Automating Initial Guess Generation for High Fidelity Trajectory Optimization Tools

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Short Abstract

Many academic studies in spaceflight dynamics rely on simplified dynamical models, such as restricted three-body models or averaged forms of the equations of motion of an orbiter. In practice, the end result of these preliminary orbit studies needs to be transformed into more realistic models, in particular to generate good initial guesses for high-fidelity trajectory optimization tools like Mystic. This paper reviews and extends some of the approaches used in the literature to perform such a task, and explores the inherent trade-offs of such a transformation with a view toward automating it for the case of ballistic arcs. Sample test cases in the libration point regimes and small body orbiter transfers are presented.

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Extended Abstract

Motivations: While the design process of a space mission trajectory does not necessarily follow a single approach, many studies rely on simplified dynamical models to provide an overview of a large set of orbit options before proceeding to more detailed design [1]. These models include, for example, the circular restricted three-body problem (CR3BP) for multi-body dynamics [9], the augmented Hill three body problem (AH3BP) for distant small body orbits [7], or even more complex models, such as higher order rotating gravity fields [5]. The list is in fact not restrictive and typically tailored to the problem at hand.

By their nature, these models present dynamical features not existing in a more realistic setting. Of these are periodic orbits (which do not exist as soon as the model becomes time-varying), integrals of motion, or symmetries. The questions of which feature is preserved or destroyed (and to which extent) when the force field is slightly perturbed is addressed in the field of bifurcation analysis [4]. In spaceflight, this situation occurs each time a baseline trajectory computed in a simplified model is "exported" into a more realistic model.

While bifurcation analysis represents a large and technical domain of research that can be difficult to apply, the practical nature of the needs of spaceflight problems may not warrant for an exhaustive mathematical theory. Rather a variety of practical approaches have been used in the literature to tackle different situations. This paper aims at reviewing some of these methods and at extracting a common framework to achieve this task semi-automatically. In particular, we assume that a "good" design is already obtained in a simplified model and the paper explores the task of transforming this design in a realistic model to be used as initial guess in a higher fidelity optimization tool such as Mystic. Examples of such transfers are given in the case of libration point orbits and small body orbiter transfer scenarios.

Previous work: The transformation of a trajectory between a simplified and higher fidelity models has been addressed in various ways, with emphasis on libration point missions. In particular, for quasi-periodic orbits in these regions, Gómez et al. [3] proposed a multiple-shooting method coupled with a step by step transformation between a sequence of model to achieve the desired result. More recently, Kolemen et al. [6] proposed a collocation method. In [1], a two level differential correction process is described, where impulsive maneuvers at selected patch points along a trajectory are used to correct for the small changes in the force model. This approach in particular has been implemented in the LTOOL software and used for the trajectory design of the Genesis mission. In this approach, the level I corrector consists of a simple shooting method to achieve position continuity between arcs, while the level II corrector enforces imposed algebraic constraints at the patch points by varying the patch point positions and times using a minimum norm update. While the approach proved to perform well in practice, it requires a fairly close initial guess and closeness of the dynamical models (which occurs naturally for libration point missions where forces are small in this region of space). Along the same lines, Lo et al. [8] used a least square approach to transition a Europa Orbiter trajectory from a realistic model to a CR3BP model. In all these approaches, the challenge of these transformation is to keep the trajectory "shape" (or some property) as close as possible as the initial trajectory and often requires the user continued interaction to gauge of the results an update the number of patch points as appropriate.

A different set of viewpoints has also been explored in relation to the analysis of family of orbit and transfers. One approach used to compute periodic orbit families consists of continuation. While in the case of periodic orbits in the CR3BP or similar conservative systems, the continuation parameter is intrinsic to the dynamics (for example and integral of motion or the period), the theory applies as well on "external" model parameters and has been considered for ballistic arcs around small bodies in [2]. This allows the deformation of a particular orbit arc across dynamical

model and will be further considered in this paper.

Research goals: By extracting key features of the previous methods, this research aims at providing a systematic framework to transform ballistic arcs from simplified to higher fidelity dynamical models and explore the inherent trade-offs stemming from the dynamics. While the application of the techniques developed in this paper still require the user to select the desired goals of the transformation, this input is kept to a minimum, allowing a semi-automation of the transformation process. A software supporting this method with sample scenarios will also be described.

