Welcome

Lumped Parameter Modeling for Rapid Vibration Response Prototyping and Test Correlation for Electronic Units

Dr. Michael B. Van Dyke
Jet Propulsion Laboratory

Spacecraft Mechanical Section/Dynamic Environments
& Aerospace Nuclear Safety Engineering

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Overview

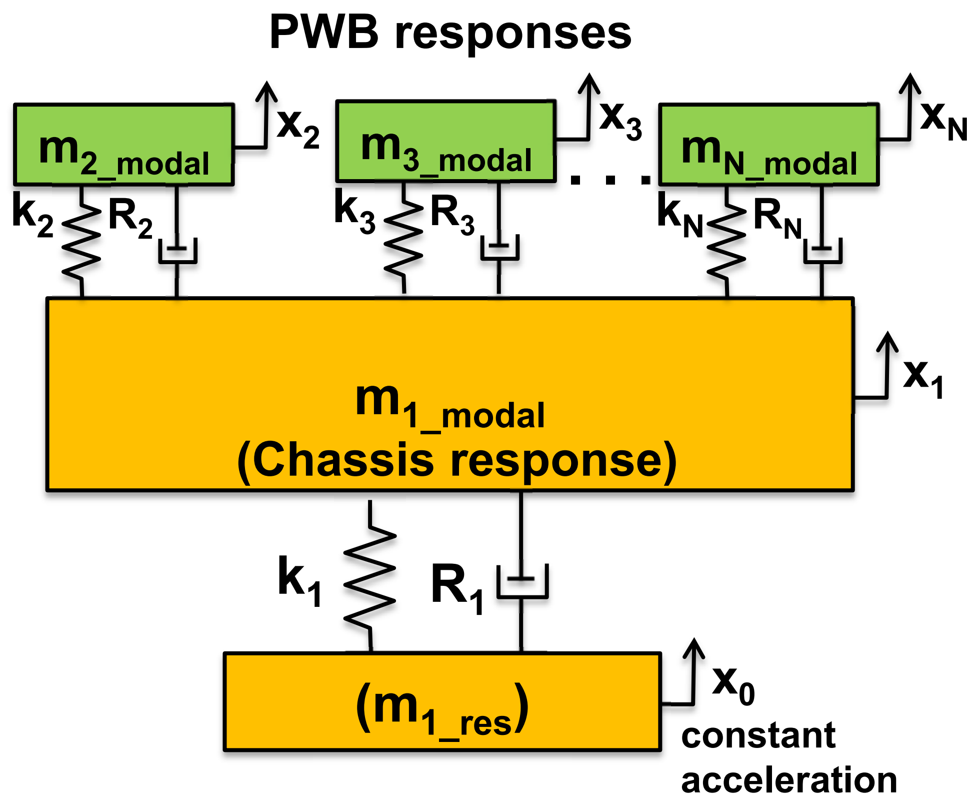
- Present preliminary work using lumped parameter models to approximate dynamic response of electronic units to random vibration
- Derive a general N-DOF model for application to electronic units
- Illustrate parametric influence of model parameters
- Implication of coupled dynamics for unit/board design
- Demonstrate use of model to infer printed wiring board (PWB) dynamics from external chassis test measurement

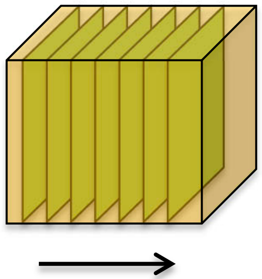
Motivation

- Quick prototyping of electronics unit dynamics for early design
- Indirect observability of unit internal PWB dynamics from external test measurement
- Forensic interpretation of test data (external measurement)
  - *Anomaly resolution*
  - *Design validation*
- This approach supplements but does not replace a detailed finite element model (FEM)

Assumptions and Limitations

- Assumptions
  - *Fixed base shake normal to PWBs*
  - *PWBs (printed wiring boards) independently coupled with chassis*
  - *Negligible mutual coupling between PWBs*
  - *Damping assigned to individual elements rather than modal damping*
  - *Model elements approximate PWB center of gravity responses*





- Limitations
  - *Approximates element fundamental modal response*
  - *Approximates linear response only*
  - *Methodology is still a work in progress*

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Model Parameters

- DOF = number of boards to represent + chassis
- Bulk unit parameters
  - *Total unit weight*
  - *Fundamental modal frequency of loaded chassis (normal to boards)*
    - Initial approximation with infinitely stiff boards
  - *Mass participation factor for loaded chassis fundamental mode*
  - *Q estimate for loaded chassis fundamental modal response*
- Board parameters
  - *Board weight (exclude attach hardware, e.g., wedge locks)*
  - *Bending mode natural frequency (*in situ *boundary conditions)*
  - *Mass participation factor (based on mode shape, BCs, mass distribution)*
  - *Q estimate for fundamental bending mode*
- Input PSD spectrum
  - *Interpolate to desired frequency resolution for model output*

