Performance Evaluation of Large Aperture “Polished Panel” Optical Receivers Based on Experimental Data

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Goal: demonstrate the feasibility of using large aperture “Polished Panel” optical receivers for hybrid RF/Optical deep-space communications via 34-meter DSN antennas

• Installed polished aluminum panels on the main reflector of the 34 meter research antenna at DSS-13, for field evaluation

• Fabricated and installed a weather-proof remotely controlled camera enclosure on the subreflector support structure
  • Contains a large-sensor camera from Finger Lakes Instruments (FLI)
  • Enclosure and camera are computer controlled from alidade

• Imaged the point-spread-function (PSF) generated by the planets Jupiter, Venus and several bright stars

• Evaluated optical communications performance based on PSF data
  • determined quantum limited error probabilities for OOK and 2PPM signals
  • evaluated photon-counting receiver performance in high background
  • developed a model for easily computing block-coded performance
Mathematical model of Focal-Plane Intensity Distribution and Photon-counting statistics

Gaussian model of signal intensity distribution

\[ I_s(x, y \mid x_0, y_0) = I_s(2\pi \sigma_s^2)^{-1} \times \exp\left\{ -\left[ \frac{(x - x_0)^2}{2\sigma_s^2} + \frac{(y - y_0)^2}{2\sigma_y^2} \right] \right\} \text{ watts/cm}^2 \]

Average signal energy over detector element \( i, j \)

\[ \lambda_s(i, j \mid x_0, y_0) = \int_0^T P_s(i, j \mid x_0, y_0) dt \]

\[ \approx T\Delta^2 I_s(i\Delta, j\Delta \mid x_0, y_0) \]

Photo-count probability density from detector element \( i, j \)

\[ p(k_{ij} \mid x_0, y_0) = [\lambda_s(i, j \mid x_0, y_0)]^{k_{ij}} \times \exp\left\{ -\lambda_s(i, j \mid x_0, y_0) \right\} / k_{ij}! \]

Joint probability density of array photo-counts

\[ p(k \mid x_0, y_0) = \prod_{i,j} [\lambda_s(i, j)]^{k_{ij}} \exp\left\{ -\lambda_s(i, j) \right\} / k_{ij}! \]

a) Focal-plane model of pixel array, and elliptical PSF with pointing offsets, motivated by: b) experimentally determined point-spread function (PSF) for the high-quality Vertex panel, photographed on the JPL mesa test range.
The Coherent-State Representation of Optical Signals

Coherent state $|\alpha\rangle$ represented in terms of the number states $|n\rangle$: $|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{(n!)^{1/2}} |n\rangle$

The number states are orthonormal, hence their “overlap” is a delta function:

$$\langle n | m \rangle = \delta_{nm} = \begin{cases} 1 & n = m \\ 0 & n \neq m \end{cases}$$

Coherent states are not orthonormal, hence the squared overlap of any two coherent states is not zero, but given by:

$$|\langle \alpha | \beta \rangle|^2 = e^{-((|\alpha|^2+|\beta|^2)/2} \sum_n \sum_m \frac{\alpha^n}{\sqrt{n!}} \frac{(\beta^*)^m}{\sqrt{m!}} |\langle n | m \rangle|^2$$

$$= e^{-|\alpha|^2} e^{-|\beta|^2} e^{-2|\alpha\beta^*|} = e^{-|\alpha-\beta|^2}$$

The probability that a coherent state $|\alpha\rangle$ contains $n$ photons is Poisson distributed:

$$p(n) = |\langle \alpha | n \rangle|^2 = e^{-|\alpha|^2} \frac{|\alpha|^{2n}}{n!}$$
Quantum limit for binary signals

For any pair of binary signals $|\psi_0\rangle, |\psi_1\rangle$, the quantum limited error probability depends only on their overlap [1]:

$$P^*(E) = 1 - P^*(C) = \frac{1}{2} \left[ 1 - \sqrt{1 - |\langle \psi_0 | \psi_1 \rangle|^2} \right]$$

Quantum limit for the special case of OOK and binary PPM signals

For OOK signals represented by $|0\rangle$ and $|\alpha\rangle$, with overlap $|\langle 0 | \alpha \rangle|^2 = e^{-|\alpha|^2}$, the quantum limit is:

$$P^*_{OOK}(E) = \frac{1}{2} \left[ 1 - \sqrt{1 - |\langle \psi_0 | \psi_1 \rangle|^2} \right] = \frac{1}{2} \left[ 1 - \sqrt{1 - e^{-|\alpha|^2}} \right] = \frac{1}{2} \left[ 1 - \sqrt{1 - e^{-2K_s}} \right]$$

with average symbol energy $K_s = \frac{1}{2} |\alpha|^2$, hence $|\alpha|^2 = 2K_s$ (average power constraint)

