Eliminating Obliquity Error From the Estimation of Ionospheric Delay In a Satellite-Based Augmentation System

Lawrence Sparks, Jet Propulsion Laboratory, California Institute of Technology

BIOGRAPHY

Lawrence Sparks, Ph.D., is a member of the Ionospheric and Atmospheric Remote Sensing Group in the Tracking Systems and Applications Section at NASA's Jet Propulsion Laboratory. He received his Ph.D. in Applied Physics from Cornell University and has pursued research in various fields including fusion plasma physics, solar magnetohydrodynamics, atmospheric radiative transfer, and ionospheric modeling. He is a member of the WAAS Integrity and Performance Panel and is currently working on issues concerning the impact of the ionosphere on satellite-based augmentation systems.

ABSTRACT

Current satellite-based augmentation systems estimate ionospheric delay using algorithms that assume the electron density of the ionosphere is non-negligible only in a thin shell located near the peak of the actual profile. In its initial operating capability, for example, the Wide Area Augmentation System incorporated the thin shell model into an estimation algorithm that calculates vertical delay using a planar fit. Under disturbed conditions or at low latitude where ionospheric structure is complex, however, the thin shell approximation can serve as a significant source of estimation error. A recent upgrade of the system replaced the planar fit algorithm with an algorithm based upon kriging. The upgrade owes its success, in part, to the ability of kriging to mitigate the error due to this approximation. Previously, alternative delay estimation algorithms have been proposed that eliminate the need for invoking the thin shell model altogether. Prior analyses have compared the accuracy achieved by these methods to the accuracy achieved by the planar fit algorithm. This paper extends these analyses to include a comparison with the accuracy achieved by kriging. It concludes by examining how a satellite-based augmentation system might be implemented without recourse to the thin shell approximation.

INTRODUCTION

Signals broadcast by global navigation satellite systems (GNSS) experience delay when they propagate through the ionosphere. For airline navigation, satellite-based augmentation systems (SBAS) have been implemented to guarantee the accuracy and integrity of user position estimates based upon GNSS signals. These systems provide accurate calibration of ionospheric delays. Systems such as the United States' Wide Area Augmentation System (WAAS) and the European Geostationary Navigation Overlay Service (EGNOS) employ the *thin shell model* of the ionosphere when estimating ionospheric delay, a crude model where the electron density is assumed to be non-negligible only in an infinitesimally thin layer located at a shell height representative of the altitude where the electron density attains its peak value. Such a crude approximation necessarily introduces error into delay estimation. In its initial operational capability, WAAS performed vertical delay estimation by incorporating the thin shell model into a planar fit algorithm. A recent upgrade of WAAS replaced the planar fit algorithm with an algorithm based upon a geo-statistical technique known as kriging [1]. The success of kriging is partly due to its ability to mitigate the error due to the thin shell approximation. Previous papers [2][3], however, have proposed alternative methods of delay estimation that eliminate the need for adopting the thin shell approximation altogether. These papers have compared the accuracy achieved to the accuracy attained by the planar fit algorithm. This paper extends these analyses by comparing the accuracy achieved to that attained by kriging, and it examines the implications of these results for SBAS.

The primary reason for adopting the thin shell approximation is that it greatly simplifies the process of ionospheric delay estimation. In particular, the thin shell model serves two very useful purposes which account for its popularity in ionospheric analyses: (1) it associates each signal raypath with an *ionospheric pierce point* (IPP) where the raypath intersects the ionospheric shell, and (2)

it provides a means of converting a slant delay measurement to a vertical delay estimate at the IPP using a simple geometrical conversion factor.

These benefits are not achieved cost-free, however. It is well known that the thin shell approximation can also serve as a significant source of estimation error [4]. This error is designated *obliquity error*. The ionosphere at low latitude poses a particularly critical challenge for any SBAS that relies upon the thin shell model. Here the structure of the ionosphere is complex. Slant delay values display large magnitudes. Nearly every day, for example, large gradients in electron density occur as a manifestation of the presence of the equatorial (Appleton) anomaly. Previous studies [5] of slant delay residual errors based upon a planar fit have, in fact, concluded that the equatorial ionosphere exhibits such irregular structure under nominally quiet conditions that the occurrence of a major magnetic storm does not dramatically alter the statistical nature of the fit residuals. To base an SBAS at low latitude upon the thin shell model is clearly problematic.

In prior papers [2][3] that address delay estimation, alternative methods have been proposed that eliminate the need for adopting the thin shell approximation, thereby removing obliquity error as a source of delay estimation error. The conical domain model [2] avoids the thin shell approximation by restricting each fit of GNSS measurements to a spatial domain encompassing signals from only a single satellite, that is, the fit domain is a cone with a satellite at the vertex. An SBAS based upon the conical domain model would broadcast fit parameters associated with each satellite. A user would then determine her line-of-sight delay to a given satellite by evaluating a model fit centered on her location, using the parameters associated with the cone of the satellite in question.

In the context of SBAS, the conical domain model suffers from a major limitation: each fit is comprised of relatively few measurements, far fewer than are used in the corresponding standard planar fit algorithm. Each planar fit incorporates measurements from multiple satellites. Given a spatial distribution of receivers similar to that of WAAS, there are, in practice, typically thirty or more vertical delay observations processed per fit. In contrast, for a conical domain fit, the number of measurements available from a single satellite is typically only four or five. Since the reliability of the fit is dependent upon the number of observations fit, a small number represents a threat to the integrity of the delay values estimated.

The multi-cone model [3] addresses this limitation by adopting a two-phase estimation scheme. In the first phase, a set of locations on the surface of the earth is identified, and the conical domain approach is used to estimate delays for raypaths that connect each of these earth points to each of the GNSS satellites that are locally visible. The signals that follow these raypaths are designated pseudo-observations.

In the second phase of the multi-cone algorithm, the conical domain model is inverted; the spatial domain for each estimation is restricted to pseudo-observations whose raypaths intersect the earth at a single point, that is, the vertex of the conical fit domain now resides on the surface of the earth. A fit of pseudo-observations is henceforth designated a pseudo-fit. A given delay estimate will thus depend upon all the measurements used to define each of the pseudo-observations in the pseudo-fit, a number generally comparable to the number of observations used in the planar fit algorithm. In this fashion the multi-cone algorithm combines the accuracy of the conical domain model with the integrity of the planar fit algorithm.