Representation of element modal and residual mass

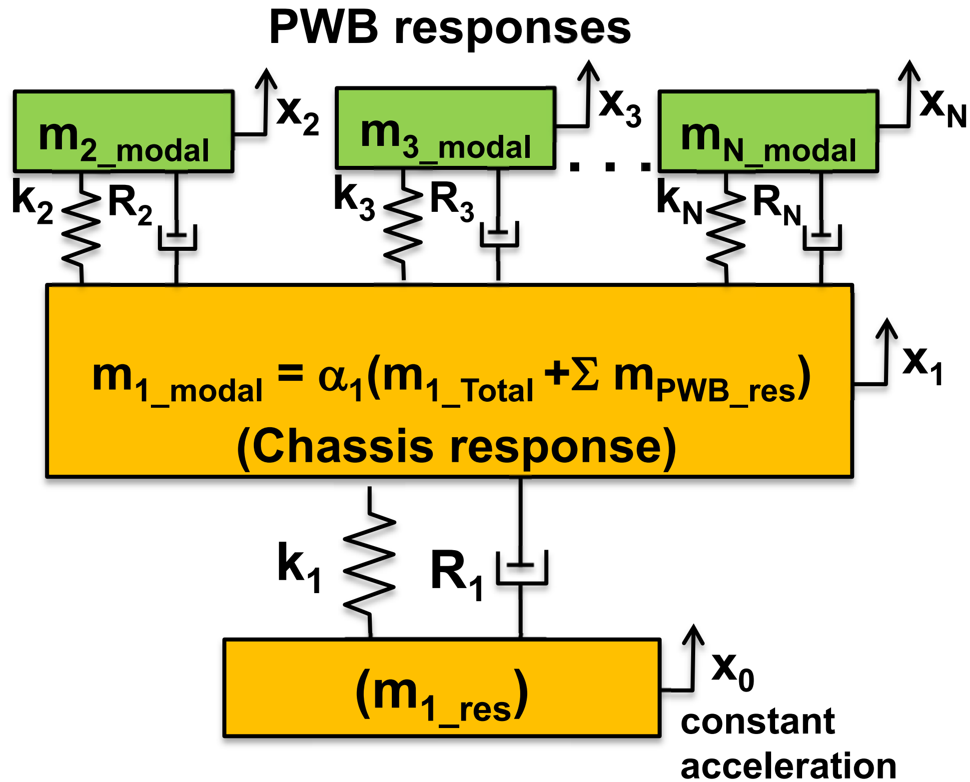
Element Total mass

- **Modal mass**

  Dynamically participating in proportion to mode shape

- **Residual mass**

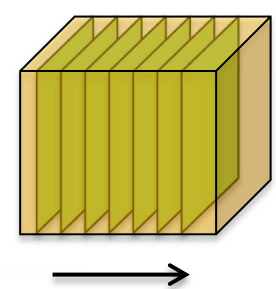
  Moves along with input in rigid body motion



$$m_{\text{modal}} = \alpha\left(m_{\text{total}}\right); \quad m_{\text{residual}} = m_{\text{total}}\left(1-\alpha\right)$$

α = *modal mass participation factor*

- *Based on total mass, approximate mode shape, mass distribution of represented structural element*



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Derivation of N-DOF Model

- System of simultaneous dynamic equations
  - *Modal mass designated by* m *for brevity*

$$
\begin{aligned}
m_1\ddot{x}_1 + R_1\left(\dot{x}_1 - \dot{x}_0\right) + k_1\left(x_1 - x_0\right) &- R_2\left(\dot{x}_2 - \dot{x}_1\right) - k_2\left(x_2 - x_1\right) \\
&- R_3\left(\dot{x}_3 - \dot{x}_1\right) - k_3\left(x_3 - x_1\right) \\
&\cdots \\
&- R_N\left(\dot{x}_N - \dot{x}_1\right) - k_N\left(x_N - x_1\right) = 0
\end{aligned}
$$

$$
\begin{aligned}
m_2\ddot{x}_2 + R_2\left(\dot{x}_2 - \dot{x}_1\right) + k_2\left(x_2 - x_1\right) &= 0 \\
m_3\ddot{x}_3 + R_3\left(\dot{x}_3 - \dot{x}_1\right) + k_3\left(x_3 - x_1\right) &= 0 \\
&\vdots \\
m_N\ddot{x}_N + R_N\left(\dot{x}_N - \dot{x}_1\right) + k_N\left(x_N - x_1\right) &= 0
\end{aligned}
$$

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Derivation of N-DOF Model (cont.)

- Mass normalize and convert to simple harmonic motion displacement

$$
\begin{aligned}
-\omega^2 x_1 + \left(j\omega \frac{R_1}{m_1} + \frac{k_1}{m_1} \right)(x_1 - x_0) - \left(j\omega \frac{R_2}{m_1} + \frac{k_2}{m_1} \right)(x_2 - x_1) \\
- \left(j\omega \frac{R_3}{m_1} + \frac{k_3}{m_1} \right)(x_3 - x_1) \\
\cdots \\
- \left(j\omega \frac{R_N}{m_1} + \frac{k_N}{m_1} \right)(x_N - x_1) = 0
\end{aligned}
$$

$$-\omega^2 x_2 + \left(j\omega \frac{R_2}{m_2} + \frac{k_2}{m_2} \right)(x_2 - x_1) = 0$$

$$-\omega^2 x_3 + \left(j\omega \frac{R_3}{m_3} + \frac{k_3}{m_3} \right)(x_3 - x_1) = 0$$

$$\vdots$$

$$-\omega^2 x_N + \left(j\omega \frac{R_N}{m_N} + \frac{k_N}{m_N} \right)(x_N - x_1) = 0$$

Derivation of N-DOF Model (cont.)

