Performance Evaluation of Large Aperture "Polished Panel" Optical Receivers Based on Experimental Data

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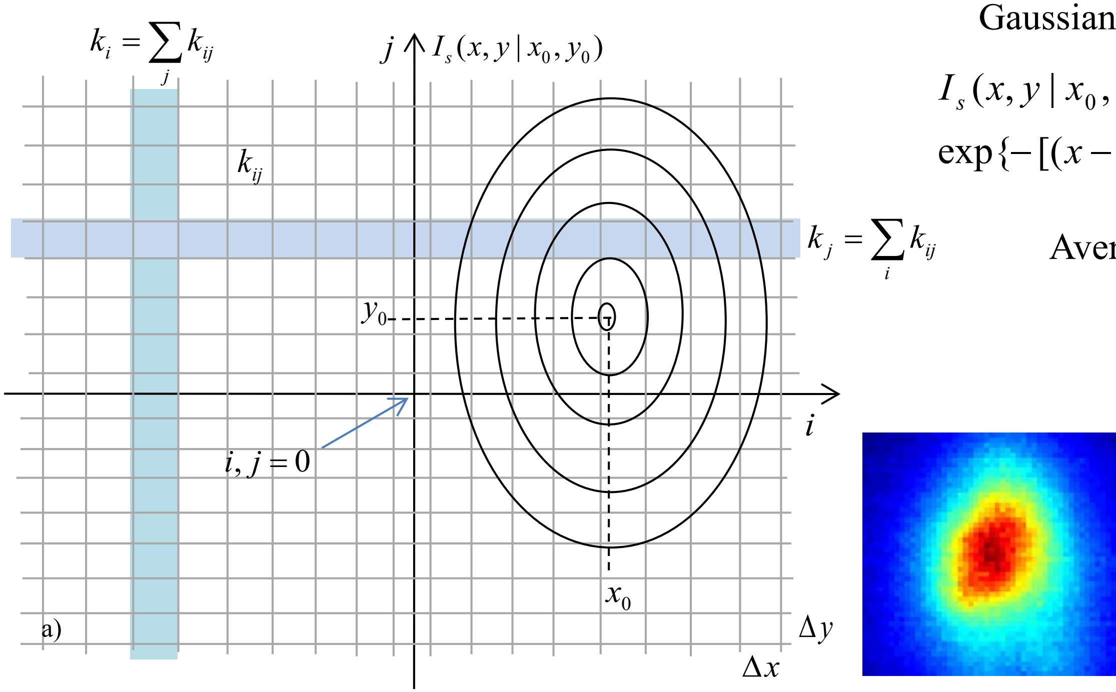
Goal: demonstrate the feasibility of using large aperture "Polished Panel" optical receivers for hybrid RF/Optical deep-space communications via 34-meter DSN antennas

- Installed polished aluminum panels on the main reflector of the 34 meter research antenna at DSS-13, for field evaluation
- Fabricated and installed a weather-proof remotely controlled camera enclosure on the subreflector support structure
  - Contains a large-sensor camera from Finger Lakes Instruments (FLI)
  - Enclosure and camera are computer controlled from alidade
- Imaged the point-spread-function (PSF) generated by the planets Jupiter, Venus and several bright stars
- Evaluated optical communications performance based on PSF data
  - determined quantum limited error probabilities for OOK and 2PPM signals
  - evaluated photon-counting receiver performance in high background
  - developed a model for easily computing block-coded performance

Mathematical model of Focal-Plane Intensity Distribution and Photon-counting statistics



Let $\Delta x = \Delta y = \Delta$, with no loss in generality

a) Focal-plane model of pixel array, and elliptical PSF with pointing offsets , motivated by: b) experimentally determined point-spread function (PSF) for the high-quality Vertex panel, photographed on the JPL mesa test range.

Gaussian model of signal intensity distribution

$$I_s(x, y \mid x_0, y_0) = I_s\,(2\pi\sigma_s^2)^{-1} \times \exp\{-[(x-x_0)^2/2\sigma_x^2 + (y-y_0)^2/2\sigma_y^2]\} \quad \text{watts/cm}^2$$

Average signal energy over detector element i, j

$$\lambda_s(i, j \mid x_0, y_0) = \int_0^T P_s(i, j \mid x_0, y_0)\,dt \cong T\Delta^2 I_s(i\Delta, j\Delta \mid x_0, y_0)$$

Photo-count probability density from detector element i, j

$$p(k_{ij} \mid x_0, y_0) = [\lambda_s(i, j \mid x_0, y_0)]^{k_{ij}} \times \exp[-\lambda_s(i, j \mid x_0, y_0)]/k_{ij}!$$

Joint probability density of array photo-counts

$$p(\mathbf{k} \mid x_0, y_0) = \prod_{i,j} [\lambda_s(i, j)]^{k_{ij}} \exp[-\lambda_s(i, j)]/k_{ij}!$$

The Coherent-State Representation of Optical Signals

Coherent state $|\alpha\rangle$ represented in terms of the number states $|n\rangle$:

$$|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2} \sum_{n=0}^{\infty} \frac{\alpha^n}{(n!)^{1/2}} |n\rangle$$

