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LIAISON-SUPPLEMENTED NAVIGATION OF A CREWED VEHICLE IN A LUNAR HALO ORBIT

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This paper offers an early examination of the challenges of navigating a crewed vehicle, with all of the associated unmodeled accelerations that arise from the crew's activities, in an orbit about the Earth-Moon L_2 point. The combination of the unstable nature of libration orbits with the lack of acceleration knowledge makes the station keeping strategy challenging. It is found that a combination of ground tracking and satellite-to-satellite tracking produces the most favorable navigation accuracy. This paper examines the costs and benefits of applying LiAISON (Linked Autonomous Interplanetary Satellite Orbit Navigation) to a crewed mission in an unstable L_2 orbit.

INTRODUCTION

Navigating a spacecraft near the Earth-Moon L_2 (EML-2) point has always been considered a challenge: the dynamics are unstable enough that an unguided spacecraft will depart the L_2 orbit within a few weeks at most. Mission designers have argued that robotic spacecraft should perform station keeping maneuvers every 4-7 days to keep the fuel use low.[1,2] The two ARTEMIS spacecraft successfully navigated several libration orbits about the Earth-Moon L_1 and L_2 points using little fuel by performing maneuvers once every seven days.[3,4] The ARTEMIS team proved that navigating an unstable libration orbit in the Earth-Moon system can be done; the next big challenge will be to demonstrate the navigation of a *crewed* vehicle in such an orbit. Crewed vehicles have historically been very noisy from a dynamical perspective: they vent gases, dump wastewater, re-orient frequently via unbalanced thrusters, etc. The collection of these poorly modeled and unpredictable activities has been known as FLAK (un**F**ortunate **L**ack of **A**cceleration **K**nowledge).[5,6] FLAK's effects drive a spacecraft away from a reference trajectory rapidly and degrade the accuracy of station keeping maneuvers. This paper presents a study on the effects of FLAK on the navigation and state uncertainty of a spacecraft such as Orion traversing an unstable EML-2 orbit. The paper presents the benefits of adding LiAISON (Linked Autonomous Interplanetary Satellite Orbit Navigation) to the navigation system. Previous studies have found that LiAISON is a significant benefit to the navigation of robotic or crewed vehicles near the Earth and Moon.[7–14] It is shown here that LiAISON is indeed a benefit to the navigation of a crewed vehicle in a lunar libration orbit.

Figure 1 illustrates the architecture being studied in this paper, viewed in the Earth-Moon rotating frame from a side perspective. The crew's vehicle is shown in a large EML-2 halo orbit for visualization, though in the paper the crew is in a smaller halo orbit: one that just avoids being occulted by the Moon as viewed from the Earth. The navigation satellite, referred to in this paper as the *NavSat*, is shown in an Earth-Moon L_1 (EML-1) halo orbit; the paper surveys the navigation results for cases where the NavSat is in any of 30 different halo orbits, including EML-1 and EML-2 orbits.

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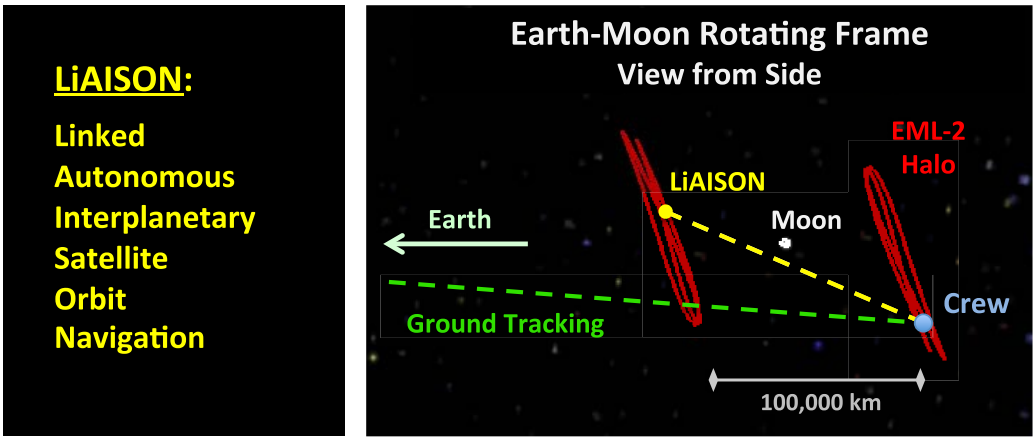

Figure 1. A scenario studied in this paper where a crewed vehicle in a lunar L_2 halo orbit is tracked from ground stations as well as from a satellite in orbit about the Earth-Moon L_1 point.

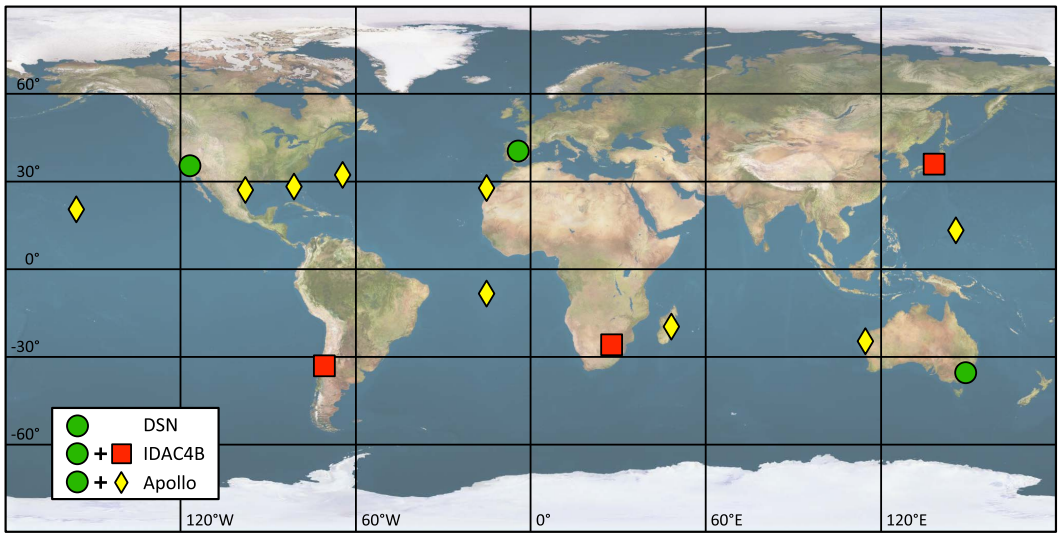

Figure 2. Different ground tracking networks. The DSN and IDAC4B networks are studied in this paper; the Apollo network is illustrated for reference.

The Apollo lunar missions were tracked using 1-2 dozen ground stations located around the Earth, including ships and aircraft. This was an intense and expensive campaign. It will be beneficial from a cost and complexity perspective if a future crewed lunar mission requires fewer ground stations. Figure 2 illustrates the main ground stations in the Apollo network as well as two smaller networks. The three main sites of the Deep Space Network (DSN) are shared in each network. A proposed *IDAC4B* configuration includes these three DSN sites as well as three supplementary sites, marked with squares in the figure. An additional three sites may be included for further redundancy. A navigation satellite near the Moon could replace several of these stations or offer improved accuracy and redundancy. Previous studies have made these conclusions when studying the navigation of a crewed vehicle in a trans-lunar cruise[14] or in a low lunar orbit.[13] The navigation of a crewed vehicle in a libration orbit presents one new significant challenge: the effects of FLAK on the spacecraft's navigation grow exponentially when the spacecraft is in an unstable orbit. It is expected

that significant navigation efforts will be required to successfully navigate the crewed vehicle; LiAISON is a strong candidate to achieve that success.

