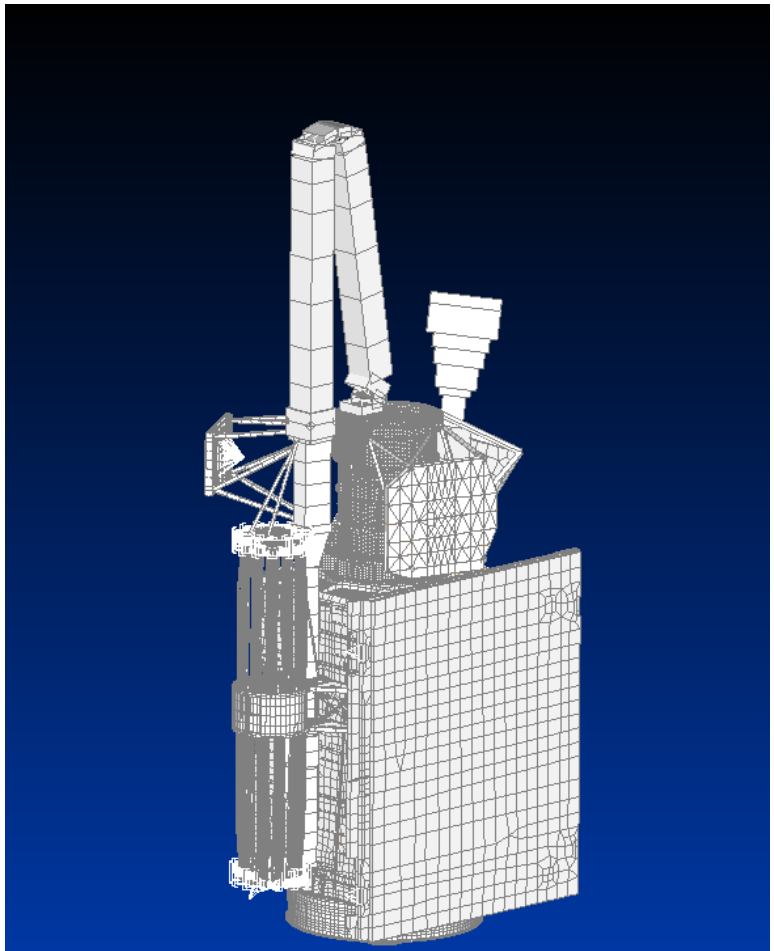
An Energy Method for Determining the Most Dynamically Responsive Axis of a Structure

Walter S. Tsuha
Jet Propulsion Laboratory
California Institute of Technology

Mechanical Systems Engineering Division
Spacecraft Structures and Dynamics Group
June 4 – 6, 2013

