Abstract

This work concentrates the modeling efforts presented in last year’s VSGC conference paper, “Model Development for Cable-Harnessed Beams.” The focus is narrowed to modeling of space-flight cables only, as a reliable damped cable model is not yet readily available and is necessary to continue modeling cable-harnessed space structures. New experimental data is presented, eliminating the low-frequency noise that plagued the first year’s efforts. The distributed transfer function method is applied to a single section of space flight cable for Euler-Bernoulli and shear beams. The work presented here will be developed into a damped cable model that can be incorporated into an interconnected beam-cable system. The overall goal of this work is to accurately predict natural frequencies and modal damping ratios for cabled space structures.

Background

Sending mass into space is expensive, so there has been great incentive to develop strong and lightweight materials for use in constructing satellites and other space structures. While materials science has developed lighter materials, the space-flight cables that are used have seen virtually no mass reduction. In addition, the ever-increasing sophistication of space technology means that the power and signal requirements have not diminished, and just as many, if not more, cables are required for each structure. Therefore, cable mass is making up a much larger percentage of the entire structure, typically 4-15% and as high as 30% in some cases (Babuska, Ardelean, Robertson and Spak, 2010). Another source finds cables comprising 4 to 20 percent of the total structure’s mass (Goodding, Babuska, Griffith, Ingram, & Robertson, 2007), and the author has found 10% to be the typical design guideline at a space flight facility.

Precision control of space structures is affected by the dynamic response of the structure, which is affected in turn by the mass and mass distribution of the structure. In addition, the natural frequencies and damping effects of the structure are important considerations when determining whether the structure can survive the extreme vibrations of launch.

In the past, cables were treated as a lumped mass; the total mass of the cable was calculated and added as a lump at the center of gravity of the structure. Research conducted by the Air Force Research Laboratory Space Vehicles Directorate has shown that this is no longer adequate, and that wiring harnesses should be included as structural mass. Models for space structures should be as accurate as possible and include cables as structural, rather than lumped, mass to determine whether the structure will fail and how it will respond to controls or movement instructions.

The long-term goal of this research is to produce models that can accurately predict the dynamic response of space structures, including the effects of damping or interaction due to cables and wiring harnesses on the structure. At this time, there is not yet a reliable damped cable model that can accurately predict the natural frequencies and damping characteristics of space flight cables. Thus, an important step in this research is to develop a model that can accurately describe the dynamic behavior of space flight cables.
of cable-harnessed beams and models of the
cabled-beam system (Spak & Inman, 2012),
but it was soon evident that a model for the
cable specifically would be required that could
include the cable's inherent damping effects
and unique non-homogenous and non-
isotropic properties.

Modeling

After investigating different types of
cable modeling, the author concluded that a
beam model would be the most appropriate
choice, combining ease of calculation of the
dynamic response with accuracy if the cable
properties could be determined carefully.
Although the beam model assumes a
homogenous and isotropic material, by
“smearing” the cable properties over the cable
area, it should be possible to determine
equivalent cable properties that can give
reasonable results. Regardless of the type of
model used, a method to include the internal
damping inherent in the cable was also
necessary. The authors hypothesized that
hysteretic damping, in which the energy lost is
due to the motion between molecules within
the structure, would be an appropriate way to
model the internal damping of the cable. The
hysteretic damping added computational time
to the model solutions, but not as much time
as the increase in matrix entries due to the
inclusion of shear and rotational inertia
effects.

Both the cable and beam were
modeled as Euler-Bernoulli beams initially,
although research from AFOSR indicates that
the cable is modeled more accurately as a
shear beam (Babuska, Ardelean, Robertson, &
Lane, 2010). Based on this research and the
flexibility of the cables, a shear beam model
and Timoshenko model (including both shear
and rotary inertial) were created. The
Timoshenko model was computationally
intense once damping terms were included
and was rejected in favor of the shear model,
which still takes shear effects into account but
contains fewer entries in the transfer function
matrix. Results from the Timoshenko model
with hysteretic damping showed that the
hysteretic damping model would require more
dissipation coordinates to effectively describe
the physical damping present (Spak, Agnes &
Inman, 2013).

