Radio Science Measurements with Suppressed Carrier

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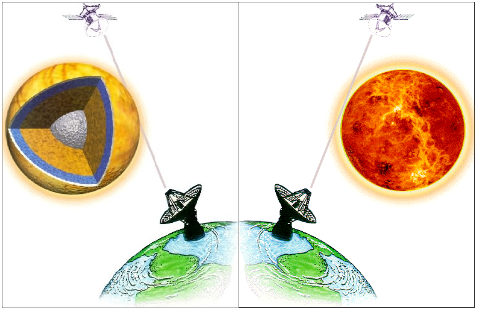
- Planetary atmospheres, planetary rings, planetary surfaces, planetary interiors,
- Solar corona and wind, comet mass flux, and fundamental physics.
- The measurements are conventionally made at the Earth station.

Concept of Radio Science Measurements using RF

The concept of Radio Science Measurements using RF is shown in the following figure



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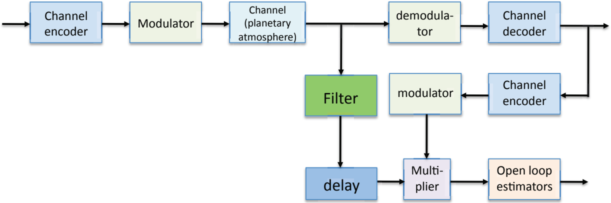
- In this paper we consider a pure suppressed carrier system and we restrict the modulation to be Binary Phase Shift Keying (BPSK).
- We present a method to remove the data from the received suppressed carrier and effectively generate a residual carrier for Radio Science from the received suppressed carrier data, for estimating the amplitude and phase variations.

Proposed System

Proposed system for Radio Science Measurements using suppressed carrier



The vector parameter $\boldsymbol{\theta} = (\theta_1, \theta_2, \theta_3) = (a, \psi_0, \psi_1)$ are estimated given the received complex observation samples

$$r_k = a d_k e^{j(2\pi\psi_1 k\Delta + \psi_0)} + n_k \tag{1}$$

Analysis

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- Using the genie argument (hiring a genie) simplifies our theoretical analysis.
- In a real implementation of the system we replace the genie (firing the genie) with the channel decoder output that provides the same information for non-ideally wiping out the data.

Cramer-Rao bound

Cramer-Rao bound provides a lower bound on achievable performance. Consider estimating a vector parameter $\boldsymbol{\theta} = (\theta_1, \theta_2, \theta_3)^T$. Using the Cramer-Rao bound we have

$$var(\hat{\theta}_i) \geq [I^{-1}(\boldsymbol{\theta})]_{i,i} \quad (2)$$

where $I(\boldsymbol{\theta})$ is the 3×3 Fisher information matrix where the (ij) component of $I(\boldsymbol{\theta})$ is defined as

$$[I(\boldsymbol{\theta})]_{i,j} = -E_{\mathbf{r}}\{\frac{\partial^2 \ln p(\mathbf{r}|\boldsymbol{\theta})}{\partial\theta_i \partial\theta_j}\} \quad (3)$$

for $i, j = 1, 2, , 3$.

Derivations

The vector $\mathbf{r}$ in CRB represents the N received observation samples r_k in equation (1). The received observation vector $\mathbf{r}$ depends on the unwanted nuisance data vector $\mathbf{d}$, in estimation of $\boldsymbol{\theta}$. The $\ln p(\mathbf{r}|\boldsymbol{\theta})$ in CRB can be obtained as

$$\ln p(\mathbf{r}|\boldsymbol{\theta}) = \ln E_{\mathbf{d}}\{p(\mathbf{r}|\boldsymbol{\theta}, \mathbf{d})\} = \sum_{k=0}^{N-1} \ln E_{d_k}\{p(r_k|\boldsymbol{\theta}, d_k)\} \qquad (4)$$

Derivations

Next we need to compute

$$\begin{aligned}\frac{\partial^2}{\partial\theta_i\partial\theta_j} \quad & \ln E_{d_k}\{p(r_k|\boldsymbol{\theta}, d_k)\} = -\frac{\partial^2}{\partial\theta_i\partial\theta_j}\frac{a^2}{2\sigma^2} \\ & +\frac{\partial^2 z_k(r_k;\boldsymbol{\theta};p)}{\partial\theta_i\partial\theta_j}\tanh(z_k(r_k;\boldsymbol{\theta};p)) \\ & +\frac{\partial z_k(r_k;\boldsymbol{\theta};p)}{\partial\theta_i}\frac{\partial z_k(r_k;\boldsymbol{\theta};p)}{\partial\theta_j}(1-\tanh^2(z_k(r_k;\boldsymbol{\theta};p)))\end{aligned} \tag{5}$$

where

$$z_k(r_k;\boldsymbol{\theta};p) = \frac{a}{\sigma^2}\Re\{r_k\beta_k^*\} + \ln\sqrt{\frac{1-p}{p}} \tag{6}$$

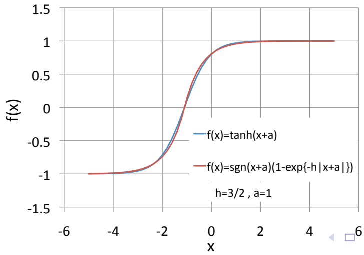
$$\beta_k = e^{j(2\pi\psi_1 k\Delta+\psi_0)} \tag{7}$$

Derivations

Next we need to take expectation with respect to $\mathbf{r}$ to compute the components of Fisher Information matrix

$$[I(\boldsymbol{\theta})]_{i,j} = -E_{\mathbf{r}}\{\frac{\partial^2 \ln p(\mathbf{r}|\boldsymbol{\theta})}{\partial\theta_i\partial\theta_j}\} \tag{8}$$

However closed form solution is not possible due to *tanh*. So we made the following tight approximation shown in the Figure.



