(Preprint) AAS 13-XXX

EXPLICIT LOW-THRUST GUIDANCE FOR REFERENCE ORBIT TARGETING

Try Lam* **and Firdaus E. Udwadia**†

The problem of a low-thrust spacecraft controlled to a reference orbit is addressed in this paper. A simple and explicit low-thrust guidance scheme with constrained thrust magnitude is developed by combining the fundamental equations of motion for constrained systems from analytical dynamics with a Lyapunov-based method. Examples are given for a spacecraft controlled to a reference trajectory in the circular restricted three body problem.

EXTENDED ABSTRACT

The issue of controlling a spacecraft to a desired reference orbit has many applications. An objective may be to maintain a particular periodic orbit, for example, in an unstable Halo orbit. Another example may be to command back to a desired design reference trajectory after being perturbed or placed into an incorrect orbit. A multitude of methods have been presented for the low-thrust control of a spacecraft: some methods use feedback (linear or nonlinear) approaches (References 1, 2, 3, and 4), and others use gradient based numerical optimization techniques (References 5, 6, and 7). More recently, the use of evolutionary algorithms and hybrids approaches have been proposed (Reference 8 and 9). Many of the proposed approaches employ either linearized approximations of the dynamics or the controller, or have unstable tracking (or control) of the errors. Thus, for systems with large perturbations or un-modeled external forces, or for states far from the reference, many of these control approaches will not be adequate. This is especially true for systems where the dynamics are highly unstable (i.e., orbits around the L1 and L2 points in the circular restricted three body problem (CR3BP)), and constant control is required to maintain the spacecraft's trajectory.

In this paper, a closed form constraint-based method is developed that provides the control force necessary to track a spacecraft to a reference orbit using the fundamental equations of motion combined with a Lyapunov-based approach. In analytical dynamics, the constraint-based method using the fundamental equations of motion was originally used as a general approach of obtaining the equations of motion of simple mechanical systems with kinematic constraints (References 10 and 11). Here we adopt the method to find the control force recquired to meet a set of mission objectives, specifically, controlling the spacecraft to some reference orbit. In this paper we give an example of a spacecraft controlled to an arbitrary reference trajectory and to a periodic Halo orbit in the CR3BP. A mechanism for controlling the thrust magnitude to better model a "real"

*Jet Propulsion Laboratory, California Institute of Technology, M/S 264-820 , 4800 Oak Grove Dr., Pasadena, CA 91109, 818.354.6901, try.lam@jpl.nasa.gov.

†Professor, University of Southern California, Department of Aerospace and Mechanical Engineering, 430K Olin Hall, Los Angeles, CA 90089, 213.740.0495, fudwadia@usc.edu.

engine is also introduced, and a simple equation for the applied thrust direction to target a reference trajectory is revealed.

In developing the guidance law necessary to control a spacecraft to a reference orbit we formulate the control objectives as constraint(s). The behavior of the imposed constraint is one that will drive the position and velocity differences between the spacecraft and the target orbit to zero. That is, we want the spacecraft state q to approach a desired state q_t over time, i.e., $\lim_{t\to\infty} q(t) = q_t(t)$.

The selected constraint which will achieve the above goal is

$$\dot{V} = -\kappa V \tag{1}$$

or

$$\phi \equiv \dot{V} + \kappa V = 0 \tag{2}$$

where $\kappa > 0$ and V is a candidate Lyapunov function of the form

$$V = \frac{a_1}{2}\Delta\mathbf{x}^T\Delta\mathbf{x} + \frac{a_2}{2}\Delta\dot{\mathbf{x}}^T\Delta\dot{\mathbf{x}} + a_3\Delta\mathbf{x}^T\Delta\dot{\mathbf{x}} \tag{3}$$

or

$$V = \frac{1}{2}\begin{pmatrix}\Delta\mathbf{x} & \Delta\dot{\mathbf{x}}\end{pmatrix}\begin{pmatrix}a_1 & a_3\\ a_3 & a_2\end{pmatrix}\begin{pmatrix}\Delta\mathbf{x}\\ \Delta\dot{\mathbf{x}}\end{pmatrix} \tag{4}$$

