
Adaptive Periodic-Correlation Algorithm for Extended-Scene Shack-Hartmann Wavefront Sensing

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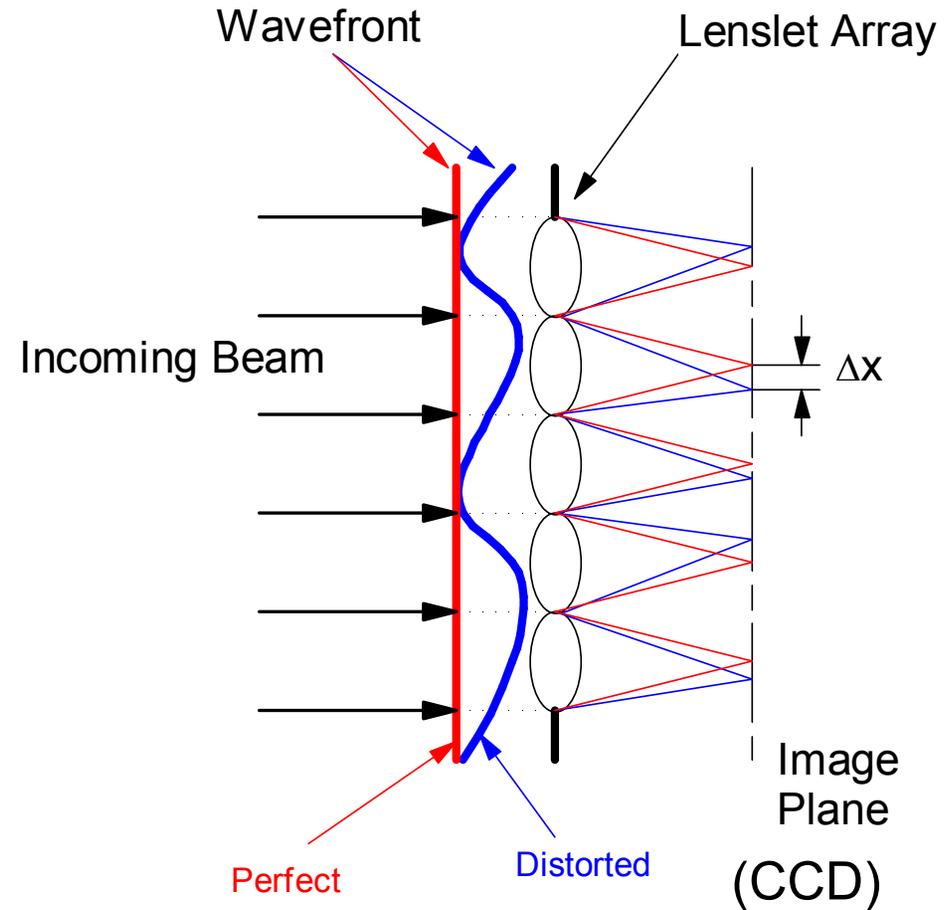
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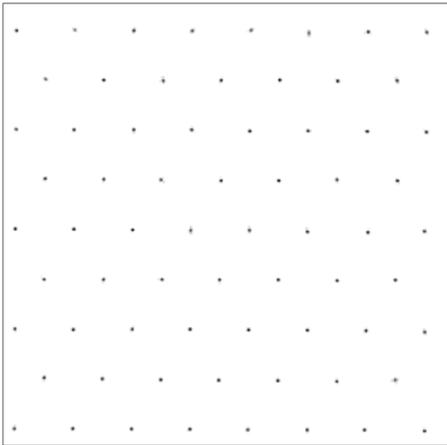
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Principle of Conventional SH-WFS

- A Shack-Hartmann sensor places a lenslet array at a plane conjugate to the WF error source
- Each sub-aperture lenslet samples the WF in the corresponding patch of the WF
- When observing a star, the image is an array of spots, each of which is a sub-aperture PSF
 - Δx is proportional to local wavefront tilt
 - Wavefront-sensing \rightarrow Finding Δx for all sub-images
 - Use centroiding (center-of-mass) method to find Δx