The number states are orthonormal, hence their "overlap" is a delta function:

$$\langle n | m \rangle = \delta_{nm} = \begin{cases} 1 & n = m \\ 0 & n \neq m \end{cases}$$

Coherent states are not orthonormal, hence the squared overlap of any two coherent states is not zero, but given by:

$$|\langle \alpha | \beta \rangle|^2 = \left| e^{-(|\alpha|^2+|\beta|^2)/2} \sum_n \sum_m \frac{\alpha^n}{\sqrt{n!}} \frac{(\beta^*)^m}{\sqrt{m!}} \langle n | m \rangle \right|^2$$

$$= \left| e^{-(|\alpha|^2+|\beta|^2-2\alpha\beta^*)/2} \right|^2 = e^{-|\alpha-\beta|^2}$$

The probability that a coherent state $|\alpha\rangle$ contains n photons is Poisson distributed:

$$p(n) = |\langle \alpha | n \rangle|^2 = e^{-|\alpha|^2} \frac{|\alpha|^{2n}}{n!}$$

Quantum limit for binary signals

For any pair of binary signals $|\psi_0\rangle, |\psi_1\rangle$, the quantum limited error probability depends only their overlap [1]:

$$P^*(E) = 1 - P^*(C) = \tfrac{1}{2}\left[1 - \sqrt{1 - |\langle \psi_0 | \psi_1 \rangle|^2}\right]$$

Quantum limit for the special case of OOK and binary PPM signals

For OOK signals represented by $|0\rangle$ and $|\alpha\rangle$, with overlap $|\langle 0 | \alpha \rangle|^2 = e^{-|\alpha|^2}$, the quantum limit is:

$$P^*_{OOK}(E) = \tfrac{1}{2}\left[1 - \sqrt{1 - |\langle \psi_0 | \psi_1 \rangle|^2}\right] = \tfrac{1}{2}\left[1 - \sqrt{1 - e^{-|\alpha|^2}}\right] = \tfrac{1}{2}\left[1 - \sqrt{1 - e^{-2K_s}}\right]$$

with average symbol energy $K_s = \tfrac{1}{2}|\alpha|^2$, hence $|\alpha|^2 = 2K_s$ (average power constraint)

For PPM signals represented by $|\varphi_1\rangle = |\alpha'\rangle|0\rangle$ and $|\varphi_2\rangle = |0\rangle|\alpha'\rangle$ the squared overlap becomes

$$|\langle \varphi_1 | \varphi_2 \rangle|^2 = |\langle 0 | \langle \alpha' | 0 \rangle | \alpha' \rangle|^2 = |\langle \alpha' | 0 \rangle \langle 0 | \alpha' \rangle|^2 = |\langle \alpha' | 0 \rangle|^2 |\langle 0 | \alpha' \rangle|^2 = e^{-|\alpha'|^2} e^{-|\alpha'|^2} = e^{-2|\alpha'|^2} = e^{-2K_s}$$

$$P^*_{2PPM}(E) = \tfrac{1}{2}\left[1 - \sqrt{1 - |\langle \varphi_1 | \varphi_2 \rangle|^2}\right] = \tfrac{1}{2}\left[1 - \sqrt{1 - e^{-2K_s}}\right]$$

with average signal power $K_s = |\alpha'|^2$. If $|\alpha'|^2 = \tfrac{1}{2}|\alpha|^2$ (equal avg. power), then $P^*_{2PPM}(E) = P^*_{OOK}(E)$

Photon Counting Receiver: OOK signals

Maximum likelihood decision strategy, OOK signals:

Obtain the photon-counts, k_0, within each T-second symbol-interval, and compare to the optimal threshold $\eta = 2K_s / \log(1+2K_s/K_b)$. If $k_0 < \eta$ declare H_0; if $k_0 > \eta$ declare H_1.

With no background, $K_b = 0, \rightarrow \eta = 0$. Hypotheses H_0 and H_1, equilikely case: $P(H_0) = P(H_1) = \frac{1}{2}$

$$P(n \mid H_0) = \begin{cases} 1, & n = 0 \\ 0, & n \geq 1 \end{cases}; \qquad P(n \mid H_1) = \frac{|\alpha|^{2n}}{n!} e^{-|\alpha|^2}$$

Conditional detection probabilities: $P(C \mid H_0) = P(0 \mid H_0) = 1, \quad P(C \mid H_1) = P(n \geq 1 \mid H_1) = 1 - e^{-|\alpha|^2}$

$$P(C) = \sum_{i=1}^{2} P(C \mid H_i) P(H_i) = 1 - \tfrac{1}{2} e^{-|\alpha|^2} \qquad P(E) = 1 - P(C) = \tfrac{1}{2} e^{-|\alpha|^2} \qquad P_{OOK}(E) = \tfrac{1}{2} e^{-2K_s}$$

With background, K_b, $\rightarrow \eta = 2K_s / \log(1+2K_s/K_b)$, probability of correct detection becomes

$$P_{OOK}(E) = 1 - \tfrac{1}{2} \left(\sum_{k_0=0}^{\text{floor}(\eta)} \frac{K_b^{k_0} e^{-K_b}}{k_0!} + \sum_{\text{ceil}(\eta)}^{\infty} \frac{(K_b + K_s)^{k_0} e^{-(K_b+K_s)}}{k_0!} \right)$$

Photon Counting Receiver: binary PPM signals

Maximum likelihood decision strategy, binary PPM signals:

Obtain the photon-counts k_1 and k_2 in the first and second slots, respectively, and compare: if $k_1 > k_2$ declare H_0 ; if $k_2 > k_1$, declare H_1. In case of a tie, toss a fair coin to determine the outcome.