Derivations

This tight approximation allows us to get closed form solutions for variance of estimators. Before presnting the results for suppressed carrier. Here we present the results for residual carrier as a reference to compare. For the residual carrier case $p = 0$, the CRB reduces to

$$var(\hat{\theta}_1) \geq \frac{\sigma^2}{N} \triangleq (\mathrm{CRB})_{\mathrm{res},1} \tag{9}$$

$$var(\hat{\theta}_2) \geq \frac{4\sigma^2}{Na^2} \triangleq (\mathrm{CRB})_{\mathrm{res},2} \tag{10}$$

$$var(\hat{\theta}_3) \geq \frac{12\sigma^2}{Na^2(2\pi\Delta N)^2} \triangleq (\mathrm{CRB})_{\mathrm{res},3} \tag{11}$$

Results for Suppressed carrier

For suppressed carrier then

$$var(\hat{\theta}_1) \geq \gamma_1(p, a^2, \sigma^2)(\mathrm{CRB})_{\mathrm{res,1}} \triangleq (\mathrm{CRB})_{\mathrm{sup,1}} \tag{12}$$

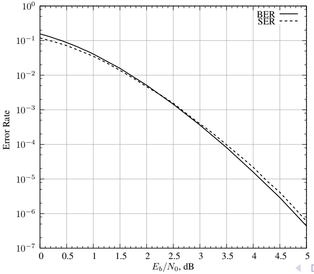
$$var(\hat{\theta}_2) \geq \gamma_2(p, a^2, \sigma^2)(\mathrm{CRB})_{\mathrm{res,2}} \triangleq (\mathrm{CRB})_{\mathrm{sup,2}} \tag{13}$$

$$var(\hat{\theta}_3) \geq \gamma_3(p, a^2, \sigma^2)(\mathrm{CRB})_{\mathrm{res,3}} \triangleq (\mathrm{CRB})_{\mathrm{sup,3}} \tag{14}$$

where $\gamma_i(p, a^2, \sigma^2)$, for $i = 1, 2, 3$ represents the amount of degradation for the estimation of parameters for the suppressed carrier with respect to the residual carrier $(\mathrm{CRB})_{\mathrm{res,i}}$.

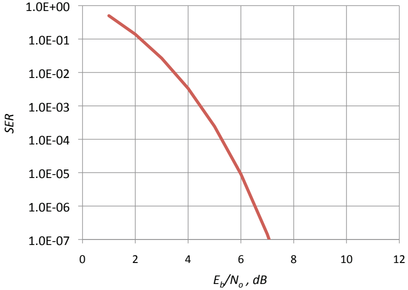
Example

Consider a rate $1/2$ $K = 7$ standard convolutional code with BPSK modulation. The output of Viterbi decoder is re-encoded by the same convolutional encoder. The coded symbol error rate (SER) of re-encoded bits at the output of a Viterbi decoder under perfect synchronization is shown in Figure below and it is compared with the bit error rate at the output of the Viterbi decoder.



Example

The SER performance of the Viterbi decoder including quantization, synchronization and amplitude variations losses is shown in Figure below. Compared to the ideal SER performance of Viterbi decoder in the previous Figure. This loss is approximately 1.75 dB.



Example

- Based on the performance of Viterbi decoder, we computed the degradation functions $\gamma_i(p, a^2, \sigma^2)$ in dB for $i = 1, 2, 3$ and are shown in the next slide (note that $\gamma_3(p, a^2, \sigma^2) = \gamma_2(p, a^2, \sigma^2)$).

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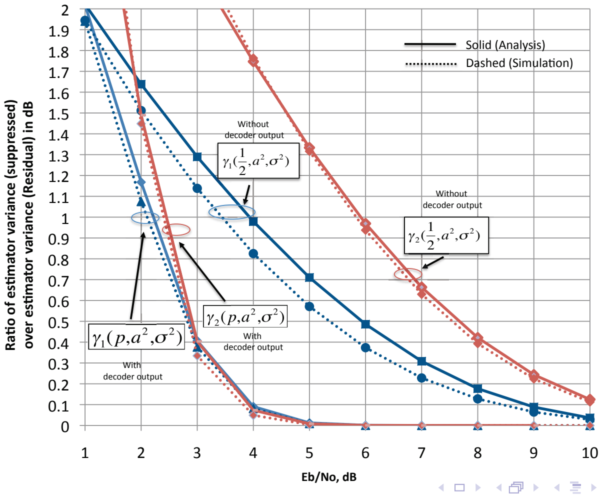
- We also computed the degradation functions directly using true tanh function and perform expectations by numerical simulations which are shown by dashed curves (simulation) in the next slide.

Example


Ratio of estimator variance (suppressed) over estimator variance (Residual) in dB
2
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1
2
3
4
5
6
7
8
9
10
Eb/No, dB
Solid (Analysis)
Dashed (Simulation)
Without decoder output
$\gamma_1(\frac{1}{2},a^2,\sigma^2)$
Without decoder output
$\gamma_2(\frac{1}{2},a^2,\sigma^2)$
$\gamma_2(p,a^2,\sigma^2)$
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- In this paper we used a convolutional coded system as an example.
- Concatenated Reed-Solomon code with convolutional code or modern coding schemes such as turbo codes or LDPC codes could be used with much better performance.

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- Future Work, we will concentrate on obtaining the optimum estimators meeting the theoretical results.