© The Aerospace Corporation 2012

Introduction

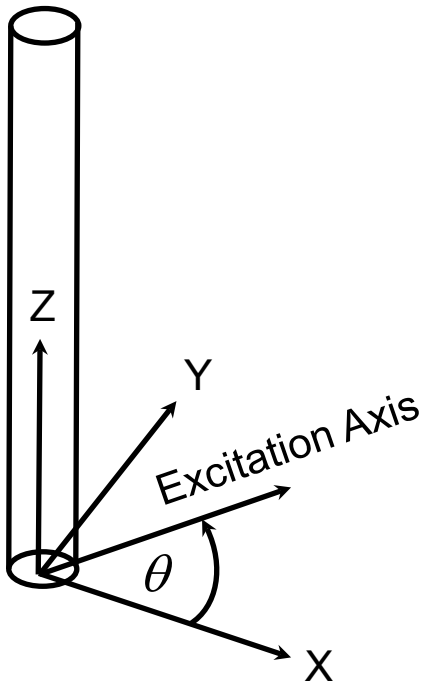


SMAP Observatory FEM

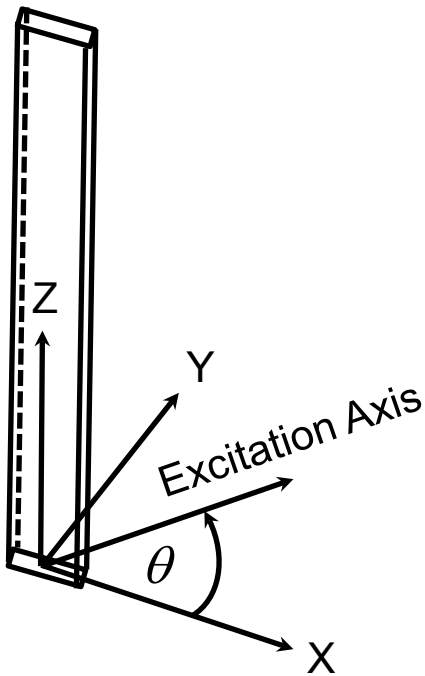
- **Problem Statement**
  - *Given a complex spacecraft system, how does one identify the most responsive axis to a base sine input?*
- **Problem Considerations**
  - *Typically, many (100s) response measurements are taken during a sine test.*
  - *How does one combine the response measurements into a single scalar that quantifies the amount of response for a given axis?*
- The work presented herein is a first step for addressing this problem.

AEROSPACE

Intuitive for Simple Structures



For an axisymmetric structure, all lateral directions are equally responsive.



For the beam shown above, it appears that the y-axis is the most responsive axis. Later, we'll show that this is indeed the case.

AEROSPACE

Overview of Approach

- In this study, energy related quantities are utilized to identify the most responsive axis to a base sine input.
- Two energy norms considered
  - *Kinetic Energy*
  - *Total Work*
- Approach
  - *Drive system with base sine input in several lateral directions*
  - *Compute*
    - Kinetic energy of system
    - Total work done by shaker forces
  - *Axis that has the largest kinetic energy or total work is considered the most responsive (or energetic) axis.*

AEROSPACE

Kinetic Energy (1 of 2)

- Kinetic Energy (i^{th} Mass at Clock Angle k)

$$\left(KE_i(f)\right)_k = \frac{1}{2} m_i \left(v_{ix}^2(f) + v_{iy}^2(f) + v_{iz}^2(f)\right)_k$$

where $v_{ix}(f),\ v_{iy}(f),\ v_{iz}(f)$ are the x, y & z velocity FRFs

- Average Kinetic Energy (i^{th} Mass at Clock Angle k)

$$\left(KE_i\right)_{avg,k} = \frac{1}{2} m_i \left[\frac{1}{(f_2 - f_1)} \int_{f_1}^{f_2} \left(v_{ix}^2(f) + v_{iy}^2(f) + v_{iz}^2(f)\right)_k df\right]$$

- Total Kinetic Energy (Clock Angle k)

$$\left(KE_{total}\right)_{avg,k} \sum_{i=1}^{n} \left(KE_i\right)_{avg,k}\ ,\quad \text{where } n \text{ is the number of nodes considered}$$

AEROSPACE

Kinetic Energy (2 of 2)

- Normalized Total Kinetic Energy (Clock Angle k)

$$\left(KE_{total}\right)_{avg,\,k,\,normalized} = \frac{\left(KE_{total}\right)_{avg,\,k}}{\left(KE_{total}\right)_{avg,\,0\,\mathrm{deg}}}$$

- Normalized Nodal Kinetic Energy (Clock Angle k)

$$\left(KE_{i}\right)_{avg,\,k,\,normalized} = \frac{\left(KE_{i}\right)_{avg,\,k}}{\max\limits_{1\le i\le n,\,0\le k\le \theta_{\max}} \left(KE_{i}\right)_{avg,\,k}}$$

AEROSPACE

Work Done by Shaker Drive Forces

- Work Done by Shaker (Clock Angle k)

$$\left(W_s(f)\right)_k = \left|F_s^T(f)D_s(f)\right|_k$$

where,

$F_s(f)$ is the 6×1 FRF of shaker drive forces and moments

$D_s(f)$ is the 6×1 FRF of shaker displacements and rotations

for a prescribed shaker output acceleration profile

- Average Work Done by Shaker (Clock Angle k)