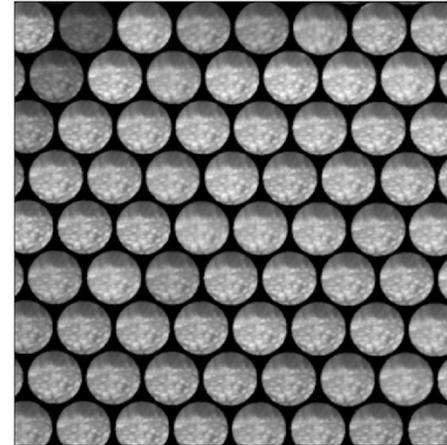
Extended-Scene SH-WFS



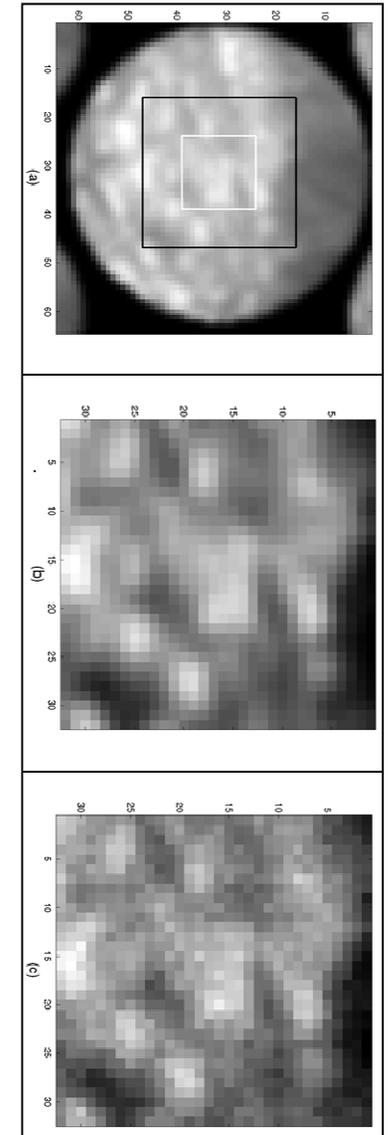
(a)



(b)



(c)



- The Shack-Hartmann Camera produces images as the convolution of the conventional image (limited by a tight field stop) with a regular grid of subaperture PSFs, as above
- Each subaperture is much blurrier than the main image, as its diffraction limit is defined by the subaperture, not the full aperture
- Subaperture image shown at right at full size (64x64)
- APC algorithm uses the central 32x32 pix box, and then identifies the multi-pixel shift of the features in the inner 16x16 pix cell with respect to a reference cell
- The subaperture-to-subaperture cell shifts give a measure of subaperture tilt