In the sections that follow, we begin by reviewing the planar fit algorithm for estimating ionospheric delay. In particular, we focus on how the thin shell approximation is used and how estimation errors arise as a consequence. Next we provide an overview of the manner in which the multi-cone algorithm eliminates the need to invoke the thin shell approximation. Subsequently, we describe briefly how estimation based upon kriging serves to mitigate obliquity error. Next, we report the results of computations that compare the accuracy of the planar fit, kriging, and the multi-cone algorithms. We assess accuracy in terms of the residual differences between the equivalent vertical delay of actual measurements and that of estimates based upon model fits. Using a Mexican data set from a day when the ionosphere becomes highly disturbed, we find that the multi-cone model reduces the RMS error of the planar fit algorithm on average by a factor of 1.9 and that of the kriging algorithm by a factor of 1.7. We conclude with a discussion of alternative means for implementing the multi-cone algorithm in an SBAS.

PLANAR FIT MODEL

This section provides a brief review of various aspects of the planar fit algorithm for estimating vertical delay from a given set of slant delay observations, namely: the ionospheric model, the vertical delay model, the estimation algorithm, and the sources of estimation error. All of these aspects have been described in more detail elsewhere [1].

Ionospheric model

Free electrons along the raypath impede the passage of a GNSS signal through the ionosphere. The total delay is proportional to the slant total electron content (STEC) along the raypath, which may be written as a path integral

of electron density along the line-of-sight from the receiver to the satellite:

$$STEC_{rs} = \int_{\ell_r}^{\ell_s} d\ell\, n_e\left(\mathbf{x}_{rs}(\ell)\right),$$

where the subscripts r and s identify, respectively, the receiver and satellite associated with the signal, n_e is the ionospheric electron density, and $\mathbf{x}_{rs}(\ell)$ identifies points along the raypath as parameterized by the path length ℓ. This integral may be rewritten as an integral over altitude h:

$$STEC_{rs} = \int_{h_r}^{h_s} dh\, F(\alpha,h)\, n_e\left(\mathbf{x}_{rs}(h)\right),$$

where

$$F(\alpha,h) \equiv \left[1-\left(\frac{R_e\cos(\alpha)}{R_e+h}\right)\right]^{-1/2},$$

R_e is the earth radius, α is the elevation angle of the satellite as seen from the receiver, and $\mathbf{x}_{rs}$ is now treated as a function of the altitude h.

By invoking the thin shell model, *i.e.*, by assuming the electron density to be non-negligible only within a thin layer at a specified ionospheric reference height h_i, an IPP can be associated with each observation in a given data set. The reference height h_i is generally chosen to represent a typical value for the daytime peak of the electron density in the F-layer. (In WAAS, h_i is specified to be 350 km.) Under the thin shell approximation the equation for STEC becomes.

$$STEC_{rs} = F(\alpha,h_i)\,VTEC_{IPP},$$

where

$$VTEC_{IPP} = \int_{h_r}^{h_s} dh\;\; n_e\left(\mathbf{x}_{rs}(h)\right)$$

is the vertical total electron content at the IPP. Thus, each slant delay observation can be converted to vertical by dividing STEC by the function $F(\alpha,h_i)$. This function is designated the *obliquity factor* of the observation raypath.

Vertical delay model

To obtain estimates of vertical delay in the neighborhood of a particular reference IPP (that may or may not be associated with an actual measurement), it is necessary to assume a model that describes the behavior of the vertical delay in the vicinity of this IPP. Over spatial scales where this behavior is approximately linear, we can define the vertical ionospheric delay I_{true} at $\mathbf{x}$ near the IPP at $\mathbf{x}_{IPP}$ as

$$I_{true}(\Delta\mathbf{x}) \equiv a_0 + a_{east}\,\Delta\mathbf{x}^T\cdot\hat{e}_{east} + a_{north}\,\Delta\mathbf{x}^T\cdot\hat{e}_{north} + r(\Delta\mathbf{x})$$

where $\Delta\mathbf{x} \equiv \mathbf{x}-\mathbf{x}_{IPP}$ is the Euclidean vector describing the distance separating the location at which a delay estimate is sought from the location of the reference IPP in ECEF Cartesian coordinates, the coefficients a_0, a_{east}, and a_{north} specify the planar trend of the ionospheric delay, and $r(\Delta\mathbf{x})$ is a scalar field describing small irregularities that are superimposed on the planar trend. The scalar field characterizes the correlation between neighboring measurements. The vertical ionospheric delay measured at $\mathbf{x}$ can then be expressed as

$$I_{meas}(\Delta\mathbf{x}) = I_{true}(\Delta\mathbf{x}) + \varepsilon$$

where ε represents the measurement error. The measurement noise is specific to each receiver-satellite pair and does not depend upon the ionosphere.

Slant delay estimation

As an example of how the ionospheric delay model can be used to estimate the vertical delay in the vicinity of a specified IPP, we now briefly review the planar fit algorithm (see reference [1] for details) used by WAAS in its initial operational capability. (As noted in the introduction, the recent upgrade of WAAS has replaced this planar fit algorithm with an algorithm based upon kriging [1], which we discuss in more detail below.)

First, the locations of IPPs are identified corresponding to a set of slant delay measurements belonging to a single epoch. An estimate of the vertical delay at each of these IPPs is obtained by dividing the slant delay by the appropriate thin shell obliquity factor. The search algorithm that selects the IPPs to be incorporated into each fit is defined by four system parameters: N_{target}, the targeted number of fit points, N_{min}, the minimum number of fit points, R_{max}, the maximum fit radius, and R_{min}, the minimum fit radius. All measurements that have IPPs residing within a minimum distance R_{min} of a given reference point are incorporated into a least-squares planar fit, providing an estimate of the vertical delay at the reference point. In this fit the vertical delay at each IPP is weighted equally. If the number of such measurements is less than N_{target}, the fit radius is expanded until it defines a circle that encompasses N_{target} points. If the fit radius fails to surround N_{target} points after attaining a maximum value of R_{max}, the fit is performed with just those points enclosed by this radius, provided they number at least N_{min}.

Sources of estimation error due to modeling

The limitations of this vertical delay model must be kept in mind. It should be valid over spatial scales where the variation of the delay is, in fact, approximately planar. Over larger scales, however, the magnitude of the scalar field $r(\mathbf{\Delta x})$ will no longer necessarily be small.