MODELING FLAK, THE UNFORTUNATE LACK OF ACCELERATION KNOWLEDGE

Past experiences have shown that crewed vehicles, such as Apollo, experience a large number of small forces that are not well-modeled, originating from frequent activities aboard the vehicle. It is expected that future crewed vehicles will also experience the same sets of disturbances. These activities and their corresponding disturbances are known as FLAK and may include any of the following, and likely others:[5,6,13]

- Wastewater venting, which results in a translational acceleration that may be performed in a particular direction, but the timing and magnitude of which is often not well predicted.
- Venting of gasses, such as CO_2, which also may be performed in a known direction, but the timing and magnitude of acceleration is not well predicted.
- Heat rejection via sublimation of water and associated venting.
- Attitude adjustments via reaction control engines that are not perfectly coupled.
- Angular momentum desaturation maneuvers. These maneuvers are also performed via the reaction control system and result in small translational accelerations.
- Other small effects such as unmodeled attitude changes that result in changes to radiation pressure on the spacecraft.

The Apollo missions provide the primary source of experience to ascertain the effects of FLAK on the uncertainty of a crewed spacecraft's state while in orbit near the Moon. The Apollo navigators observed that when a crewed vehicle was in a low near-circular orbit about the Moon and not being tracked, the vehicle's position uncertainty grew by approximately 500 meters per hour.[5,6] Some of this growth may be attributed to large errors in the lunar gravity field, which will no longer be present for future missions. Other constituents in the growth in the vehicle's estimated state depends on the activities being performed, whether or not the astronauts are awake and active, and what environment the vehicle is in. It is expected that FLAK will grow far more during an active period in the astronaut's day. It is also expected that FLAK will grow when a crewed vehicle, such as Orion, is attached to a station of some kind, since the crewed vehicle cannot then re-orient itself for its own thermal needs but must accommodate the station's needs; the station's mass may balance the increase in small force activity though that is not clear. Further, it is expected that FLAK will grow faster in an orbit like a low lunar orbit, on account of varying thermal conditions. However, it is not clear how much FLAK's effects will diminish when in a more thermally stable environment. This paper takes a very conservative approach and uses the same FLAK model for a vehicle in a libration orbit as Apollo observed in low lunar orbit.

This paper remains consistent with literature[6,13,14] and models FLAK as a source of uncertainty that grows in a spherically-symmetric fashion, yielding a position uncertainty growth of approximately 500 meters (1-σ) every hour during all waking periods. The FLAK model diminishes to only 10% during sleeping periods, namely, 8 hours of every 24.[6,13] This model is expected to be a very conservative model.

Previous studies have found that the position uncertainty of the crewed vehicle scales very linearly with the level of FLAK.[13,14] Hence, the results presented in this paper may be scaled proportionally with any improved FLAK models, presuming that they remain spherically symmetric.

MISSION ORBITS

This paper studies the scenario where a crewed vehicle is in an EML-2 halo orbit with a z-axis amplitude of 10,000 km.[15–17] This orbit has been identified as a good candidate for the proposed Orion vehicle because it has continuous access to the far side of the Moon and near-continuous access to most scientifically-interesting areas on the far side and in the southern hemisphere, such as the South Pole – Aitken Basin and the Schrödinger Crater. Further, a spacecraft in such an orbit is always visible from Earth. The NavSat will be located in one of several different locations to test out different geometries, including EML-1 and EML-2

orbits with z-axis amplitudes of 10,000 km, 20,000 km, and 35,500 km. Its phasing in its orbit compared with the crewed vehicle's phasing will be tested as well. The 10,000 km L_2 orbit is very similar to the crewed vehicle's orbit, such that the NavSat could be used as another communication relay satellite for the far side of the Moon; the 35,500 km L_2 orbit has been identified as another good candidate for a communication relay satellite about L_2.

Figure 3 illustrates the geometry of these scenarios, where the crewed vehicle's orbit is highlighted in black and the possible NavSat orbits are each shown in gray. The figure is shown in the Earth-Moon rotating reference frame, which rotates with the motion of the Earth-Moon system. Since the Earth and Moon are not in circular orbits about their barycenter, the bodies pulse back and forth; one can see this pulsation in the halo orbits in the figure. One can clearly see how the NavSat's geometry is always significantly different from ground-based tracking.

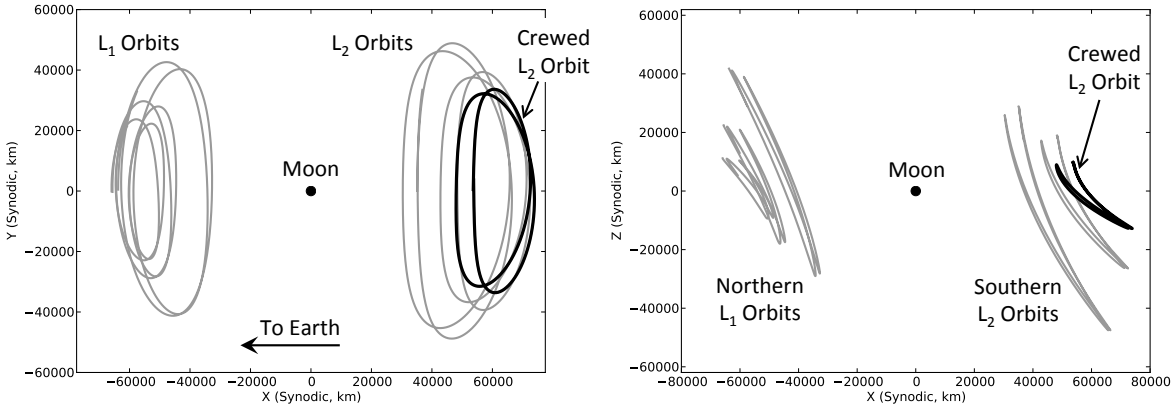

Figure 3. An illustration of the orbits studied in this paper, viewed in the Earth-Moon rotating frame from above (left) and from the side (right). The orbits pulsate with the Moon's motion. The NavSat is only located in one orbit per scenario, though each of the locations is visualized.

One major challenge of navigating a spacecraft in a halo orbit is the unstable nature of the orbit. Any deviation in the spacecraft's state away from a reference trajectory will exponentially grow, doubling approximately every 1.6 days.[17] Robotic spacecraft such as ARTEMIS combatted this exponential growth by performing station keeping maneuvers every seven days.[4] It is reasonable to assume that a crewed vehicle will have to execute maneuvers far more often due to the significant state deviations caused by FLAK. The scenarios examined in this paper implement a 4-day station keeping period.

One significant advantage in using a lunar halo orbit as the location for the NavSat is that the satellite may be inserted into it using very little fuel via a low-energy lunar transfer. In fact, the NavSat does not require any sizable propulsion system at all to insert into any useful libration orbit.[11,17–19]

SPACECRAFT NAVIGATION

Navigating a crewed vehicle on a mission near the Moon in the near future will undoubtedly involve the use of ground-based tracking stations using radiometric tracking data, such as observations of a tracking signal's round trip light time and Doppler shift – yielding information about the range and range-rate between the ground station and the spacecraft. A crewed mission will likely use additional observation data for navigation purposes, either as a backup system in case the radiometric ground-based tracking fails, and/or to improve the navigation accuracy of the spacecraft. Other observations may include optical tracking of landmarks on the Earth, Moon, or elsewhere as viewed by an observer on the spacecraft; or optical tracking of the spacecraft as viewed from an observer on the surface of the Earth. Another potential tracking system that

may be designed to be independent from ground-based tracking is the use of satellite-to-satellite radiometric tracking. Crosslink data is useful from a navigation satellite located anywhere in the Earth-Moon system, but there are particular benefits if the satellite is located in a three-body orbit, such as a halo or other libration point orbit about either the Earth-Moon L_1 or L_2 points.