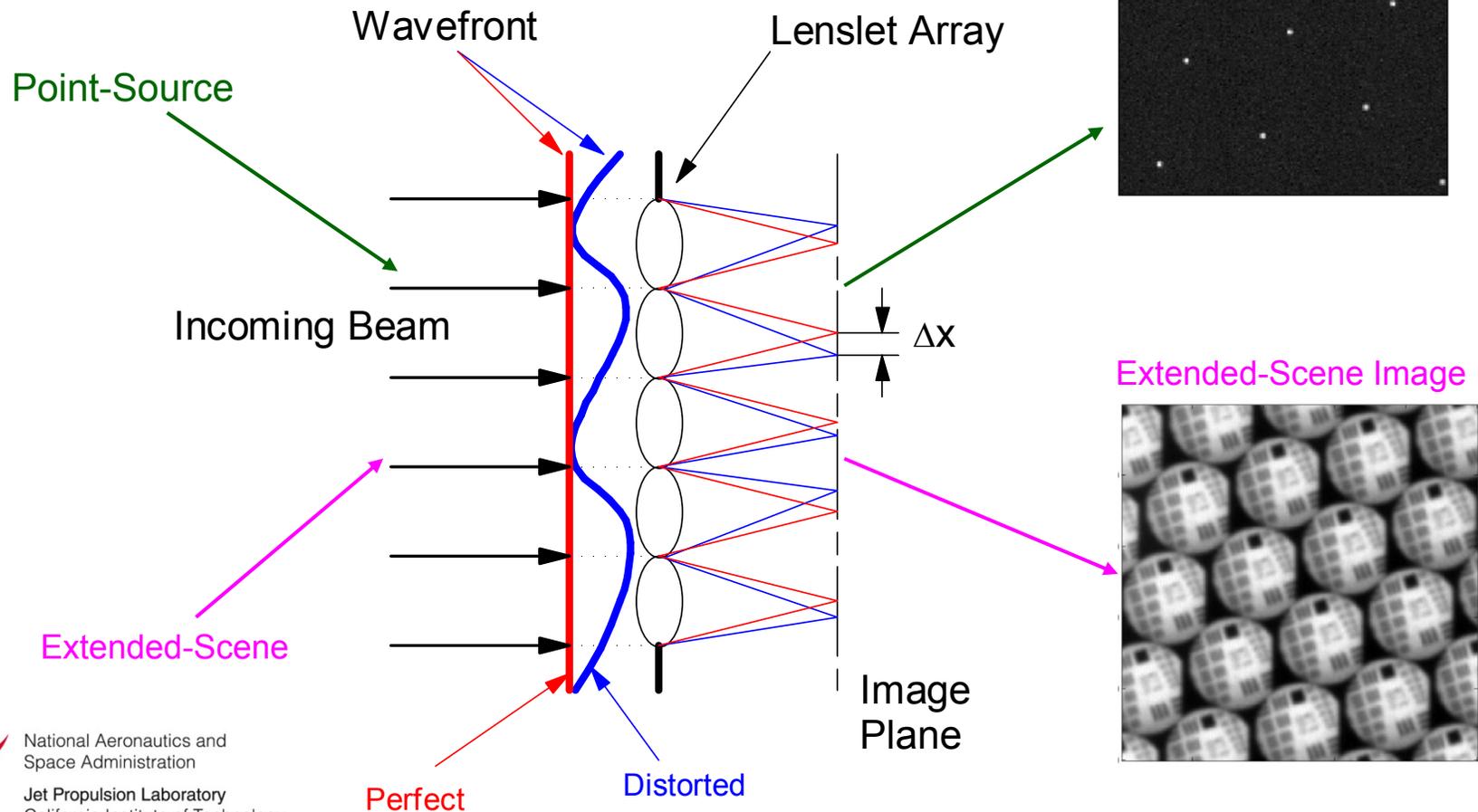


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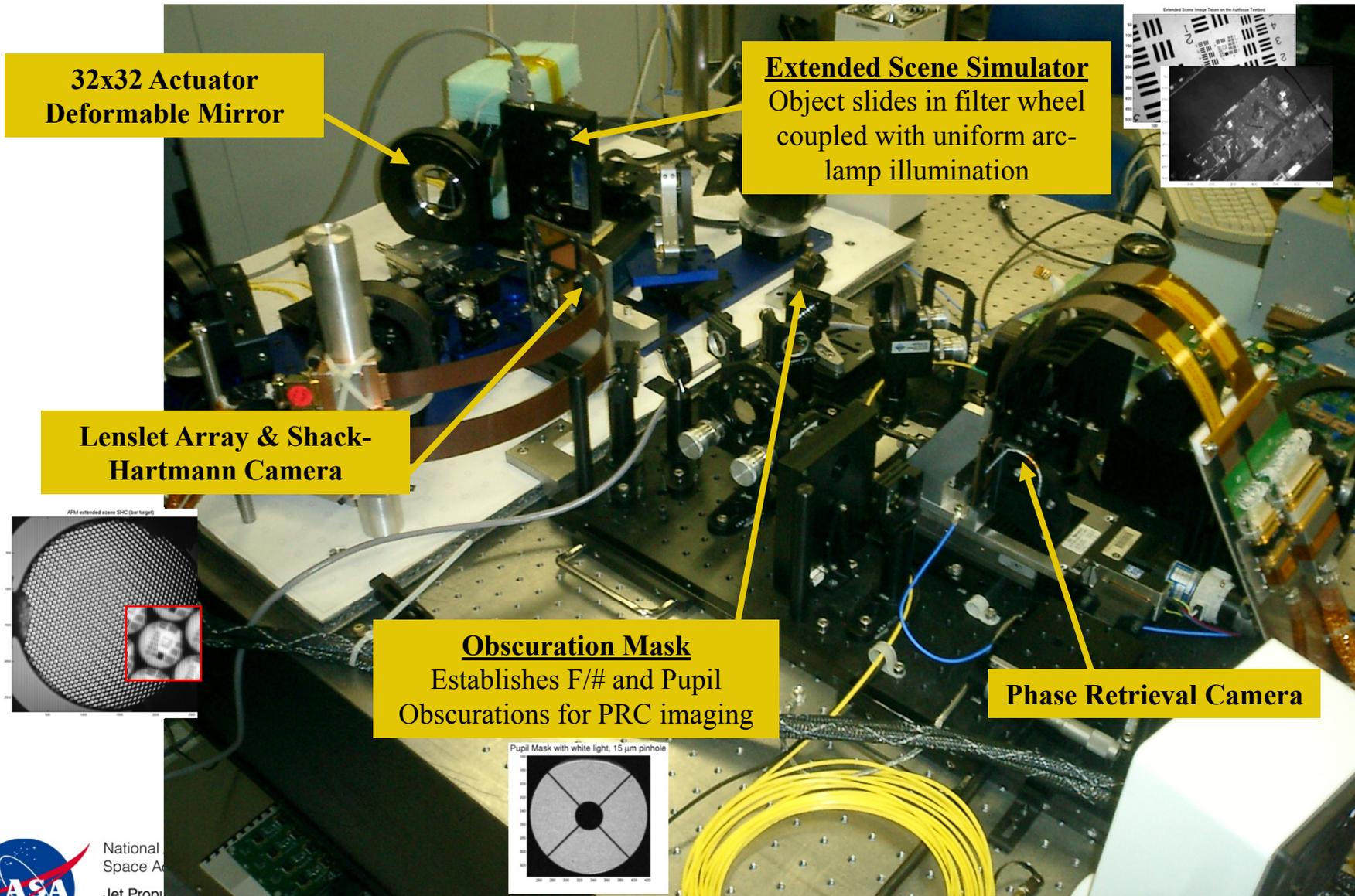
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Point-Source (Star) versus Extended-Scene

