GRAIL Science Data System
Orbit Determination:
Approach, Strategy, and Performance

Eugene Fahnestock, Sami Asmar, Daniel Kahan, Alex Konopliv, Gerhard Kruizinga, Kamal Oudrhiri, Meegyeong Paik, Ryan Park, Dmitry Strekalov, Dah-Ning Yuan

Jet Propulsion Laboratory, California Institute of Technology

23rd AAS/AIAA Spaceflight Mechanics Meeting, Kauai, Hawaii, Feb 2013
Copyright 2013 California Institute of Technology. Government sponsorship acknowledged.
Introduction

- Iterative “Bootstrapping Strategy” described previously (Kruizinga paper AAS 13-270)

- This involved close coordination between “Level 1” and “Level 2” within GRAIL Science Data System (SDS) team
  - L1 = orbit determination (OD) passes conducted to get epoch-state to propagate, for making corrections during pre-processing into instantaneous inter-spacecraft KBRR
  - L2 = actual gravity field estimation, using adjusted KBRR
Introduction

- Iterative “Bootstrapping Strategy” described previously (Kruizinga paper AAS 13-270)

- This involved close coordination between “Level 1” and “Level 2” within GRAIL Science Data System (SDS) team

  L1 = orbit determination (OD) passes conducted to get epoch-state to propagate, for making corrections during pre-processing into instantaneous inter-spacecraft KBRR

  L2 = actual gravity field estimation, using adjusted KBRR

- Several L1 OD passes made, this paper describes details of methodology, and flight results
Setup, Arcs Strategy, Etc.

- For all OD, employed **Multiple Interferometric Ranging Analysis** using **GPS Ensemble (MIRAGE)** software
  - Trajectory and partials integration
  - Observations processing
  - Filtering

- Used dual-core JPL flight-ops machine, later JPL cluster

- Chose 36 hr data arcs centered on each day, \( n \)
  - Nominally 18:00:00 UTC day \( n-1 \) to 06:00:00 UTC day \( n+1 \)
  - 12 hr overlaps
  - but broke arcs at “large” (\( \Delta V > 10 \) mm/s) maneuvers, not AMDs

- Size of lunar gravity fields used increased over mission, started with LP150Q

Majority of computational burden here for L1, vs. L2 (fewer sol’n params.)
Dynamics Modeling, Integration, & Solution Method

- Dynamics formulation:

\[
\dot{\mathbf{r}} = \mathbf{a} = a_{gPM} + a_{goblate} + a_{srp} + a_{lrc} + a_{rel} + a_{str} + a_{misc.} + a_{uf}
\]

- All propagation done in lunicentric, inertial frame & TDB time
- Legacy DIVA numerical integrator → used to propagate state, STM, partials matrix
- SRIF variety of least-squares filtering technique applied
- Size of lunar gravity fields used varied for first 2 “quicklook” OD passes; consistent for later “final” passes