With no background, $K_b = 0$, an error is made only if no photons are observed, in which case a fair coin is tossed leading to correct detection half the time. Since the probability of observing no photons under either hypothesis is e^{-K_s} , it follows that the average probability of error for binary PPM is

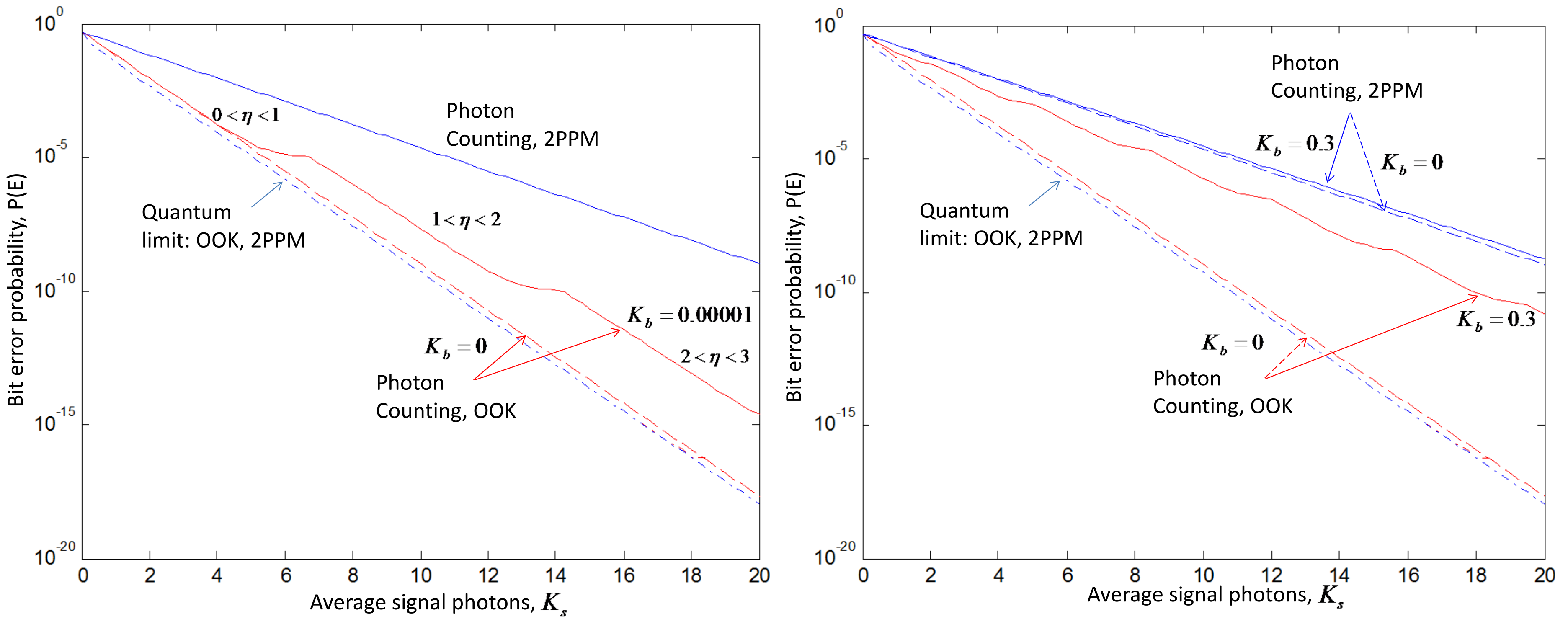
$$P_{2PPM}(E) = \tfrac{1}{2} e^{-K_s}$$

With background, K_b , the probability of bit error can be expressed as [2]

$$P(E) = \sum_{k_1=0}^{\infty} \sum_{k_2=k_1}^{\infty} \gamma(k_1,k_2)\,(K_s + K_b)^{k_1} K_b^{k_2} e^{-(K_s+2K_b)} / k_1! k_2!$$

