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THRUST DIRECTION OPTIMIZATION: SATISFYING DAWN'S ATTITUDE AGILITY CONSTRAINTS

Gregory J. Whiffen*

The science objective of NASA's Dawn Discovery mission is to explore the giant asteroid Vesta and the dwarf planet Ceres, the two largest members of the main asteroid belt.[1] Dawn successfully completed its orbital mission at Vesta. The Dawn spacecraft has complex, difficult to quantify, and in some cases severe limitations on its attitude agility.[2] The low-thrust transfers between science orbits at Vesta required very complex time varying thrust directions due to the strong and complex gravity and various science objectives. Traditional low-thrust design objectives (like minimum ΔV or minimum transfer time) often result in thrust direction time evolutions that cannot be accommodated with the attitude control system available on Dawn. This paper presents several new optimal control objectives, collectively called thrust direction optimization that were developed and turned out to be essential to the successful navigation of Dawn at Vesta.

INTRODUCTION

The Dawn spacecraft has two long solar arrays which together span 20 meters (see Figure 1.) The solar arrays provide power for the spacecraft including three ion thrusters. The arrays are sized to allow both the spacecraft bus and one ion thruster to operate at heliocentric distances approaching 3 AU at Ceres. The long arrays result in a large moment of inertia along axes orthogonal to the arrays. The attitude of the spacecraft for any given thrust direction is determined by an algorithm

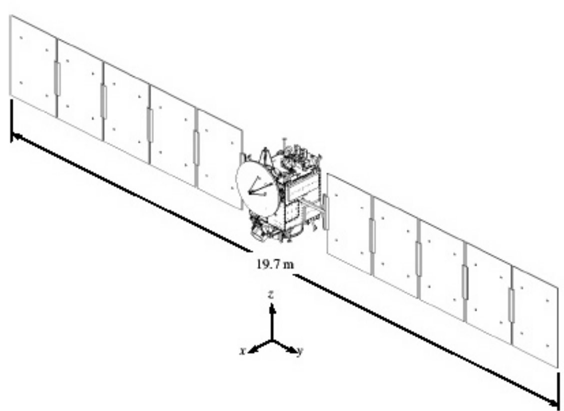

Figure 1. The Dawn spacecraft showing the fully deployed solar array. The ion thrusters are located under (-z direction) the spacecraft in this view.

*Mission Design Engineer, Mission Design and Navigation Section, Jet Propulsion Laboratory, California Institute of Technology, Mail Stop 264-820, 4800 Oak Grove Drive, Pasadena, CA 91109

called "power steering[2]". The purpose of the power steering algorithm is to always maximize the solar array power output for spacecraft operations. Power steering requires the attitude to be chosen so that the solar array surface is exactly orthogonal to the Sun for any thrust or instrument pointing direction. This is accomplished by rotating the spacecraft around the chosen thrust or instrument direction. For example, when thrusting using the ion thruster pointing in the -Z axis direction, this amounts to a rotation about the Z axis. This rotation is complete when the Y axis (solar array axis) is orthogonal to the Sun line. The solar array surface can then be made orthogonal to the Sun line by rotating the arrays relative to the spacecraft about the y axis (see Figure 1 for a definition of the axes).

The power steering algorithm has two poles in attitude space that can require arbitrarily large attitude angular accelerations and rates. For thrusting, the two poles correspond to thrusting in the Sun or anti-Sun direction. If a thrust profile requires thrusting to move near a pole then a rapid, nearly 180 degree flip is required by the solar array. This flip is a rotation around the chosen direction (thrust direction) and is orthogonal to the solar array (Y axis). Rotations around any axis orthogonal to the solar array is associated with the highest moment of inertia of the spacecraft.

Dawn's attitude is controlled during ion thrusting by thrust vectoring (this controls the two axes perpendicular to the thrust direction) and by reaction wheels or hydrazine thrusters (this controls the roll around the thrust direction). Thrust Vector Control or TVC is achieved by gimbals on the ion thrusters that have a maximum excursion of about 5 degrees. A steering torque is created by using the gimbals to thrust through points away from the center of mass. The gentle thrust provided by Dawn's ion thrusters (on the order of 50 [mN] for the 1000 [kg] spacecraft) and the small gimbal excursion (< 5 degrees) provide little torque and hence limited attitude agility. The limited agility combined with the power steering algorithm results in the Dawn spacecraft not being able to keep up with many potential thrust profiles. The agility constraints become severely limiting when thrusting or pointing near the Sun and anti-Sun directions.

Often two separately flyable thrust profiles become un-flyable if the spacecraft cannot move through the intermediate attitudes required during the coasting between each maneuver. This commonly occurred when the first maneuver ends near the Sun or anti-Sun direction and the second maneuver starts in a thrust direction on the opposite side of the Sun or anti-Sun direction and there is little time between the maneuvers. When the spacecraft is not thrusting the attitude was controlled either with reaction wheels or hydrazine thrusters.

The actual agility limitations are difficult to predict[2] in part because the momentum state of the spacecraft (reaction wheel rates) can not be exactly known or even approximately predicted until a momentum management plan is developed for the thrust profile in question. This is a chicken and egg problem. The agility constraints are therefor not analytic and cannot be directly incorporated into methods to design thrust profiles for orbital transfers. The feasibility of any given thrust profile is best predicted by a Monte Carlo analysis of possible momentum management plans and reaction wheel momentum states.[2] Despite the fact the agility constraints are non-analytic, general rules can be used to modify the thrusting directions to reduce the agility requirements. Moving thrusting away from the Sun or anti-Sun direction and/or reducing the time variation of the thrust direction will both obviously improve the likelihood of meeting agility constraints. However, providing the same angular distance from the Sun or anti-Sun directions and thrust direction rates will in some cases work and in others it will not work. This is a result of the interplay of the history of reaction wheel momentum build up and spacecraft body momentum.

Given the difficulty in quantifying the agility constraints, it is necessary to have the ability to shift the thrusting directions and change the thrust directions rates in various ways and test the resulting thrust profiles for feasibility using a Monte Carlo analysis. Of course, the purpose of the maneuver (state targeting) has to be preserved. If a minimum thrust-on time (or propellant optimal) maneuver is found to violate agility constraints, then any acceptable thrust profile will require thrusting in non-time-optimal or mass-optimal directions and therefore will require more thruster on time. The extra thruster on time is called "**slack**" in this investigation. A goal is to reduce the slack needed to satisfy spacecraft agility constraints.