Distributed Transfer Function Method

The distributed transfer function
method yields an exact solution, and its only
computationally intense step is the
determination of the exponential matrix
required for the eigenvalue calculation of

\[
\det(M(s) + N(s) * e^{F(s)})^{-1} = 0
\]

where M(s) and N(s) are matrices that
represent the boundary conditions and F(s) is a
matrix based on the Laplace transform of the
equations of motion (Yang, 2005, Yang &
Tan, 1994). For the system of a single bare
cable with pinned-pinned end conditions
modeled as an Euler-Bernoulli beam or shear
beam with applied axial tension T,

\[
M = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & EI & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
N = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & EI & 0
\end{bmatrix}
\]

\[
F_{EulerBernoulli} = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\rho A s^2 & T & 0 & EI
\end{bmatrix}
\]

\[
\frac{\rho A s^2}{EI} & 0 & T & 0
\]
The transfer function matrix $F(s)$ changes depending on the equation of motion, with the more complex shear and Timoshenko models having more entries, and damping and tension terms adding complexity to those entries. This model can also be extended easily to interconnected systems (Yang, 1994), lending support to its use for cable-harnessed structures.

The beam parameters used in the three models include density ($\rho$), area ($A$), modulus of elasticity ($E$), moment of inertia ($I$) and modulus of rigidity ($G$). Each of these could be measured or calculated using the rule of mixtures to describe the copper wire in a matrix of the jacketing material, but the rule of mixtures gives an upper and lower bound, not an exact value. These calculations were performed and used and gave a range of frequencies that bounded the experimental data collected. Thus, it was decided that using a statistical approach on the cable data to determine the cable parameters would likely lead to a better understanding of the cable properties. The interaction between the twisted wires, shielding material, jacketing and insulation material, and cable layers could not be easily described by a simple equation.

Experimentation

Since little research has been done on space flight cables specifically, it was important to set up the test fixture in a way that would both mimic the model and reality. Cable ties and TC105 tabs were used to attach the cables to the test fixture and the excitation method to the cable. This is the same attachment method used on many spacecraft for cable management.

A load cell was attached to the cable to measure the input force from the modal shaker and a laser vibrometer was set up opposite the shaker to measure the dynamic response of the cable. Figure 1 shows the test set up. The shaker is suspended to prevent vibration from traveling through the inertial table, and the cable is attached to the test fixture with TC105 tabs and cable ties. The shaker connects to the cable via a tensioned string that ends in a screw end that connects to the load cell attached to the cable.

Figure 1. Test set up for space flight cable test.
types of cable ties fastened to either tight or loose configurations.

The first round of testing used a single section of helically twisted 1 by 18 cable made of M27500 wire, tie-laced every 4-6” and overwrapped with Kapton tape, as is typical for space flight cables. Figure 2 shows the cable used for experimental testing.

![Figure 2. 1 by 18 space flight cable used for experimental testing.](image)

To ensure comparability as different cables were tested, a “standard run” was developed based on observations from the initial experiments. The standard run was a cable length of 0.254 m with two buffer zones of 0.203 m on either side to reduce end effects, all secured with tight cable ties and TC105 tabs. The cable tension was 8.9 N, and tight TyRap cable ties were used. White noise excited the cable via a tensioned string attached to the shaker at 0.24 m away at an amplitude of 0.3 volts. A low-pass filter at 5kHz was applied and the frequency range of interest was 0 to 250 Hz.

The first section of cable was tested in a standard run 14 times, removed from the test fixture and reattached each time. Additional sections from the same cable were tested and compared as well. Full laser scans of the cable test sections were conducted on each day of testing so that the mode shapes could be identified and visualized.

Results

The initial cable testing showed clear results for what factors needed to be controlled. Cable tension, zip tie tightness, cable orientation in the test fixture, and differences in the cable sections all had large effects on the response, while excitation method, excitation string length and tension, and zip tie type had little to no effect.

The cables tested had a first natural frequency around 55 Hz and a second natural frequency around 175 Hz, with damping ratios of 2.94% and 3.18% respectively. Figure 3 shows the first mode shape from a full test section scan; it is clear that the cable is forming an arc typical of the first mode shape.

![Figure 3. First mode shape of cable test section; node at center and anti-nodes at ends.](image)

Figure 4 shows the frequency response for 14 standard test runs of a single section of 1 by 18 cable. The first natural transverse frequency, indicated by the largest peak around 55 Hz, shows good agreement, while the second natural transverse frequency around 175 Hz shows more variation. Combination modes around 90 Hz that may include torsional and longitudinal vibration and mode interaction are much less
predictable, but also have much smaller amplitudes and are therefore less likely to cause structural failure due to large amplitude. Figure 5 shows the experimental data for multiple runs of five different sections of the same 1 by 18 cable. It is clear that there is some variation in the frequency response even for the same type of cable, which indicates that a statistical analysis could be a good approach.