- In order to express in terms of M_n, Q_n, and ω_n, note that:

$$\frac{k_n}{m_n} = \omega_{0n}^2$$

$$\frac{k_n}{m_1} = \omega_{0n}^2 \frac{m_n}{m_1}$$

$$\frac{R_n}{m_n} = \frac{\omega_{0n}}{Q_n}$$

$$\frac{R_n}{m_1} = \frac{\omega_{0n}}{Q_n} \frac{m_n}{m_1}$$

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Derivation of N-DOF Model (cont.)

- System of equations in terms of M_n, Q_n, and ω_n :

$$
\begin{aligned}
-\omega^2 x_1 + \left(j\omega \frac{\omega_{01}}{Q_1} + \omega_{01}^2 \right)(x_1 - x_0) - \frac{m_2}{m_1}\left(j\omega \frac{\omega_{02}}{Q_2} + \omega_{02}^2 \right)(x_2 - x_1) \\
- \frac{m_3}{m_1}\left(j\omega \frac{\omega_{03}}{Q_3} + \omega_{03}^2 \right)(x_3 - x_1) \\
\cdots \\
- \frac{m_N}{m_1}\left(j\omega \frac{\omega_{0N}}{Q_N} + \omega_{0N}^2 \right)(x_N - x_1) = 0
\end{aligned}
$$

$$-\omega^2 x_2 + \left(j\omega \frac{\omega_{02}}{Q_2} + \omega_{02}^2 \right)(x_2 - x_1) = 0$$

$$-\omega^2 x_3 + \left(j\omega \frac{\omega_{03}}{Q_3} + \omega_{03}^2 \right)(x_3 - x_1) = 0$$

$$\vdots$$

$$-\omega^2 x_N + \left(j\omega \frac{\omega_{0N}}{Q_N} + \omega_{0N}^2 \right)(x_N - x_1) = 0$$

Michael.B.VanDyke@jpl.nasa.gov
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General Transfer Function Equation for N DOF

- Let transfer function $z_n = \dfrac{x_n}{x_0}$ and let $a_n(\omega) = j\omega\dfrac{\omega_{0n}}{Q_n} + \omega_{0n}^2$; then express in matrix form:

$$
\begin{bmatrix}
-\omega^2 + a_1(\omega) + \sum_{2}^{N}\frac{m_n}{m_1}a_n(\omega) & -\frac{m_2}{m_1}a_2(\omega) & -\frac{m_3}{m_1}a_3(\omega) & \cdots & -\frac{m_N}{m_1}a_N(\omega) \\
-a_2(\omega) & -\omega^2 + a_2(\omega) & 0 & \cdots & 0 \\
-a_3(\omega) & 0 & -\omega^2 + a_3(\omega) & \cdots & 0 \\
\vdots & 0 & 0 & \ddots & \vdots \\
-a_N(\omega) & 0 & 0 & \cdots & -\omega^2 + a_N(\omega)
\end{bmatrix}
\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ \vdots \\ z_N \end{bmatrix}
=
\begin{bmatrix} a_1(\omega) \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}
$$

- Solve for z_n to get element response complex transfer functions relative to input

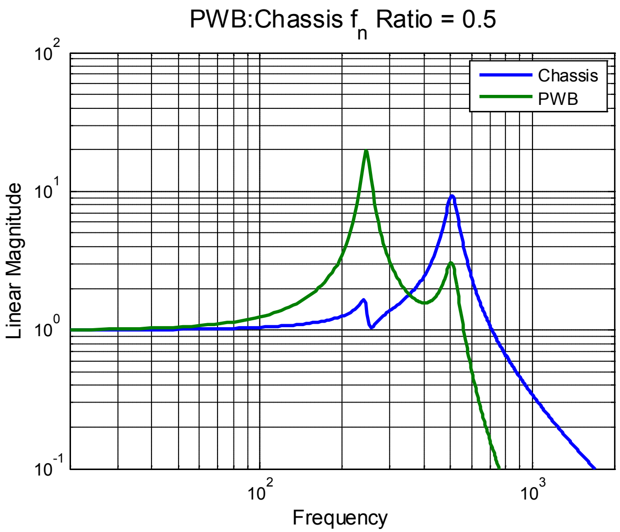
2 DOF Parametric Studies

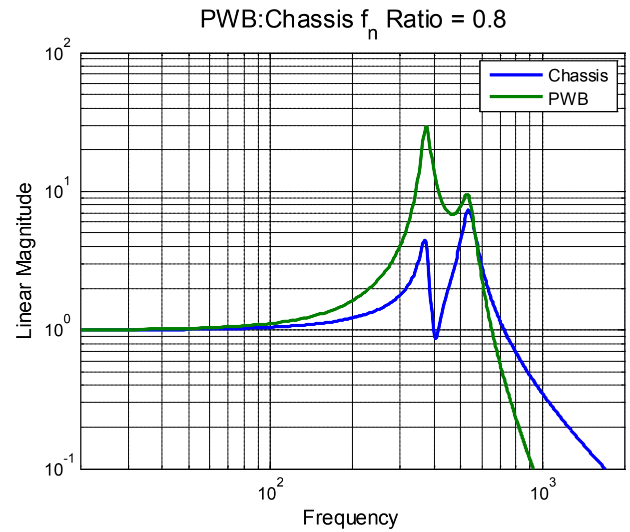
Illustrate influence of model parameters