A successful maneuver design must employ "slack" thrusting in a way that both achieves the required state target and meets agility constraints. One agility requirement is that both thrust directions and *rates* do not change discontinuously or too rapidly: this is the "slow-continuity" requirement. The slow-continuity requirement rules out maneuver design approaches that employ hard constraints on thrusting directions because hard constraints result in very rapid attitude accelerations when the constraint becomes active or inactive. Further, hard constraints often result in attitude discontinuities when thrusting "jumps" over the attitude directions that are excluded by the constraint. What is needed is design optimization objectives that achieve slow-continuity, avoid the attitude poles in a smooth way, and meet the maneuver state targets while using slack thrusting efficiently.

Time-optimal and propellant-optimal maneuvers frequently violate agility constraints because these objectives are blind to the attitude poles where constraints are most severe. They can also fail far from the attitude poles when the required attitude rates or accelerations are too high. Several new optimal control objectives, collectively called thrust direction optimization, were developed to overcome the shortcomings of time- and mass-optimal maneuvers. The new objectives are described in this paper and were employed extensively and successfully during Dawn's year long orbital operations at Vesta.

The software used to design all of the maneuvers for the Dawn spacecraft is a tool set called Mystic.[3] The optimization algorithm used in Mystic is the Static Dynamic optimal Control or SDC algorithm.[4] The SDC algorithm is based on Bellman's Principle of Optimality.[5] Mystic was originally designed for low-thrust (typically electric propulsion) mission design but has been extended to provide a very high-fidelity maneuver design capability for mission operations.

THRUST DIRECTION OPTIMIZATION

Mystic's main function is low-thrust trajectory optimization. The optimization variables include thrust as a function of time, launch or start date, flight time (or equivalently arrival date), and up to seven initial condition parameters, including the initial spacecraft mass. The formulation also allows for state variables or indirectly controlled variables. The general problem that Mystic's optimization algorithm solves is find the optimal control v^* and w^* such that the objective $J(v, w)$ is minimized:

$$J^* \equiv J(v^*, w^*) = min_{v(t),w} J(v, w). \tag{1}$$

The SDC algorithm recognizes two types of controls vectors. The first, w, is the so-called "static control" or list of decisions that are not time dependent. The second type of control, $v(t)$, is the "dynamic control" or list of controls that are functions of time. The definition of the objective is:

$$J(v, w) = \int_{t_o=w_1}^{t_f=f(w)} F(x(t), v(t), w, t)dt + \sum_{i=1}^{M} H_i(x(t_i), v(t_i), w, t_i) + G(x(t_f), v(t_f), w, t_f), \tag{2}$$

where $x(t)$ represents the system state variables subject to a state equation of the form

$$\frac{dx(t)}{dt} = T(x(t), v(t), w, t), \tag{3}$$

and an initial condition of the form

$$x(t_0) = \Gamma(w). \tag{4}$$

The trajectory start time, t_o, is the first component of the static control variable vector w and the trajectory end time t_f is a function f of the static control vector (typically a sum of a subset of the components of w). The functions F, H, and G are user selectable, once continuously differentiable objective functions. The function T represents the trajectory equations of motion including propellant mass flow. The function Γ returns the initial system state as a function of the static control variables. In the current implementation for Dawn, the dynamic control vector $v(t)$ is the three components of ion propulsion thrust as a function of time. The "static" control parameters w are defined to be the start time t_o, the initial spacecraft position, velocity, mass, and the time of flight for each trajectory phase. For most maneuver designs, the spacecraft initial state is fixed (not subject to optimization). The initial state used to design maneuvers is the current orbit determination and estimated spacecraft mass. Therefore the function Γ is a constant vector with no dependence on the static control w.

The equation used to describe the time evolution of the state T includes ion thruster mass flow, attitude control thruster mass flow, multi-body gravity, gravity harmonics, thrust, and solar radiation pressure. Both linear and non-linear constraints are also used on the variables $x(t)$, $v(t)$, and w to account for thruster operational limitations, transfer targets, and power available constraints. For example, a commonly used six-state target has the form

$$x(t_f) = \left\{ \begin{array}{c} target \;\; x \;\; position \\ target \;\; y \;\; position \\ target \;\; z \;\; position \\ target \;\; x \;\; velocity \\ target \;\; y \;\; velocity \\ target \;\; z \;\; velocity \end{array} \right\} \tag{5}$$

The objective J in equation (2) that was initially used to design transfer thrusting at Vesta was

$$J = G(x(t_f), u_N, w, t_f) = \min \left[K - x_7(t_f)\right]^2 . \tag{6}$$

The functions F and H_i in equation (2) were set to 0. The scaler constant K was selected to be the spacecraft initial mass before any maneuvering. The objective specified by equation (6) is to minimize the square of the difference between the spacecraft's start mass $K = x_7(t_0)$ and its ending mass $x_7(t_f)$. The objective (6) is equivalent to minimizing propellant consumption. The operational transfer maneuver designs at Vesta involved a fixed flight time for each designed maneuver, therefore the solutions can be described as "mass optimal" as opposed to being "time-optimal" or minimum time transfers.

It is important to note that the thrust vector variations required to achieve the necessary state targets were very complex. Simple spiral control laws, such as thrust parallel or anti-parallel to the Vesta relative velocity direction were not adequate. Figure 2 illustrates a thrust vector profile that was optimized and executed during a portion of a transfer between the medium and low-altitude