where

$$\Delta\mathbf{x} = (x - x_t, \quad y - y_t, \quad z - z_t)^T \tag{5}$$

$$\Delta\dot{\mathbf{x}} = (\dot{x} - \dot{x}_t, \quad \dot{y} - \dot{y}_t, \quad \dot{z} - \dot{z}_t)^T \tag{6}$$

and x, y, and z is the position of the spacecraft and $(\ )_t$ is the position of the targeted reference orbit tracked at some time, t. From Eq. (1) it is noted that with $\kappa > 0$ the Lyapunov function, V, exponentially approaches zero as a function of time, where κ is the decay constant.

In addition, Eq. (3) is positive definite if $a_1 a_2 > a_3^2$. If this is satisfied and if $\kappa > 0$, then $\dot{V}$ in Eq. (1) is always negative definite, i.e., $\dot{V} < 0$, and thus, the function V has an asymptotically stable solution $\Delta\mathbf{x} = 0$ and $\Delta\dot{\mathbf{x}} = 0$. Furthermore, V is radially unbounded, i.e., $V \to \infty$ as $\Delta q \to \infty$, where q is $\Delta\mathbf{x}$ and/or $\Delta\dot{\mathbf{x}}$, thus, the proposed constraint is *globally* asymptotically stable to the reference target. Therefore, the constraint proposed in Eqs. (2) and (3) will drive the position and velocity differences between the spacecraft and its reference target trajectory to zero as time progresses asymptotically, even when the initial conditions are very far apart. This paper refers to this behavior as *stable* control, where the control will always take you to the reference target.

The explicit closed form control acceleration that will maintain a spacecraft to a reference orbit is

$$\mathbf{a^c} = \mathbf{A}^+\left(-a_1\Delta\mathbf{x}^T\Delta\dot{\mathbf{x}} - a_3\Delta\dot{\mathbf{x}}^T\Delta\dot{\mathbf{x}} - \mathbf{A}\left(\ddot{\mathbf{r}} - \ddot{\mathbf{r}}_\mathbf{t}\right) - \kappa V\right) \tag{7}$$

where $\mathbf{A}$ and b are

$$\mathbf{A} = a_3\Delta\mathbf{x}^T + a_2\Delta\dot{\mathbf{x}}^\mathbf{T} \tag{8}$$

$$b = -a_1\Delta\mathbf{x}^T\Delta\dot{\mathbf{x}} - a_3\Delta\dot{\mathbf{x}}^T\Delta\dot{\mathbf{x}} - \kappa V + \mathbf{A}\ddot{\mathbf{r}}_\mathbf{t} \tag{9}$$

and (9), V is the Lyapunov function described in Eq. (3). This simple explicit equation will provide the necessary control acceleration needed by the spacecraft to maintain or track a particular reference orbit. This equation does not discriminate on the dynamical environment (2-body problem, 3-body problem, n-body problem, etc.) and can track any reference orbit if the position, velocity, and acceleration of the target is given. In addition, this guidance scheme is globally asymptotically stable for $\kappa > 0$ and $a_1 a_2 > a_3^2$.

To better model a "real" thruster system on board a spacecraft we have implemented a simple limitation to the applied control force, where the applied control force is limited to two operational thrust levels: 0 (thrusters off) or a^c_{max} (thrusters on), where a^c_{max} is the maximum thrust acceleration capability of the thrusters. The implementation of this switching criteria is accomplished in two steps.

Step 1: Allowing the thrust level to be "on" if the magnitude of Eq. (7) is such that $a^c \geq \eta\, a_{max}$, where η is some constant. For example, if $\eta = 0$, then the thruster will be continuously "on". If $\eta = 1$, then the thruster will be "on" if the computed control acceleration such that $a^c \geq a_{max}$.