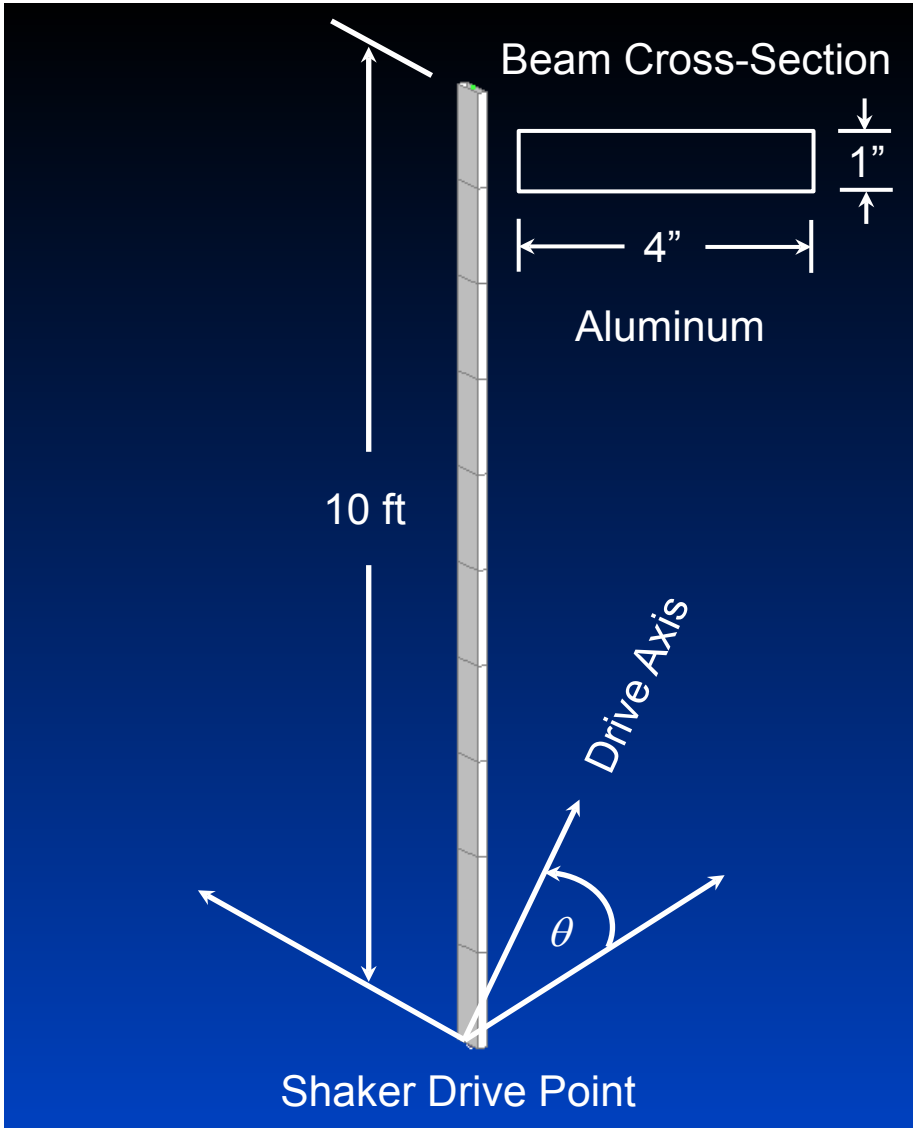
$$\left(W\right)_{avg,k} = \frac{1}{f_2 - f_2}\int_{f_1}^{f_2}\left(W_s(f)\right)_k df$$

- Normalized Shaker Work

$$\left(W_s\right)_{avg,k,normalized} = \frac{\left(W_s\right)_{avg,k}}{\left(W_s\right)_{avg,0\,\mathrm{deg}}}$$

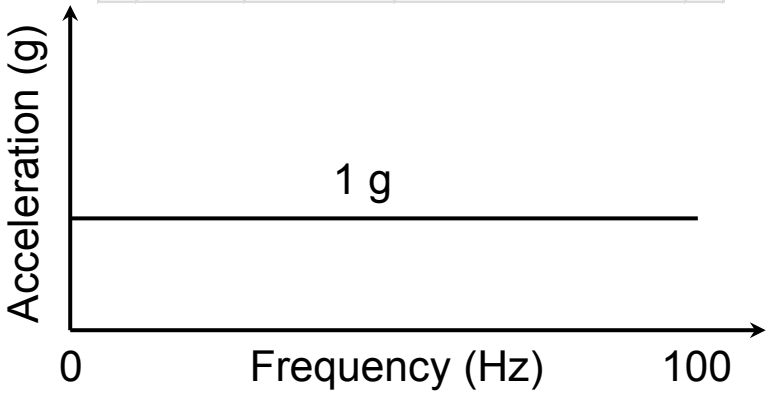
AEROSPACE

Beam Example (1 of 3)