Most soak-up parameters in here
## Quicklook Vs. Final OD

<table>
<thead>
<tr>
<th>OD pass #</th>
<th>F1* weight (mm/s)</th>
<th>F2* weight (mm/s)</th>
<th>KBRR (version, sample times)</th>
<th>KBRR weight (μm/s)</th>
<th>Attitude data used?</th>
<th>Antenna location corr. to Doppler?</th>
<th>PCI corr. to KBRR?</th>
<th>Other corr. to KBRR?</th>
<th>Gravity Field Used</th>
<th>Time span processed</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.2</td>
<td>1.0</td>
<td>None</td>
<td>n/a</td>
<td>No, used apriori model</td>
<td>No, since no vectors input</td>
<td>n/a</td>
<td>n/a</td>
<td>Varied, from LP150q thru GL0420A</td>
<td>Full PM &amp; Full XM</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
<td>1.0</td>
<td>Version 0 5 s PM, 10 s XM</td>
<td>10.0</td>
<td>No, used apriori model</td>
<td>Yes, w/ apriori att. model</td>
<td>No, not done</td>
<td>No, not done</td>
<td>Varied, from LP150q thru GL0420A</td>
<td>Full PM &amp; Full XM</td>
</tr>
<tr>
<td>3</td>
<td>0.2</td>
<td>1.0</td>
<td>Version 1 5 s PM, 2 s XM</td>
<td>10.0</td>
<td>Yes</td>
<td>Yes, w/ real attitude</td>
<td>Yes</td>
<td>Yes</td>
<td>210x210</td>
<td>PM start until 4/19/2012</td>
</tr>
<tr>
<td>4</td>
<td>0.2</td>
<td>1.0</td>
<td>Version 1 5 s PM, 2 s XM</td>
<td>10.0</td>
<td>Yes</td>
<td>Yes, w/ real attitude</td>
<td>Yes</td>
<td>Yes</td>
<td>270x270</td>
<td>PM start until 5/09/2012</td>
</tr>
<tr>
<td>5</td>
<td>0.2</td>
<td>1.0</td>
<td>Version 1 5 s PM, 2 s XM</td>
<td>10.0</td>
<td>Yes</td>
<td>Yes, w/ real attitude</td>
<td>Yes</td>
<td>Yes</td>
<td>420x420</td>
<td>Full PM</td>
</tr>
<tr>
<td>6</td>
<td>0.2</td>
<td>1.0</td>
<td>Version 1 5 s PM, 2 s XM</td>
<td>10.0</td>
<td>Yes</td>
<td>Yes, w/ real attitude</td>
<td>Yes</td>
<td>Yes</td>
<td>420x420 = GL0420A</td>
<td>Full PM &amp; Full XM</td>
</tr>
<tr>
<td>7</td>
<td>0.3</td>
<td>0.3</td>
<td>Version 1 5 s PM, 2 s XM</td>
<td>1.0</td>
<td>Yes</td>
<td>Yes, w/ real attitude</td>
<td>Yes</td>
<td>Yes</td>
<td>540x540</td>
<td>Full XM</td>
</tr>
<tr>
<td>8</td>
<td>0.3</td>
<td>0.3</td>
<td>Version 1 5 s PM, 2 s XM</td>
<td>1.0</td>
<td>Yes</td>
<td>Yes, w/ real attitude</td>
<td>Yes</td>
<td>Yes</td>
<td>660x660 (trun. 600x600)</td>
<td>11/06/2012 to end of XM</td>
</tr>
<tr>
<td>9</td>
<td>0.3</td>
<td>0.3</td>
<td>Version 2 5 s PM, 2 s XM</td>
<td>PM 0.06</td>
<td>Yes</td>
<td>Yes, w/ real attitude</td>
<td>Yes</td>
<td>Yes</td>
<td>660x660 from PM only</td>
<td>Full PM</td>
</tr>
<tr>
<td>10</td>
<td>0.3</td>
<td>0.3</td>
<td>Version 2 5 s PM, 2 s XM</td>
<td>PM 0.06, XM 0.2 until 10/3, 1.0 after 10/3</td>
<td>Yes</td>
<td>Yes, w/ real attitude</td>
<td>Yes</td>
<td>Yes</td>
<td>660x660 from PM &amp; XM</td>
<td>Full PM &amp; Full XM</td>
</tr>
</tbody>
</table>

* Here F1 = closed-loop X-band one-way Doppler; F2 = closed-loop S-band two-way Doppler; All OD passes used both with 60 s compression time

---

23rd AAS/AIAA Spaceflight Mechanics Meeting, Kauai, Hawaii, Feb 2013
## Parameterization