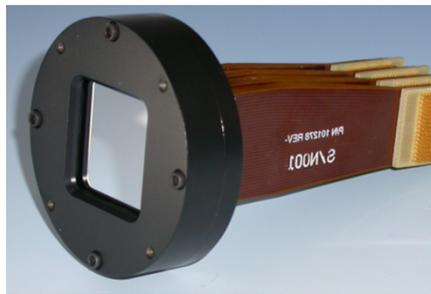
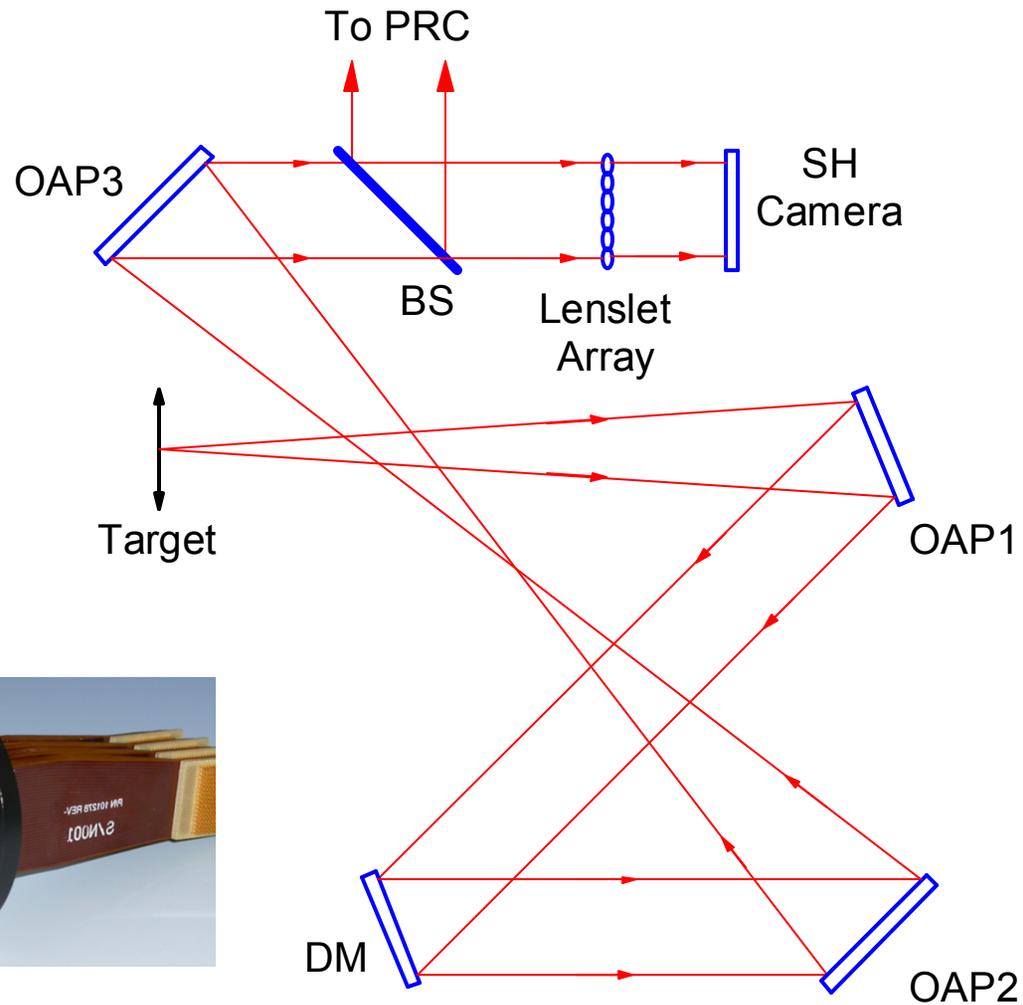
- Following images were measured at JPL SH-WFS Testbed
 - Can be used with both point-source and extended-scene
 - Each spot-image is replaced by a sub-image in extended-scene SH-WFS
 - Local wavefront distortion causes a sub-image to shift from ideal position
 - SH camera provides large capture range WFS&C