Step 2: For each integration step compute the thrust direction by computing a value for κ such that the magnitude $\mathbf{a^c}$ in Eq. (7) is equal to the acceleration magnitude produced by the engine, $a^c = a_{engine}$. Since $\mathbf{A}$ is a 1x3 matrix with linearly independent columns Eq. (7) can be written as

$$\mathbf{a^c} = \frac{\mathbf{A}^T\left(\mathbf{b} - \mathbf{A}\mathbf{a}\right)}{\mathbf{A}\mathbf{A}^T} \tag{10}$$

The value for κ can be explicitly computed using Eqs. (8), (9), and (10), and is found to be

$$\kappa = \frac{1}{V}\left(a_{engine}\sqrt{\mathbf{A}\mathbf{A}^T} - a_1 \Delta\mathbf{x}^T \Delta\dot{\mathbf{x}} - a_3 \Delta\dot{\mathbf{x}}^T \Delta\dot{\mathbf{x}} - \mathbf{A}\left(\ddot{\mathbf{r}} - \ddot{\mathbf{r}}_\mathbf{t}\right)\right) \tag{11}$$

After finding the κ in Eq. (11) the thrust direction and magnitude can now be solved computed which will ensure that $a^c = a_{engine}$. In this fixed thrust magnitude formulation κ as an adaptive term per integration step, but it is noted that the described guidance law still requires that that $\kappa > 0$. Substituting Eq. (11) into Eq. (7), we have

$$\mathbf{a^c} = -\frac{a_{engine}\mathbf{A}^T}{\sqrt{\mathbf{A}\mathbf{A}^T}} \tag{12}$$

where $\mathbf{A}$ is from Eq. (8) and a_{engine} is the available acceleration magnitude of the engine or thruster.

Equation (12) is the explicit closed form solution for fixed thrust magnitude control acceleration which satisfies the constraint in Eq. (2). A caveat of using Eq. (12) is the assumption that the control limitation of the engine has enough control authority to bring the spacecraft to the desire target.

Numerical simulations are performed for the Jupiter-Europa system in the CR3BP. The x-axis is fixed and points from the barycenter of Jupiter and Europa toward Europa. The z-axis points along Europas angular momentum vector with respect to Jupiter, and the y-axis completes the right-handed coordinate system. The mass ratio used is μ_1 = 2.52337464717194e-05. In this system the transformations from normalized units to un-normalized units are DU = 670,900 km and TU = 48,822 seconds (orbit period of Europa around Jupiter is 3.551 days). Here we assume the applied control acceleration is a continuous low-thrust system, where thrust magnitudes are allowed to vary.

In our example, the desired control is to match the position and velocity of the target reference trajectory as described in the above sections. The initial states for the spacecraft and the reference

trajectory in this example are listed in Table 1, where the target reference trajectory is on a trajectory toward the L2 point, while the spacecraft trajectory is deviated slightly (369.58 km in position and 1.11 m/sec in velocity) from the reference trajectory. The state of the reference trajectory was found by backward propagating the Halo orbit by $t = 2P$, where $P = 3.0984224791541$ TU. This is to ensure that the reference orbit will approach the L2 Halo orbit from its stable manifold, $W^s_{L2-Halo}$(Reference 12). States are then forwardly propagated.

Table 1. Initial state of the reference target orbit and the spacecraft (S/C) orbit in normalized units.

$\mathbf{r}_t$	1.06674183675193	0.275238653754246	-0.0116349382699965
$\mathbf{v}_t$	0.0218965193245572	-0.160539673664387	0.00285062288048665
$\mathbf{r}_{S/C}$	1.06620846583355	0.275101034427369	-0.0116291208008615
$\mathbf{v}_{S/C}$	0.0218855710648949	-0.160459403827555	0.00284919756904641

Figure 1 shows both the uncontrolled trajectory (Figure 1a) and the controlled trajectory (Figure 1b) using the continuous applied acceleration described in Eq. (7). In Figure 1a we see the spacecraft's trajectory if uncontrolled. In this plot we note that the spacecraft will approach the L2 point then orbit around $x \approx 1$, where Europa is located, and head towards the L1 point. Figure 1b shows the controlled trajectory; the reference trajectory and the spacecraft trajectory are visually indistinguishable as they approaches the Halo orbit. The selected coefficients used in Eqs. (8) and (9) are $a_1 = a_2 = 1, a_3 = 0.5$, and $\kappa = 0.7$.