Beam Modes Below 100 Hz

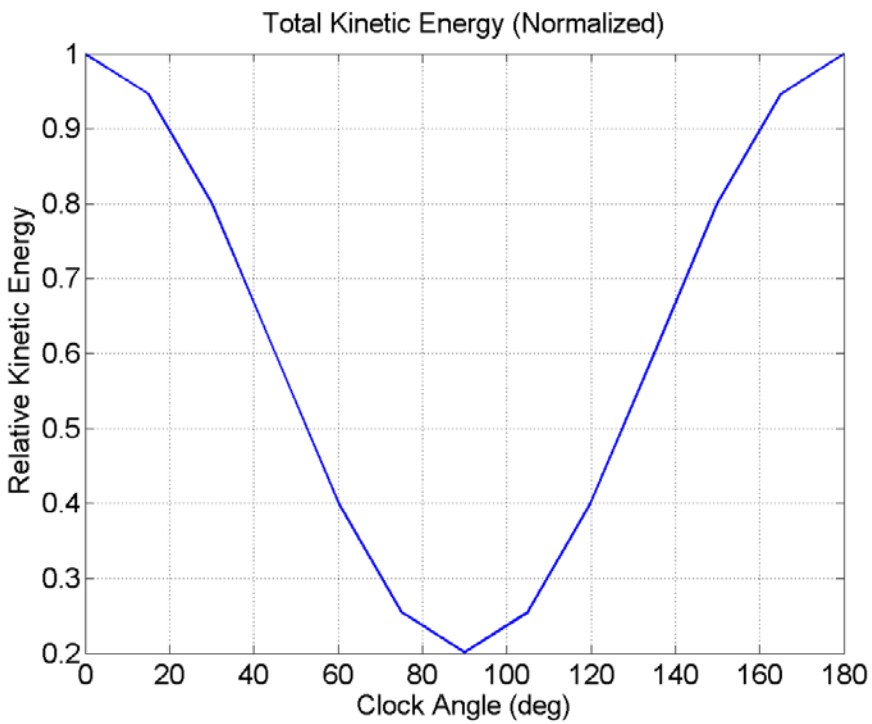
Mode	Freq (Hz)	Description
1	3.2	1st X-Bending
2	12.6	1st Y-Bending
3	19.6	2nd X-Bending
4	54.2	3rd X-Bending
5	77.8	2nd Y-Bending