Extended Scene SHS Testbed



Extended-Scene SHS Testbed — Schematic Diagram



Deformable-Mirror (DM)



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Why Need Adaptive Periodic-Correlation (APC) Algorithm?

- Previously reported Adaptive Cross-Correlation (ACC) algorithm is based on the following property of Fourier-transform:
 - Shift in time-domain \leftrightarrow Linear-phase in frequency-domain
 - In Fourier optics, $t \rightarrow (x, y)$ and $f \rightarrow (u, v)$
- Fourier-transform pair—Shown as one-dimensional for simplicity:

$$s(x) \leftrightarrow \hat{s}(u)$$

$$s(x - \Delta x) \leftrightarrow \hat{s}(u) e^{-j2\pi\Delta x u}$$

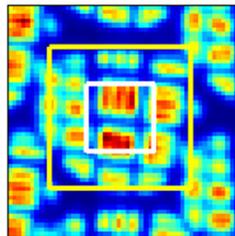
- Sub-image shifts are obtained from the linear-phase of cross-correlation spectrum
- Works well when sub-images only shifted but not distorted
 - This is the case when the number of cells is small: 20 to several hundred cells
- Some applications require >1000 cells
 - Sub-images are not only shifted, but also distorted—Sub-images are not uniform
 - In such cases, linear-phase can become zero when the actual shift is not zero
 - Need an alternative method that works with >1000 cells \rightarrow APC algorithm



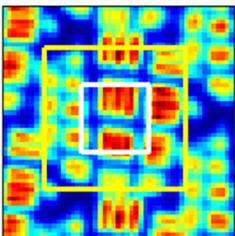
An Example of Non-Uniform Sub-Images in an SHC Frame

- The SHC image was taken on an ES-SHS testbed at JPL
- Yellow- and white-square boxes have sizes of 32x32 pix & 16x16 pix, respectively
- The main cause of this sub-image non-uniformity is vignetting