LiAISON Navigation

Ordinarily, satellite-to-satellite tracking data provides information only about the relative differences between the two satellites' orbits. If the two satellites are located in typical orbits about the Earth, then one can take both orbits and rotate them freely about the Earth without modifying either the range or range-rate time-series at all. Hence, there is no information in the range or range-rate data indicating the absolute, or inertial, orientations of the two orbits. However, if one or both of the satellites are located in three-body orbits, then their dynamics are strictly tied to both massive bodies and one cannot simply rotate their states about any coordinate origin. The dynamics are tied to the locations of both bodies, e.g., both the Earth and the Moon. Since the positions of the Earth and Moon are very well known, the absolute locations of both satellites becomes known. This has been simulated and verified in numerous studies.[7–12,14,20–22]

Covariance Study

As the designs of crewed vehicles such as Orion improve, navigators will be able to model FLAK more accurately. Until the models mature, FLAK is best modeled as a spherically-symmetric constant growth in the uncertainty of the velocity of the spacecraft. When FLAK models improve it will be possible to perform full navigation simulations of the crewed vehicle. At present, the best steps to take involve studying the covariance of the vehicles, illustrating how well navigators may expect to know the spacecraft's position and velocity relative to the reference trajectory. While this doesn't provide a station keeping budget, it illustrates which combination of tracking data is needed to keep the uncertainty in the system to a minimum, which will reduce the cost of station keeping.

The covariance studies performed here construct the Cramér-Rao lower bound (CRLB) for a particular navigation scenario.[23–26] The CRLB describes the best a navigation system can ever be expected to perform in that scenario. If the CRLB for a scenario indicates an unacceptably high level of uncertainty, then one knows that there is no way that a linear navigation filter can process that data and achieve more accuracy in the state estimate – one simply requires more tracking data or tracking data of a different type to improve the navigation accuracy. The details of the algorithms used here are described by Parker, et al.,[13] with additional information about the navigation processes available in the literature, notably Tapley, Schutz, and Born.[27]

The covariance studies performed here build reference trajectories for both the crewed vehicle and the NavSat, build simulated observations, and then process them using a Kalman filter.[11,27] The simulated tracking data includes white noise and several sources of measurement biases. The Kalman filter is initialized using an *a priori* covariance matrix that represents the initial uncertainty in the state of the system. As observations are processed, this state estimate covariance matrix shrinks according to the information present in the observations. The covariance matrix is also inflated systematically to simulate the station keeping maneuvers and FLAK model. The time series of this covariance matrix yields the information that is of interest.

The state vector $\mathbf{X}$ used in the analyses performed here includes a simple set of 12 parameters: the position and velocity of both the NavSat and the crewed spacecraft:

$$\mathbf{X} = \left[\, \mathbf{R}^T_{NavSat},\ \mathbf{V}^T_{NavSat},\ \mathbf{R}^T_{Crew},\ \mathbf{V}^T_{Crew} \,\right]^T .$$

The *a priori* covariance, $\overline{P}_0$, used in each simulation, unless otherwise noted, is set to approximate a situation where the NavSat is somewhat better known than the crewed vehicle, partly because the NavSat will have presumably been tracked ahead of time and also because the NavSat is expected to be a quieter vehicle. The *a priori* covariance is a 12×12 diagonal matrix with the following values on the diagonal:

$$\overline{P}_0 = Diag\left(\, \sigma^2_{x,NavSat},\ \sigma^2_{y,NavSat},\ \sigma^2_{z,NavSat},\ \sigma^2_{\dot{x},NavSat},\ \sigma^2_{\dot{y},NavSat},\ \sigma^2_{\dot{z},NavSat},\ \cdots \right.$$
$$\left. \sigma^2_{x,Crew},\ \sigma^2_{y,Crew},\ \sigma^2_{z,Crew},\ \sigma^2_{\dot{x},Crew},\ \sigma^2_{\dot{y},Crew},\ \sigma^2_{\dot{z},Crew} \right),$$

where each uncertainty has been set to the following:

- 100 meter 1-σ uncertainty on each position component of the NavSat;
- 1 cm/s 1-σ uncertainty on each velocity component of the NavSat;
- 1 km 1-σ uncertainty on each position component of the crewed vehicle;
- 1 m/s 1-σ uncertainty on each velocity component of the crewed vehicle.

Process noise is introduced to compensate for unmodeled accelerations in both the crewed vehicle and the NavSat. Of course FLAK is a large unmodeled acceleration and dominates the unmodeled accelerations of the crewed vehicle; the NavSat on the other hand is likely to be a relatively well-modeled spacecraft in a very benign environment. The process noise covariance matrix applied to the NavSat is a 3×3 diagonal matrix with diagonal elements of $\sigma^2 = (5 \times 10^{-15}\ \text{km/s}^2)^2$. Empirical testing[13] estimates that the process noise covariance matrix should have diagonal elements of $\sigma^2 \approx (1.195 \times 10^{-5}\ \text{m/s}^2)^2$ in order to permit the position uncertainty of the crewed vehicle to grow by about 500 meters in an hour while in low lunar orbit, approximately matching the Apollo experiences. Once again, the magnitude of FLAK drops by an order of magnitude during sleep periods.

Tracking Network

A previous study led by the Constellation program identified three locations that could supplement the three DSN sites and provide substantial North-South baselines as well as strong East-West baselines: enough to be sufficient for supporting modern lunar exploration. These sites were identified in the *integrated design and analysis* (IDAC) 4B analysis cycle, hence the name "IDAC4B". The three new sites include stations located in Santiago, Chile, in Hartebeesthoek, South Africa, and in Usuda, Japan, as illustrated in Figure 2. Notice that each of the three DSN sites has a counterpart in the opposite hemisphere. The three new IDAC4B sites would likely be receive-only stations, cooperating with the DSN sites as three-way receiving stations. The locations of each of the ground stations in the DSN and IDAC4B networks used in the current simulations is presented in Table 1. These are approximate locations of each station and do not correspond with any particular antenna.

Table 1. The latitude, longitude, and altitude of each of the three DSN sites and the remaining three IDAC4B sites used in this research, relative to a spherical Earth with a radius of 6378.14 km. Longitudes are presented in Eastward coordinates.

Station	Latitude	Longitude	Altitude
Goldstone	35.247°	243.205°	1.071 km
Madrid	40.427°	4.251°	0.835 km
Canberra	-35.398°	148.982°	0.692 km
Santiago	-33.450°	289.333°	0.731 km
Hartebeesthoek	-25.890°	27.687°	1.414 km
Usuda	36.139°	138.363°	1.456 km

For the purpose of the simulations presented here, each of these stations has a 10 degree elevation mask. The IDAC4B sites are indeed receive-only and perform three-way tracking with whichever DSN site is in view. If multiple DSN sites are in view of the spacecraft, only one site tracks the spacecraft: Goldstone is prioritized if it is available, followed by Madrid and then Canberra. Each of the IDAC4B receive-only stations are assigned to provide complementary services with one DSN site; for the purpose of this study, Santiago and Goldstone work together, Hartebeesthoek and Madrid work together, and Usuda and Canberra work together. Hence, only one DSN site and at most one IDAC4B receive-only station are active at any given time.

Tracking Data

The tracking data includes 2-way range and range-rate observations between the NavSat and the crewed vehicle; 2-way range and range-rate observations between the DSN and each vehicle; and 3-way range and range-rate observations when one of the IDAC4B ground stations is in view, since they are only designed to receive data and not to transmit it. The NavSat will have DSN tracking passes, but only the crewed vehicle will be tracked by the IDAC4B stations. The actual radiometric tracking data likely consists of observations of the Doppler shift of the carrier signal and an estimate of the phase angle of the given waveform being observed. Time-tags may be used to estimate the ambiguity in the number of waveforms that have passed between transmission and reception of the signal. It is assumed that the NavSat's clock is reasonably stable and that the IDAC4B stations' clocks are synchronized with the DSN clocks. The result is that these measurements may be treated as observations of range and range-rate in the link.

