Ultimate limits to resource efficienc in photonic communication

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Abstract: We review resource-efficien y metrics for transferring classical information with photons, present an overview of the ultimate limits to photon and dimensional efficien y permitted by quantum mechanics, as well as that achieved with structured transmitter-receiver pairs.

The ultimate limits of resource-efficien photonic communication is dictated not only by the efficien y of the modulation, coding and decoding schemes, but also by the optical states into which the information is encoded and the measurement performed at the receiver. Photonic communication links almost universally adhere to the following architecture: at the transmitter the message bits are encoded into a symbol string with structured redundancy to allow error correction at the receiver. The encoded bits are then modulated onto optical states (photons). The photons propagate through the physical medium and map to a set of corresponding states at the receiver. A measurement is performed at the receiver—possibly collectively over multiple information symbols—the outcomes of which are processed to estimate the transmitted message. The aforementioned communication task consumes resources in two fundamental classes: (1) energy (i.e., photons) must be available to the transmitter as the physical information carrier; and (2) the signaling constellation must map to a subset of photonic degrees of freedom (i.e., spatial, temporal, polarization, and wavelength modes of light). Consequently, the photon information efficien y (PIE) c_p is given by

$$c_p \equiv C/E \qquad \text{(bits per photon)}, \tag{1}$$

where C is the capacity of the link in bits per channel use and E is the average photon number utilized per channel use. The dimensional information efficien y (DIE) c_d, on the other hand, is given by

$$c_d \equiv C/D \qquad \text{(bits per dimension)}, \tag{2}$$

where D is the number of signal dimensions utilized per use of the channel. Suppose we consider L-meter paraxial propagation with the transmitter and receiver having aperture areas A_T and A_R respectively. In addition, let us assume T-second symbol durations, and nonoverlapping modulation bandwidths B around N center wavelengths, denoted by λ_i. Finally, assuming independent modulation of the two polarizations, the maximum number of dimensions is [1]

$$D = 2 \times BT \times \sum_{i=1}^{N} A_T A_R / (\lambda_i L)^2 \,. \tag{3}$$

Here, the factor 2 is due to the two polarization states, BT is the time-bandwidth product, and each term in the sum is the area-bandwidth product in transverse space. Therefore, D is fundamentally represents the dimension of the subspace spanned by all waveforms that can be generated within the temporal, spatial, polarization and wavelength constraints specifie by the link parameters.

We now turn our attention to the achievable (c_p, c_d) pairs in the pure loss channel, with various modulation and measurement pairs, as shown in Fig. 1(a). The ultimate quantum limit to resource efficien y is given by the outermost gold curve, and is approached only by optimal selection of all building blocks discussed in the previous paragraph. Several important conclusions are derived from this curve. First, there is an inherent fundamental tradeoff between the best possible PIE and DIE, i.e., improving PIE will come at the expense of reduced DIE and vice versa. Second, there is no brick-wall maximum to either PIE or DIE. However, high values of PIE or DIE results in exponential penalty for the alternative variable, e.g., when $c_p \gg 1$ we have $c_d \approx e c_p 2^{-c_p}$ [1].

While the ultimate limits show that unbounded PIE is achievable, structured systems may not have brick-wall asymptotes. In particular, Fig. 1(a) shows that heterodyne and homodyne receivers coupled with ideal coherent-state modulation both hit a c_p asymptote at 1.44 bits/photon and 2.89 bits/photon respectively. This is due to the signal-independent and additive nature of the fundamental quantum noise resulting from measuring either a single quadrature