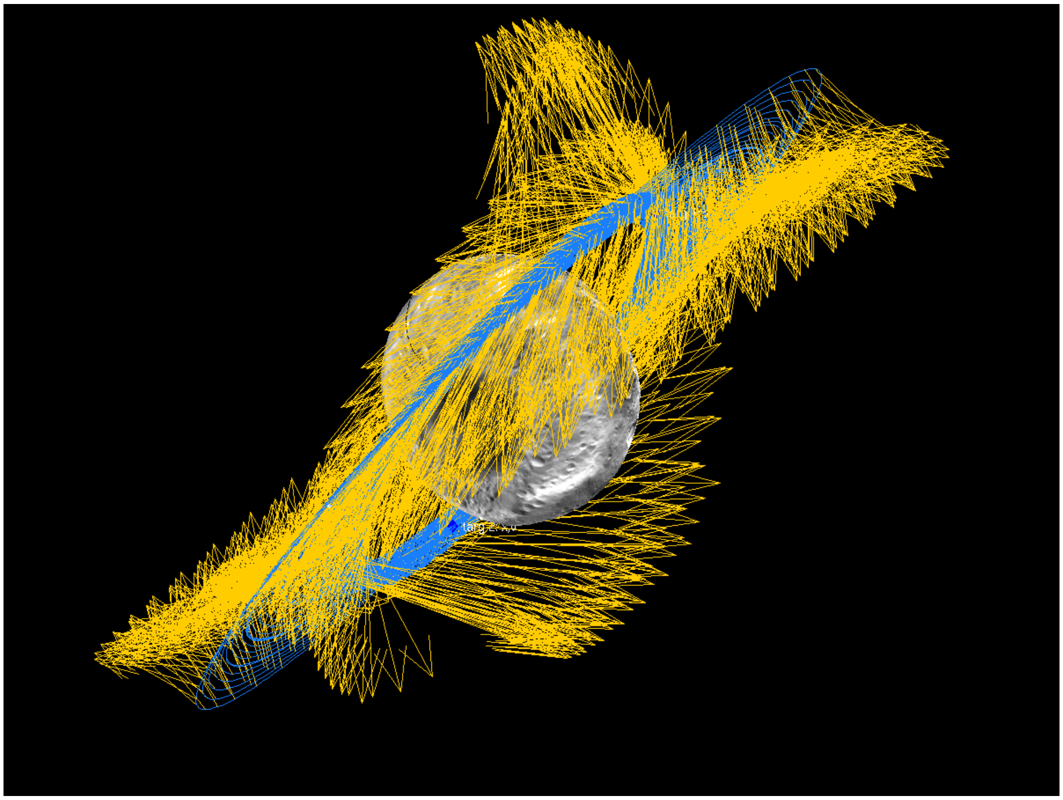

Figure 2. An example of the complex evolution of the thrust direction required for transfers around Vesta. Vesta resides at the center of the illustration. The spacecraft trajectory is indicated in blue and the thrust acceleration direction is indicated by yellow arrows. The view is inertial and is above the north pole of Vesta.

science orbits at Vesta. The strong and highly non-spherical gravity of Vesta combined with precise science orbit state targets and trajectory safety constraints resulted in complex maneuvers, as is evident in Figure 2. Generally, the closer Dawn got to Vesta, the more complex and dynamic the thrust direction evolution became. The maneuver illustrated in Figure 2 is an optimal control solution for the Optimal Control Problem or OCP outlined above.

Mass optimal solutions obtained by solving the OCP described above typically have smoothly varying thrust directions. Smooth variation is desirable and necessary for flight system feasibility. However, mass optimal solutions do not necessarily avoid the attitude poles and/or high attitude rates that the flight system cannot achieve. Direct constraints on thrust directions, for example, a cone angle exclusion constraint around the attitude poles result in attitude rate discontinuities and often high rates occur when the constraints are active.

An undesirable aspect of mass optimal solutions is that for many spiraling transfer problems, the thruster is cycled on and off one or more times per revolution. It is generally true that it is mass optimal to coast during times that correspond to very high attitude (thrust direction) rates which is good. However, this often moves the agility violation from the thrusting portion of the maneuver to the coasting portion of the maneuver. As a result, even when the spacecraft can keep up with the attitude requirements for the individual thrust arcs comprising a mass optimal solution, the spacecraft could not achieve attitude changes required during the optimal coasting periods between active thrust arcs. Formulating constraints that account for attitude when not thrusting is not possible in a "three degree" of freedom formulation. The three degrees of freedom are the thrust vector components. When the thrust magnitude is zero (during optimal or forced coasting) the attitude is obviously not constrainable given a three degree of freedom OCP definition. More variables that explicitly define attitude need to be defined to constrain attitude during coasting. Formulations that track spacecraft attitude as state variables are called "six degree of freedom" formulations.

The development of a six degree of freedom formulation, including three attitude angles as part of the spacecraft state, can account for attitude even when not thrusting and provides a way to control attitude rates. However, the complexity of the formulation leads to the difficulty of constructing useful initial guesses. Specifically, the enormous increase in the resulting local minima from spacecraft revolutions overwhelms the advantage of the 6 degree of freedom approach for this application. As a result, a near feasible solution would need to be known before attempting to solve the optimal control problem. This was determined to be impractical given the maneuver complexity and the short design periods allowed during operations.

An approach that provided limited success in designing maneuvers at Vesta while using the mass optimal objective (6) involved having the designer introduce forced coasting during periods when the mass optimal solution had either high rates or ventured close to an attitude pole. As long as the introduced coast was long enough, there would be enough time to move between arbitrary attitudes during the coast. Forcing coasting in places where it is optimal to thrust most often increases attitude rates during thrusting at other times. This may be acceptable if the higher rates occur further from an attitude pole. The approach was found to be too unreliable and time consuming to be applied in general.

As a result, a new formulation was sought that would provide the maneuver designer with the ability to satisfy attitude agility constraints while not requiring sophisticated initial guesses for the OCP. Not requiring sophisticated initial guesses is equivalent to having a formulation that does not result in a large number of superfluous local minima. Specifically, the new OCP formulation must be a three degree of freedom formulation in order to avoid the attitude revolution local minima of six-degree formulations. The new OCP formulation or objectives need to provide control over thrust direction (to avoid attitude poles), and attitude rates and accelerations in general. A further requirement is that the resulting solutions exhibit slow-continuity of thrust directions - a feature of well behaved mass optimal solutions. The new formulation that was developed to achieve these objectives is called "Direction Optimization."