<table>
<thead>
<tr>
<th>Parameter description (units)</th>
<th>Max #</th>
<th>Sub-arc length, $\Delta t$</th>
<th>nominal</th>
<th>apriori $\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>FOR EACH SPACECRAFT</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>X, Y, Z, cartesian s/c epoch-state position (km)</td>
<td>3</td>
<td>n/a</td>
<td>initialize from previous sol’n</td>
<td>$10^4$ for pass #1, $10^{-2}$ after</td>
</tr>
<tr>
<td>DX, DY, DZ, cartesian s/c epoch-state velocity (km/s)</td>
<td>3</td>
<td>n/a</td>
<td>initialize from previous sol’n</td>
<td>$10$ for pass #1, $10^{-5}$ after</td>
</tr>
<tr>
<td>SRP scale factor parallel to sun-to-spacecraft vector</td>
<td>1</td>
<td>whole arc</td>
<td>1.0</td>
<td>$10^{-2}$</td>
</tr>
<tr>
<td>SRP scale factors perp. to sun-to-spacecraft vector</td>
<td>2</td>
<td>whole arc</td>
<td>0</td>
<td>$5 \times 10^{-3}$</td>
</tr>
<tr>
<td>one-way Doppler bias (Hz)</td>
<td>50</td>
<td>whole arc</td>
<td>0</td>
<td>$10^{20}$ for pass #1, $10^{2}$ after</td>
</tr>
<tr>
<td>one-way Doppler drift (Hz/s)</td>
<td>50</td>
<td>whole arc</td>
<td>10$^{20}$ for pass #1, $10^{-7}$ after</td>
<td>$10^{20}$ for pass #1, $10^{-12}$ after</td>
</tr>
<tr>
<td>one-way Doppler drift rate (Hz/s$^2$)</td>
<td>50</td>
<td>whole arc</td>
<td>10$^{20}$ for pass #1, $10^{-12}$ after</td>
<td>$10^{20}$ for pass #1, $10^{-12}$ after</td>
</tr>
<tr>
<td>bias, amplitude of $\cos \theta$, $\sin \theta$, in $R$ (km/s$^2$)</td>
<td>3x24</td>
<td>6813 s for PM, 6630 s for XM</td>
<td>0</td>
<td>$10^{-20}$ (effectively not estim.)</td>
</tr>
<tr>
<td>amplitude of $\cos(2\theta)$, $\sin(2\theta)$, in $R$ (km/s$^2$)</td>
<td>2x24</td>
<td>6813 s for PM, 6630 s for XM</td>
<td>0</td>
<td>$10^{-20}$ (effectively not estim.)</td>
</tr>
<tr>
<td>bias, amplitude of $\cos \theta$, $\sin \theta$, in $T$ (km/s$^2$)</td>
<td>3x24</td>
<td>6813 s for PM, 6630 s for XM</td>
<td>0</td>
<td>$10^{-11}$</td>
</tr>
<tr>
<td>amplitude of $\cos(2\theta)$, $\sin(2\theta)$, in $T$ (km/s$^2$)</td>
<td>2x24</td>
<td>6813 s for PM, 6630 s for XM</td>
<td>0</td>
<td>$10^{-20}$ (effectively not estim.)</td>
</tr>
<tr>
<td>bias, amplitude of $\cos \theta$, $\sin \theta$, in $N$ (km/s$^2$)</td>
<td>3x24</td>
<td>6813 s for PM, 6630 s for XM</td>
<td>0</td>
<td>$10^{-11}$</td>
</tr>
<tr>
<td>amplitude of $\cos(2\theta)$, $\sin(2\theta)$, in $N$ (km/s$^2$)</td>
<td>2x24</td>
<td>6813 s for PM, 6630 s for XM</td>
<td>0</td>
<td>$10^{-20}$ (effectively not estim.)</td>
</tr>
<tr>
<td>x, y, z components of small force (thruster) event accelerations in frame of integration (km/s$^2$)</td>
<td>3x4</td>
<td>n/a</td>
<td>0</td>
<td>$10^{-5}$</td>
</tr>
<tr>
<td><strong>FOR BOTH SPACECRAFT / WHOLE SYSTEM</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>KBRR bias</td>
<td>50</td>
<td>whole arc</td>
<td>0</td>
<td>$10^{-7}$</td>
</tr>
<tr>
<td>KBRR drift</td>
<td>50</td>
<td>whole arc</td>
<td>0</td>
<td>$10^{-10}$</td>
</tr>
<tr>
<td>KBRR cosine and sine periodic amplitudes</td>
<td>2x50</td>
<td>whole arc</td>
<td>0</td>
<td>$10^{-20}$ (effectively not estim.)</td>
</tr>
<tr>
<td>KBRR time tag bias</td>
<td>3</td>
<td>whole arc**</td>
<td>0</td>
<td>100 (widely unconstrained)</td>
</tr>
</tbody>
</table>

