km. The time of flight to transfer the HST from its original parking orbit to this HEO is about 448 days. Then, it will be its new high Earth orbit having an eccentricity of 0.033 with a period of about 11.3 days. The thermal power obtained to bring the HST to this HEO is about 17.9 kW. This power can be achieved using General Purpose Heat Source (GPHS)-Radiosotope Thermal Generators (RTGs) of Plutonium (Pu-238) to provide electricity to the entire propulsion system in the HST. The use of 4 RTGs (Cassini spacecraft has 3 RTGs aboard) can supply all electrical power in the HST. Each RTG can supply a power at the beginning of life (BOL), $P_{BOL}$, of about 300 W. Assuming that the conversion efficiency is 6.7%, then the BOL power is $17.9 \times 0.067 = 1,199$ W, which corresponds to the actual power, $300 W \times 4 RTGs = 1,200$ W, that the 4 RTGs can supply to the HST. Assuming a specific power of 0.56 W/g for the isotope Pu-238, the total mass of Pu-238 needed is $\frac{17.9}{0.56} W/g$ or 32 kg. The mass of each RTG is about 55 kg, therefore the RTG system makes a total mass of 252 kg.

![Diagram](image)

Figure 12. Low-thrust trajectory of the Hubble Space Telescope from a 559-km low Earth orbit to a high Earth orbit with semimajor axis of 183,556 km.

The specific impulse delivered by the Hall thrusters was assumed to be 1,600 seconds. This performance can be achieved with a Stationary Plasma Thruster (SPT)-100 which has electric propulsion lifetimes of about 7,000 hours. This specific impulse yields a mass flow rate of $3.066 \times 10^{-5}$ kg/s and the power required of about 4,200 W assuming 90% efficiency. However, this power is larger than the actual power that the HST can supply. The Hubble Space Telescope has aboard a SPT-140 with an electric power performance of 2,700 W and lifetime of 10,000 hours ($\approx$424 days). This power of 4,150 W would produce a thrust of 0.5288 N. With this propulsion specifications, the HST will take a very long time to escape Earth’s sphere of influence in a permitted time. Therefore, our solution to this problem was to increase the thrust to 2.28 N bringing the HST to the HEO in about 14 months and using 4 RTGs that meet our power requirement of 17.9 kW.
DISCUSSION AND SUMMARY

 Trojan orbits and sub-$L_5$ orbits were previously found as potential locations for space weather monitoring since they can anticipate weather up to one week in advance before solar events actually arrive at Earth. In this paper, we have investigated new orbit geometries, such as heteroclinic and homoclinic connections between the triangular points and the collinear points in the Sun-Earth system.

 Some of these trajectories are heteroclinic connections linking $L_4$ and $L_5$ for a total $\Delta V$ (departure and arrival) of less than 1.8 km/s. This type of trajectory loops around the Earth so material could be transported not only between the triangular points but also between these trajectories and the quasi-satellite orbits (QSO) described in this work. Other heteroclinic connections are also possible between the $L_5$ and the collinear point $L_3$ in the Sun-Earth system. The total $\Delta V$ required for some of these trajectories can be as low as 299 m/s and as high as 730 m/s, depending on the time of transfer and the size of the initial orbit around $L_5$. These orbits are very attractive from the science perspective because a probe could monitor space weather without interruptions once it orbits the vicinity of $L_3$ with amplitudes of about 0.01 AU (1,500,000 km) to 0.18 AU (27,000,000 km) and therefore, could be used during the end of life of the spacecraft. The spacecraft would not need any deterministic maneuvers to be captured into the $L_3$ vicinity because it will be captured without a maneuver. The probe will stay in the vicinity of $L_3$ for many years before being ejected out of this neighborhood either towards the $L_4$ point (similar to the motion of asteroid 2010 TK7) or back to the $L_5$ vicinity again. These are examples of heteroclinic and homoclinic connections that can be used in future space missions for reconnaissance purposes where the spacecraft will tour different libration points.

 Other orbit geometries (QSO) were also investigated as promising orbits in which to place a spacecraft for solar observations purposes. These orbits are in 1:1 resonance with the motion of Earth around the Sun from where several small spacecraft will be able to provide continuous solar observations from a sub-$L_1$ perspective and be able to anticipate space weather many hours in advance from a $L_1$ position. Quasi-satellite orbits can also be displaced either upstream or downstream from the Sun-Earth line and therefore could be tailored accordingly to the mission requirements.

 We are constantly intrigued by the different family of orbits that we can find and we may be able to find in the future. Besides the Trojans orbits around $L_5$, sub-$L_5$ orbits, tadpole orbits, horseshoe orbits described in past work, this work describes very attractive heteroclinic and homoclinic connections between the triangular points and collinear points in the CRTBP of the Sun-Earth system and quasi-satellite orbits for solar observations.

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