General Formulation of Direction Objectives

A thrust profile is said to be "Direction Optimal" if the thrust profile is closest to, or furthest from, a thrust direction target and the thrust profile meets all constraints and achieves the required state targets. The direction target can be either inertially fixed for the duration of the transfer or the direction can be a function of time. The direction objective can be applied to part or all of the transfer. Direction objectives can be used in combination with each other and with mass objectives to create a pareto optimal front. The general form of a direction "attractor" objective is

$$min_{v,w} \int_{t\in\tau} \left[1 - \hat{v}(t) \cdot \hat{D}(x(t), v(t), w, t)\right]^2 dt, \tag{7}$$

where $\hat{v}(t)$ is a unit vector pointing in the same direction as the thrust acceleration and $\hat{D}$ is unit vector function that provides the target thrust direction at time t. The set of times τ need not be continuous. The general form of a direction "pole repulsor" objective is

$$min_{v,w} \int_{t\in\tau} \left[1 - (\hat{v}(t) \times \hat{S}(x(t), v(t), w, t)) \cdot (\hat{v}(t) \times \hat{S}(x(t), v(t), w, t))\right]^n dt, \tag{8}$$

where $\hat{S}$ is unit vector function that provides a pole that is to be avoided in thrust direction space and n is a positive integer. Again, the set of times τ need not be continuous. It is easy to construct many other general types of thrust direction objectives including a direction repulsor objective and a pole attractor objective.

Numerically Tested Direction Objectives

Six different specific thrust direction objectives were developed and tested. Since, at the outset, it was not at all clear which objective or objectives would be most useful, several different objectives were simultaneously constructed and tested. Each specific objective that was built had a plausible hypothesis of success associated with it. The first objective uses a single inertial direction attractor target

$$min_{v,w} \int_{t\in\tau} \left[1 - \hat{v}(t) \cdot \hat{D}\right]^2 dt, \tag{9}$$

where $\hat{D}$ is a fixed inertial direction. This objective is the simplest meaningful type of direction objective. It was hypothesized that thrusting could be drawn away from the Sun and anti-Sun direction attitude poles if the inertial direction target is chosen far from these directions. The second objective is a generalization of the first. The second objective uses a list of inertial directions (time dependence) as an attractor target

$$min_{v,w} \int_{t\in\tau} \left[1 - \hat{v}(t) \cdot \hat{D}(t)\right]^2 dt, \tag{10}$$

where $\hat{D}(t)$ is an inertial direction that depends only on time - not state or any other variable.

The third thrust direction objective uses the spacecraft-central body (Vesta) relative velocity vector as an attractor target,

$$min_{v,w} \int_{t\in\tau} \left[1 - \hat{v}(t) \cdot \hat{V}(x(t), t)\right]^2 dt, \tag{11}$$

where $\hat{V}$ is the spacecraft-central body relative velocity unit vector. The fourth objective is the same as the third objective (11) except that the target direction is the anti-velocity direction

$$min_{v,w} \int_{t\in\tau} \left[1 + \hat{v}(t) \cdot \hat{V}(x(t), t)\right]^2 dt. \tag{12}$$

The objectives (11) and (12) are useful during maneuvers that are spiraling out or in respectively. The bulk of the thrusting is expected to occur near the relative velocity or anti-velocity directions during spiraling in or out from a body. It was hypothesized that applying these objectives with sufficient slack would dampen troublesome, out of plane and radial thrusting. Specifically, these objectives should tend to keep the minimum angle between the thrust directions and the attitude poles (Sun and anti-Sun direction) close to the orbit β angle. The orbit β angle is the angle between the orbital plane and the Sun direction.

The fifth thrust direction objective is a fixed inertial pole repulsor objective of the form

$$min_{v,w} \int_{t\in\tau} \left[1 - (\hat{v}(t) \times \hat{S}) \cdot (\hat{v}(t) \times \hat{S})\right]^n dt, \tag{13}$$

where $\hat{S}$ is fixed inertial direction unit vector defining the attitude pole to be avoided. It was hypothesized that this objective could be applied to avoid thrusting near the Sun and anti-Sun directions by defining the unit vector $\hat{S}$ to be the Sun direction. Note that the Sun direction varied little during any single maneuver at Vesta so using a fixed inertial direction would suffice. The integer n in objective (13) can be varied to make the objective influence strong over a small range of angles (when n is large) or weaker and longer ranged (when n is small.)

The sixth thrust direction objective is an attractor target specified as a fixed direction in a rotating frame called the Radial-Normal-Transverse or RTN frame. This objective turned out to be the most useful for Vesta operations. The RTN frame is defined based on the position and velocity of the spacecraft relative to the center of mass of the central body (Vesta). The radial axis is parallel to the central body - spacecraft direction. The transverse axis is the local horizontal direction of the spacecraft relative to the central body that is closest to the relative velocity of the spacecraft with respect to the body. Another way to define the transverse axis is the direction that is orthogonal to the radial direction that is closest to the spacecraft - central body relative velocity. The normal axis completes the right handed coordinate system. The RTN frame is non-inertial, rotating once every orbit. The rotation rate is not constant for elliptical orbits and orbits around non-spherical bodies like Vesta. A fixed direction in the RTN frame is therefore time varying in an inertial frame. The RTN attractor objective is

$$min_{v,w} \int_{t\in\tau} \left[1 + \hat{v}(t) \cdot (a\hat{r} + b\hat{t} + c\hat{n})\right]^2 dt. \tag{14}$$

The radial, transverse, and normal unit vector directions are defined as

$$\hat{r} = (X_{sc} - X_{body})/||X_{sc} - X_{body}||,$$

$$\hat{t} = (\hat{V} - (\hat{r} \cdot \hat{V})\hat{r})/||\hat{V} - (\hat{r} \cdot \hat{V})\hat{r}||,$$

and

$$\hat{n} = \hat{r} \times \hat{t},$$

where the body relative spacecraft velocity is defined as

$$\hat{V} = (V_{sc} - V_{body})/||V_{sc} - V_{body}||.$$

The variables X_{sc} and V_{sc} are the spacecraft position and velocity in an inertial frame and X_{body} and V_{body} are the central bodies position and velocity in the same inertial frame.

The constants, a, b, and c in equation (14) are chosen to define the fixed direction objective in the RTN frame. Selection of a, b, and c are subject to the constraint $||a\hat{r} + b\hat{t} + c\hat{n}|| = 1$ or equivalently $a^2 + b^2 + c^2 = 1$. Since there are only two degrees of freedom in specifying the direction, it is convenient to specify the objective direction as a right ascension angle and a declination angle in the RTN frame. The direction $a\hat{r} + b\hat{t} + c\hat{n}$ clearly has fairly complex dependence on both the spacecraft state and time. This made the derivation of the analytic first and second derivatives of equation (14) considerably more difficult compared to the objectives (9) - (13). Analytic expressions for derivatives are always preferable, whenever practical, to improve optimization robustness.