The applied control acceleration, Eq. (7), to control the spacecraft in Figure 1b is shown in Figure 2a. The differences in position, velocity, and Jacobian constant between the spacecraft and the reference trajectory are shown in Figure 2b - 2d. We note desired decreases in differences in the position, velocity, and the Jacobi Constant as time progress.

For the case of a fixed magnitude control accleration, Eq. (12), we set $a_{engine} = 1\ mm/s^2$ (or $0.00355282111193919\ DU/TU^2$) and $a_1 = a_2 = 1$, and $a_3 = 0.5$. In this case, the controlled trajectory plot is visually indistinguishable from Figure 1b and, thus, not plotted. However, Figure 3a shows the applied thrust for a fixed control acceleration with $\eta = 0$, i.e., continuous thrusting. From this figure we note that the black solid line (acceleration magnitude) is constant over the controlled time period. The differences in position, velocity, and Jacobian constant between the spacecraft and the reference trajectory for the fixed acceleration magnitude case are shown in Figure 3b-3d.

REFERENCES

[1] A. E. Petropoulos, "Simple Control Laws for Low-Thrust Orbit Transfers," *AAS/AIAA Astrodynamics Specialist Conference*, AAS Paper 03-630, Big Sky, Montana, August 3-7, 2003.

[2] J. Jie, M. Kemao, and Y. Yu, "Nonlinear Geometry Control of Spacecraft for Halo Orbit Maintenance," *Proceedings of the 30th Chinese Control Conference*, Yantai, China, July 22-24, 2011.

[3] G. Gomez, K. Howell, J. Masdemont, and C. Simo, "Station-Keeping Strategies for Translunar Libration Point Orbits," *Advances in the Astronomical Sciences*, AAS Paper 98-168, 1998.

[4] H. Wong and V. Kapila, "Adaptive Nonlinear Control of Spacecraft Near Sun-Earth L2 Lagrange Point," *Proceedings of the American Control Conference*, Denver, Colorado, June 4-6, 2003.

[5] J. T. Betts, "Survey of numerical methods for trajectory optimization," *Journal of Guidance, Control, and Dynamics*, Vol. 21, No. 2, 2012.

[6] G. J. Whiffen and J. A. Sims, "Application of a Novel Optimal Control Algorithm to Low-Thrust Trajectory Optimization," *AAS/AIAA Astrodynamics Specialist Conference*, AAS Paper 01-209, Santa Barbara , California, February 11-14, 2001.

[7] I. Ross, Q. Gong, and P. Sekhavat, "Low-thrust, high-accuracy trajectory optimization," *Journal of Guidance Control and Dynamics*, Vol. 30, No. 4, 2007, pp. 921–933.