For PPM signals represented by $|\varphi_1\rangle = |\alpha'| |0\rangle$ and $|\varphi_2\rangle = |0\rangle |\alpha'\rangle$ the squared overlap becomes

$$|\langle \varphi_1 | \varphi_2 \rangle|^2 = |\langle 0 | \langle \alpha' | 0 \rangle | \alpha' \rangle|^2 = |\langle \alpha' | 0 \rangle |\langle 0 | \alpha' \rangle|^2 = |\langle \alpha' | 0 \rangle|^2 = e^{-|\alpha'|^2} = e^{-|\alpha|^2} = e^{-2K_s}$$

$$P^*_{PPM}(E) = \frac{1}{2} \left[ 1 - \sqrt{1 - |\langle \varphi_1 | \varphi_2 \rangle|^2} \right] = \frac{1}{2} \left[ 1 - \sqrt{1 - e^{-2K_s}} \right]$$

with average signal power $K_s = |\alpha'|^2$. If $|\alpha'|^2 = \frac{1}{2} |\alpha|^2$ (equal avg. power), then $P^*_{PPM}(E) = P^*_{OOK}(E)$
Photon Counting Receiver: OOK signals

Maximum likelihood decision strategy, OOK signals:

Obtain the photon-counts, $k_0$, within each $T$-second symbol-interval, and compare to the optimal threshold $\eta = 2K_s / \log(1 + 2K_s / K_b)$. If $k_0 < \eta$ declare $H_0$; if $k_0 > \eta$ declare $H_1$.

**With no background**, $K_b = 0 \Rightarrow \eta = 0$. Hypotheses $H_0$ and $H_1$, equilikely case: $P(H_0) = P(H_1) = \frac{1}{2}$

\[
P(n | H_0) = \begin{cases} 1, & n = 0 \\ 0, & n \geq 1 \end{cases}; \quad P(n | H_1) = \frac{\alpha^{2n}}{n!} e^{-|\alpha|^2}
\]

Conditional detection probabilities:

\[
P(C | H_0) = P(0 | H_0) = 1, \quad P(C | H_1) = P(n \geq 1 | H_1) = 1 - e^{-|\alpha|^2}
\]

\[
P(C) = \sum_{i=1}^{2} P(C | H_i)P(H_i) = 1 - \frac{1}{2} e^{-|\alpha|^2} \quad P(E) = 1 - P(C) = \frac{1}{2} e^{-|\alpha|^2} \quad P_{OOK}(E) = \frac{1}{2} e^{-2K_s}
\]

**With background**, $K_b \Rightarrow \eta = 2K_s / \log(1 + 2K_s / K_b)$, probability of correct detection becomes

\[
P_{OOK}(E) = 1 - \frac{1}{2} \left( \sum_{k_0=0}^{\lfloor \eta \rfloor} \frac{K_b^{k_0} e^{-K_b}}{k_0!} + \sum_{\lceil \eta \rceil}^{\infty} \frac{(K_b + K_s)^{k_0} e^{-(K_b+K_s)}}{k_0!} \right)
\]
Photon Counting Receiver: binary PPM signals

Maximum likelihood decision strategy, binary PPM signals:
Obtain the photon-counts $k_1$ and $k_2$ in the first and second slots, respectively, and compare: if $k_1 > k_2$ declare $H_0$; if $k_2 > k_1$, declare $H_1$. In case of a tie, toss a fair coin to determine the outcome.

With no background, $K_b = 0$, an error is made only if no photons are observed, in which case a fair coin is tossed leading to correct detection half the time. Since the probability of observing no photons under either hypothesis is $e^{-K_s}$, it follows that the average probability of error for binary PPM is

$$P_{2PPM}(E) = \frac{1}{2} e^{-K_s}$$

With background, $K_b$, the probability of bit error can be expressed as [2]

$$P(E) = \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{k_1} \gamma(k_1, k_2) (K_s + K_b)^{k_1} K_b^{k_2} e^{-(K_s + 2K_b) / k_1!k_2!}$$

where $\gamma(k_1, k_2) = \begin{cases} 1, & k_1 = k_2 \\ \frac{1}{2}, & k_1 \neq k_2 \end{cases}$
Bit error probabilities for OOK and 2PPM: zero to low background

Quantum limit and photon-counting detection performance of OOK and binary PPM signals as a function of average signal energy: a) extremely low average background energy per symbol-interval; b) low average background energy per symbol-interval.
Bit error probabilities for OOK and 2PPM: moderate to high background