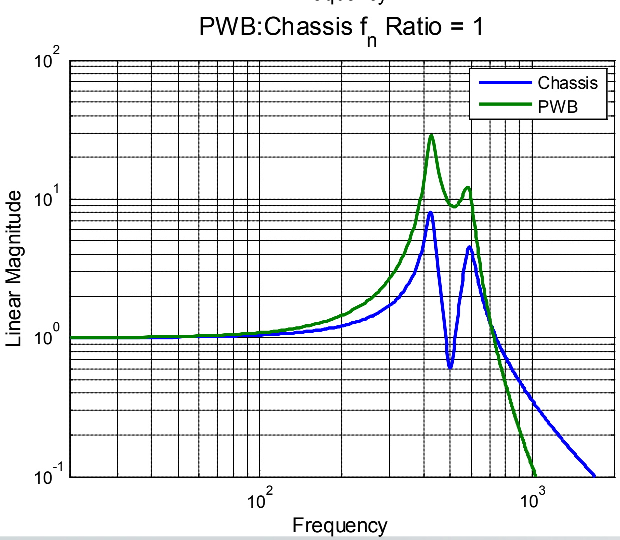
- Chassis + 1 PWB
- PWB response RMS vs PWB:chassis natural frequency ratio
  - *For unit/PWB weight ratios of 10 and 30*
- Coupled response over range of PWB Q

PWB Response vs. PWB:Chassis fn Ratio

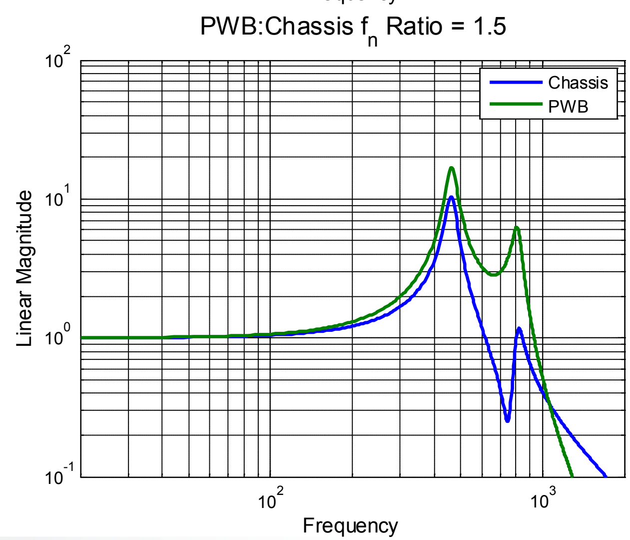
Case 1: Relatively small unit - PWB:mass ratio = 10







PWB Q = 25
Chassis Q = 10



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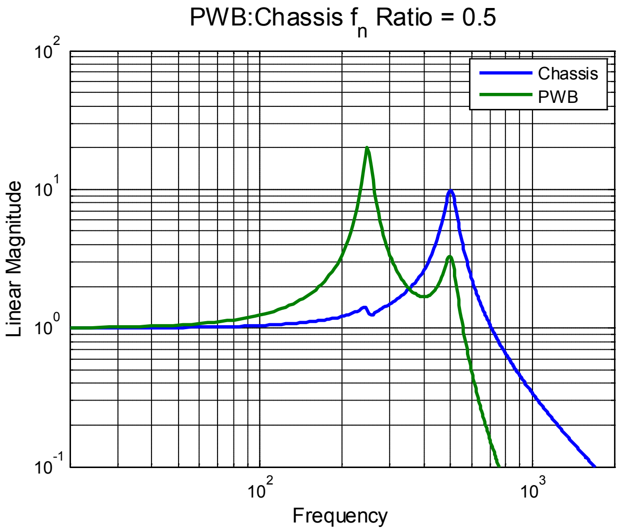
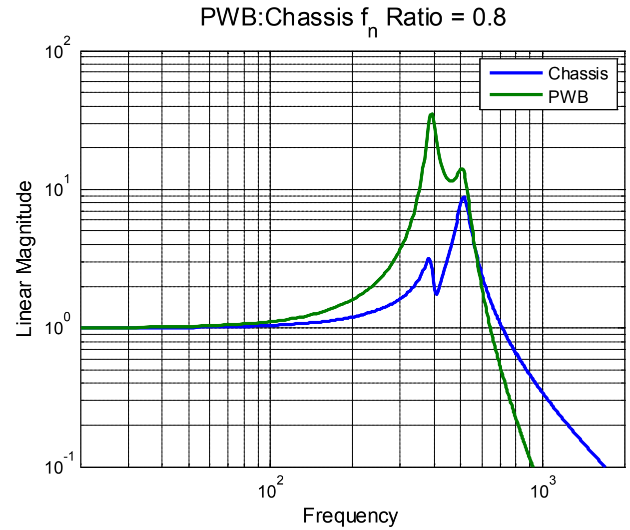
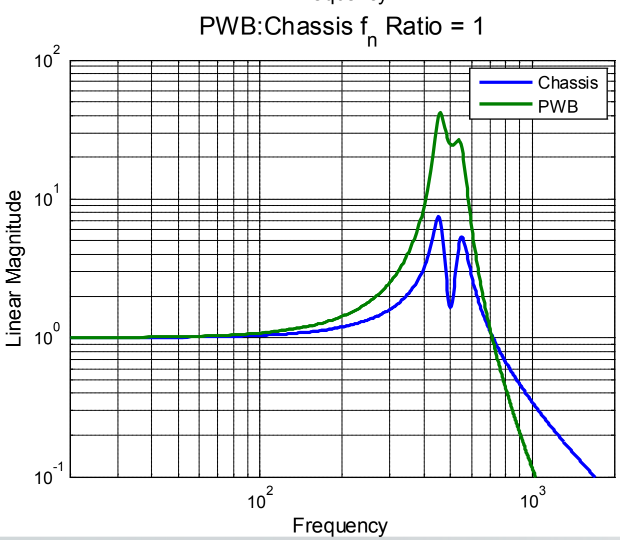
PWB Response vs. PWB:Chassis fn Ratio