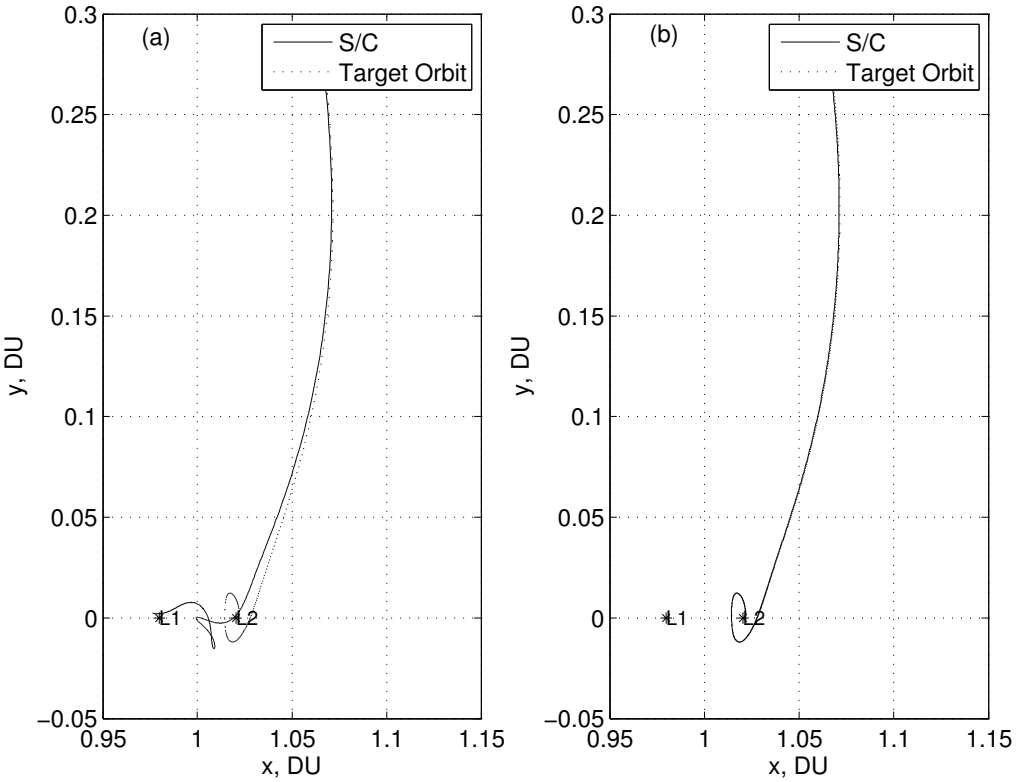

Figure 1. Integrated spacecraft and reference trajectory for $t = 2P$ for the uncontrolled spacecraft (a) and the controlled spacecraft (b). In both (a) and (b), the reference target orbits are plotted, but is indistinguishable to the actual spacecraft trajectory in (b).

[8] M. Vavrina and K. Howell, "Multiobjective Optimization of Low-Thrust Trajectories using a Genetic Algorithm Hybrid," *AAS/AIAA Space Flight Mechanics Meeting*, AAS Paper 09-151, Savannah, Georgia, February 2009.

[9] M. A. Paluszek and S. J. Thomas, "Trajectory Optimization Using Global Methods," *29th International Electric Propulsion Conference*, IEPC05-173, Princeton, New Jersey, October 31 - Novemeber 4, 2005.

[10] L. A. Pars, *A Treatise of Analytical Dynamics*. Ox Bow Press, 1965.

[11] F. E. Udwadia, "Equations of Motion for Mechanical Systems: A Unified Approach," *International Journal of Nonlinear Mechanics*, Vol. 31, No. 6, 1996, pp. 951–958.

[12] R. P. Russell and T. Lam, "Designing Ephemeris Capture Trajectories at Europa Using Unstable Periodic Orbits," *Journal of guidance, control, and dynamics*, Vol. 30, No. 2, 2007, p. 482.