Furthermore, when this model is used to perform delay estimation, there is an additional source of error - the obliquity error mentioned in the introduction, *i.e.*, the error associated with the slant-to-vertical conversion of the observations that comprise each fit. Obliquity error may in turn be considered to originate from two distinct sources: (1) the lack of azimuthal symmetry at the IPP (the thin shell model stipulates that the electron density is independent of the azimuthal angle of the raypath at the IPP serving as the fit center), and (2) an invalid obliquity factor (even under conditions where the ionosphere is, in fact, azimuthally symmetric about the IPP, error can arise, for example, from a suboptimal choice of shell height). The successful operation of WAAS over the past ten years has demonstrated that, under nominal conditions at mid-latitudes, the magnitude of the error incurred from these sources is small. At low latitudes or at mid-latitudes under disturbed conditions, however, the presence of enhanced ionization, complex ionospheric structure, and large electron density gradients can lead to a significant increase in estimation error.

MULTI-CONE MODEL

This section presents an overview of how the multi-cone model improves the accuracy of slant delay estimates. The multi-cone model and the conical domain model upon which it is based do not avail themselves of the thin shell approximation. Consequently, delay estimates based upon these models do not suffer from obliquity error.

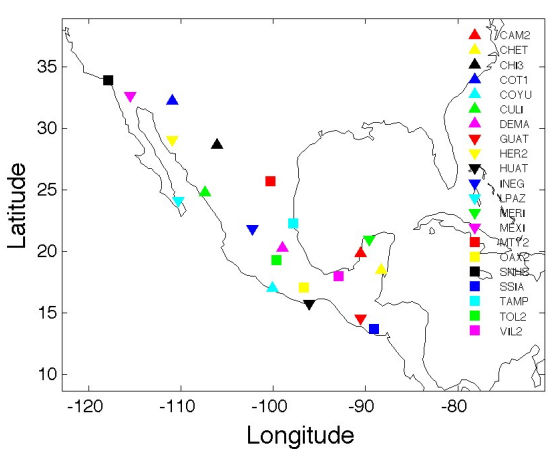

Figure 1. Locations of Mexican receivers.

The conical domain model provides delay estimates only for raypaths that originate at GNSS satellites. In contrast, the multi-cone model can, in principle, determine delay estimates for any arbitrary raypath within the region of coverage. Nevertheless, the accuracy of a given delay estimate will depend upon how well the GNSS observations sample the same portion of the ionosphere traversed by the raypath associated with the delay estimate.

Consider an arbitrary raypath that terminates at a user reference position (or, more precisely, at an *earth reference point*; for an airborne user, the raypath must be projected beyond the user to the point where it strikes the earth). We wish to use the multi-cone algorithm to estimate the slant delay associated with this test raypath. To illustrate the steps in the algorithm, we resort to computations, conducted using the *IonoSTAGE* software package [6], that process data consisting of measurements collected from a set of 21 Mexican receivers. Figure 1 labels each receiver and identifies its location. Prior to analysis these data have been post-processed (1) to eliminate interfrequency biases, (2) to remove the effects of cycle slips in carrier phase measurements, and (3) to level the carrier phase measurements to the corresponding range measurements.

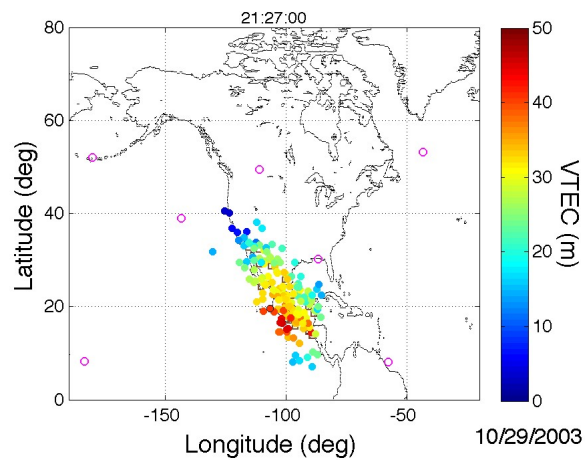

Figure 2. Equivalent vertical delay (in meters) at IPPs associated with observations recorded by Mexican receivers on October 29, 2002, at 21:27 UTC.

The first step is formally identical to the first step of the planar fit algorithm, namely, identifying IPPs associated with each observation in a given epoch and converting slant delay to vertical delay. Figure 2 shows equivalent vertical delay for a set of Mexican observations in a representative epoch. The epoch occurs on October 29, 2003, the date of one of the most intense ionospheric storms to have occurred over the last solar cycle. In a subsequent section, data from this date will be used to examine multi-cone model accuracy.

Note: the interpretation of slant delay values converted to vertical in the multi-cone model differs from their interpretation in the planar fit model. In the planar fit algorithm, each value is considered to be an approximation of the actual vertical delay at the IPP. In contrast, the slant delay values converted to vertical here are simply considered to be the equivalent vertical delay of the corresponding slant delay, *i.e.*, the algorithm never loses sight of the fact that a particular vertical delay value is associated with a particular raypath.

The objective of the first phase of the multi-cone estimation is to generate a set of slant delay values associated with raypaths (pseudo-observations) that connect the visible satellites to the earth reference point. This is accomplished by performing conical domain estimation, where the vertex of each fit domain coincides with one of the visible satellites. The algorithm used to conduct conical domain estimation is designated the *base model* of the multi-algorithm. The multi-cone formalism can incorporate alternative base models in each phase of the algorithm.

In this example, the base model is similar to the model used to perform planar fit estimation except that the IPP search parameters are generally different. In addition, the data to be fit are restricted to signals emitted from the satellite defining the vertex of the conical fit domain. Figure 3 shows the observations from Figure 2 associated with signals emitted from satellite 31. Figure 3 also shows the suborbital position of the satellite and the earth positions of all Mexican receivers. (Note: not all receivers have provided observations in Figure 3.)