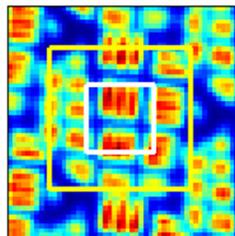
Top Cell



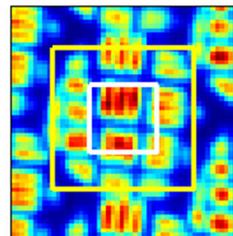
Left Cell



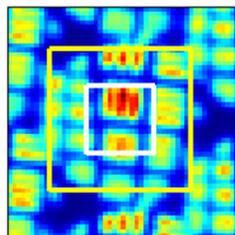
Reference



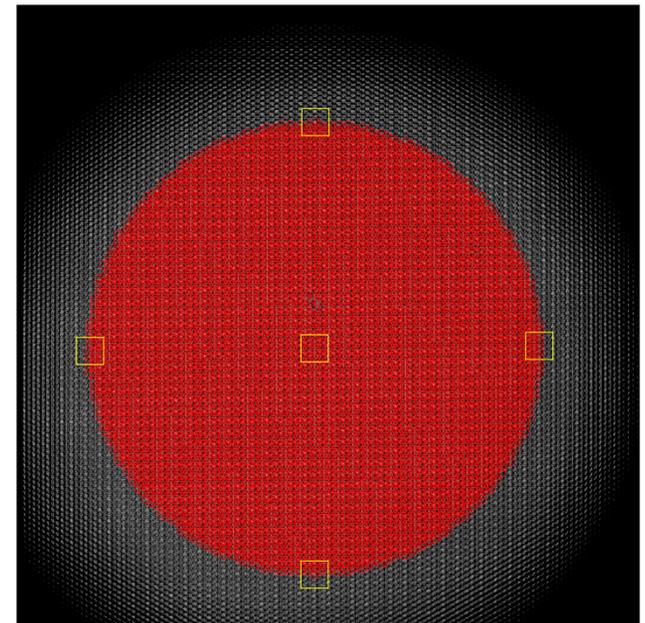
Right Cell



Bottom Cell

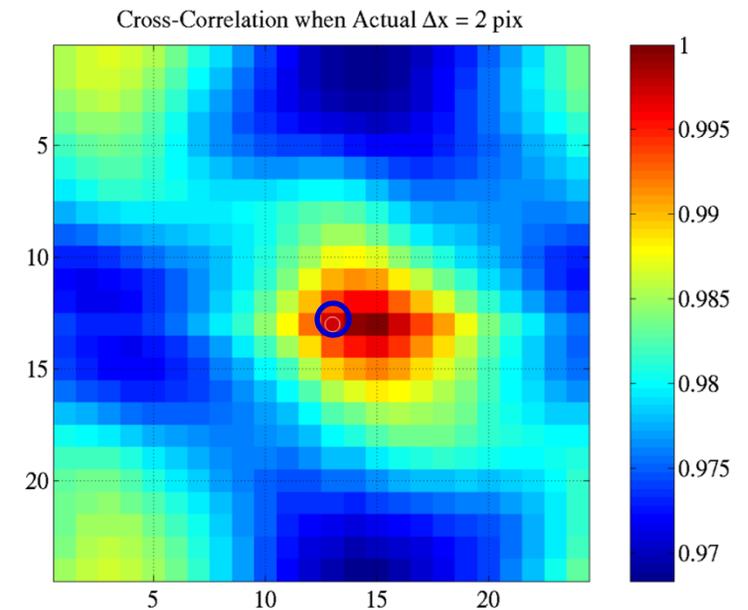
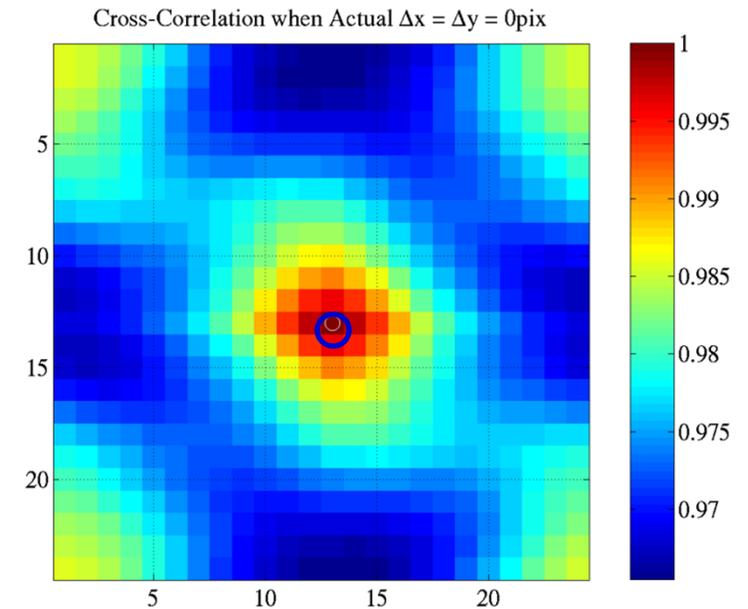
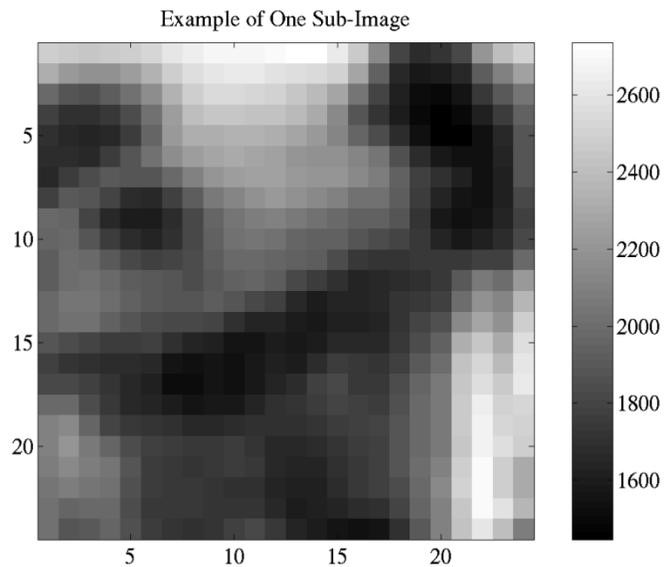