For the fidelity required at present, each measurement is modeled with Gaussian noise and a bias. Every link between two antennas has a systematic bias attached to it, representing the semi-permanent errors in modeling the paths in the hardware for a particular communication link, such as the link between Goldstone and the crewed vehicle (assuming the same antenna at Goldstone is always used). In addition, each and every tracking pass has an additional bias added on, representing the errors that correspond to locking on to that particular track, through a mis-modeled atmosphere, and with slight variations in the hardware. Therefore each observation track's bias is a sum of the systemic bias between those two antennas and a new independent bias added to that particular track.

The models for range and range-rate between antennas 1 and 2 are:

$$\rho_{12} = \sqrt{(\mathbf{r}_1 - \mathbf{r}_2) \cdot (\mathbf{r}_1 - \mathbf{r}_2)} + \rho_{12}^{bias} + \rho_{12}^{noise} \tag{1}$$

$$\dot{\rho}_{12} = \frac{(\mathbf{r}_1 - \mathbf{r}_2) \cdot (\dot{\mathbf{r}}_1 - \dot{\mathbf{r}}_2)}{\rho_{12}} + \dot{\rho}_{12}^{bias} + \dot{\rho}_{12}^{noise} \tag{2}$$

Table 2 summarizes the bias and white noise placed on each tracking link in these scenarios. Tracking data are processed every 10 minutes; the uncertainties may be reduced further by adding more data into each scenario. We stress that these values are incredibly conservative, partly to accommodate for any spacecraft hardware uncertainties, for any ground system anomalies, and for any atmospheric effects. The DSN's experiences have repeatedly demonstrated much more accurate 2-way tracking, especially with regards to Doppler biases. With a crew involved, these initial studies have opted to consider conservative designs.

Table 2. The parameters used to define the errors in the tracking data. The bias added to each link and the white noise added to each observation are drawn from Gaussian distributions with zero mean and the given standard deviations.

Tracking Link	Const. Bias 1σ	Pass Bias 1σ	White Noise 1σ	Comments
NavSat - Crew 2-way range	1 m	3 m	1 m	LiAISON range SST
NavSat - Crew 2-way range-rate	0.5 mm/s	1 mm/s	1 mm/s	LiAISON range-rate SST
DSN - NavSat 2-way range	10 m	30 m	10 m	DSN ground tracking of
DSN - NavSat 2-way range-rate	0.5 mm/s	1 mm/s	0.5 mm/s	the NavSat halo orbiter
DSN - Crew 2-way range	10 m	30 m	10 m	DSN ground tracking of
DSN - Crew 2-way range-rate	0.5 mm/s	1 mm/s	0.5 mm/s	the crewed vehicle
IDAC4B - Crew 3-way range	10 m	30 m	10 m	IDAC4B ground tracking
IDAC4B - Crew 3-way range-rate	0.5 mm/s	1 mm/s	0.5 mm/s	of the crewed vehicle

Figure 4 illustrates the timeline of observations that are simulated for a crewed vehicle in a 10,000 km halo orbit about EML-2 and for a NavSat in a 35,500 km halo orbit about EML-1. The timeline for a scenario that places the NavSat at any other orbit is nearly identical, since each other orbit remains near the Moon and always in view of the crew. Of course, not all scenarios process all observations. As mentioned earlier,

there may be overlaps where two DSN stations and/or two IDAC4B stations can view either of the vehicles, but the simulations process data from only one DSN station and one IDAC4B station at a time. It is assumed that each DSN site has antennas available to track both the NavSat and the crewed vehicle, if needed. It is assumed that the IDAC4B sites never track the NavSat.

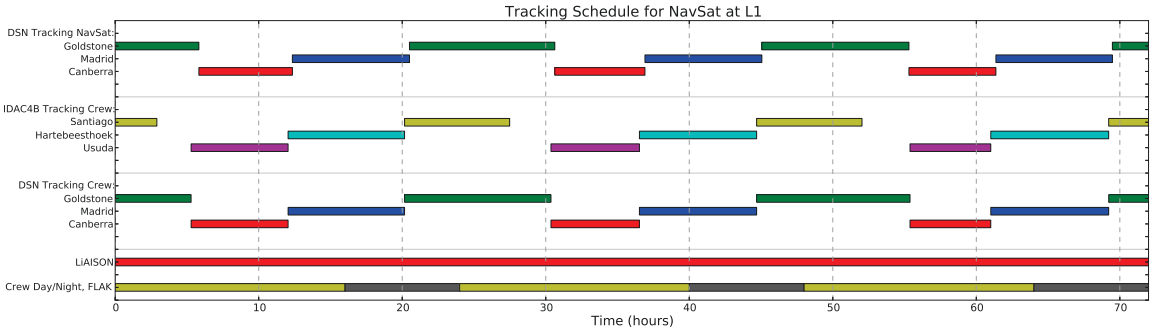

Figure 4. An illustration of the schedule of tracking observations for the case of an L_1 halo orbiter. Not all observations are used in all scenarios.

Figure 5 illustrates example observation tracking errors in the links between the DSN and the crewed vehicle. While observation errors are shown during DSN overlap periods, only one DSN station's observations are processed in these simulations.

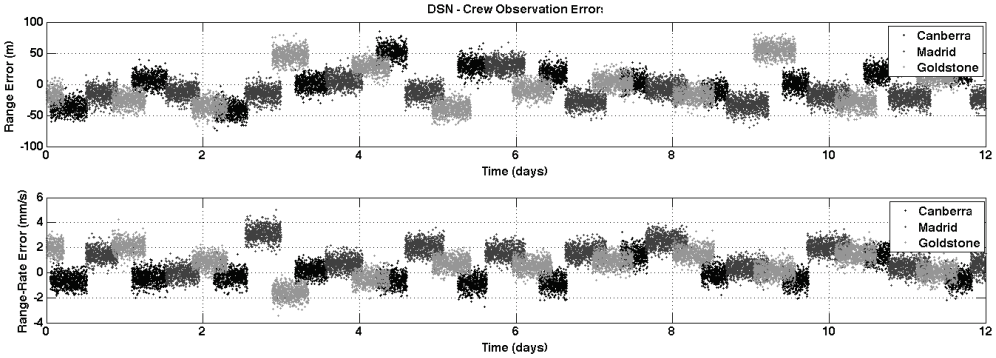

Figure 5. Example observation errors applied to the DSN–Crew links.

Since the NavSat is in continuous view of the crewed vehicle, continuous tracking is available, depending on whether or not the hardware on the crewed vehicle can support that. It is assumed that the link between the two vehicles loses lock at regular intervals to simulate a change in the range and range-rate biases in the tracking data. These scenarios assume the loss occurs every two hours, as illustrated in Figure 6. This corrupts the data a small amount in a realistic way and effectively raises the level of noise on the signal.

MODELS

The dynamical model used to propagate each vehicle in these scenarios includes the force of gravity of all of the planets in the Solar System, using the Jet Propulsion Laboratory's Planetary and Lunar Ephemerides DE405.[28]. The planets' masses are each modeled as point-masses, which is a very good approximation for modeling gravity in halo orbits. Both vehicles have been assigned an area-to-mass ratio of 0.01 for the purpose of modeling solar radiation pressure. The solar radiation pressure is modeled using a flat-plate approximation with a nominal C_R value of 1.0. Neither vehicle ever enters the shadow of the Earth or Moon.