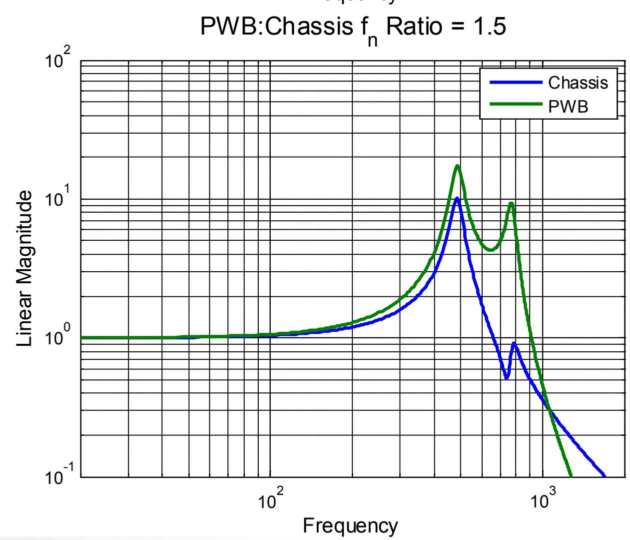
Case 2: Medium-sized unit - PWB:mass ratio = 30







PWB Q = 25
Chassis Q = 10



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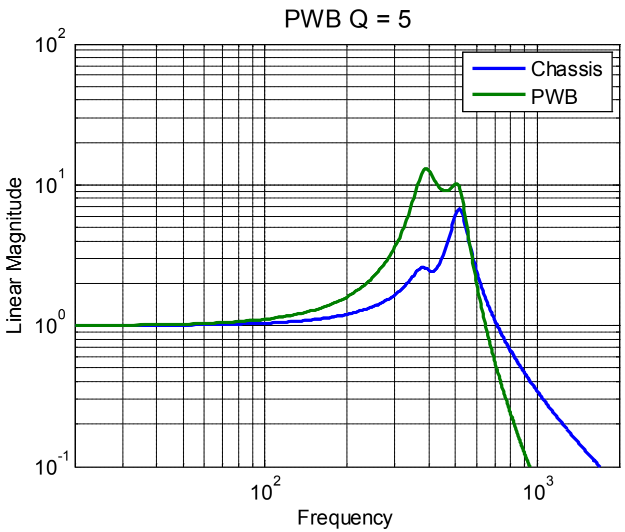
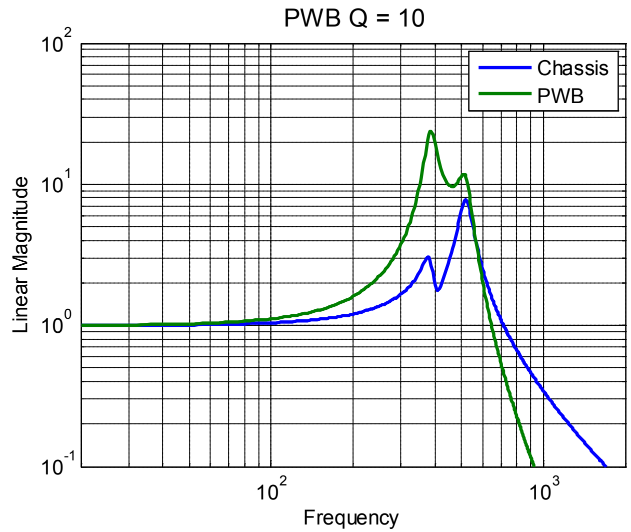
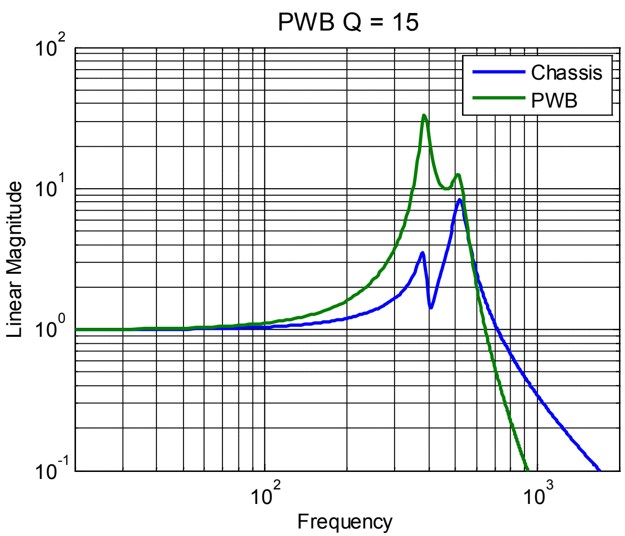
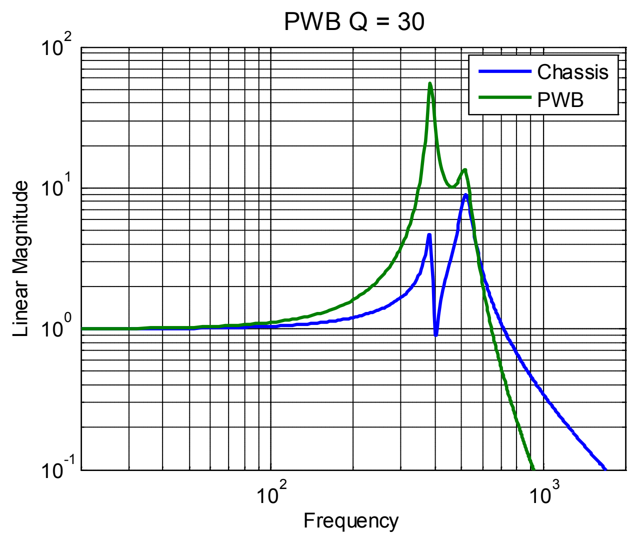
PWB Response vs. PWB Q