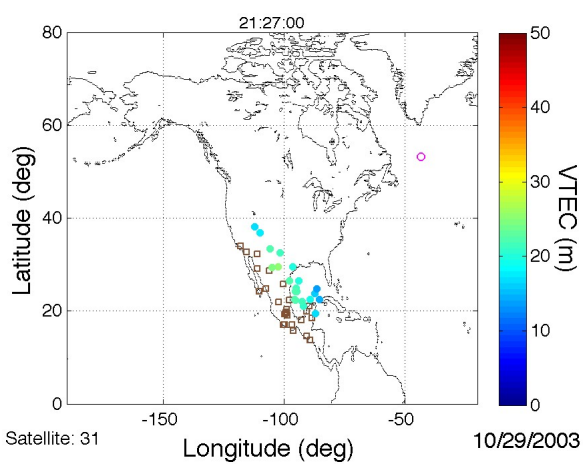

Figure 3. Observations fit in phase 1 of the multi-cone algorithm. Colored disks identify the IPPs of the observations in Figure 2 associated with satellite 31. Brown squares indicate receiver locations. The pink circle identifies the satellite sub-orbital position.

Each fit produces a vertical delay value corresponding to a raypath that links a satellite at the cone vertex to the earth reference point. To achieve maximum accuracy, the fit center for the fit should coincide with the IPP associated with this raypath. The vertical delay value generated may be regarded as specifying the equivalent vertical delay of the pseudo-observation associated with this raypath. Only those measurements sufficiently near the reference raypath, as determined by the search parameters, are incorporated into the fit.

In phase 2 of the multi-cone algorithm, the conical domain algorithm is turned upside down, that is, the earth reference point becomes the vertex of the conical fit domain, and the vertical delay values generated for distinct satellites in phase 1 provide the data to be fit. The search parameters that govern this fit can be different from the search parameters used in the fits of phase 1. The result of this fit is an estimate of the equivalent vertical delay associated with the test raypath. The slant delay along this raypath is then evaluated by multiplying this equivalent vertical delay by the appropriate obliquity factor.

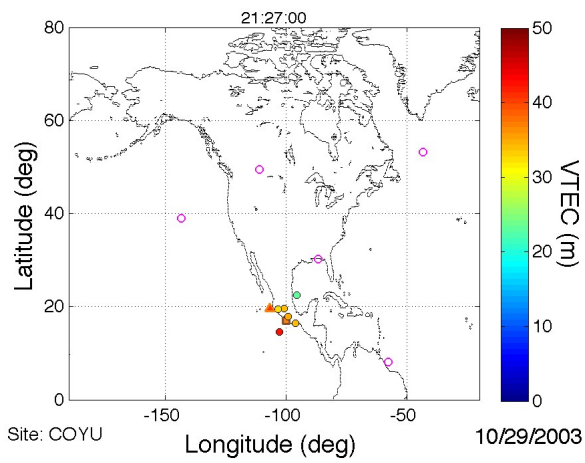

Figure 4. Pseudo-observations fit in phase 2 of the multi-cone algorithm. The square indicates the earth reference point. The triangle identifies the test IPP. The six colored disks identify pseudo-observations generated in phase 1. The pink circles identify five of the six satellite sub-orbital positions associated with the pseudo-observations.

Figure 4 illustrates this process. The partially hidden, square marker identifies the earth reference point, and the triangular marker identifies the IPP associated with the test raypath. The six colored disks show the equivalent vertical delay values estimated in phase 1 for the raypaths that link the earth reference point to visible satellites. (The blue-green disk northeast of the other disks identifies the vertical delay value associated with the fit of the data displayed in Figure 2.) The five pink circles identify the suborbital positions of five of the six visible satellites (the position of the satellite associated with the red southern-most disk lies further south). The color in the interior of the triangle (orange) indicates the observed value of the

equivalent vertical delay for the test raypath. The edge color of the triangle (tan) indicates the estimated value as determined by a pseudo-fit of the six equivalent vertical delay values of the neighboring pseudo-raypaths. (Note: we have chosen here an example where the disparity in color is large, indicative of a poor estimate.)

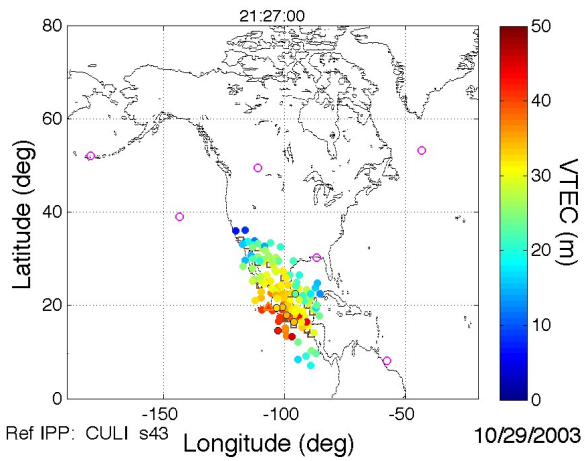

Figure 5. Equivalent vertical delay (in meters) at IPPs associated with pseudo-observations generated by multi-cone estimations for October 29, 2002, at 21:27 UTC.

The accuracy of the multi-cone algorithm can be evaluated by comparing the equivalent vertical delay value of a measured slant delay to the value estimated for its raypath when the observation is excluded from the data fit. Figure 5 displays the results of repeating this process for all the observations in the epoch represented in Figure 2. Comparing Figure 5 to Figure 2 shows good qualitative agreement. In a subsequent section we provide a more quantitative analysis of model accuracy, based upon this approach.

Note that several data points appear around the edges of coverage in Figure 2 in regions that are devoid of points in Figure 5. This is due to the fact that multi-cone delay estimates cannot be generated when the local density of observations is insufficient to meet the requirements imposed by the choice of search parameters. Generally, it is easier to meet the search requirements for a planar fit than for a multi-cone fit.

It should be emphasized that the use of the thin shell obliquity factor, both initially to define the equivalent vertical delay of slant delay observations and finally to convert the vertical delay estimate to slant, does not incur the error inherent in the thin shell model as discussed above. Throughout conical domain estimation, in both phase 1 and phase 2 of the multi-cone algorithm, the equivalent vertical delay at each measurement IPP maintains an azimuthal spatial dependence; no equivalent vertical delay value is ever treated as an approximation of the true vertical delay at its IPP. Signals emitted from distinct satellites that share the same IPP can (and generally will) have different equivalent vertical delays, but this fact never results in conflicting estimates of vertical delay at a given location since signals from distinct satellites in phase 1 (and distinct earth reference points in phase 2) are never included in the same fit.