The objectives (11) - (14) all have an interesting property that the objectives (9) and (10) do not have. Specifically, the direction targets in the former are non-trivial, explicit functions of the spacecraft state. As a result, both a direct (control variable) dependance through the thrust vector $v(t)$ and an indirect state variable $x(t)$ dependence will influence the objective value. This means that there generally exists advantages to both alter the thrust directions to directly minimize the objective and also alter thrust directions at earlier times to change the state at later times to further minimize the objective. For long transfers the indirect influence could result in significant changes in the intermediate trajectory states depending on the objective used. This property can be useful because the optimal control problem solution will now favor placing the spacecraft in future states where thrust direction evolution is presumably more benign (assuming the direction objective is chosen well).

EXAMPLES OF DIRECTION OPTIMAL SOLUTIONS

Direction objectives are properly formulated when a fixed amount of thrusting time is selected in advance. Thrusting time is not a control, rather it is a parameter that is adjusted as needed to achieve the desired thrust direction deviation. The selected amount of thrusting time must be equal to or be greater than the amount of thrusting time required for the mass optimal solution, otherwise the problem will obviously be infeasible (i.e. the end state target will not be reachable.) The time

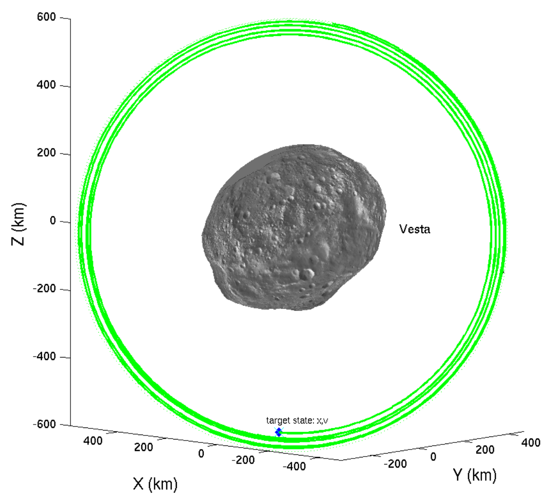

Figure 3. The basis of the numerical examples is a spiral-in maneuver used during Dawn's Vesta orbital operations. The average orbital radius decreases from 640 [km] to 580 [km] during the spiral-in.

in excess of the mass optimal thrusting time is called slack time. The more slack time, the more freedom the thrust profile has to react to the thrust direction objective. Further, it is necessary to employ a non-zero thrust magnitude constraint to avoid direction degeneracy when the thrust magnitude approaches zero during optimal coasting times. When the thrust magnitude numerically approaches zero during optimal coasting, the thrust direction will have a vanishing effect on the trajectory state evolution - therefore the minuscule thrust will exactly line up with the objective. This leads to numerical problems and less effective control of the thrust direction when the thrust magnitude is large. A properly formulated direction optimization problem does not have any optimal coasting as a result of a constraint of the form

$$||v(t)|| = T_M \qquad \forall t \in \tau \tag{15}$$

where T_M is the fixed "on" magnitude of thrust. All of the numerical examples provided in this paper satisfy the constraint (15) during all given thrust times. The numerical examples presented in this paper are based on the Vesta spiral-in maneuver plotted in Figure 3. The spacecraft revolves around Vesta 7 times during the maneuver. The average orbital radius decreases from 640 [km] to 580 [km]. The total flight time is 1.93 days. This example is drawn from part of a designed maneuver used during Vesta operations in mid-November 2011. The trajectory begins at a fixed position and velocity based on orbit determination and is constrained to end on a given position and velocity. The ending state is determined by the transfer reference trajectory at that epoch.

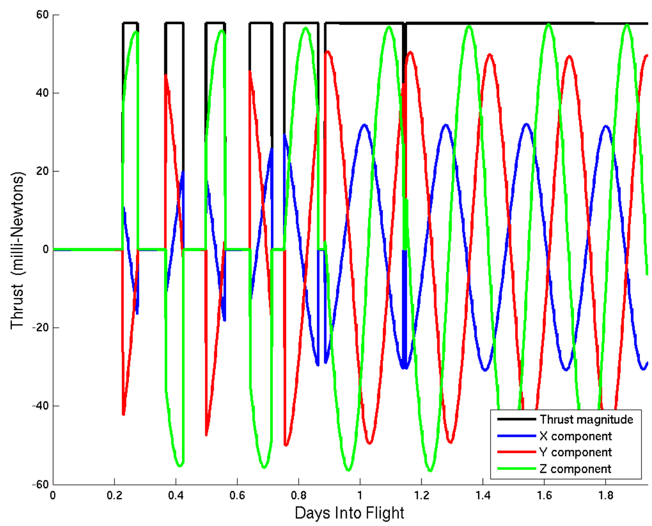

Figure 4. The mass optimal thrust magnitude and inertial thrust components in the Earth Mean Orbit and equinox of the J2000 epoch frame or EMO-2000 frame for the Vesta spiral-in maneuver example.

The mass optimal solution to the maneuver design requires 1.3932 total days of thrusting time. The mass optimal solution consists of 7 individual thrust arcs with varying amounts of optimal coasting in between - see Figure 4. The large number of optimal coasting arcs result in an undesirably high number of engine on/off cycles. Forcing coasting at strategic times during the transfer results in fewer separate thrust arcs but somewhat longer total thrust times and often higher attitude rates. Direction optimization provides an alternative way to reduce thruster on/off cycles by forcing thrusting via the constraint (15) while avoiding high rates when the direction objective is chosen correctly. The gravity of Vesta is modeled using a harmonic field of order 8. The Sun and planets were modeled as point masses. The capability of the ion propulsion system at the time of this maneuver was 57 [mN] and the spacecraft mass was 952 [kg].