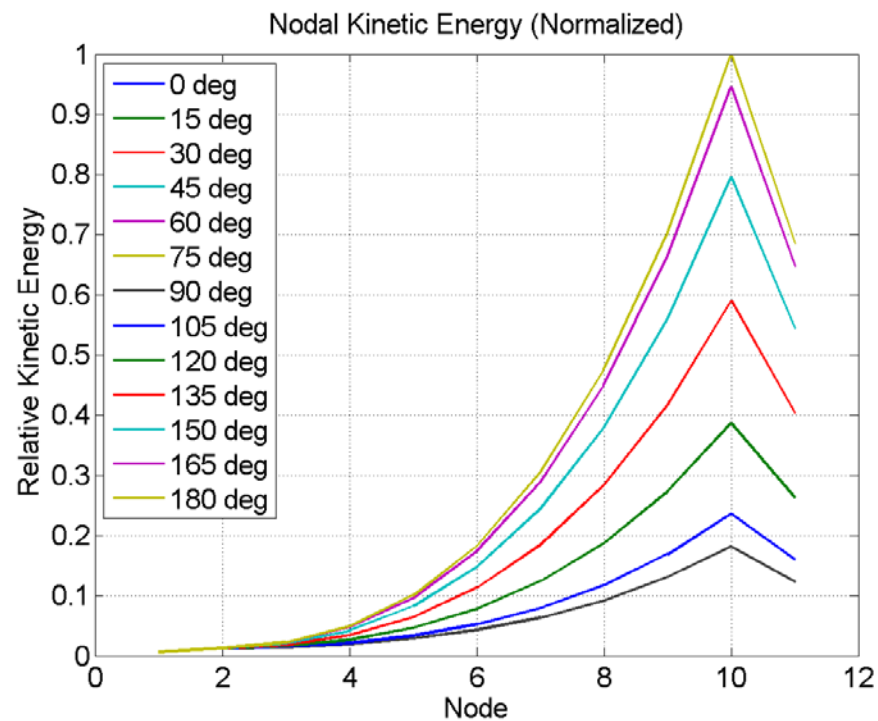


- Drive beam with lateral sine input at angle θ, from 0 to 180 degrees
- Compute kinetic energy and work for each driven axis
- Axis with largest kinetic energy or work is defined to be the most responsive axis.

AEROSPACE

Beam Example (2 of 3)





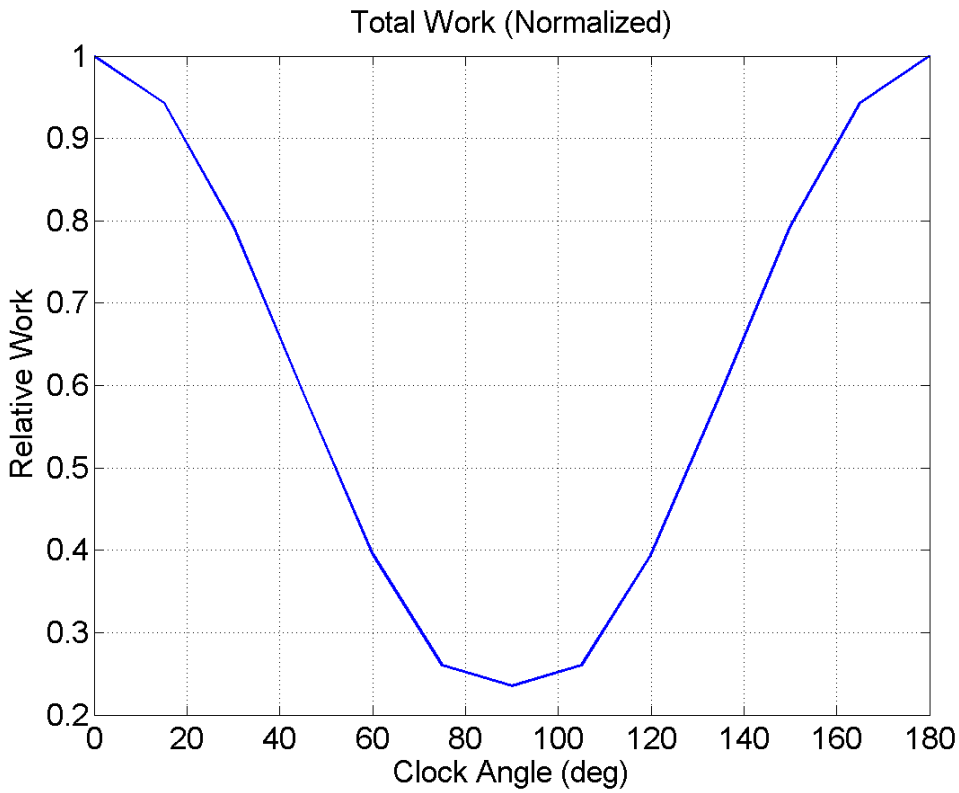
- Total kinetic energy plot indicates
  - *0 degree is most energetic axis*
  - *90 degree is least energetic axis*
  - *Consistent with intuition*

Reduced response for node 11 is because only half the mass is present

- Nodal kinetic energy plot provides relative energies at the nodal level
  - *Results are consistent with intuition and total kinetic energy results*