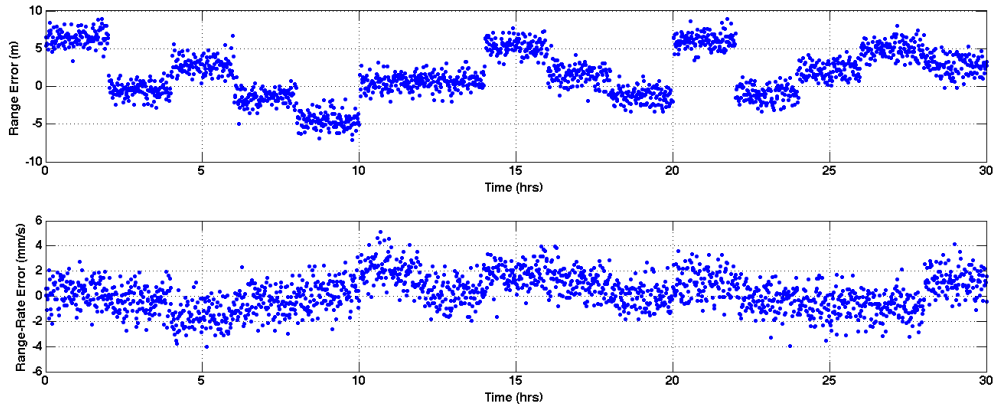

Figure 6. The first 30 hours of simulated LiAISON tracking data errors.

Both the crewed spacecraft and the NavSat execute simulated station keeping maneuvers in order to remain on their respective halo orbits. The simulations presented here implement a strategy where both spacecraft execute station keeping maneuvers at the same time every four days. Since these studies do not design the maneuvers themselves, no information is available about their orientations; hence a spherically-symmetric error is added to each spacecraft's velocity uncertainty to model the maneuver. Each velocity component's variance is inflated by $\sigma^2 = (1 \text{ mm/s})^2$.

SIMULATIONS

Ground Only

As a reference, the first simulations that are studied include only ground-based tracking. This provides a foundation for future studies, since other engineers may use slightly different parameters for the noise, bias, elevation mask, and other parameters of each navigation simulation. Each of these simulations begins with large *a priori* values to illustrate the simulations clamping down on the position uncertainties of each vehicle.

Figure 7 illustrates the 3σ position uncertainty over time as the DSN tracks each vehicle. This scenario involves no LiAISON observations, no IDAC4B observations, and also no FLAK. It is very apparent that the navigation system converges quickly to a steady state. One can see a subtle, but distinct, growth in the position uncertainty of both vehicles due to the station keeping maneuvers at the 4th and 8th day of the time series. This growth is far more visible in the individual components of the position and velocity uncertainty for each vehicle, especially the velocity components, as illustrated in Figure 8.

One expects to see a drop in the position uncertainty for the crewed vehicle if the full IDAC4B system is used to track it. Conversely, one expects to see a significant rise in the position uncertainty if FLAK is included. Figure 9 compares all four of these combinations to illustrate the effects of FLAK and of a varying number of ground stations on the position uncertainty of the crewed vehicle. One should also note that the NavSat's position uncertainty does not change in any of these scenarios, since it is not communicating with the crewed vehicle, nor with the IDAC4B receive-only stations. Its position uncertainty does change slightly depending on which halo orbit it is placed in.

This paper will include a large number of scenarios with the intention of directly comparing them to identify the advantages and disadvantages of each. This study has formulated a cost function that summarizes the results of a navigation simulation using just two numbers that are based on the magnitudes of the position uncertainty in the navigation time series. First we take the RSS of the position uncertainty after the simulation has reached or come close to steady-state, nominally 4 days into the simulation, and collect the data through

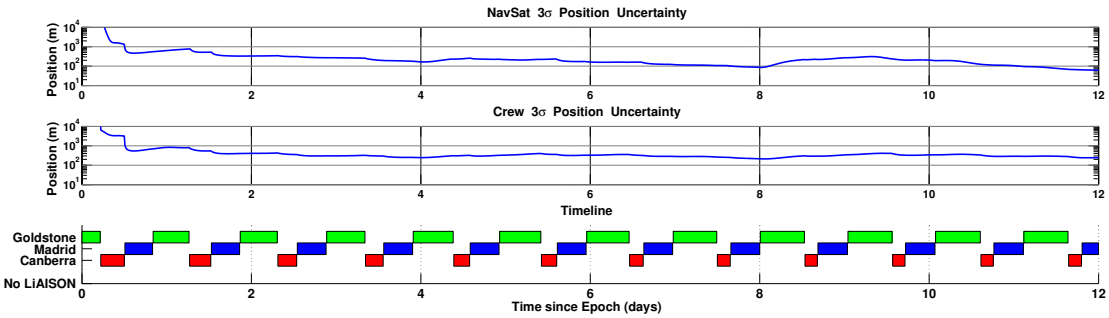

Figure 7. The position uncertainty over time of both the NavSat and the crewed vehicle as they are each independently tracked by the three main DSN ground stations. No FLAK or LiAISON observations are included.

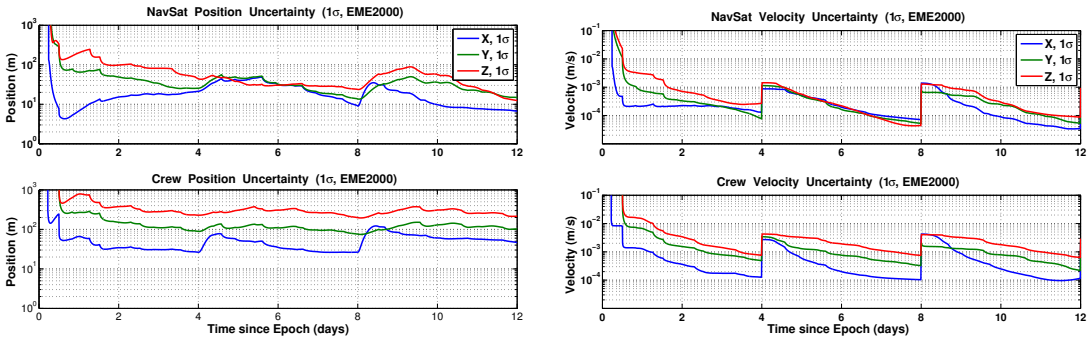

Figure 8. The position (left) and velocity (right) uncertainty components over time of both the NavSat and the crewed vehicle as they are each independently tracked by the three main DSN ground stations. No FLAK or LiAISON observations are included.

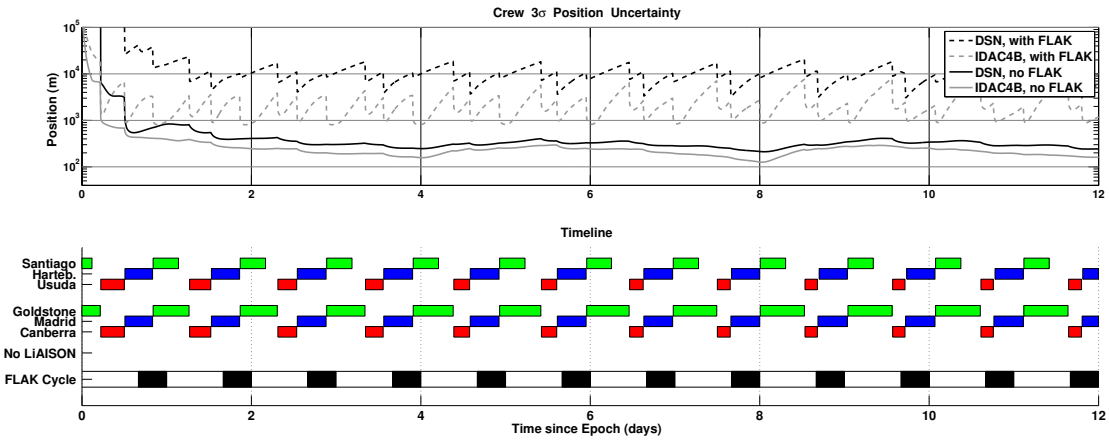

Figure 9. A comparison of the position uncertainty over time of the crewed L_2 vehicle as it is tracked by either the DSN or IDAC4B in the presence or not of FLAK.

the 12th day of the simulation. We take care to collect data for a multiple of 24 hours to capture a complete number of astronaut activity cycles as well as two full station keeping cycles. Further, this amounts to approximately one half of a revolution about the halo orbit. We then evaluate the mean and 99% values of the position uncertainty time-series. The mean is useful to understand what the expected uncertainty in position will be; the 99% is useful to know how uncertain it can be. Table 3 captures these values for the ground-only simulations illustrated above.