KRIGING

In our original implementation of the multi-cone algorithm [3], the standard planar fit algorithm was selected as the base model for each phase of vertical delay estimation. As mandated by this algorithm, the fit domain consisted of a set of observations (in phase 1) or pseudo-observations (in phase 2) intersecting at a specified vertex (satellite in phase 1; earth reference point in phase 2). Recently, however, we have implemented the option of changing the base model from the planar fit algorithm to an algorithm based upon *kriging*, a geo-statistical technique that originated in the mining industry in the 1950's. Kriging is a type of minimum mean square estimator adapted to spatial data. This change in our code was motivated, in part, by the fact that WAAS has also recently implemented delay estimation based upon kriging [1]. Kriging has been shown to produce significantly more accurate delay estimates than the planar fit algorithm, with a consequent improvement in system availability. This section describes how kriging serves to reduce obliquity error. A discussion that provides more details can be found elsewhere [1].

In practice kriging differs from planar fit estimation by assigning non-uniform weights to the observations in the fit, weighting more heavily those closest to the fit center. The magnitudes of the weights are dependent upon an empirical model of the variogram that characterizes the underlying spatial structure of the random scalar field $r(\mathbf{\Delta x})$ in the ionospheric model, defined as follows:

$$\gamma\left(\Delta\mathbf{x}_\kappa, \Delta\mathbf{x}_\zeta\right) \equiv \frac{1}{2}\left\langle\left[r\left(\Delta\mathbf{x}_\kappa\right) - r\left(\Delta\mathbf{x}_\zeta\right)\right]^2\right\rangle.$$

where the outer brackets indicate expectation value. The variogram is a measure of the rate at which the values of $r(\mathbf{\Delta x})$, evaluated at two points, become decorrelated as a function of the distance separating the two points. It is assumed that there is no deterministic underlying trend and that $r(\mathbf{\Delta x})$ is stationary.

By definition, the variogram must vanish when its two arguments are identical. Where the two points coalesce, however, the variogram may be discontinuous. The presence of this discontinuity has been designated the *nugget* effect, a legacy of the origin of kriging in the mining industry. (For example, soil grade can be discontinuous due to the presence of gold nuggets.) Just

such a discontinuity appears at the origin in variograms that have been derived empirically from WAAS observational data sets [7]. Obliquity error appears to serve, at least in part, as the physical basis for this discontinuity. In delay estimation based upon the thin shell model, the assumed azimuthal symmetry about the fit center is, at best, only approximate. Two measurements with the same IPP but recorded at distinct receivers will not, in general, even under quiet ionospheric conditions, generate the same vertical delay estimate. By accommodating the nugget effect in this fashion, kriging helps to mitigate obliquity error, but it does not entirely remove it.

MODEL ACCURACY

In this section we examine the accuracy of the planar fit, kriging, and the multi-cone algorithms. We consider the multi-cone algorithm using as the base model both the planar fit and kriging algorithms. The kriging results are new. The planar fit results are similar to those reported previously [3] and are provided to serve as a basis for comparison.

To determine model accuracy, we evaluate the residual differences between measured delay values and their corresponding estimates as computed by each estimation scheme. We work with equivalent vertical delay rather than slant delay, *i.e.*, we convert slant delay to vertical delay using the appropriate obliquity factor for each observation. This serves to remove the raypath geometry from the results and permits the plotting of comparable delay values. When we estimate a delay value for a test observation, we first remove that observation from the data to be fit. For every observation in the data set, we attempt to calculate one estimate for each estimation algorithm. Computations are performed using the *IonoSTAGE* package [6].

Parameter	*Planar fit*	*Kriging*	*Multi-cone phase1*	*Multi-cone phase 2*
h_{iono}	350 km	350	350 km	350 km
N_{target}	30	30	6	5
R_{max}	2100	2100	1000 km	1000 km
R_{min}	800	800	0	0
N_{min}	10	10	4	4

Table 1. Search parameters used in analysis.

The search parameters used in the study are recorded in Table 1. Note: the kriging algorithm requires the specification of three additional parameters that are determined by fitting the empirical variograms discussed in the previous section. In this analysis we have adopted the values currently used by WAAS [1].

Our assessment of model accuracy uses the Mexican data set of October 29, 2003, described previously. These data provide a particularly challenging environment for evaluating model accuracy, given the high level of ionospheric disturbance that occurred on that date. While we have also, on occasion, used WAAS data to study the accuracy of the multi-cone model (and will no doubt do so again in the future), it is preferable to adopt the Mexican data as truth for model evaluation. In general we expect the accuracy of the multi-cone model to increase with the density of observations, and the density of IPPs in a given epoch is significantly higher for this Mexican data set than it is for WAAS data. Note: these are the same observational data that were used in the prior evaluation of multi-cone accuracy [3].

Selecting the data of October 29, 2003, allows us to examine both quiet and highly disturbed ionospheric conditions within a single 24-hour period. On this day began one of the most severe storms to take place during the last solar cycle. Storm onset occurred at approximately at 18:00 UTC. Prior to this hour, the ionosphere exhibited nominal (quiet) behavior. The severity of the storm that followed generated highly complex and irregular ionospheric structure. Such a large disturbance presents an unusually challenging set of conditions under which to model ionospheric delay.

Figure 6 shows the residual error between estimated and observed equivalent vertical delay values at six Mexican sites for data at 90-second intervals. These are the same six sites for which results have been reported previously [3]. (Indeed these plots differ from the corresponding plots in [3] only in that (1) points are plotted at 90 rather than 30 second intervals, (2) different values for N_{target} are used in the computation, and (3) data from redundant sites have been omitted, *e.g.*, we now use only one receiver located in Mexico City.)

In each plot, the multi-cone model residuals are depicted in red, and the thin shell model residuals are in blue. A point is plotted only when both models have been successful at generating an estimate for the observation in question. In other words, if the number of points inside the fit radius for either model does not meet the required minimum, no residual is plotted for either model.

The corresponding set of plots comparing the WAAS kriging algorithm and the multi-cone algorithm with kriging used as the base model is similar to the set shown in Figure 6. As expected, kriging helps to reduce the magnitudes of the fit residuals, but this effect is much more pronounced comparing pure kriging to the planar fit algorithm than it is comparing the multi-cone results with different base models. This is not surprising. Since the weights given to fit observations are non-uniform under kriging, its impact will tend to increase with the number

of points in the fit. From Table 1, we note that the number of fit observations under the planar fit and kriging algorithms ranges from 10 to 30. In contrast for each phase of the multi-cone algorithm this number ranges from 4 - 6.