The thrust directions obtained from solving the mass-optimal problem are illustrated in Figure 5. The mass-optimal thrust directions plotted on the unit sphere are plotted in two different frames

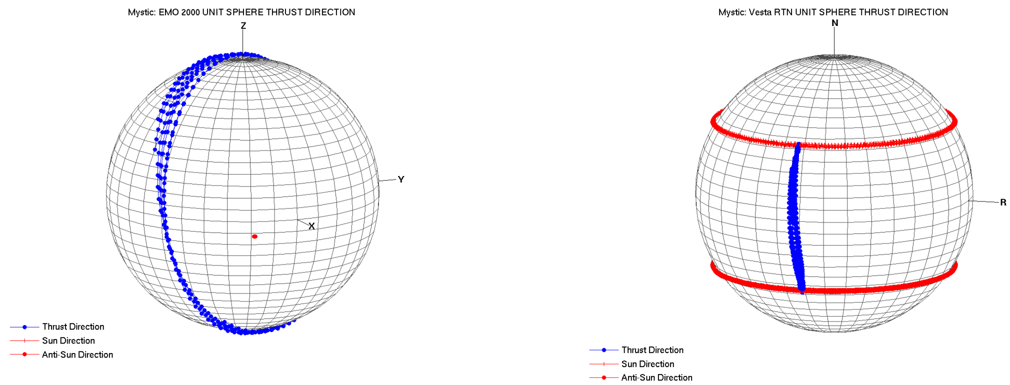

Figure 5. The mass-optimal solution thrust directions plotted on the unit sphere in the inertial Earth Mean Orbit and equinox of the J2000 epoch frame or EMO-2000 frame (on the left) and in the Radial, Transverse, Normal, or RTN rotating frame on the right.

in Figure 5. The unit sphere on the left is the Earth Mean Orbit and equinox of the J2000 epoch frame (EMO-2000 frame). The unit sphere on the right is the RTN rotating frame. Most of the thrusting is in the negative "T" axis direction which corresponds approximately to thrusting in the anti-relative velocity direction as expected for a spiral-in maneuver. Since the maneuver is aiming for a particular position and velocity at the end, the thrusting is more complex than exactly thrusting in the anti-relative velocity direction. The Sun and anti-Sun directions are indicated by red "+" and red dot symbols in Figure 5. The Sun and anti-Sun directions wrap around the unit sphere at two equal and opposite latitudes in the rotating RTN frame.

Examples of Inertially Fixed Direction Objectives

The thrust directions of the mass optimal solution to the Vesta spiral-in maneuver design problem are compared to a fixed direction objective solution in Figure 6. The mass-optimal thrust directions

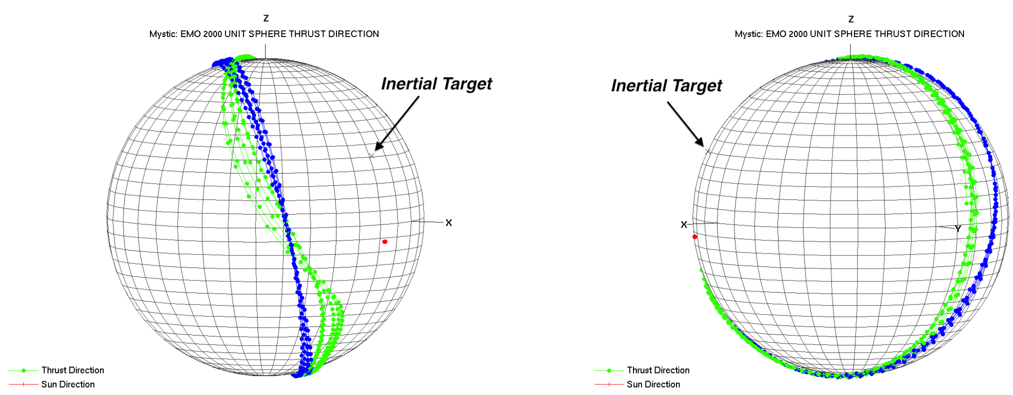

Figure 6. Two views of the thrust directions for the mass-optimal solution (blue) and an inertial direction optimal solutions (green) plotted on the unit sphere in the EMO-2000 frame. The inertial direction target $\hat{D}$ is marked with a black "X" and labeled.

are plotted in blue on the unit sphere in the inertial EMO-2000 frame. If we introduce a fixed direction constraint of the form of equation (9) and provide 50 minutes of slack thrusting time relative to the mass-optimal solution then the thrusting shifts to the green trace in Figure 6. All thrusting time during the direction optimization occurs during a single thrust arc that ends at the end of the trajectory. This means all coasting was forced to occur at the start of the trajectory. The direction objective used is indicated by a black "X" in Figure 6. The direction target $\hat{D}$ has a right ascension of -20^o and a declination of +25^o in the EMO-2000 frame. The optimal solutions presented in Figure 6 satisfy both the first and second order conditions of optimality. Several common behaviors of direction optimal solutions are evident in Figure 6. First, overall, the thrusting directions have moved closer to the target, however some thrust has moved further away as is evident on the left-hand unit sphere in Figure 6. Where thrusting has moved away from the target, the direction rate has increased. The direction rate is indicated by the spacing of the points that mark equal amounts of time along the thrust direction traces in Figure 6. Since the objective (9) is an integral over time, spending a short amount of time (with high angular rates) is inexpensive and advantageous if lower rate thrusting can be moved closer to the target $\hat{D}$. Recognizing these behaviors allows the analyst to better select a direction target to achieve the desired goals.

The effect of increasing the slack thrusting time is illustrated in Figure 7. In this example, the direction target $\hat{D}$ is held constant at a right ascension of -20^o and a declination of 0^o in the EMO-2000 frame and the slack is 0, 50, or 116 minutes. "Zero minutes of slack" is equivalent to the mass-optimal solution. Increasing the amount of slack does not necessarily decrease the thrust direction

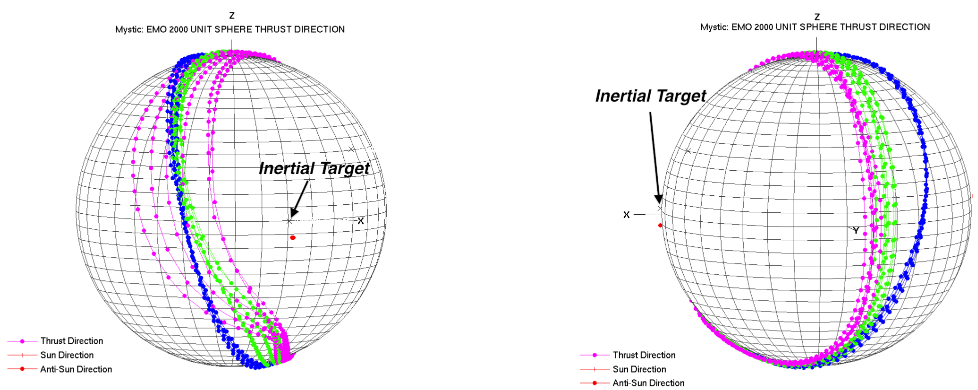

Figure 7. Two views of the thrust directions for the mass-optimal solution or zero slack solution (blue), an inertial direction optimal solution with 50 minutes of slack (green), and an inertial direction optimal solution with 116 minutes of slack (magenta) plotted on the EMO-2000 frame unit sphere. The inertial direction target $\hat{D}$ is marked with a black "X" and labeled.