Table 3 illustrates how significant FLAK's effects are on the navigation of a crewed vehicle in an unstable libration orbit. The IDAC4B network reduces the crew's position uncertainty by a large amount, but it would be largely beneficial to drop this uncertainty further.

Table 3. The mean and 99% position uncertainty 3-σ values for the ground-only simulations of a crewed vehicle in an L_2 halo orbit after the simulation has reached a nearly steady state.

Scenario	Mean Pos Uncertainty (3σ)	99% Pos Uncertainty (3σ)
DSN Tracking, No NavSat, With FLAK	9.850 km	18.501 km
IDAC4B, No NavSat, With FLAK	2.430 km	7.665 km
DSN Tracking, No NavSat, No FLAK	0.308 km	0.409 km
IDAC4B, No NavSat, No FLAK	0.219 km	0.292 km

LiAISON Only

The next set of simulations studies the case of tracking the crewed vehicle using only LiAISON tracking via a NavSat located in either an L_1 or an L_2 halo orbit. Several halo orbits are surveyed about each Lagrange point to get a feel for what geometry works the best.

Previous studies have demonstrated that LiAISON is sufficient to reduce each vehicle's uncertainties down to any level over time, provided that the vehicles do not experience noisy behavior, e.g., FLAK, and provided that there is sufficient tracking data.[9] Our investigations yield the same result if the effects of FLAK and station keeping maneuvers are removed. It takes about a month of LiAISON tracking to reduce both vehicles' position uncertainties from the kilometer level down to the meter level.

While this is a useful and academically interesting result, the timescale exceeds most crew objectives. Further, both vehicles experience growth in their uncertainties due to the effects of maneuvers and FLAK. We have found that FLAK is such a dominant disturbance that no realistic simulations have successfully achieved a stable position uncertainty below 100 km, using only LiAISON tracking, without reducing FLAK. Nevertheless, results are presented here to illustrate the costs and benefits of LiAISON between the crewed vehicle and a single other NavSat.

The LiAISON-only simulations presented here have surveyed many configurations. The NavSat has been placed in six different halo orbits: three about L_1 and three about L_2. The L_2 orbit with a z-axis amplitude of 10,000 km is almost identical to the crewed vehicle's orbit. Other orbits are further out of plane. In addition, the NavSat is placed at different locations in its halo orbit relative to the crewed vehicle's location in its L_2 orbit. The phase difference between these orbits is indicated by a $\Delta\tau$ value, where τ is a parameter that describes where a spacecraft is in its halo orbit. A $\Delta\tau$ of 60° means that the NavSat is 1/6th of a revolution ahead of the crewed vehicle at the start of the simulation. We have surveyed the results of many other configurations and are comfortable that no configurations exist that are particularly better than those presented.

Previous work has shown that the navigation results are not expected to be as successful if the $\Delta\tau$ is set to 0° or 180°.[9] Further, the navigation results are expected to be very similar in symmetric scenarios, such as $\Delta\tau$ vs. $360° - \Delta\tau$ and to a lesser extent $\Delta\tau$ vs. $180° - \Delta\tau$.[9] Hill illustrated that the best navigation performance is obtained from scenarios where the $\Delta\tau$ between the two vehicles in their own halo orbits falls

between 50° and 150° as well as the symmetric $\Delta\tau$ range between 210° and 310°. If a navigator were to select an ideal value, it would likely be around 90°, though navigators do not always have the privilege of making that selection. It may be the case that the worst geometry is the only one available. The simulations studied here consider what is expected to be the best and the worst, setting $\Delta\tau$ to values of 0°, 45°, 60°, 90°, and 120°. To survey all possible options, one should also consider the differences between northern and southern orbits; this paper only considers southern L_2 orbits, due to their communication benefits with the southern lunar hemisphere, and northern L_1 orbits, due to their navigation benefits with a crew in a southern orbit about L_2.

Table 4 summarizes the results of each of these scenarios. One immediate conclusion is that LiAISON-only tracking is not sufficient to keep the position uncertainty of the crewed vehicle in this scenario below 100 km (3σ). The results indicate that the position uncertainty of the crew when the NavSat is in orbit about

Table 4. The mean and 99% position uncertainty 3-σ values for the LiAISON-only simulations, where the crewed vehicle is in a 10,000 km L_2 halo orbit and the NavSat is in one of many halo orbits as indicated. The results are shown after each simulation has reached a nearly steady state.

NavSat	A_z	$\Delta\tau$	Crew Pos Uncertainty (3σ)		NavSat Pos Uncertainty (3σ)	
Location	(km)	(deg)	Mean (km)	99% (km)	Mean (km)	99% (km)
L_1	10,000	0	255.615 km	399.072 km	67.568 km	116.154 km
L_1	10,000	45	257.337 km	358.146 km	72.097 km	169.901 km
L_1	10,000	60	264.900 km	362.003 km	75.757 km	190.092 km
L_1	10,000	90	260.768 km	338.923 km	79.653 km	217.581 km
L_1	10,000	120	250.069 km	336.869 km	77.572 km	189.715 km
L_1	20,000	0	210.594 km	270.384 km	80.609 km	149.338 km
L_1	20,000	45	226.625 km	369.461 km	94.249 km	271.130 km
L_1	20,000	60	235.699 km	395.241 km	103.853 km	311.627 km
L_1	20,000	90	238.603 km	386.025 km	108.427 km	316.839 km
L_1	20,000	120	233.942 km	356.510 km	100.781 km	311.439 km
L_1	35,500	0	228.752 km	449.765 km	137.446 km	309.629 km
L_1	35,500	45	265.593 km	638.391 km	176.938 km	535.386 km
L_1	35,500	60	269.041 km	658.821 km	189.755 km	580.990 km
L_1	35,500	90	318.265 km	982.919 km	239.598 km	940.161 km
L_1	35,500	120	310.584 km	719.071 km	228.039 km	710.786 km
L_2	10,000	0	LiAISON Ineffective: not enough separation			
L_2	10,000	45	267.862 km	602.170 km	173.665 km	567.057 km
L_2	10,000	60	243.631 km	476.491 km	146.710 km	451.879 km
L_2	10,000	90	212.272 km	344.969 km	120.662 km	304.243 km
L_2	10,000	120	178.914 km	275.691 km	98.739 km	191.683 km
L_2	20,000	0	464.241 km	917.314 km	215.565 km	762.875 km
L_2	20,000	45	316.111 km	592.005 km	169.082 km	559.982 km
L_2	20,000	60	275.637 km	500.623 km	151.460 km	483.095 km
L_2	20,000	90	225.349 km	377.150 km	120.328 km	312.137 km
L_2	20,000	120	185.450 km	309.763 km	104.063 km	211.686 km
L_2	35,500	0	621.691 km	1235.625 km	168.394 km	544.832 km
L_2	35,500	45	471.928 km	1024.344 km	140.630 km	462.895 km
L_2	35,500	60	318.909 km	571.837 km	134.733 km	408.770 km
L_2	35,500	90	227.058 km	440.482 km	103.462 km	265.223 km
L_2	35,500	120	181.372 km	370.286 km	102.552 km	227.899 km

L_1 depends very little on the values of A_z or $\Delta\tau$, though there is some improvement for higher values of A_z and lower values of $\Delta\tau$. The story is very different for the cases when the NavSat is in orbit about L_2. In those cases, the position uncertainty of both vehicles benefits if A_z is low and $\Delta\tau$ is high. These results are not consistent with Hill's findings.[9]

Ground and NavSat Support

Investigations have now considered the ground-only and LiAISON-only scenarios, which are both useful as baselines to help judge the performance of each new navigation scenario. In reality, a system may have better or worse radiometric tracking; it may have other ground networks; it may even use other navigation data types. These scenarios may all be compared to the ground-only and LiAISON-only scenarios presented earlier to judge the benefits of a certain combination of ground and space-based tracking.