AEROSPACE

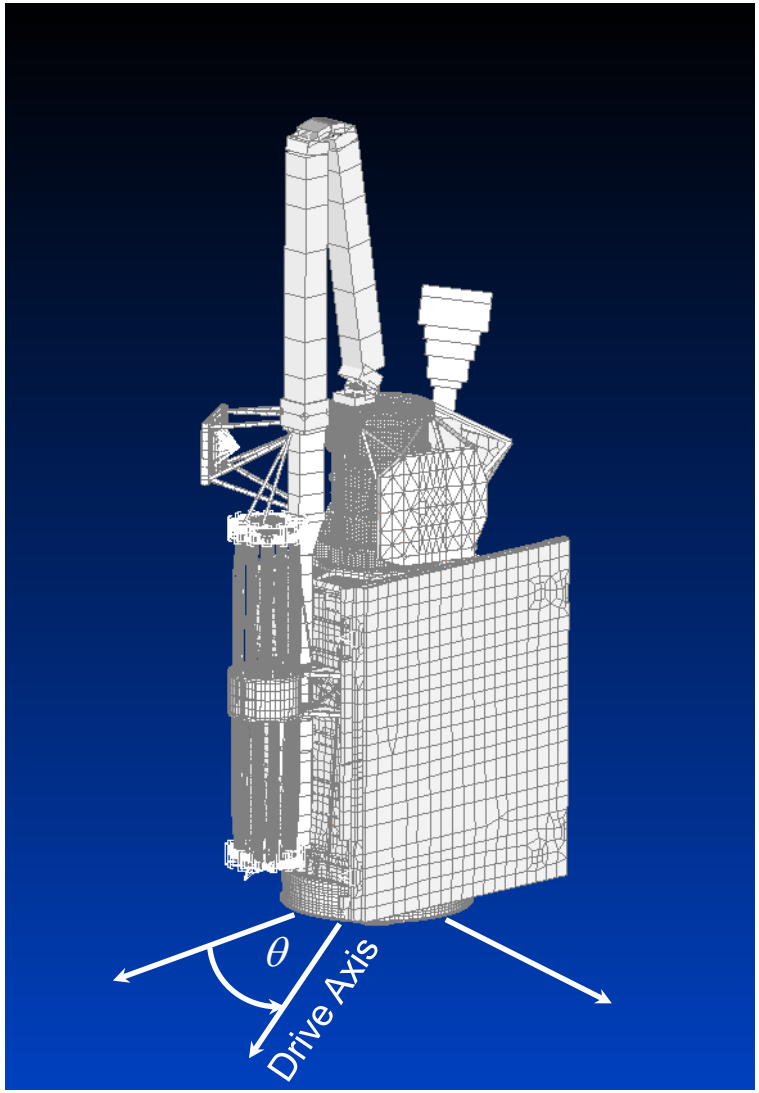
Beam Example (3 of 3)

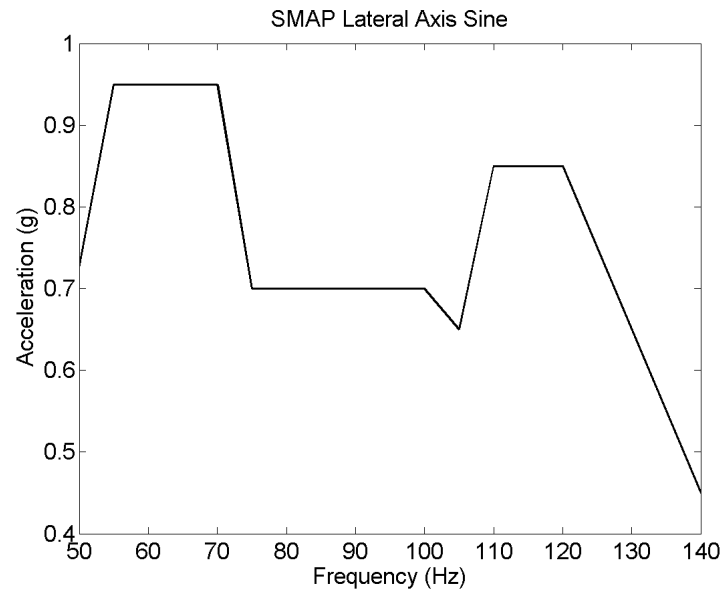


- Total work also shows that
  - *0 degree is the most responsive axis*
  - *90 degree is the least responsive axis*
  - *Consistent with kinetic energy results*
- Unlike kinetic energy, nodal level information is absent

AEROSPACE

Soil Moisture Active Passive (SMAP) Example (1 of 3)