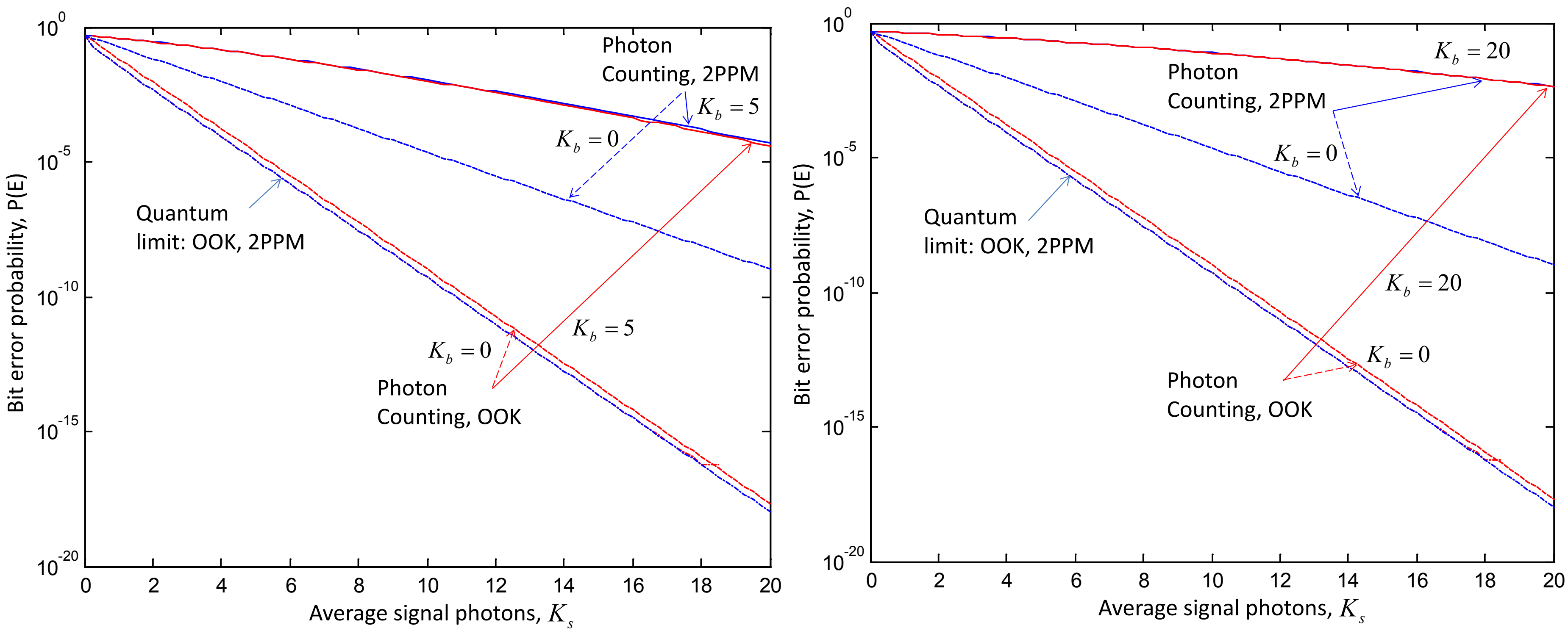
$$\text{where} \quad \gamma(k_1,k_2) = \begin{cases} 1, & k_1 = k_2 \\ \frac{1}{2}, & k_1 \neq k_2 \end{cases}$$

Bit error probabilities for OOK and 2PPM: zero to low background



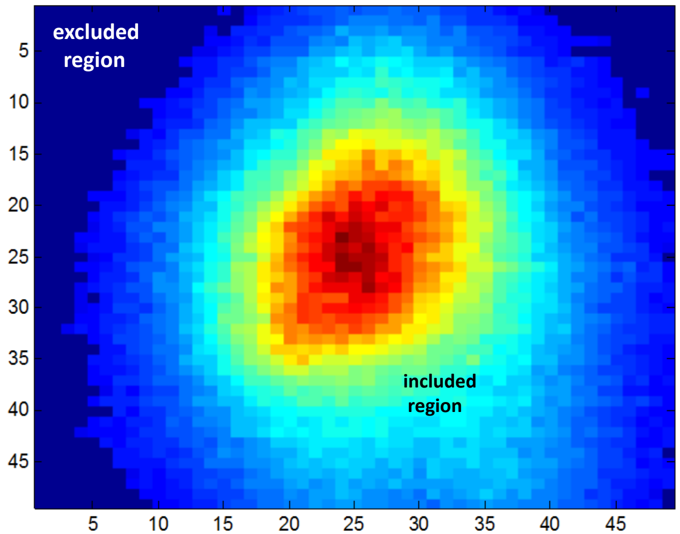
Quantum limit and photon-counting detection performance of OOK and binary PPM signals as a function of average signal energy : a) extremely low average background energy per symbol-interval; b) low average background energy per symbol-interval.

Bit error probabilities for OOK and 2PPM: moderate to high background



Quantum limit and photon-counting detection performance of OOK and binary PPM signals as a function of average signal energy : a) moderate average background energy; b) high average background energy.

Spatial filtering to improve detection performance in the presence of background



Polished panel PSF generated by the star Vega, showing the low signal energy region excluded by spatial filtering.