The scenarios presented here use a combination of ground-based tracking and space-based tracking. It is assumed that the NavSat is always tracked by the DSN. This ensures that the NavSat's state is well-known, which helps to anchor it as a navigation asset to the crewed vehicle. Previous studies have illustrated that the NavSat does not need to be tracked by the ground, and that it only helps the crewed spacecraft's navigation a small amount;[13] nevertheless it is expected that the NavSat will be tracked. Alternatively, the NavSat could be providing navigation services to other vehicles, each link of which helps reduce its state uncertainties.

Tables 5 – 7 summarize the position uncertainties of both vehicles in each scenario, where each scenario is identical to those presented in Table 4, except now with ground tracking as well as LiAISON. Table 5 adds DSN tracking to just the NavSat, leaving the crewed vehicle with only LiAISON tracking data from the NavSat. One can see that the position uncertainties of both vehicles have improved, though only the NavSat's position uncertainty is remarkably better. Table 6 adds DSN tracking to both the NavSat and the crewed vehicle. This provides an impressive improvement to the position uncertainty of the crewed vehicle. Finally, Table 7 illustrates the position uncertainty of both vehicles when the DSN is tracking the NavSat and both the DSN and IDAC4B are tracking the crewed vehicle. This brings the 3σ uncertainty of the crewed vehicle's position below 1 kilometer in several scenarios.

Comparisons and Conclusions

Table 8 summarizes the range of navigation uncertainties that have been observed in each major navigation configuration studied in this paper. The ranges include everything from the best mean position uncertainty through the worst-case 99% position uncertainty from the results presented in this paper.

The data presented in this paper yield several interesting conclusions. First of all, navigating a noisy vehicle on an unstable trajectory such as a lunar L_2 halo orbit requires careful planning. Previous studies have indicated that LiAISON is sufficient to navigate a crewed vehicle on a trans-lunar cruise,[14] in a low lunar orbit,[13], or in a halo orbit without FLAK;[9] this study demonstrates that LiAISON is generally not sufficient by itself to navigate a crewed vehicle in a lunar halo orbit with meter-level or even kilometer-level precision. The results summarized in Table 8 show that with the assumed FLAK level, the LiAISON-only navigation achieves position uncertainties on the order of 180 – 1000 km (3σ).

The results also indicate that ground-only tracking does not achieve kilometer-level precision, but achieves anything from 2.4 – 18.5 km 3σ position uncertainty. It requires both ground tracking and LiAISON to achieve better than 1 kilometer position uncertainty, and even that is challenging.

This paper has demonstrated that LiAISON provides a benefit to the navigation of a crewed vehicle in any circumstance, except for the case that the NavSat is too close to the crewed vehicle in the same orbit. The paper has shown that a navigation configuration that includes a combination of LiAISON and ground-based tracking is not sensitive to the particular orbit or phase angle of the NavSat. The location of the NavSat is more important in a scenario where the crewed vehicle loses its ground tracking passes. In those cases, it is

Table 5. The navigation performance for the configuration that includes continuous LiAISON tracking between the crew at L_2 and the NavSat, as well as continuous DSN tracking of the NavSat; there is no ground tracking of the crewed vehicle.

Ground Network	NavSat Location	A_z (km)	$\Delta\tau$ (deg)	Crew Pos Uncertainty (3σ) Mean (km)	Crew Pos Uncertainty (3σ) 99% (km)	NavSat Pos Uncertainty (3σ) Mean (km)	NavSat Pos Uncertainty (3σ) 99% (km)
LiAISON+	L_1	10,000	0	234.303 km	397.279 km	0.195 km	0.314 km
LiAISON+	L_1	10,000	45	225.342 km	354.658 km	0.188 km	0.316 km
LiAISON+	L_1	10,000	60	226.422 km	357.727 km	0.192 km	0.316 km
LiAISON+	L_1	10,000	90	221.250 km	329.278 km	0.191 km	0.314 km
LiAISON+	L_1	10,000	120	215.562 km	332.095 km	0.193 km	0.317 km
LiAISON+	L_1	20,000	0	182.334 km	257.089 km	0.184 km	0.320 km
LiAISON+	L_1	20,000	45	177.488 km	222.001 km	0.170 km	0.308 km
LiAISON+	L_1	20,000	60	173.766 km	216.644 km	0.172 km	0.309 km
LiAISON+	L_1	20,000	90	175.952 km	205.117 km	0.173 km	0.309 km
LiAISON+	L_1	20,000	120	188.624 km	219.674 km	0.175 km	0.310 km
LiAISON+	L_1	35,500	0	149.344 km	185.203 km	0.186 km	0.379 km
LiAISON+	L_1	35,500	45	135.732 km	151.454 km	0.167 km	0.304 km
LiAISON+	L_1	35,500	60	129.514 km	145.643 km	0.167 km	0.306 km
LiAISON+	L_1	35,500	90	131.470 km	149.936 km	0.167 km	0.310 km
LiAISON+	L_1	35,500	120	143.060 km	161.803 km	0.170 km	0.310 km
LiAISON+	L_2	10,000	0	LiAISON Ineffective: not enough separation			
LiAISON+	L_2	10,000	45	146.893 km	320.652 km	0.306 km	0.407 km
LiAISON+	L_2	10,000	60	146.922 km	337.945 km	0.304 km	0.405 km
LiAISON+	L_2	10,000	90	140.042 km	319.765 km	0.290 km	0.400 km
LiAISON+	L_2	10,000	120	129.175 km	274.223 km	0.278 km	0.386 km
LiAISON+	L_2	20,000	0	328.945 km	537.197 km	0.291 km	0.399 km
LiAISON+	L_2	20,000	45	201.478 km	444.084 km	0.298 km	0.411 km
LiAISON+	L_2	20,000	60	176.111 km	425.506 km	0.295 km	0.411 km
LiAISON+	L_2	20,000	90	157.224 km	375.961 km	0.277 km	0.404 km
LiAISON+	L_2	20,000	120	135.738 km	308.594 km	0.263 km	0.395 km
LiAISON+	L_2	35,500	0	496.337 km	927.115 km	0.284 km	0.406 km
LiAISON+	L_2	35,500	45	394.555 km	1022.961 km	0.292 km	0.417 km
LiAISON+	L_2	35,500	60	243.265 km	570.872 km	0.286 km	0.414 km
LiAISON+	L_2	35,500	90	182.272 km	439.300 km	0.267 km	0.406 km
LiAISON+	L_2	35,500	120	142.889 km	369.877 km	0.253 km	0.402 km

Table 6. The navigation performance for the configuration that includes continuous LiAISON tracking between the crew at L_2 and the NavSat, as well as continuous DSN tracking of both vehicles.