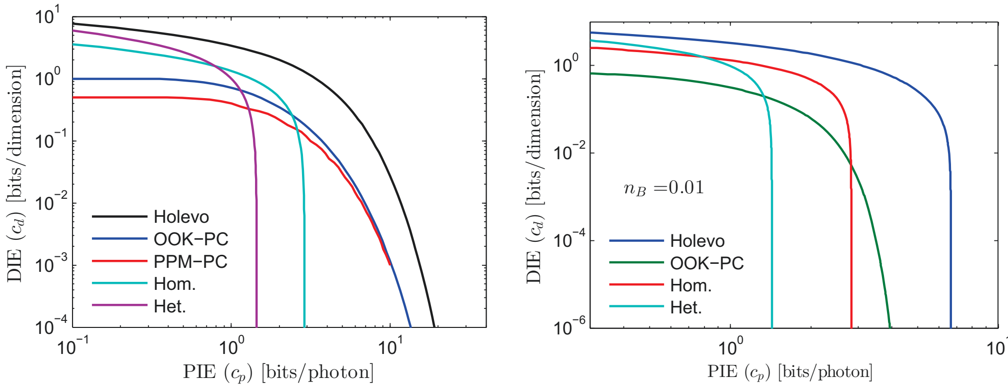

Fig. 1. Photon versus dimensional information efficien y tradeoffs for (a) the noiseless (pure-loss) channel; and (b) thermal-noise channel with $n_b > 0$.

(homodyne) or both quadratures (heterodyne) of a fiel mode in a coherent state. On the other hand, in the $c_d \gg 1$ regime heterodyne detection and coherent-state modulation approaches the Holevo bound, confirmin its optimality in dimensionally-constrained links operating with an abundance of photons (e.g., fibe optic communications). Using homodyne detection results in a factor-of-$1/2$ worse c_d due to the fact they measure a single quadrature of the field

On-off keyed coherent-state modulation coupled with a photon-counting detector (OOK-PC) asymptotically approaches the Holevo bound in photon efficien y when $c_p \gg 1$. This is achieved by having a low-duty-cycle modulation, resulting in high peak-to-average photon-flu ratio. While not shown in the figure the same asymptotic performance is attained by a more convenient implementation to OOK-PC, namely pulse-position modulation paired with photon-counting (PPM-PC) [1]. Unfortunately, the DIE of OOK-PC is $c_d \approx (2\log_2 e/e)2^{-c_p}$, which is suboptimal to the Holevo bound by a factor proportional to c_p. This observation has spurred interest in recent years to fin communication systems that have the same asymptotic scaling as the Holevo bound [2,3]. OOK-PC and PPM-PC have $c_d < 1$ because they are binary modulations, and the best (c_p, c_d) tradeoff achievable with higher-order intensity modulation and photon-counting (IM-PC) has long been an open problem. Tight bounds derived in the $c_d \gg 1$ limit have shown that IM-PC systems achieve c_d that is approximately a factor 0.33 worse than that of heterodyne detection, and a factor 0.66 worse than that of homodyne detection.

Whereas unbounded PIE is achievable in pure-loss channels, this is no longer true when noise is present. In high-sensitivity optical systems this noise can be a result of photodetector dark current, signal independent background radiation generated in the propagation medium, or signal-dependent unwanted excess radiation (e.g., finit extinction of laser source). While the noise statistics for each category varies [4], all of them result in effective asymptotes to the achievable PIE. In particular, Fig. 1(b) shows an example when an average of $n_b = 0.01$ background photons are in a thermal state. The (conjectured) Holevo bound in this case yields a brick-wall PIE asymptote of $log_2(1+1/n_b)$. Homodyne and heterodyne detection with coherent-state modulation hit the asymptotes $1/(n_b+1/2)$ and $1/(n_b+1)$ respectively. OOK-PC and PPM-PC systems (with a Poisson approximation to the noise statistics) have $c_d \approx e^{-e^{c_p}}$, yielding an effective bound to PIE when $c_p > 1$.

To summarize, we have provided a brief review of the ultimate resource efficien y limits of photonic communications. We have define energy and signal dimensionality as two important resource classes and we have briefl reviewed the high PIE and high DIE asymptotes for pure-loss systems, as well as systems with background noise.

The research described in this paper was supported by the DARPA InPho, PROP. 97-15402, and was carried out by the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration.

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