Figure 4. Frequency response of multiple test runs of the same section of the same 1 by 18 cable.

Figure 5. Frequency response of five different sections of the same 1 by 18 cable.

The models were run with properties of solid copper, solid Tefzel jacketing material, and the calculated mixtures based on the fractional area of each component. The resulting frequencies did bound the experimental results. Undamped models were used as a baseline.

Table 1. Values used for undamped DTFM models yielding minimum frequency bounds, calculated using the rule of mixtures for copper wires and shielding in a matrix of Tefzel (ETFE)

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
<th>Units</th>
</tr>
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<tbody>
<tr>
<td>Density, $\rho$</td>
<td>2.547 E 3</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>Area, $A$</td>
<td>7.369 E -5</td>
<td>m$^2$</td>
</tr>
<tr>
<td>Modulus of Elasticity, $E$</td>
<td>6.972 E 8</td>
<td>Pa</td>
</tr>
<tr>
<td>Moment of Inertia, $I$</td>
<td>8.935 E -10</td>
<td>m$^4$</td>
</tr>
<tr>
<td>Modulus of Rigidity, $G$</td>
<td>1.205 E 8</td>
<td>Pa</td>
</tr>
<tr>
<td>Tension, $T$</td>
<td>8.89</td>
<td>N</td>
</tr>
<tr>
<td>Length, $L$</td>
<td>0.254</td>
<td>m</td>
</tr>
</tbody>
</table>

With values shown in Table 1, the first natural frequency from the Euler-Bernoulli beam DTFM model was 46.38 Hz, and the shear model calculated 41.99 Hz. By adjusting the values in Table 1 to the other end of the rule of mixtures spectrum, first frequency values of 81.8 Hz and 74.5 Hz were obtained for the Euler-Bernoulli and shear beam models, respectively. The shear beam model values of 41.99 Hz and 74.5 Hz certainly bound the experimentally determined first frequency of 55Hz. The second natural frequency was bounded by 171 and 303 Hz in the shear beam model, which also contains the experimentally determined second mode of 175 Hz. Table 2 summarizes the model results and compares the experimental frequencies. Torsional frequencies were possible at 80-90 Hz a location of activity in the frequency response function.
Table 2. Summary of model results and experimental comparison

<table>
<thead>
<tr>
<th></th>
<th>First Transverse Frequency</th>
<th>Second Transverse Frequency</th>
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<tbody>
<tr>
<td></td>
<td>Lower Bound</td>
<td>Upper Bound</td>
</tr>
<tr>
<td>E-B</td>
<td>46.83 Hz</td>
<td>81.8 Hz</td>
</tr>
<tr>
<td>Shear</td>
<td>41.99 Hz</td>
<td>74.5 Hz</td>
</tr>
<tr>
<td>Experimental</td>
<td>54.3 Hz</td>
<td></td>
</tr>
</tbody>
</table>

The calculation of the cable properties based on material assumptions and geometrical measurements is vague at best, so the bounding of the experimental data with the model results is a good indication that the property values are in the correct range. It is also noteworthy that the second natural frequency falls within the shear beam model range, but not the Euler-Bernoulli range, lending support to the use of the shear beam model. The experimental frequencies being on the low edge of the model range is significant; adding damping terms should further reduce the natural frequencies found by the models, keeping the experimental data well within the model range. The next step will be refining these property values to more closely match equivalent beam properties, and then adding damping terms to mimic the damping mechanisms that are physically acting on and in the cable. Ideally, experimental testing will validate the calculation methods used for the determination of the properties used to model the non-isotropic, non-homogenous cable as an isotropic, homogenous beam. A statistical analysis of the experimental data should be performed due to the variation in the data for different sections of the same cable.

Conclusions and Future Work

The distributed transfer function models incorporate shear effects and damping, and are able to bound the natural frequencies of the tested space flight cable accurately for upper and lower bounds of cable parameters calculated simply by measuring the cable and making some material assumptions. More research is necessary to incorporate the cable properties more exactly in a way that reflects the modeling of the cable as an isotropic and homogenous beam.

In addition, investigation of the inclusion of damping in a variety of forms (building off the work of Spak, Agnes & Inman, 2013) must continue, with a goal of determining how to incorporate damping to successfully reflect the physical damping characteristics. Once cable properties are determined and the damping mechanisms identified and incorporated into the models, a reasonable cable model will result which can be used in DTFM models for a cabled-beam system.

The author looks forward to incorporating these cable models into cable-harnessed beam models for further development.

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