Ground Network	NavSat Location	A_z (km)	$\Delta\tau$ (deg)	Crew Pos Uncertainty (3σ) Mean (km)	Crew Pos Uncertainty (3σ) 99% (km)	NavSat Pos Uncertainty (3σ) Mean (km)	NavSat Pos Uncertainty (3σ) 99% (km)
DSN	None	-	-	9.879 km	18.606 km	-	-
DSN	L_1	10,000	0	2.561 km	7.978 km	0.183 km	0.301 km
DSN	L_1	10,000	45	2.612 km	8.025 km	0.178 km	0.281 km
DSN	L_1	10,000	60	2.654 km	8.157 km	0.182 km	0.282 km
DSN	L_1	10,000	90	2.686 km	8.335 km	0.183 km	0.290 km
DSN	L_1	10,000	120	2.717 km	8.574 km	0.186 km	0.295 km
DSN	L_1	20,000	0	2.333 km	7.604 km	0.173 km	0.304 km
DSN	L_1	20,000	45	2.569 km	7.773 km	0.162 km	0.273 km
DSN	L_1	20,000	60	2.590 km	7.784 km	0.164 km	0.276 km
DSN	L_1	20,000	90	2.670 km	7.990 km	0.165 km	0.284 km
DSN	L_1	20,000	120	2.815 km	10.869 km	0.169 km	0.289 km
DSN	L_1	35,500	0	2.386 km	7.812 km	0.178 km	0.371 km
DSN	L_1	35,500	45	2.553 km	7.779 km	0.159 km	0.271 km
DSN	L_1	35,500	60	2.577 km	7.723 km	0.160 km	0.275 km
DSN	L_1	35,500	90	2.590 km	7.779 km	0.160 km	0.285 km
DSN	L_1	35,500	120	2.635 km	8.071 km	0.163 km	0.289 km
DSN	L_2	10,000	0	LiAISON Ineffective: not enough separation			
DSN	L_2	10,000	45	2.715 km	8.289 km	0.291 km	0.396 km
DSN	L_2	10,000	60	2.741 km	8.351 km	0.290 km	0.391 km
DSN	L_2	10,000	90	2.721 km	8.497 km	0.279 km	0.391 km
DSN	L_2	10,000	120	2.834 km	8.546 km	0.269 km	0.384 km
DSN	L_2	20,000	0	2.650 km	7.637 km	0.264 km	0.374 km
DSN	L_2	20,000	45	2.586 km	7.471 km	0.281 km	0.408 km
DSN	L_2	20,000	60	2.600 km	7.843 km	0.280 km	0.406 km
DSN	L_2	20,000	90	2.672 km	7.932 km	0.266 km	0.391 km
DSN	L_2	20,000	120	2.643 km	8.085 km	0.256 km	0.391 km
DSN	L_2	35,500	0	2.649 km	7.751 km	0.260 km	0.382 km
DSN	L_2	35,500	45	2.556 km	7.321 km	0.278 km	0.406 km
DSN	L_2	35,500	60	2.534 km	7.420 km	0.271 km	0.409 km
DSN	L_2	35,500	90	2.632 km	7.713 km	0.253 km	0.395 km
DSN	L_2	35,500	120	2.598 km	7.914 km	0.245 km	0.391 km

Table 7. The navigation performance for the configuration that includes continuous LiAISON tracking between the crew at L_2 and the NavSat, as well as continuous ground tracking of both vehicles: DSN for the NavSat and IDAC4B for the crewed vehicle.

Ground Network	NavSat Location	A_z (km)	$\Delta\tau$ (deg)	Crew Pos Uncertainty (3σ) Mean (km)	Crew Pos Uncertainty (3σ) 99% (km)	NavSat Pos Uncertainty (3σ) Mean (km)	NavSat Pos Uncertainty (3σ) 99% (km)
IDAC4B	None	-	-	2.449 km	7.695 km	-	-
IDAC4B	L_1	10,000	0	0.971 km	3.852 km	0.167 km	0.279 km
IDAC4B	L_1	10,000	45	0.944 km	3.192 km	0.166 km	0.248 km
IDAC4B	L_1	10,000	60	0.929 km	2.955 km	0.169 km	0.248 km
IDAC4B	L_1	10,000	90	1.007 km	5.018 km	0.168 km	0.272 km
IDAC4B	L_1	10,000	120	0.945 km	2.935 km	0.170 km	0.288 km
IDAC4B	L_1	20,000	0	0.989 km	3.279 km	0.159 km	0.266 km
IDAC4B	L_1	20,000	45	0.979 km	3.569 km	0.152 km	0.246 km
IDAC4B	L_1	20,000	60	0.956 km	3.272 km	0.154 km	0.238 km
IDAC4B	L_1	20,000	90	1.017 km	5.048 km	0.154 km	0.259 km
IDAC4B	L_1	20,000	120	1.142 km	6.124 km	0.156 km	0.280 km
IDAC4B	L_1	35,500	0	1.066 km	3.991 km	0.163 km	0.316 km
IDAC4B	L_1	35,500	45	1.055 km	4.599 km	0.150 km	0.257 km
IDAC4B	L_1	35,500	60	1.008 km	3.654 km	0.151 km	0.237 km
IDAC4B	L_1	35,500	90	1.041 km	5.185 km	0.150 km	0.257 km
IDAC4B	L_1	35,500	120	1.003 km	3.567 km	0.150 km	0.271 km
IDAC4B	L_2	10,000	0	LiAISON Ineffective: not enough separation			
IDAC4B	L_2	10,000	45	0.944 km	3.306 km	0.272 km	0.370 km
IDAC4B	L_2	10,000	60	0.919 km	2.933 km	0.273 km	0.377 km
IDAC4B	L_2	10,000	90	0.978 km	4.688 km	0.263 km	0.382 km
IDAC4B	L_2	10,000	120	0.902 km	2.802 km	0.257 km	0.380 km
IDAC4B	L_2	20,000	0	1.212 km	4.327 km	0.234 km	0.330 km
IDAC4B	L_2	20,000	45	0.962 km	3.284 km	0.259 km	0.393 km
IDAC4B	L_2	20,000	60	0.968 km	4.003 km	0.257 km	0.394 km
IDAC4B	L_2	20,000	90	0.959 km	4.181 km	0.245 km	0.367 km
IDAC4B	L_2	20,000	120	0.904 km	2.824 km	0.243 km	0.378 km
IDAC4B	L_2	35,500	0	1.201 km	4.193 km	0.232 km	0.336 km
IDAC4B	L_2	35,500	45	1.021 km	2.992 km	0.251 km	0.367 km
IDAC4B	L_2	35,500	60	1.029 km	4.684 km	0.247 km	0.392 km
IDAC4B	L_2	35,500	90	0.941 km	3.405 km	0.231 km	0.357 km
IDAC4B	L_2	35,500	120	0.908 km	2.854 km	0.230 km	0.369 km

Table 8. A summary of the approximate ranges of position uncertainty levels observed in several different navigation configurations. FLAK and station keeping maneuvers are included in these estimates.

Scenario	LiAISON Tracking	DSN Tracking		IDAC4B Crew	Expected Position Uncertainty Range (3σ)	
		NavSat	Crew		Crew	NavSat
DSN-only, NavSat	-	Yes	-	-	-	0.17 – 0.42 km
DSN-only, Crew	-	-	Yes	-	9.8 – 18.5 km	-
IDAC4B-only, Crew	-	-	-	Yes	2.4 – 7.7 km	-
LiAISON-only	Yes	-	-	-	180 – 1000 km	70 – 600 km
LiAISON+DSN	Yes	Yes	-	-	140 – 600 km	0.16 – 0.41 km
LiAISON+DSN	Yes	Yes	Yes	-	2.4 – 10.8 km	0.16 – 0.41 km
LiAISON+IDAC4B	Yes	Yes	Yes	Yes	0.9 – 5.1 km	0.15 – 0.39 km

generally better if the NavSat is in an L_1 orbit with a large z-axis amplitude or in an L_2 orbit with a large $\Delta\tau$, i.e., 120°.

Table 8 shows that the "IDAC4B-only" expected position uncertainty range is approximately the same as the "LiAISON+DSN" uncertainty range when the DSN is tracking the Crew. The NavSat's tracking data may be used as a substitute for the IDAC4B tracking stations to achieve a 3σ position uncertainty on the order of 5 km for the crewed vehicle. These simulations have shown that even in the best circumstances, without LiAISON the crewed vehicle's 3σ position uncertainty is never below 2 km. Only by combining satellite tracking, within the architectures studied in this paper, can the crew be tracked to better than a kilometer (3σ).

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