angular rate as is evident in the highest rates occurring in the magenta trace or 116 minutes of slack case in Figure 7. High rates are indicated by large spacing between the dots on each thrust direction trace. The thrusting that is most distant from the target is moved preferentially toward the target with increasing slack (see the right-hand view in Figure 7.) This is a result of the quadratic aspect of objective equation (9). Thrusting that occurs twice as far from the target $\hat{D}$ is four times as expensive. Since the thrusting in this example approximately follows great circles in any inertial frame, a single fixed inertial target is a poor type of objective to achieve attitude agility constraints. The thrusting pattern in a rotating frame like the RTN frame is generally more compact (recall Figure

5.) More compact patterns behave in simpler ways when direction optimization is employed.

Examples of Polar Repulsor Direction Objectives

It was hypothesized that a pole repulsor type objective could be applied to avoid thrusting near the Sun and anti-Sun directions by defining the unit vector $\hat{S}$ to be the Sun direction. However, the pole repulsor objective in an inertial frame results in solutions that often increase rates near the pole due to the time-integrated nature of the objective equation (13). Not surprisingly, pole repulsor solutions exhibit similar, but opposite behaviors exhibited by inertial attractors. Figure 8 illustrates the result of the application of a pole repulsor objective of the form of objective equation (13). The pole $\hat{S}$ is defined by the right ascension of -20^o and a declination of 0^o in the EMO-2000 frame. The Pole is near the Sun, anti-Sun line. The parameter n in objective equation (13) was selected to be 2. The pole repulsor solution succeeds in moving the thrusting further away from both the Sun and

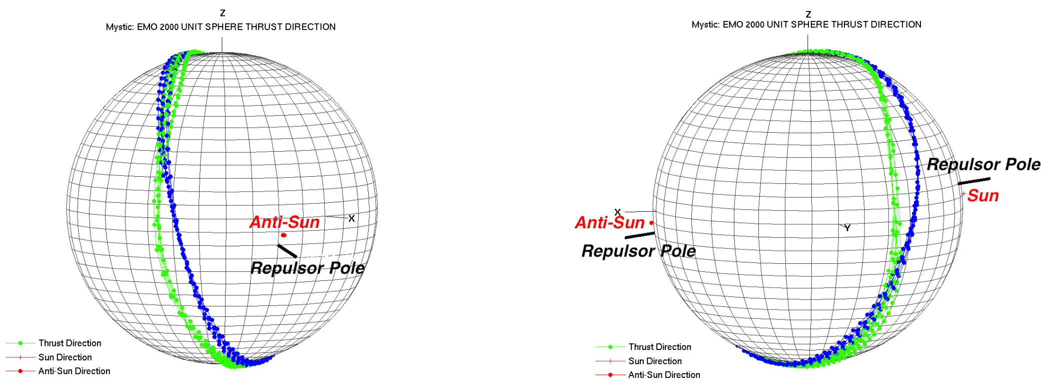

Figure 8. Two views of the thrust directions for the mass-optimal solution or zero slack solution (blue) and a pole repulsor optimal solution with 50 minutes of slack (green) plotted on the EMO-2000 frame unit sphere. The pole direction $\hat{S}$ is indicated by a black line passing through the sphere.

anti-Sun directions (compare the blue and green thrust direction traces in Figure 8). A more subtle effect is that where the thrusting has deviated the most from the mass optimal solution, the direction rate has increased the most. This may or may not be acceptable from an attitude agility standpoint. On one hand, the spacecraft is more agile further from the Sun direction, but on the other hand, the spacecraft is required to rotate more quickly. During Vesta operation, the pole repulsor objective was used only very occasionally. By far the most effective objective for satisfying Dawn's attitude agility constraints was a fixed direction target in the rotating RTN frame.

Example of RTN Frame Direction Objectives

An example of a mass optimal versus a direction optimal thrust direction evolution in the RTN frame is provided in Figure 9. The mass optimal solution (blue) cannot be flown by Dawn as a result of attitude rates when thrusting near the anti-Sun direction. The anti-Sun direction in the rotating RTN frame corresponds to the southern latitude red band on the unit sphere in Figure 9. A RTN frame direction objective equation (14) was placed high in the northern hemisphere to draw the thrusting away from the anti-Sun direction. The target was placed at a right ascension of -110^o and a declination of +45^o in the RTN frame. The thrust direction solution is plotted in green in Figure