Methods and sample results: Out of the previous methods, a common feature is the segmentation of an orbit into several arcs that allows both sensitivity control and localized correction toward the desired result. In the approach of Gómez et al. [3] the use of intermediary models to slowly increase the complexity of the dynamics is also leveraged in the proposed approach. However, in addition to hand-picking the intermediary dynamical models, a systematic embedding of each model within a one parameter family allows the use of continuation algorithms to be applied. More precisely, if X denotes the state of a spacecraft and $F_1(X)$ the simplified dynamics in which the orbit arc $X(t)$ is computed, and $F_2(X)$ the desired complex model, the following set of models is considered: $F(X,\lambda) = (1-\lambda).F_1(X)+\lambda.F_2(X)$. When the continuation parameter λ varies from 0 to 1 the dynamical models are smoothly deformed from F_1 to F_2. The orbit arc $X(t)$ solution to $\frac{d}{dt}X(t) = F_1(X(t))$ with boundary conditions can now be deformed in a family of arcs $X(t,\lambda)$ such that $\frac{d}{dt}X(t,\lambda) = F(X(t,\lambda),\lambda)$. Figure 1, for example, shows the smooth deformation of a transfer arc between two fixed positions in the augmented Hill's problem as the magnitude of the solar radiation pressure is increased.

One benefit of this approach when compared to directly "jumping" between distant dynamical models directly is a better control of the deformation step. As compared to [2], this paper considers a larger class of constraints to impose on the trajectory rather than fixed boundary conditions or transfer time. In particular, following the collocation approach, functional representations of the orbit arcs (e.g. using polynomials) allow for the consideration of continuous constraints along an arc, and provide a generalization of the above methods. The paper will thus further explore these techniques (and their variations) in light of the previous works mentioned. These techniques will then be applied to sample scenarios, in an effort to demonstrate the strengths and limitations of these approaches.

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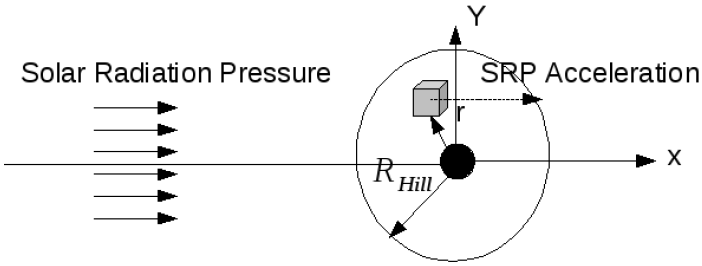
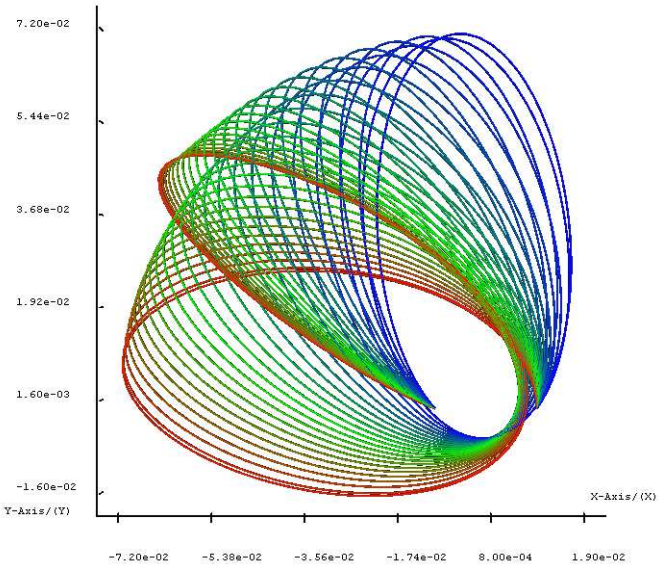


(a)



(b)

Figure 1: **(a) Schematic of the augmented Hill 3-body problem frame and forces. (b) Continuation of an orbit arc joining two opposite positions in the augmented Hill three-body problem as the solar radiation pressure varies from low (red) to high (blue).**

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