Radio Science Measurements with Suppressed Carrier

Sami Asmar, Dariush Divsalar, Kamal Oudrhiri, Jon Hamkins

Jet Propulsion Laboratory
California Institute of Technology

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Introduction

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  - Solar corona and wind, comet mass flux, and fundamental physics.
Radio Science investigations utilize the telecommunication links between spacecraft and Earth to examine changes in the phase/frequency, amplitude, and polarization of radio signals to investigate:

- Planetary atmospheres, planetary rings, planetary surfaces, planetary interiors,
- Solar corona and wind, comet mass flux, and fundamental physics.
- The measurements are conventionally made at the Earth station.
The concept of Radio Science Measurements using RF is shown in the following figure.
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Residual and Suppressed Carrier

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In this paper we consider a pure suppressed carrier system and we restrict the modulation to be Binary Phase Shift Keying (BPSK).

We present a method to remove the data from the received suppressed carrier and effectively generate a residual carrier for Radio Science from the received suppressed carrier data, for estimating the amplitude and phase variations.
Proposed System

Proposed system for Radio Science Measurements using suppressed carrier

The vector parameter $\theta = (\theta_1, \theta_2, \theta_3) = (a, \psi_0, \psi_1)$ are estimated given the received complex observation samples

$$r_k = a d_k e^{j(2\pi \psi_1 k \Delta + \psi_0)} + n_k$$  (1)
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Analysis

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In a real implementation of the system we replace the genie (firing the genie) with the channel decoder output that provides the same information for non-ideally wiping out the data.
Cramer-Rao bound

Cramer-Rao bound provides a lower bound on achievable performance. Consider estimating a vector parameter \( \theta = (\theta_1, \theta_2, \theta_3)^T \). Using the Cramer-Rao bound we have

\[
\text{var}(\hat{\theta}_i) \geq [I^{-1}(\theta)]_{i,i}
\]

where \( I(\theta) \) is the \( 3 \times 3 \) Fisher information matrix where the \((ij)\) component of \( I(\theta) \) is defined as

\[
[I(\theta)]_{i,j} = -E_r\left\{ \frac{\partial^2 \ln p(r|\theta)}{\partial \theta_i \partial \theta_j} \right\}
\]

for \( i, j = 1, 2, , 3 \).
The vector $\mathbf{r}$ in CRB represents the $N$ received observation samples $r_k$ in equation (1). The received observation vector $\mathbf{r}$ depends on the unwanted nuisance data vector $\mathbf{d}$, in estimation of $\theta$. The $\ln p(\mathbf{r}|\theta)$ in CRB can be obtained as

$$\ln p(\mathbf{r}|\theta) = \ln E_d\{p(\mathbf{r}|\theta, \mathbf{d})\} = \sum_{k=0}^{N-1} \ln E_{d_k}\{p(r_k|\theta, d_k)\} \quad (4)$$
Next we need to compute

\[
\frac{\partial^2}{\partial \theta_i \partial \theta_j} \ln E_{d_k} \{ p(r_k | \theta, d_k) \} = -\frac{\partial^2}{\partial \theta_i \partial \theta_j} \frac{a^2}{2 \sigma^2}
\]

\[
+ \frac{\partial^2 z_k(r_k; \theta; p)}{\partial \theta_i \partial \theta_j} \tanh(z_k(r_k; \theta; p))
\]

\[
+ \frac{\partial z_k(r_k; \theta; p)}{\partial \theta_i} \frac{\partial z_k(r_k; \theta; p)}{\partial \theta_j} \left(1 - \tanh^2(z_k(r_k; \theta; p))\right)
\]

(5)

where

\[
z_k(r_k; \theta; p) = \frac{a}{\sigma^2} \Re \{ r_k \beta_k^* \} + \ln \sqrt{\frac{1 - p}{p}}
\]

(6)

\[
\beta_k = e^{i(2\pi \psi_1 k \Delta + \psi_0)}
\]