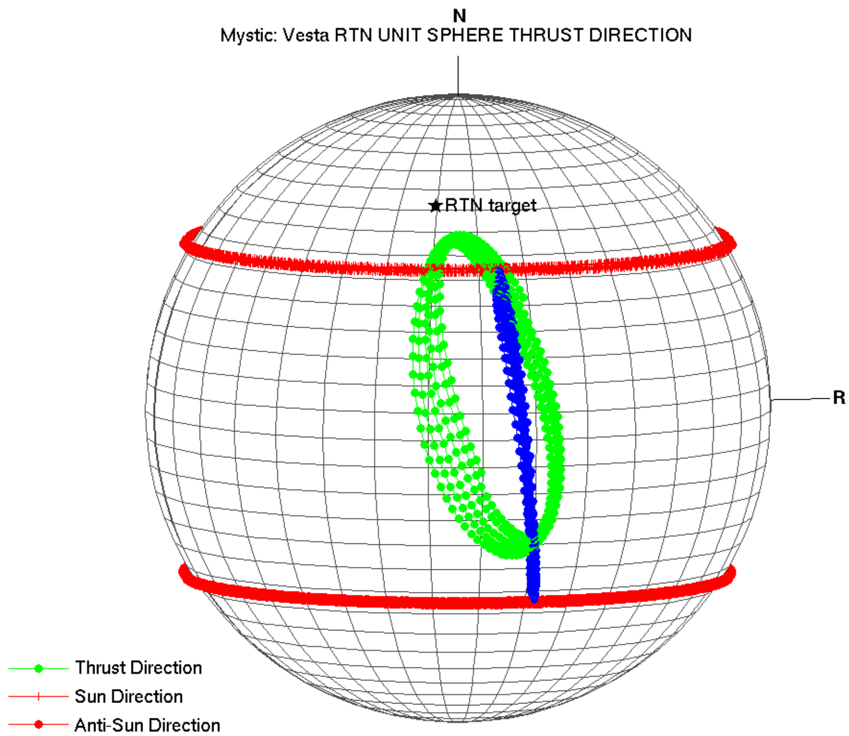

Figure 9. An example of a mass optimal versus a direction optimal thrust direction evolution in the Radial - Transverse - Normal or "RTN" frame. The mass optimal solution is indicated in blue and the direction optimal solution is indicated in green. The Sun and anti-Sun directions are indicated in red.

9. The RTN direction target $a\hat{r} + b\hat{t} + c\hat{n}$ location is labeled and indicated by a black star on the northern hemisphere of the unit sphere in Figure 9. The direction objective successfully moved the thrusting above the anti-Sun direction. It may appear that more thrusting is occurring near the Sun direction in the direction optimal solution, but this is not the case. The phasing of the Sun and thrust directions is such that thrusting still avoids the Sun direction. The RTN rotating frame plots do not provide this phasing information. Inspection of inertial frame thrust direction plots demonstrates the improvement in avoiding the anti-Sun direction - see the left-hand unit sphere in Figure 10. The right-hand plot in Figure 10 indicates that the thrusting near the Sun direction is not significantly altered by the direction optimal solution. The direction optimal solution is flyable by the Dawn spacecraft. Flight feasibility was achieved with only 50 minutes of slack thrusting in this example. The total mass-optimal thrust time was 2,031 minutes so the slack represents an increase of only 2.5%.

In general, targeting in the RTN frame is more successful because spiral thrust directions are usually more compact in this frame. When the initial mass-optimal thrusting is compact then its reaction to thrust direction objectives is generally easier to predict.

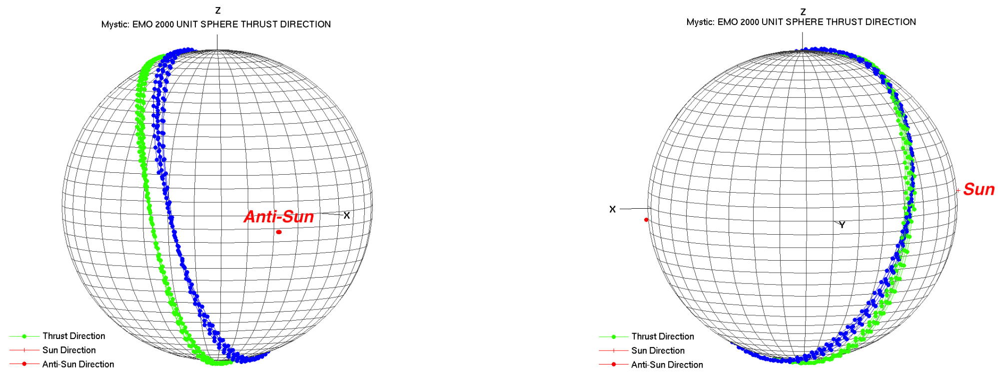

Figure 10. Two views of the thrust directions for the mass-optimal solution or zero slack solution (blue) and a RTN direction optimal solution with 50 minutes of slack (green) plotted on the EMO-2000 frame unit sphere.

CONCLUSION

Direction optimization was essential to the success of the Dawn Discovery mission's Vesta orbital operations. Flyable maneuvers were obtained in a timely manner as a result of the simplicity and robustness of direction optimization formulations described in this paper. No demonstrated local minima were found in direction optimal problems that did not have a corresponding mass-optimal local minima. This result is very important compared to the likely large number of superfluous local minima generated by a six-degree of freedom (full attitude modeling) spacecraft formulation. A lack of troublesome local minima allows very simple initial guesses to initialize the SDC optimization process. Typically, an initial guess consists of thrust parallel or anti-parallel to the Vesta relative velocity. An initial guess trajectory did not need to end near the final required position and velocity. The initial guess trajectory could even end on the side of Vesta opposite to the required position target. Robust convergence was obtained for the direction objectives described here. The concept of direction optimization may have applications outside of meeting attitude agility constraints. For example, direction optimization may be useful for some close proximity formation flight problems or for designing science acquisition during powered flight.

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NOMENCLATURE

t	Time
J	The numerical objective value (scaler)
J^*	The optimal objective value
v	The dynamic control vector in the SDC formulation, function of time
w	The static control vector in the SDC formulation
x	The state vector in the SDC formulation, function of time

x_7	The seventh component of the state vector, spacecraft mass
v^*	The optimal dynamic control vector
w^*	The optimal static control vector
t_o, t_f	The beginning and ending trajectory epoch
$F(x, v, w, t)$	The objective accumulation per time (scaler function)
$f(w)$	The trajectory end epoch as a function of the static control
$H_i(x, v, w, t)$	The objective accrued at a finite number of intermediate times t_i (scaler function)
$G(x, v, w, t)$	The portion of the objective accrued at the trajectory end time t_f (scaler function)
$T(x, v, w, t)$	The state transition equation: time rate of change of the state (scaler function)
$\Gamma(w)$	The initial spacecraft state as a vector function of the static control
K	A constant chosen to be the initial spacecraft mass
$\hat{D}$	The target thrust direction for attractor type objectives
$\hat{S}$	A direction that defines the pole for pole repulsor type objectives
τ	A subset of times during which a direction objective is applied
$\hat{V}$	A unit vector in the direction of the spacecraft - central body relative velocity
X_{sc}, X_{body}	The spacecraft and central body positions in an inertial frame
V_{sc}, V_{body}	The spacecraft and central body velocities in an inertial frame
$\hat{r}$	A unit vector in the body-spacecraft radial direction
$\hat{t}$	A unit vector in the transverse direction (local horizontal in velocity direction)
$\hat{n}$	a unit vector in the orbit normal direction
T_M	A fixed thrust magnitude

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