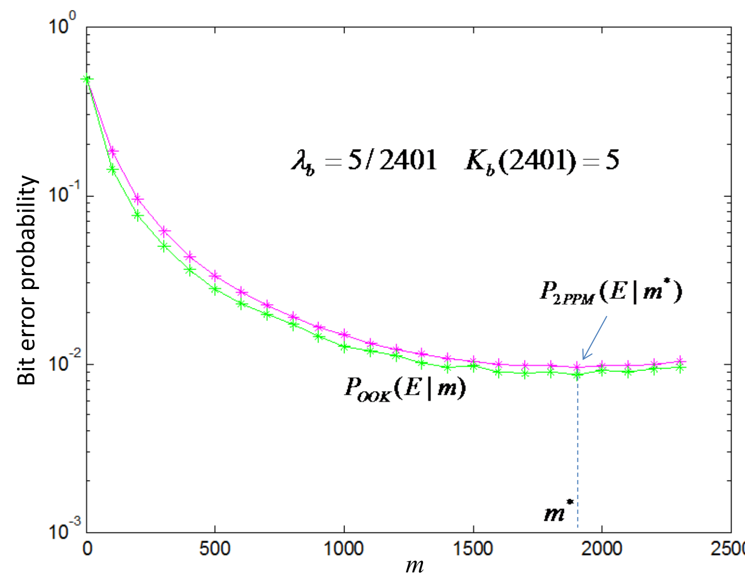
Accumulated signal energy (array detectors sorted according to signal intensity, in decreasing order)

$$K_s(m) = \sum_{n=1}^{m} \lambda_s(n)$$

Accumulated background energy (constant background intensity)

$$K_b(m) = m\lambda_b$$

$$P_{2PPM}(E) = \sum_{k_1=0}^{\infty} \sum_{k_2=k_1}^{\infty} \Big(\gamma(k_1,k_2)\,(K_s(m)+K_b(m))^{k_1} \times K_b^{k_2}(m)\, e^{-(K_s(m)+2K_b(m))} / k_1!k_2!$$



$$P_{OOK}(E) = \tfrac{1}{2}\left(\sum_{\mathrm{ceil}(\eta)}^{\infty} \frac{K_b^k(m) e^{-K_b(m)}}{k!} + \sum_{k=0}^{\mathrm{floor}(\eta)} \frac{(K_b(m)+K_s(m))^k e^{-(K_b(m)+K_s(m))}}{k!} \right)$$

Bit error probability as a function of m for OOK and binary PPM signaling, with minimum error probability achieved by m^*.

Binary PPM Simulation Results

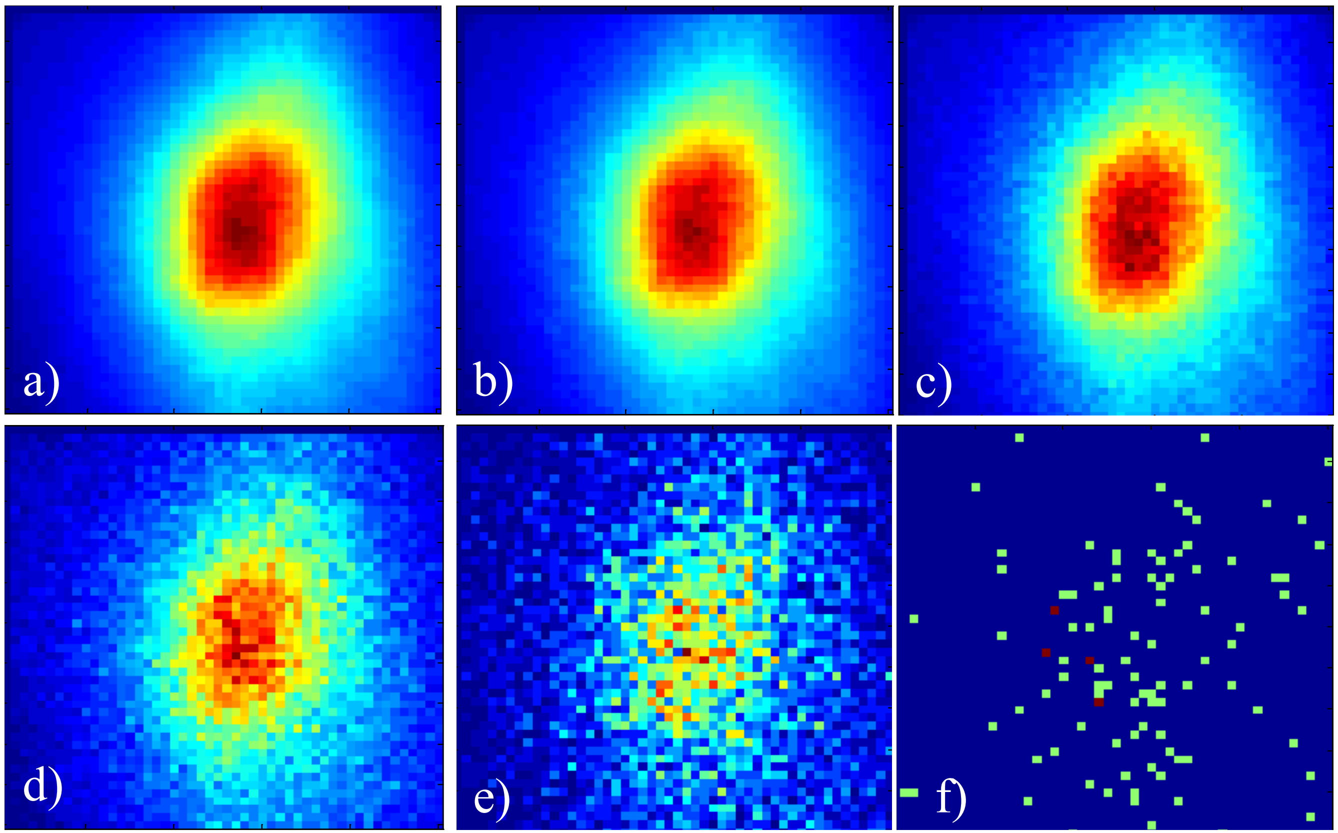
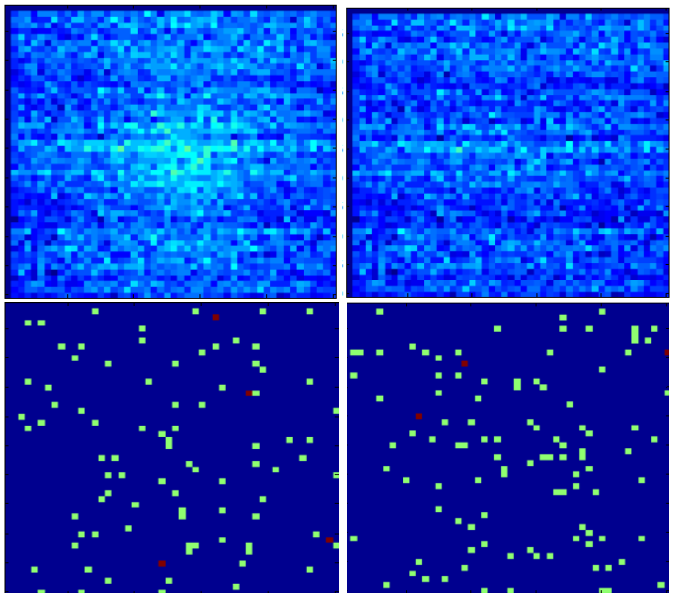