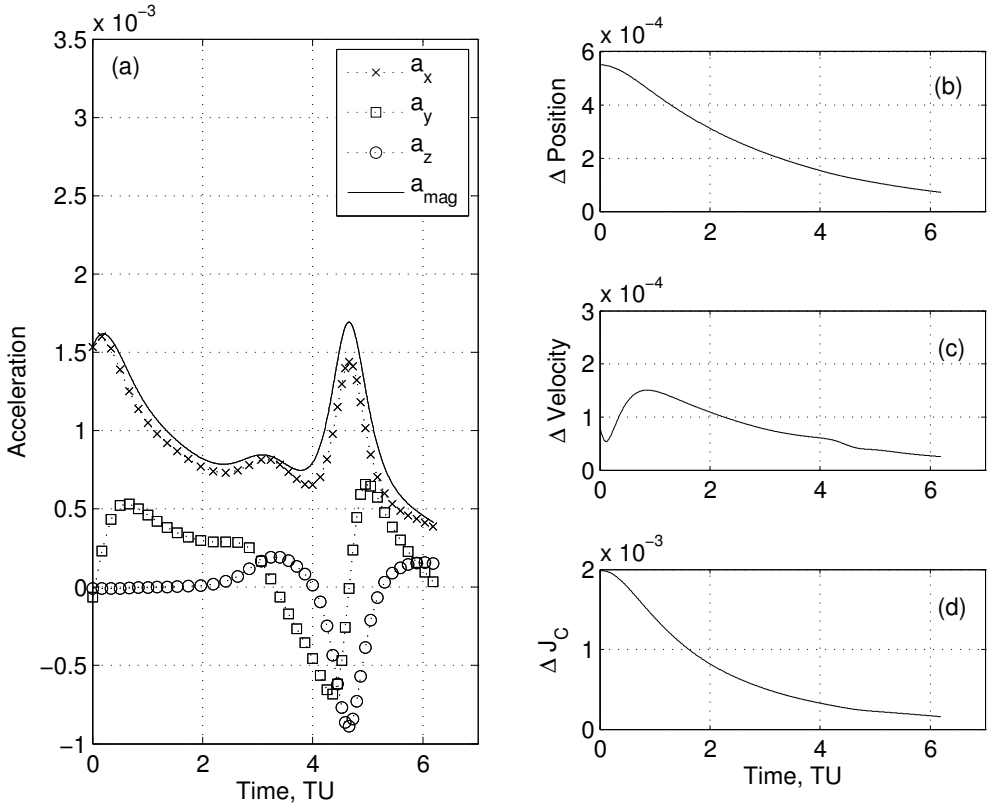

Figure 2. Applied control acceleration of the spacecraft (a), absolute differences in the position (b), velocity (c), and Jacobian Constant (d) between the spacecraft and the reference trajectory for $a_1 = a_2 = 1, a_3 = 0.5$, and $\kappa = 0.7$.

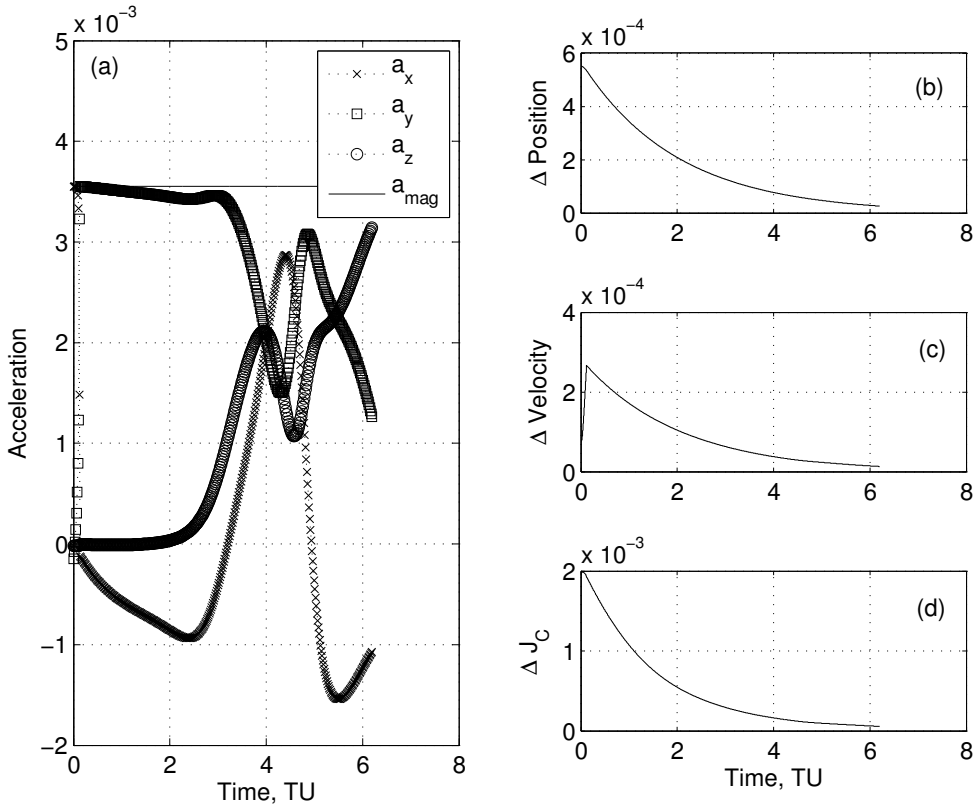

Figure 3. Applied control acceleration with fixed thrust acceleration ($1\ mm/s^2 \approx 0.00344\ DU/TU^2$) of the spacecraft (a), absolute differences in the position (b), velocity (c), and Jacobian Constant (d) between the spacecraft and the reference trajectory for $a_1 = a_2 = 1, a_3 = 0.5$, and $\eta = 0$ (continuous thrusting).