Quantum limit and photon-counting detection performance of OOK and binary PPM signals as a function of average signal energy:

a) moderate average background energy;
b) high average background energy.
Spatial filtering to improve detection performance in the presence of background

Accumulated signal energy (array detectors sorted according to signal intensity, in decreasing order)

\[ K_s(m) = \sum_{n=1}^{m} \lambda_s(n) \]

Accumulated background energy (constant background intensity)

\[ K_b(m) = m \lambda_b \]

Polished panel PSF generated by the star Vega, showing the low signal energy region excluded by spatial filtering.

\[
P_{2PPM}(E) = \sum_{k_1=0}^{\infty} \sum_{k_2=k_1}^{\infty} \gamma(k_1, k_2) \left( K_s(m) + K_b(m) \right)^{k_1} \times K_b^{k_2}(m) e^{-(K_s(m)+2K_b(m))} / k_1! k_2!
\]

\[
P_{OOK}(E) = \frac{1}{2} \left( \sum_{\text{ceil}(\eta)} K_b^k(m) e^{-K_b(m)} / k! + \sum_{k=0}^{\text{floor}(\eta)} (K_b(m) + K_s(m))^k e^{-(K_b(m)+K_s(m))} / k! \right)
\]

Bit error probability as a function of \( m \) for OOK and binary PPM signaling, with minimum error probability achieved by \( m^* \).
Binary PPM Simulation Results

Illustration of random array counts for various pixel-intensity distributions using the experimentally recorded PSF in a) as the average energy distribution, ranging from very high, b), to very low, f), total signal energy.
Signal + background intensity in slot #1

Background intensity only in slot #2

Sample of array photo-counts for each slot:
low signal intensity, ~ 100 photons/frame

Sample frames of background and signal-plus-background frames, and associated array photo-counts.

Polished Panel Receiver Performance Simulation

- Probability of bit error, $P_e$
- Average number of photons/frame/pulse, $K_s$
- Pulse-position modulation, $M=2$
- $K_b = 5$ photons/frame/slot
- $N = 10,000$ frames/simulation point

Theoretically derived binary PPM performance and simulation results.
Coding to Improve Detection Performance

For bounded distance decoders that correct all combinations of \( t \) or fewer errors, the bit error probability can be expressed approximately as:

\[
P_B(E) = n^{-1} \sum_{j=t+1}^{n} \beta_j \binom{n}{j} p^j (1-p)^{n-j}
\]

With \( \beta_j \approx j \) (a good approximation for systematic codes), and using the identity \( n^{-1} \sum_{j=2}^{n} j \binom{n}{j} p^j (1-p)^{n-j} = p - p(1-p)^{n-1} \)

the bit error probability can be further simplified as:

\[
P_B(E) \approx n^{-1} \sum_{j=t+1}^{n} j \binom{n}{j} p^j (1-p)^{n-j}
\]

\[
= p - p \sum_{j=0}^{t-1} \binom{n-1}{j} p^j (1-p)^{n-1-j}
\]

\[
p \left[ 1 - C_{bin}(t-1,n-1,p) \right] = P_b(E)
\]

where \( C_{bin}(t-1,n-1,p) \) is the binomial cumulative distribution with \( (n-1) \) degrees of freedom, from 0 to \( (t-1) \).

Conditional error probabilities for equally probable but highly asymmetrical OOK signals, validating the binary symmetric channel model for high background intensities.
BCH-coded performance and optimization

Uncoded and BCH block-coded performance of binary PPM symbols for increasing codeword-length, with $t = 2$; b) minimum error probability achieved with $n = 127$, $t = 2$. 

BCH codes, $(n, k)$

- Binary PPM
  - $t = 2$, $K_p = 40$

- BCH-coded performance and optimization
  - $t = 2$
  - $K_b = 40$, $K_s = 40$
BCH-coded performance and optimization

Uncoded and BCH block-coded performance of binary PPM symbols for increasing error correction strength, with $n = 63$; b) minimum error probability achieved with $t = 5, n = 63$. 
CONCLUSIONS

- A high quality “polished panel” (Vertex Antennentechnic GmbH.) was installed on the DSN’s 34 meter antenna at DSS-13.

- The PSF generated by the polished panel was determined by tracking the planets Jupiter and Venus, as well as several bright stars.

- Detailed mathematical model of the focal-plane energy distribution was developed, and the experimentally determined PSF was used to evaluate potential deep-space communications performance of a “polished-panel” optical receiver, with OOK and binary PPM signals.

  - It was shown that with zero background OOK can approach the quantum bound even with optically imperfect polished panel reflectors (average power constraint).
  - It was shown that deep-space communications requirements can be met even with simple codes, and parameter optimization with BCH codes was illustrated.