Illustration of random array counts for various pixel-intensity distributions using the experimentally recorded PSF in a) as the average energy distribution, ranging from very high, b), to very low, f), total signal energy

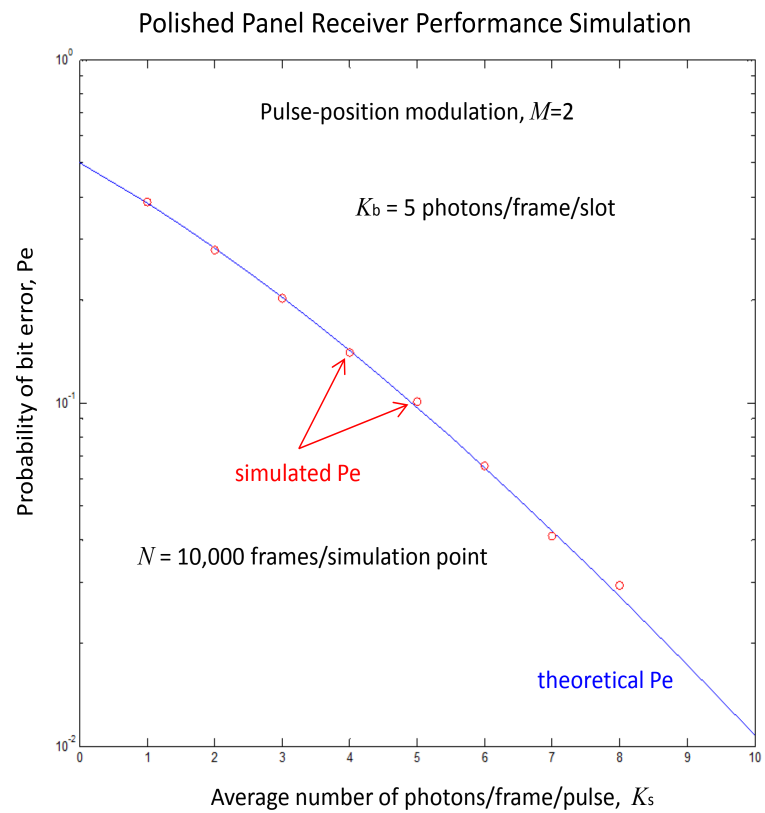
Signal + background intensity in slot #1