For each plot in Figures 6, we have calculated the root mean square (RMS) residual error over the entire day for each of the models. Note that in all cases displayed, the multi-cone model clearly outperforms both the planar fit the kriging models, substantially reducing the overall RMS error.

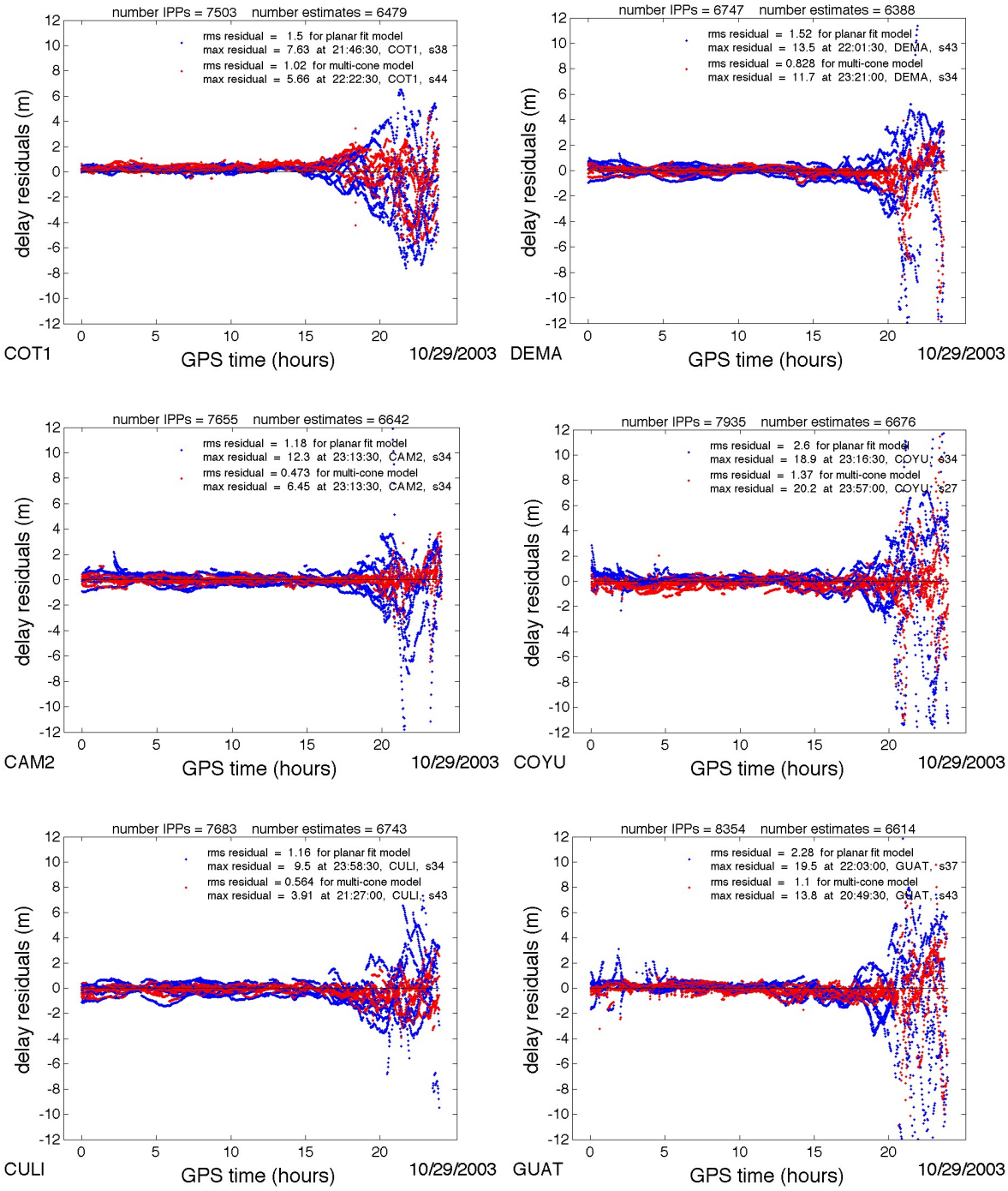

Figure 6. Equivalent vertical delay error vs. UTC time for observations on October 29, 2003, at six sites in Mexico, where fits have been performed using the planar fit algorithm (blue) and multi-cone algorithm with a planar fit base model (red).

Figure 7 shows the RMS error for each model at each Mexican receiver over the entire day. Note that the RMS error for the multi-cone is always significantly less than that for the thin shell and kriging models. After averaging over all stations, we find that the multi-cone model reduces the RMS error of the planar fit algorithm on average by a factor of 1.9 and that of the kriging algorithm by a factor of 1.7.

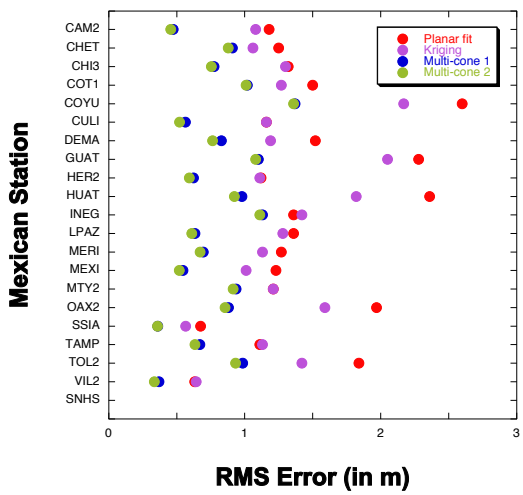

Figure 7. The root mean square (RMS) error at each Mexican receiver over the entire day using the following algorithms: planar fit (red), kriging (violet), multi-cone with a planar fit base model (blue), and multi-cone with a kriging base model (green).

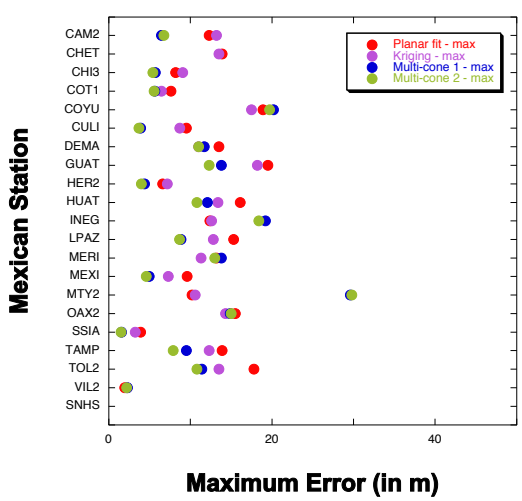

Figure 8. The maximum error at each Mexican receiver over the entire day using the following algorithms: planar fit (red), kriging (violet), multi-cone with a planar fit base model (blue), and multi-cone with a kriging base model (green).