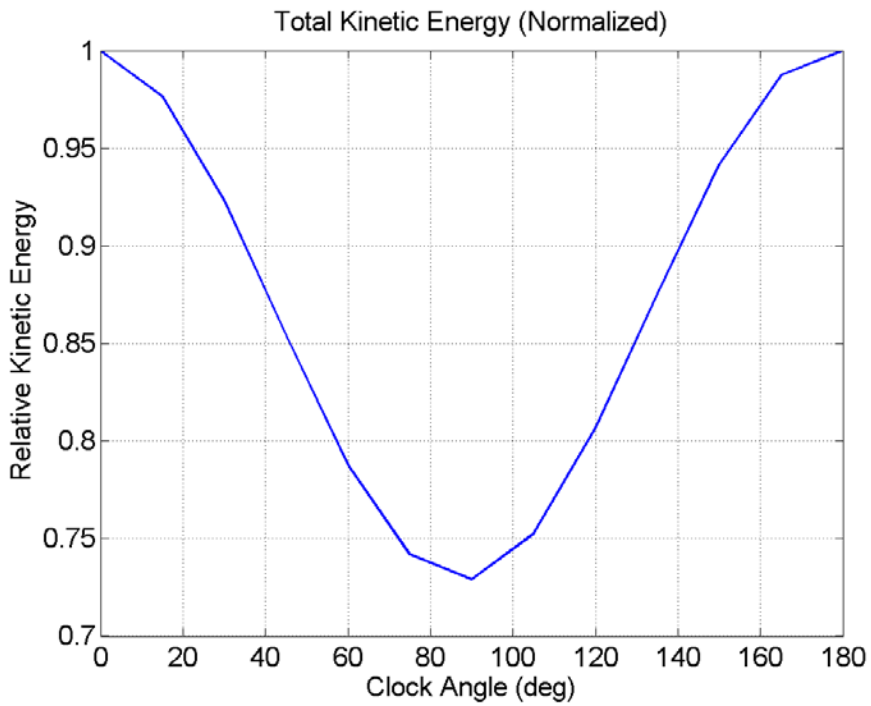


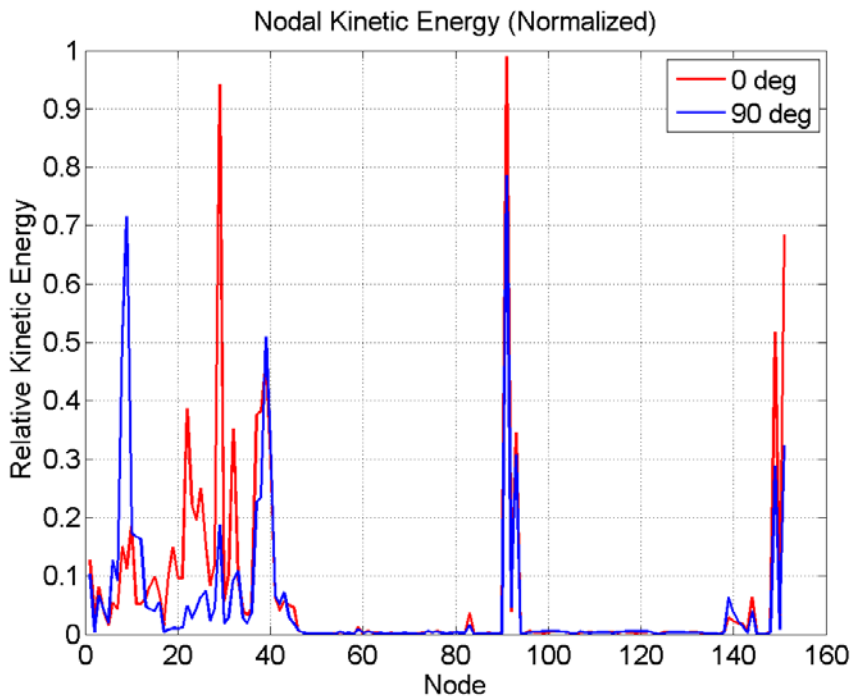


- Like the beam problem, the SMAP structure is driven, using the above sine profile, at angles ranging from 0 – 180 degrees.
- Both kinetic energy and work are computed for each driven axis.
- Axis having the largest kinetic energy or work is considered the most responsive axis.