Background intensity only in slot #2



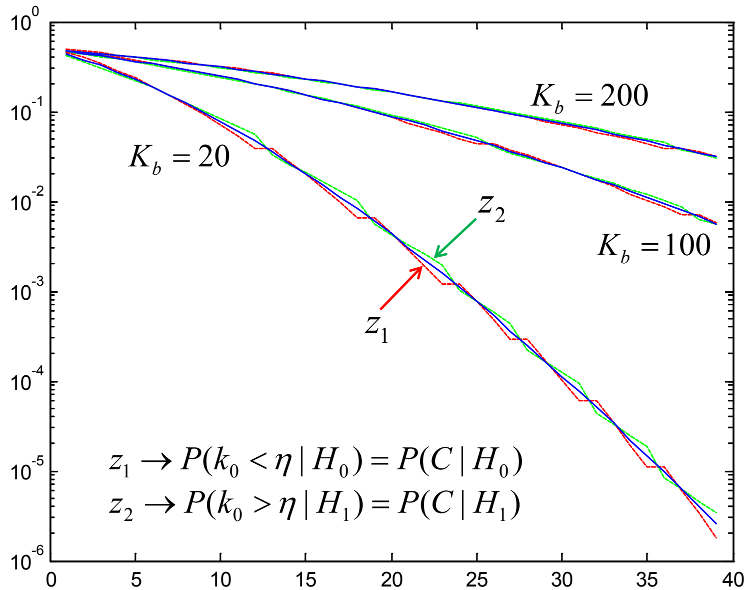
Sample of array photo-counts for each slot: low signal intensity, ~ 100 photons/frame

Sample frames of background and signal-plus-background frames, and associated array photo-counts.



Theoretically derived binary PPM performance and simulation results.

Coding to Improve Detection Performance



Conditional error probabilities for equally probable but highly asymmetrical OOK signals, validating the binary symmetric channel model for high background intensities.

For bounded distance decoders that correct all combinations of t or fewer errors, the bit error probability can be expressed approximately as:

$$P_B(E) = n^{-1} \sum_{j=t+1}^{n} \beta_j \binom{n}{j} p^j (1-p)^{n-j}$$

With $\beta_j \cong j$ (a good approximation for systematic codes),

and using the identity $n^{-1} \sum_{j=2}^{n} j \binom{n}{j} p^j (1-p)^{n-j} = p - p(1-p)^{n-1}$

the bit error probability can be further simplified as:

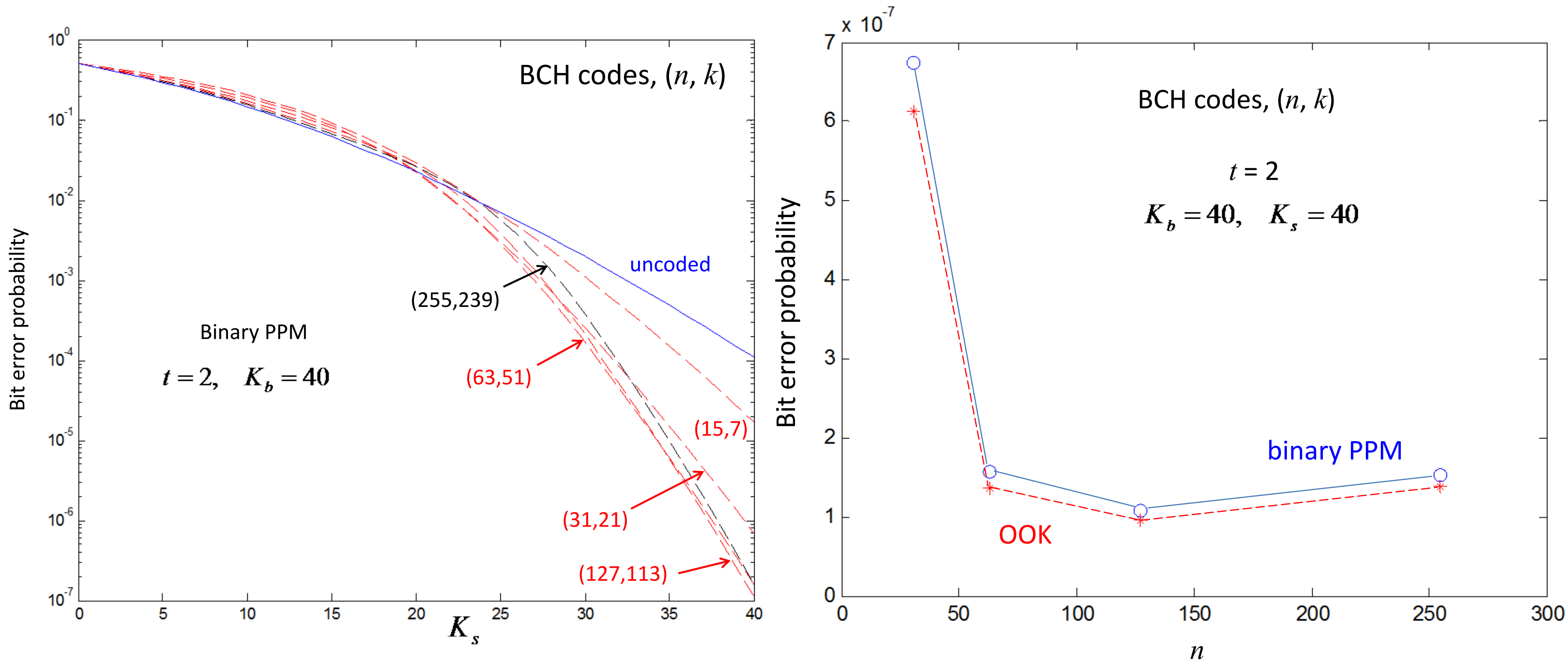
$$P_B(E) \cong n^{-1} \sum_{j=t+1}^{n} j \binom{n}{j} p^j (1-p)^{n-j}$$