Figure 8 displays a similar plot for the maximum residual error. While the results here are less consistent, multi-cone tends to outperform the other models here as well.

We now conclude this section with some comments on the convergence properties of these estimation algorithms. WAAS performance over the past decade has demonstrated that the planar fit and kriging algorithms provide accurate estimates of slant delay under nominal conditions. For these two algorithms, however, it is important to remember that there is no limiting sense in which a series of delay estimates strictly converges to a true delay value. As a user approaches a reference receiver, for example, the WAAS estimate of slant delay to that user does not, in general, converge to the value measured at that receiver. Alternatively, the estimation error that occurs when an observed signal passes through a distant irregularity, and thereby corrupts the planar fit, cannot be eliminated by clustering reference receivers increasingly close to the user. (A similar conclusion holds concerning a lack of convergence when receiver locations are adjusted so as to produce a series of measurement IPPs that approach the user's IPP.) When the density of receivers on the ground is relatively sparse, increasing this density may serve to improve estimation accuracy. However, there must necessarily be a point of diminishing returns. This conclusion stands in sharp contrast to the behavior of the multi-cone algorithm, whose delay estimates do converge to their true values in as the densities of receivers and satellites increase.

APPLICATION TO SBAS

In this section we consider various ways in which the multi-cone algorithm might be implemented in an SBAS. There are at least four distinct approaches that deserve consideration. Here we sketch an outline of each approach and discuss its various advantages and disadvantages.

Broadcasting actual observations

In principle, the multi-cone algorithm could be implemented directly in a manner very similar to that described above. In this approach the system broadcasts ionospheric data corresponding to the actual measurements collected by the receiver network. The user performs both phases of multi-cone estimation: a phase 1 fitting of actual observations to obtain pseudo-observations linking each satellite to the user's position; and a phase 2 fitting of these pseudo-observations to determine the delay along each raypath linking the user position to a visible satellite. (Note that here the role of the earth reference point, as described above, is played by the user position, not assumed to be on the surface of the earth.) This approach has the advantage of achieving the highest estimation accuracy. A disadvantage is that this approach demands an expansion of the bandwidth

currently devoted to broadcasting the data required by the user to compute delay corrections.

Broadcasting pseudo-observations

A second approach is to have the system broadcast the pseudo-observations that link the visible satellites to grid points on the surface of the earth located at regularly spaced intervals of latitude and longitude. Here the system rather than the user performs the phase 1 fitting to derive pseudo-observations from actual observations. To obtain a delay estimate along a test raypath linking a given satellite to a user's position, the user first projects that raypath to the point where it intersects the earth grid. The user then performs phase 2 fitting of pseudo-observations that link the satellite in question to the four earth grid points that define the cell in which the earth intersection point lies. These delay estimates are interpolated to estimate the delay along the test raypath. This approach should achieve nearly the accuracy of the first approach. However, it suffers from the same disadvantage, namely, requiring an expanded bandwidth for broadcasting the data to be fit.

Broadcasting the standard parameter set

We next consider how to design a multi-cone algorithm to make use of existing SBAS algorithms and technology to the maximum extent possible. According to the standards set forth in the *Minimum Operation Performance Standards for Global Positioning System/Wide Area Augmentation System Airborne Equipment (MOPS)* [8], WAAS broadcasts ionospheric delay error and confidence bounds at points on an ionospheric grid specified at regularly spaced intervals of latitude and longitude. WAAS determines these broadcast data from estimates of vertical delay calculated at each ionospheric grid point (IGP) from a fit of neighboring slant delay measurements, projected to vertical using obliquity factors derived from the thin shell model. Interpolation is required to obtain an estimate of the vertical delay at any arbitrary point on the grid. The slant delay between the satellite and the user can then be determined from the product of the interpolated vertical delay at the user's IPP and the user's thin shell obliquity factor.

The multi-cone algorithm can be implemented most easily in an existing SBAS such as WAAS by using it simply to provide improved estimates of the true vertical delay and its uncertainty at each IGP. In this approach, kriging is adopted as the base model. The improvement in delay estimation accuracy represented in Figure 7 provides an example of the potential reduction in vertical delay error that might be achieved. An advantage of this approach is that it requires no change in either the data broadcast or the user algorithms that evaluate ionospheric corrections from these data, *i.e.*, no changes to the MOPS are required. The chief disadvantage is that the final step of the algorithm - converting an estimated user vertical delay to slant delay - restores some of the obliquity error whose elimination has motivated the development of the multi-cone model in the first place. This is not likely to be a severe limitation, however. The advent of kriging in WAAS has already demonstrated that enhancing the accuracy of vertical delay at IGPs can lead to significant improvement in system availability.[9][10]

Broadcasting an expanded parameter set

Finally we wish to consider how to implement the multi-cone algorithm in an SBAS that preserves the elimination of obliquity error by broadcasting an expanded set of parameters. Like the first two approaches discussed above, this approach entails modification of the standard broadcast message, but here the required changes are more modest.

In this approach, we choose the planar fit model, rather than the kriging model, as the base model of the multi-cone algorithm. For our purposes here, the planar fit algorithm enjoys an algorithmic advantage over kriging. Kriging estimates delays directly from observations. In contrast, planar fit estimation involves an intermediate step – it generates a set of fit parameters that are subsequently used to evaluate raypath delay as a function of the location of the raypath IPP. By choosing the planar fit model as the base model, phase 2 estimation can be designed to produce a set a pseudo-fit parameters for conical domains whose vertices coincide with earth grid points. These are the parameters that the SBAS broadcasts.

Note that the results plotted in Figure 7 suggest that there is not much advantage in using kriging as the base model of the multi-cone algorithm instead of the planar fit model. In planar fits each observation is weighted equally. Kriging achieves its advantage over the planar fit algorithm by assigning greater weight to the observations whose IPPs are nearest the fit center. This advantage is mitigated, however, as the mean number of observations per fit is reduced.