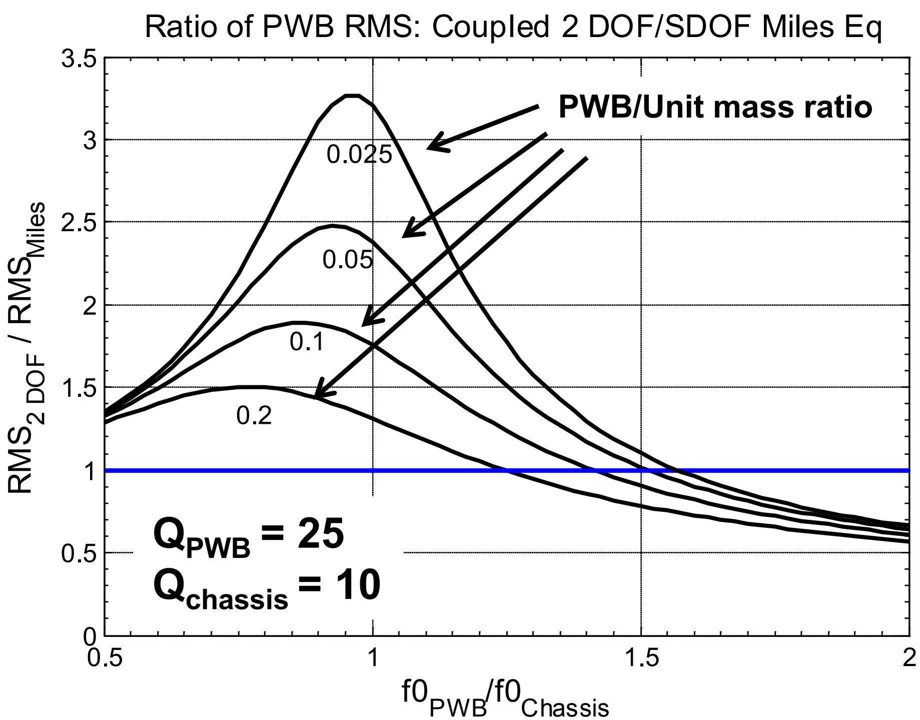


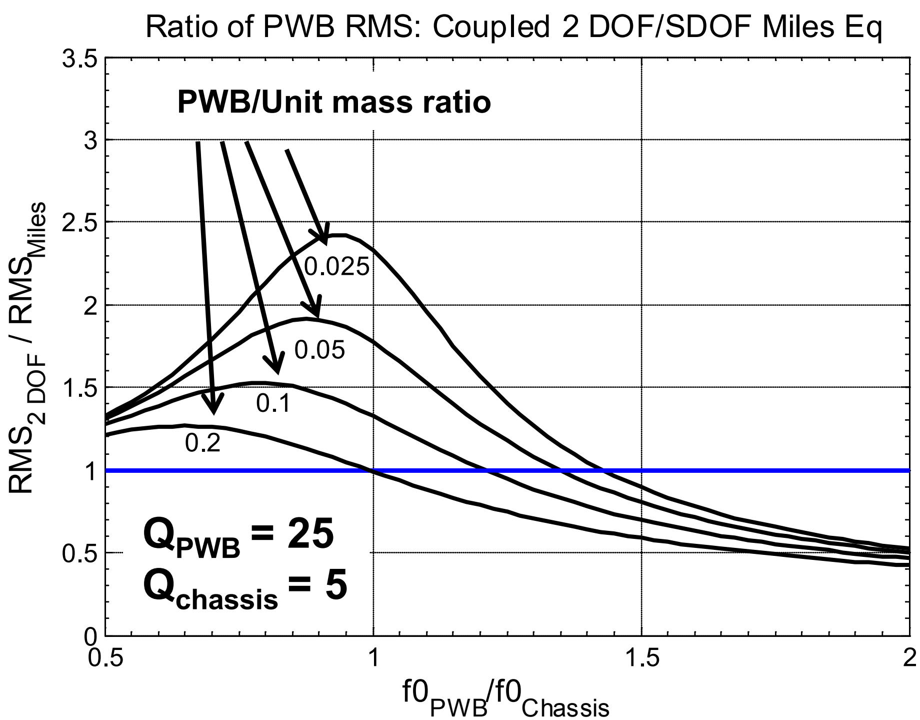




Chassis Q = 10

RMS: Coupled PWB Response Based on 2 DOF Model



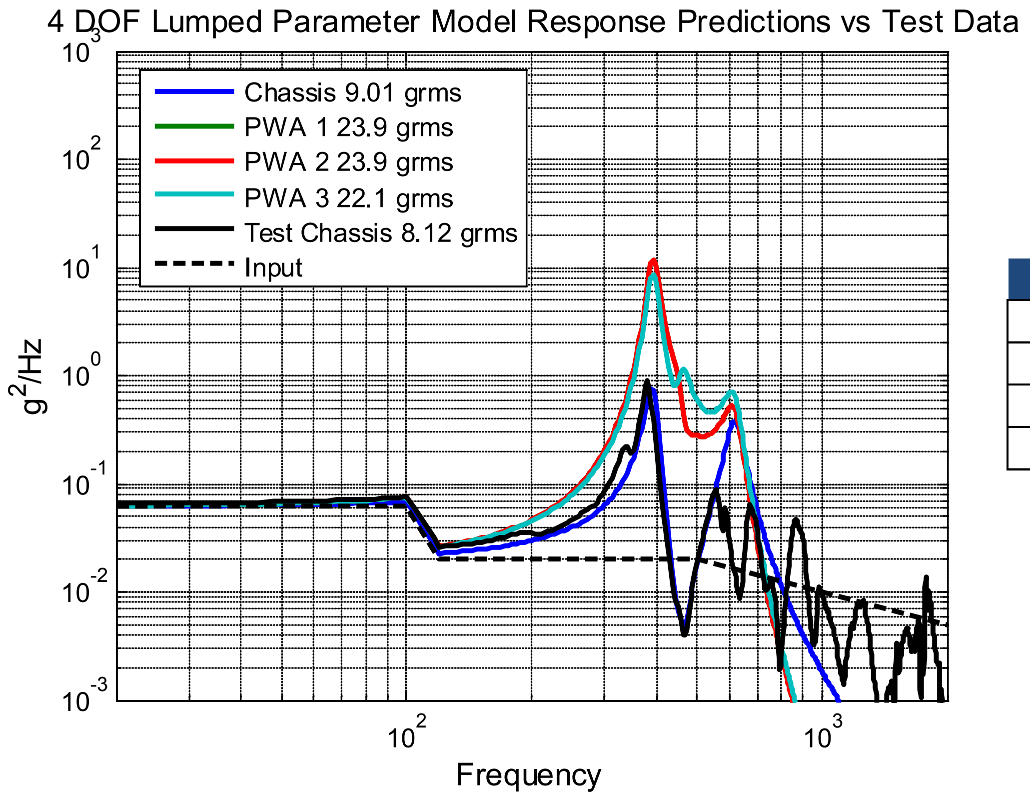


- PWB – chassis coupling is an important consideration in unit design
  - *Coupled relative RMS response of first bending mode (whether g's or mils) can be significantly greater than a simple SDOF Miles Equation would predict*
  - *Degree of amplification dependent on relative mass ratio, f0 ratio, Q*

Multi-DOF Approximation of Electronic Unit RV Response

- Premise:
  - *chassis response can be used as observable of internal (PWB) responses*
- Approach:
  - *Use multi-DOF lumped parameter model to approximate test-measured chassis response*
    - Chassis – primary mass
    - PWBs (or other internal dynamic elements) – secondary masses
  - *Secondary masses tuned for model approximation of chassis external response predict fundamental modal responses of internal elements*
- Prior confidential study showed excellent comparison of 6 DOF approximation with measured chassis and PWB responses
- Demonstration
  - *External chassis response from 28 lb unit approximated with 7 DOF model*
  - *No* a priori *knowledge of unit internal configuration*
    - Plans to compare model prediction with unit finite element model (FEM)