**TOTAL SOL’N PARAMETERS** 1265
Flight Results (Doppler Data Residuals, PM)

420x420 field from PM only (released)

23rd AAS/AIAA Spaceflight Mechanics Meeting, Kauai, Hawaii, Feb 2013
Flight Results (Doppler Data Residuals, PM)

660x660 field from PM & XM

- GRAIL-A, F1
- GRAIL-A, F2
- GRAIL-B, F1
- GRAIL-B, F2

Arc ID

01-MAR-2012k
03-MAR-2012s
06-MAR-2012s
09-MAR-2012s
12-MAR-2012s
15-MAR-2012s
18-MAR-2012s
21-MAR-2012s
24-MAR-2012s
27-MAR-2012s
30-MAR-2012s
02-APR-2012s
05-APR-2012s
08-APR-2012s
11-APR-2012s
14-APR-2012s
17-APR-2012s
20-APR-2012s
23-APR-2012s
26-APR-2012s
02-MAY-2012s
05-MAY-2012s
08-MAY-2012s
11-MAY-2012s
14-MAY-2012s
17-MAY-2012s
20-MAY-2012s
23-MAY-2012s
26-MAY-2012s

23rd AAS/AIAA Spaceflight Mechanics Meeting, Kauai, Hawaii, Feb 2013
Flight Results (Doppler Data Residuals, XM)

420x420 field from PM only (released)

RMS residuals (mm/s)

Arc ID

23rd AAS/AIAA Spaceflight Mechanics Meeting, Kauai, Hawaii, Feb 2013
Flight Results (Doppler Data Residuals, XM)

660x660 field from PM & XM

RMS residuals (mm/s)

Arc ID


23rd AAS/AIAA Spaceflight Mechanics Meeting, Kauai, Hawaii, Feb 2013
Flight Results (KBRR Data Residuals, PM)

- DSN + corrected KBRR pass, 210x210 field
- DSN + uncorrected KBRR pass, changing field
- DSN + corrected KBRR pass, 270x270 field
- DSN + corrected KBRR pass, 420x420 field
- DSN + corrected KBRR pass, 420x420 field (released)
- DSN + corrected KBRR pass, 660x660 field from PM only
- DSN + corrected KBRR pass, 660x660 field from PM & XM

RMS residuals (micron/s)

Arc ID

01-MAR-2012
03-MAR-2012
06-MAR-2012
08-MAR-2012
15-MAR-2012
18-MAR-2012
21-MAR-2012
24-MAR-2012
30-MAR-2012
02-APR-2012
05-APR-2012
08-APR-2012
11-APR-2012
14-APR-2012
17-APR-2012
20-APR-2012
23-APR-2012
26-APR-2012
02-MAY-2012
05-MAY-2012
08-MAY-2012
11-MAY-2012
14-MAY-2012
17-MAY-2012
20-MAY-2012
23-MAY-2012
26-MAY-2012

23rd AAS/AIAA Spaceflight Mechanics Meeting, Kauai, Hawaii, Feb 2013
Flight Results (KBRR Data Residuals, PM)

- DSN + corrected KBRR pass, 210x210 field
- DSN + uncorrected KBRR pass, changing field
- DSN + corrected KBRR pass, 270x270 field
- DSN + corrected KBRR pass, 420x420 field
- DSN + corrected KBRR pass, 420x420 field (released)
- DSN + corrected KBRR pass, 660x660 field from PM only
- DSN + corrected KBRR pass, 660x660 field from PM & XM