The user determines the delay along a test raypath linking a given satellite to her position in a manner similar to that of the second approach described above except that the calculation is based upon broadcast pseudo-fit parameters rather than broadcast pseudo-observations. Earthward projection of the satellite-user raypath again identifies the grid cell within which the raypath strikes the earth. The delay estimates for the raypaths connecting the satellite in question to the cell corners are evaluated using the pseudo-fit parameters broadcast for each corner. These delay estimates are then interpolated to obtain the delay along the test raypath.

A question remains, however, as to precisely what pseudo-fit parameters are to be broadcast. First consider restricting the number of pseudo-fits at each earth grid point to one. In this case the fit center for the pseudo-fit will be the IGP directly above the earth grid point. The pseudo-fit will generate three fit parameters defining a plane centered on the IGP (as discussed above): a_0, a_{east}, and a_{north}, where a_0 is an estimate of the vertical delay at the IGP and a_{east} and a_{north} govern the tilt of the plane. The parameter a_0 and an integrity bound based upon its uncertainty are the standard broadcast fit parameters defined by the MOPS.

In an SBAS such as WAAS, fit parameters analogous to a_{east} and a_{north} are also generated at the IGP by the corresponding fit of actual observations (*i.e.*, observations with IPPs neighboring the IGP, observations converted to vertical using the appropriate obliquity factors). These parameters are of little value for planar fit delay estimation, however, primarily due to obliquity error. Consequently they are neglected in WAAS.

In contrast to systems based upon planar fit or kriging algorithms, however, the parameters a_{east} and a_{north} generated in phase 2 of the multi-cone algorithm provide useful information about the variation of the equivalent vertical delay for raypaths that terminate at the earth grid point that serves as the vertex of the conical domain pseudo-fit. As long as the IPP of such a raypath is sufficiently near the fit center (*i.e.*, the IGP directly above the vertex), these parameters permit evaluation of linear corrections to an estimate of the equivalent vertical delay at the IPP based solely on a_0.

When the number of pseudo-fits at each earth grid point is restricted to one, the SBAS must broadcast three fit parameters and their full (3 x 3) covariance matrix (or, more precisely, the 3 x 3 analog of the WAAS integrity bound). Thus the total number of variables broadcast per grid point is twelve. This represents a six-fold increase in the quantity of gridded data broadcast as compared to the data currently broadcast by WAAS.

The accuracy of delay estimation in phase 2 will generally degrade as the test raypath IPP moves far enough from the fit center that curvature effects become important. Consequently, this method will not necessarily be successful in estimating delay for satellites that appear at low elevation angles from the vantage of a given earth grid point. To ensure the accuracy of delay estimates, an upper bound must be imposed on the distance that a test raypath's IPP can be from the fit center.

To overcome this limitation, systems can be designed to broadcast sets of fit parameters associated with more than one pseudo-fit per earth grid point. For example, consider a system that broadcasts delay estimates (and integrity bounds) associated with five pseudo-fits at each earth grid point. Each pseudo-fit is identified by the elevation angle, α, and the azimuthal angle, β, of its fit center on the ionospheric grid as observed from the earth grid point. For best results the fit centers should be distributed more or less uniformly over the fit domain. In addition to the fit center at the IGP directly over the earth grid point $(\alpha = \pi / 2)$, let fit centers be defined for azimuthal angles corresponding to north, south, east, and west, each with the same elevation angle $\alpha = \alpha_0$ for some specified value α_0.

The value of a_0 calculated for the pseudo-fit at one of these fit centers provides an estimate of the equivalent vertical delay for a test raypath, terminating at the earth grid point in question, whose IPP coincides with the fit center. The equivalent vertical delay of any other raypath terminating at the earth grid point in question can be obtained by interpolation (or extrapolation, depending upon the value of α_0) of the a_0 values.

Using five pseudo-fits per earth grid point, the total number of variables broadcast is ten: one value of a_0 and one integrity bound for each pseudo-fit. This represents a five-fold increase over the quantity of data currently broadcast by WAAS. Greater estimation accuracy can be achieved, at least in principle, if the number of pseudo-fits per earth IGP is increased, thereby reducing interpolation error. This comes, however, at the cost of increased bandwidth requirements. Whether this implementation of the multi-cone algorithm having ten broadcast parameters will perform better than the previously described implementation having twelve broadcast parameters should largely depend upon the magnitude of the minimum satellite elevation angle that one plans to use for positioning.

CONCLUSIONS

Current satellite-based augmentation systems perform delay estimation using algorithms that invoke the thin shell approximation of the ionosphere. Under disturbed ionospheric conditions and at low latitude where the structure of the ionosphere is complex on a routine basis, the obliquity error that arises from this approximation can significantly degrade delay estimation accuracy.

This paper has studied the accuracy of an alternative method of delay estimation, the multi-cone algorithm, that dispenses with the thin shell approximation altogether. In

particular, this paper has compared multi-cone estimation accuracy to that of the initial and current algorithms employed by WAAS, based upon planar fits and upon kriging, respectively. We have assessed estimation accuracy in terms of the root mean square (RMS) of the residual differences between the equivalent vertical delay of actual measurements and estimates based upon model fits. Using a Mexican data set under highly disturbed conditions, we find that the multi-cone model reduces the RMS error of the planar fit algorithm on average by a factor of 1.9 and that of the kriging algorithm by a factor of 1.7.

We have concluded the paper by examining several possible approaches to implementing the multi-cone algorithm in a satellite-based augmentation system. The primary drawback of each approach that eliminates obliquity error entirely is a significant increase in the bandwidth required to broadcast fit parameters compared to that required for existing SBAS.

We have also examined an approach that does not eliminate obliquity error entirely but rather eliminates it just from the estimates of vertical delay broadcast at each IGP. (Obliquity error in slant delay estimates is introduced in the final vertical-to-slant conversion that the user performs to estimate the delay to a given satellite.) The advantage of implementing this approach in an SBAS is that it requires no modification of existing broadcast data standards or user delay estimation computations.

The recent implementation of kriging in WAAS has led to improved vertical delay estimates, and this, in turn, has led to significant improvement in system availability. The reduction in vertical delay estimation error achieved by the multi-cone algorithm relative to the kriging algorithm as reported in this paper considerably exceeds the reduction achieved by the kriging algorithm relative to the original planar fit algorithm. This suggests that using a multi-cone algorithm to estimate the broadcast fit parameters might attain an even greater improvement in WAAS availability.

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