$$= p - p \sum_{j=0}^{t-1} \binom{n-1}{j} p^j (1-p)^{n-1-j} =$$

$$p\left[1 - C_{bin}(t-1, n-1, p)\right] = P_b(\mathrm{E})$$

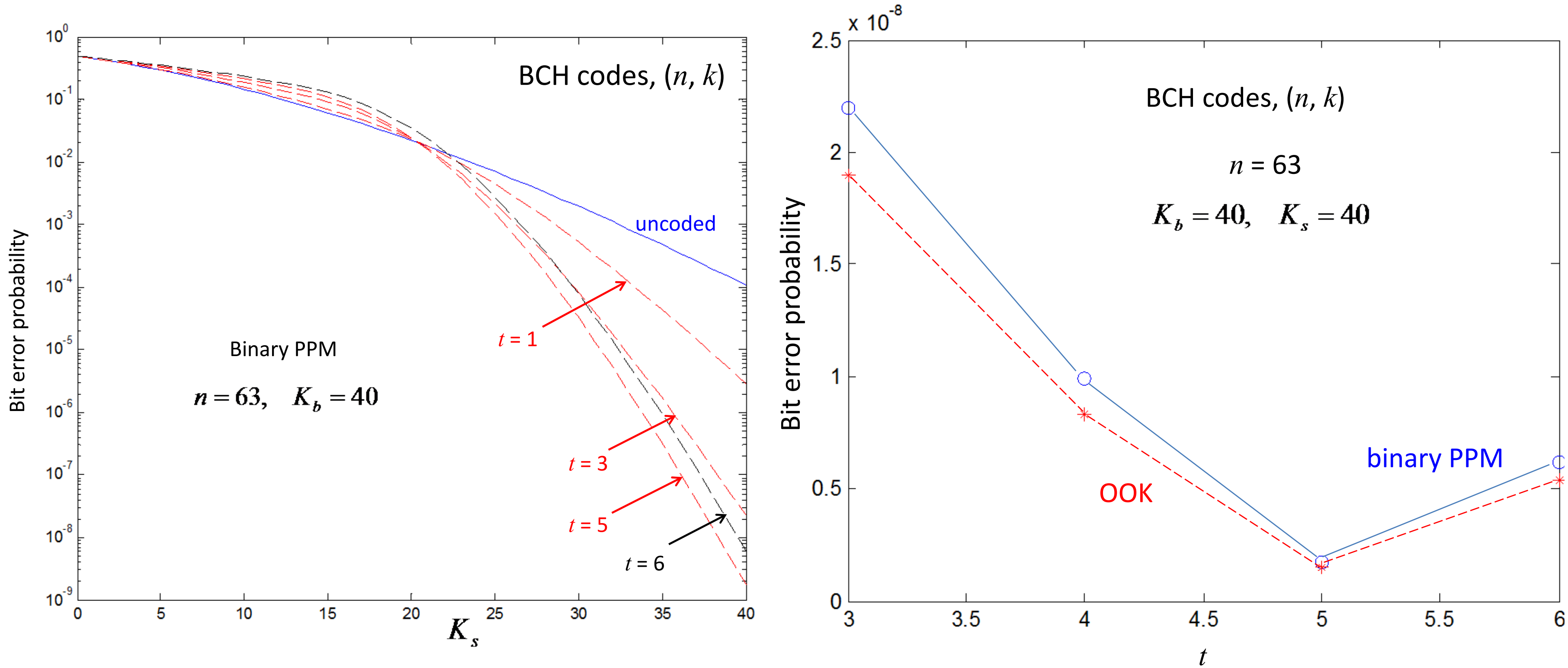
where $C_{bin}(t-1, n-1, p)$ is the binomial cumulative distribution with (n-1) degrees of freedom, from 0 to (t-1).

BCH-coded performance and optimization



Uncoded and BCH block-coded performance of binary PPM symbols for increasing codeword-length, with $t = 2$; b) minimum error probability achieved with $n = 127$, $t = 2$.

BCH-coded performance and optimization



Uncoded and BCH block-coded performance of binary PPM symbols for increasing error correction strength, with $n = 63$; b) minimum error probability achieved with $t = 5$, $n = 63$.

CONCLUSIONS

- A high quality "polished panel" (Vertex Antennentechnic GmbH.) was installed on the DSN's 34 meter antenna at DSS-13
- The PSF generated by the polished panel was determined by tracking the planets Jupiter and Venus, as well as several bright stars
- Detailed mathematical model of the focal-plane energy distribution was developed, and the experimentally determined PSF was used to evaluate potential deep-space communications performance of a "polished-panel" optical receiver, with OOK and binary PPM signals
  - It was shown that with zero background OOK can approach the quantum bound even with optically imperfect polished panel reflectors (average power constraint)
  - It was shown that deep-space communications requirements can be met even with simple codes, and parameter optimization with BCH codes was illustrated