RMS residuals (micron/s)

Arc ID

23rd AAS/AIAA Spaceflight Mechanics Meeting, Kauai, Hawaii, Feb 2013
Flight Results (KBRR Data Residuals, XM)

- 420x420 field (released)
- 540x540 field from PM & XM til 10/03
- 660x660 field from PM & XM til 10/24, truncated to 600x600
- 660x660 field from PM & XM

RMS residuals (micron/s)

Arc ID

23rd AAS/AIAA Spaceflight Mechanics Meeting, Kauai, Hawaii, Feb 2013
Flight Results (KBRR Data Residuals, XM)

- 420x420 field (released)
- 540x540 field from PM & XM til 10/03
- 660x660 field form PM & XM til 10/24, truncated to 600x600
- 660x660 field from PM & XM

RMS residuals (micron/s)

Arc ID

23rd AAS/AIAA Spaceflight Mechanics Meeting, Kauai, Hawaii, Feb 2013
Flight Results (KBRR Time Tag Bias Est., PM)

- DSN + corrected KBRR pass, 210x210 field
- DSN + uncorrected KBRR pass, changing field
- DSN + corrected KBRR pass, 270x270 field
- DSN + corrected KBRR pass, 420x420 field

Arc ID

23rd AAS/AIAA Spaceflight Mechanics Meeting, Kauai, Hawaii, Feb 2013
Flight Results (KBRR Time Tag Bias Est., XM)

- 420x420 field (released)
- 540x540 field from PM & XM til 10/03
- 660x660 field from PM & XM til 10/24, truncated to 600x600
- 660x660 field from PM & XM

Arc ID

23rd AAS/AIAA Spaceflight Mechanics Meeting, Kauai, Hawaii, Feb 2013
Primary Mission: (small further $\Delta$ with change of fields)

![Graphs showing RSS of $\Delta R$, $\Delta T$, and $\Delta N$ over time.](image)

- $RSS$ of $\Delta R (m)$
  - 0.15 m
  - 2 m
  - 0.7 m

23rd AAS/AIAA Spaceflight Mechanics Meeting, Kauai, Hawaii, Feb 2013
Flight Results (Orbit Overlap Statistics, XM)

Extended Mission:

- GRAIL-A, GL0420A
- GRAIL-B, GL0420A
- GRAIL-A, 660x660 field from PM & XM
- GRAIL-B, 660x660 field from PM & XM

Time:
- Aug 31
- Sep 09
- Sep 19
- Sep 28
- Oct 08
- Oct 17
- Oct 27
- Nov 05
- Nov 15
- Nov 24
- Dec 04
- Dec 14

RSS of Δ R (m):
- 0
- 10
- 20
- 30

RSS of Δ T (m):
- 0
- 50
- 100
- 150
- 200

RSS of Δ N (m):
- 0
- 10
- 20
- 30
- 40

30 m

~150 m

~30 m

23rd AAS/AIAA Spaceflight Mechanics Meeting, Kauai, Hawaii, Feb 2013
Flight Results (Orbit Overlap Statistics, XM)

Extended Mission: (same, zoomed to show large Δ w/ same change of fields)

- 0.8 m
- ~9.5 m
- ~1.2 m
Summary

- Key aspects of our approach:
  1) Concurrently estimating the state of both s/c together
  2) Excellent data set!
     - KBRR has observability of relative s/c motion and tie to body-fixed frame /field
     - Doppler has observability of orbits’ absolute motion, inertial frame tie
  3) Bootstrapping using successively higher degree gravity fields, to handle lowering altitudes, with more gravity signal

- Seem to have already converged to noise floor of data for PM, esp. “quiet” central portion: Yet to converge to same for XM

- More iterations, use of potentially better RSR one-way Doppler data planned

- Great science from gravity fields to date (see next talk)