4 DOF Approximation of Chassis Test Measurement

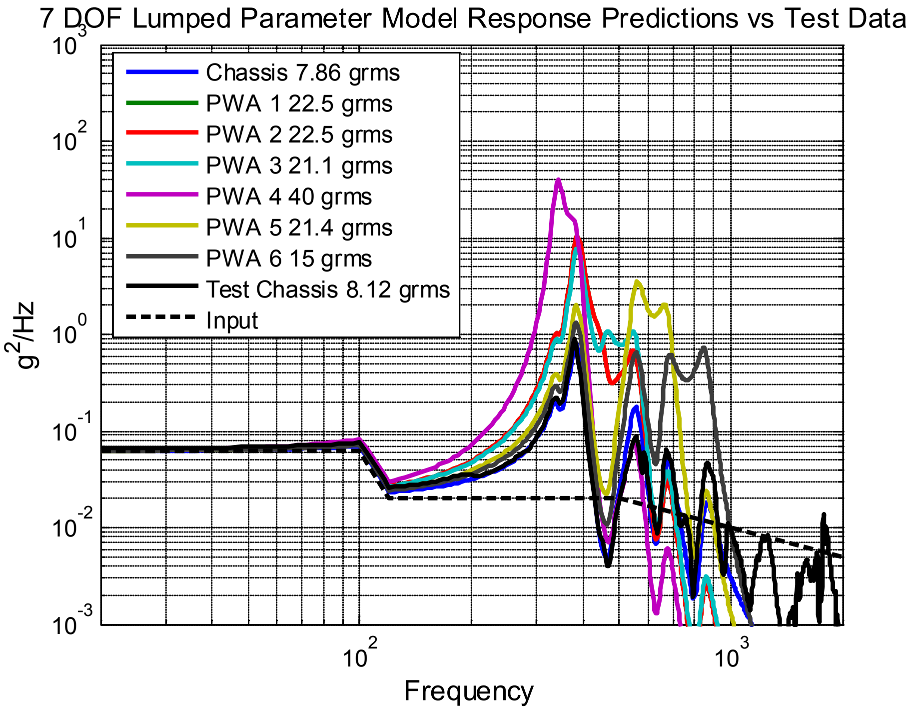


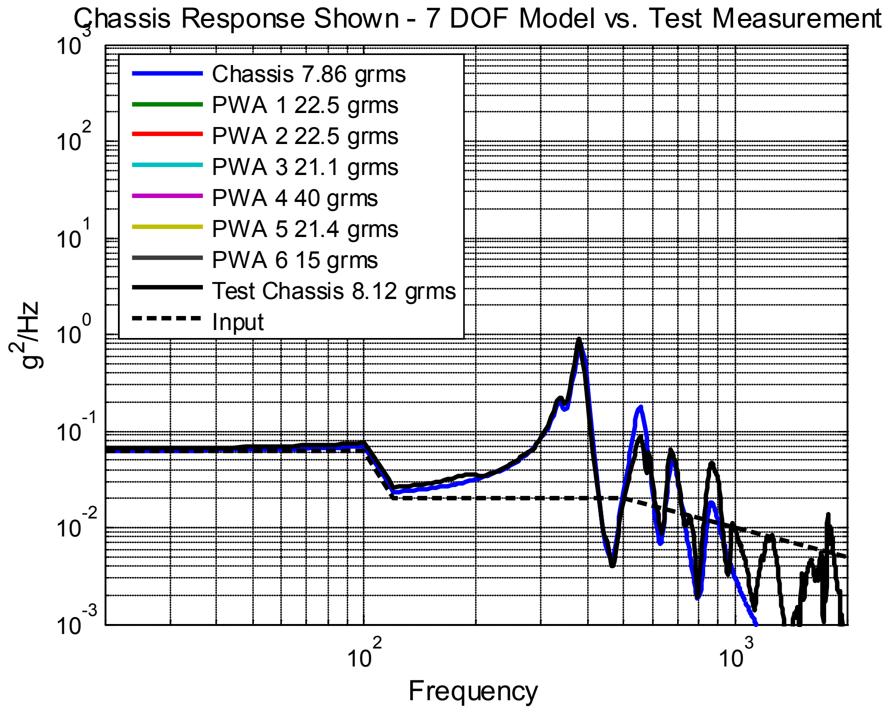
Model parameters

Element	Weight (lb)	Mass Partic.	f0 (Hz)	Q
Chassis	*	0.28	530	10
"PWA 1"	1.5	0.3	450	12
"PWA 2"	1.5	0.3	450	12
"PWA 3"	2	0.35	465	15

- Test external chassis response (black line) measured at top corner of chassis
  - *Low level test run used for model correlation*
  - *Test data normalized to nominal input spectrum to eliminate spectral noise*
- Predicted random vibe response = $|\text{transfer function}|^2$ x Input PSD (interpolated)

7 DOF Approximation of Chassis Test Measurement





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"PWA 2"	1.5	0.3	450	12
"PWA 3"	2	0.35	465	15
"PWA 4"	0.4	0.3	345	15
"PWA 5"	1.4	0.3	630	15
"PWA 6"	1.5	0.3	800	15

Further Work

- Additional correlation of approach with test data and finite element models
- Incorporate test force measurements along with chassis response as test data observables of internal dynamics
- Explore extension to transient response for application to shock test response predictions
  - *Limited to spectral range dominated by structural response (typically <1000Hz)*

Acknowledgements

- Ben Tsoi, JPL
  - *Test conductor for electronic unit used in demonstration*
- David C. Sandkulla, The Aerospace Corporation
  - *For collaboration in adapting the lumped element approach to modeling electronic box dynamics*

Thank you