AEROSPACE

SMAP Example (2 of 3)

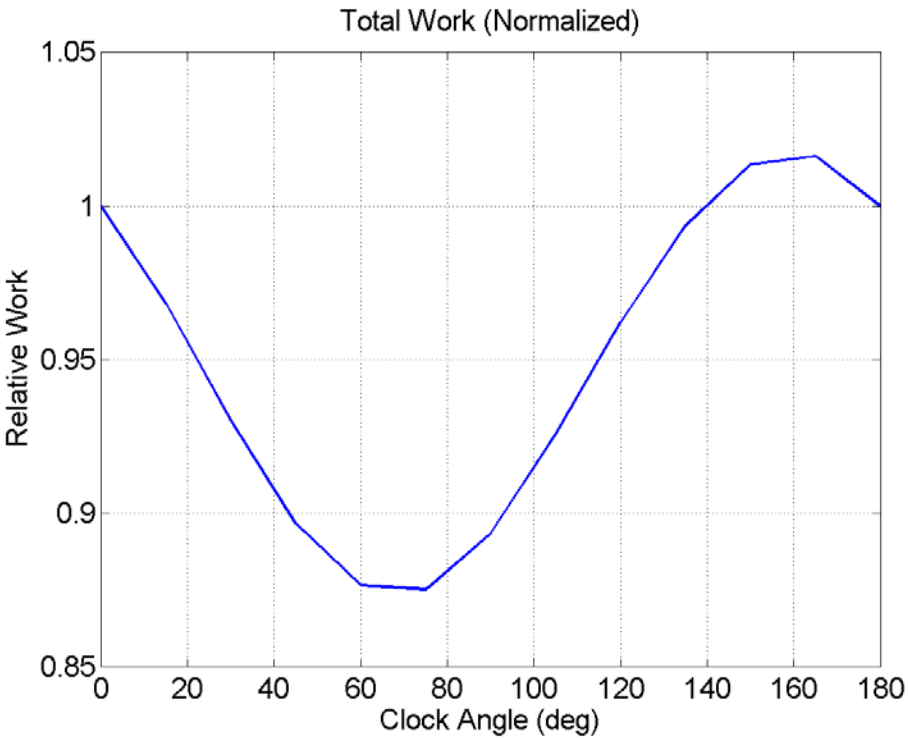




- Total kinetic energy shows
  - *Most responsive axis is at 0 degrees*
  - *Least response axis is at 90 degrees*
- Nodal kinetic energy plot compares kinetic energy at the nodal level
  - *Consistent with total kinetic energy results*

12

SMAP Example (3 of 3)



- Total work indicates
  - *Most responsive axis is at 165 degrees*
  - *Least responsive axis is at 75 degrees*
- Total work results differ from kinetic energy results by 15 degrees
- It is conjectured that difference is due to the following:
  - *Total work reflects total energy of the system*
  - *Kinetic energy only reflects total energy of the nodes selected*

13

Remarks

- **Kinetic Energy**
  - *Requires a set of nodes to be specified*
  - *Results are dependent on the nodes selected; i.e., the minimum and maximum response axes can change if the node set is altered.*
    - This may be desirable if only a select set of responses are to be considered in establishing the most responsive axis.
    - Total work does not have this flexibility
  - *Provides nodal level information that is not available for total work*
- **Total Work**
  - *Does not require a set of response nodes to be selected.*
    - Only shaker drive point FRFs are required
  - *It is conjectured that total work provides the total response of the structure, unlike kinetic energy, which is node set dependent.*
- **Kinetic Energy & Total Work**
  - *Results (i.e., minimum and maximum response axes) from both methods are dependent on the frequency range selected.*
  - *Both methods inherently accounts for the sine input profile.*

AEROSPACE

Summary & Conclusion

- Two methods are proposed for identifying the most responsive (or energetic) axis of a structure for a base sine input
  - *Kinetic Energy*
  - *Total Work*
- Both methods have their advantages and disadvantages, which were cited in the previous slide.
- Both methods appear to be viable in identifying the most responsive axis of a complex structural system.

AEROSPACE

Thank you

© The Aerospace Corporation 2012