(7)
Next we need to take expectation with respect to $r$ to compute the components of Fisher Information matrix

$$[I(\theta)]_{i,j} = -E_r\left\{ \frac{\partial^2 \ln p(r|\theta)}{\partial \theta_i \partial \theta_j} \right\}$$  (8)

However closed form solution is not possible due to $tanh$. So we made the following tight approximation shown in the Figure.
This tight approximation allows us to get closed form solutions for variance of estimators. Before presenting the results for suppressed carrier. Here we present the results for residual carrier as a reference to compare. For the residual carrier case $p = 0$, the CRB reduces to

\[
\text{var}(\hat{\theta}_1) \geq \frac{\sigma^2}{N} \triangleq (\text{CRB})_{\text{res},1} \tag{9}
\]

\[
\text{var}(\hat{\theta}_2) \geq \frac{4\sigma^2}{Na^2} \triangleq (\text{CRB})_{\text{res},2} \tag{10}
\]

\[
\text{var}(\hat{\theta}_3) \geq \frac{12\sigma^2}{Na^2(2\pi \Delta N)^2} \triangleq (\text{CRB})_{\text{res},3} \tag{11}
\]
Results for Suppressed carrier

For suppressed carrier then

\[ \text{var}(\hat{\theta}_1) \geq \gamma_1(p, a^2, \sigma^2)(\text{CRB})_{\text{res}, 1} \triangleq (\text{CRB})_{\text{sup}, 1} \]  \hspace{1cm} (12)

\[ \text{var}(\hat{\theta}_2) \geq \gamma_2(p, a^2, \sigma^2)(\text{CRB})_{\text{res}, 2} \triangleq (\text{CRB})_{\text{sup}, 2} \]  \hspace{1cm} (13)

\[ \text{var}(\hat{\theta}_3) \geq \gamma_3(p, a^2, \sigma^2)(\text{CRB})_{\text{res}, 3} \triangleq (\text{CRB})_{\text{sup}, 3} \]  \hspace{1cm} (14)

where \( \gamma_i(p, a^2, \sigma^2) \), for \( i = 1, 2, 3 \) represents the amount of degradation for the estimation of parameters for the suppressed carrier with respect to the residual carrier \((\text{CRB})_{\text{res}, i}\).
Consider a rate $1/2$ $K = 7$ standard convolutional code with BPSK modulation. The output of Viterbi decoder is re-encoded by the same convolutional encoder. The coded symbol error rate (SER) of re-encoded bits at the output of a Viterbi decoder under perfect synchronization is shown in Figure below and it is compared with the bit error rate at the output of the Viterbi decoder.
The SER performance of the Viterbi decoder including quantization, synchronization and amplitude variations losses is shown in Figure below. Compared to the ideal SER performance of Viterbi decoder in the previous Figure. This loss is approximately 1.75 dB.
Example

Based on the performance of Viterbi decoder, we computed the degradation functions $\gamma_i(p, a^2, \sigma^2)$ in dB for $i = 1, 2, 3$ and are shown in the next slide (note that $\gamma_3(p, a^2, \sigma^2) = \gamma_2(p, a^2, \sigma^2)$).
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- The degradation functions $\gamma_i(\frac{1}{2}, a^2, \sigma^2)$ in dB for $i = 1, 2, 3$ when the decoder output is not used are also shown in the next slide (note that $\gamma_3(\frac{1}{2}, a^2, \sigma^2) = \gamma_2(\frac{1}{2}, a^2, \sigma^2)$).
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- We also computed the degradation functions directly using true tanh function and perform expectations by numerical simulations which are shown by dashed curves (simulation) in the next slide.
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- In this paper we used a convolutional coded system as an example.
- Concatenated Reed-Solomon code with convolutional code or modern coding schemes such as turbo codes or LDPC codes could be used with much better performance.
These codes allow the Radio Science estimators for suppressed carrier to perform with negligible degradation at much lower signal-to-noise ratio.

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