Number of the Red Cells > 6,000

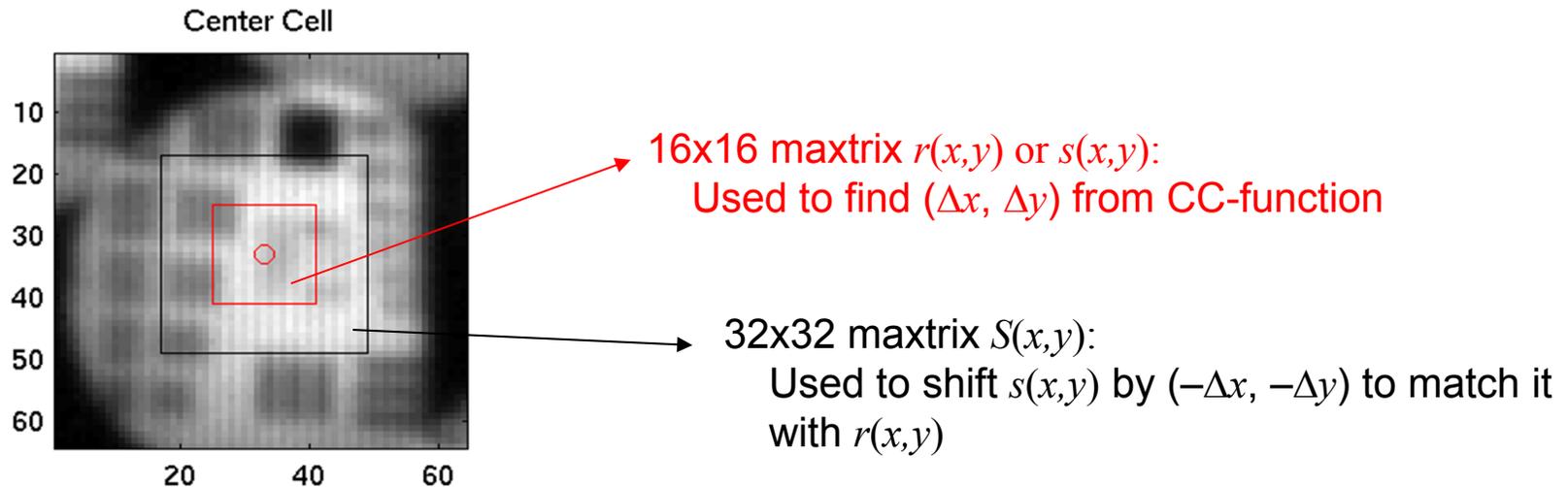


Adaptive Periodic-Correlation Algorithm—How it Works

- Target sub-image shift causes CC peak to move
- Fit a quadratic-curve to 3 points along the x-axis: Left, Peak, Right
- Do the same along the y-axis
- The peak point of each fit gives an estimate of the sub-image shift along the corresponding direction



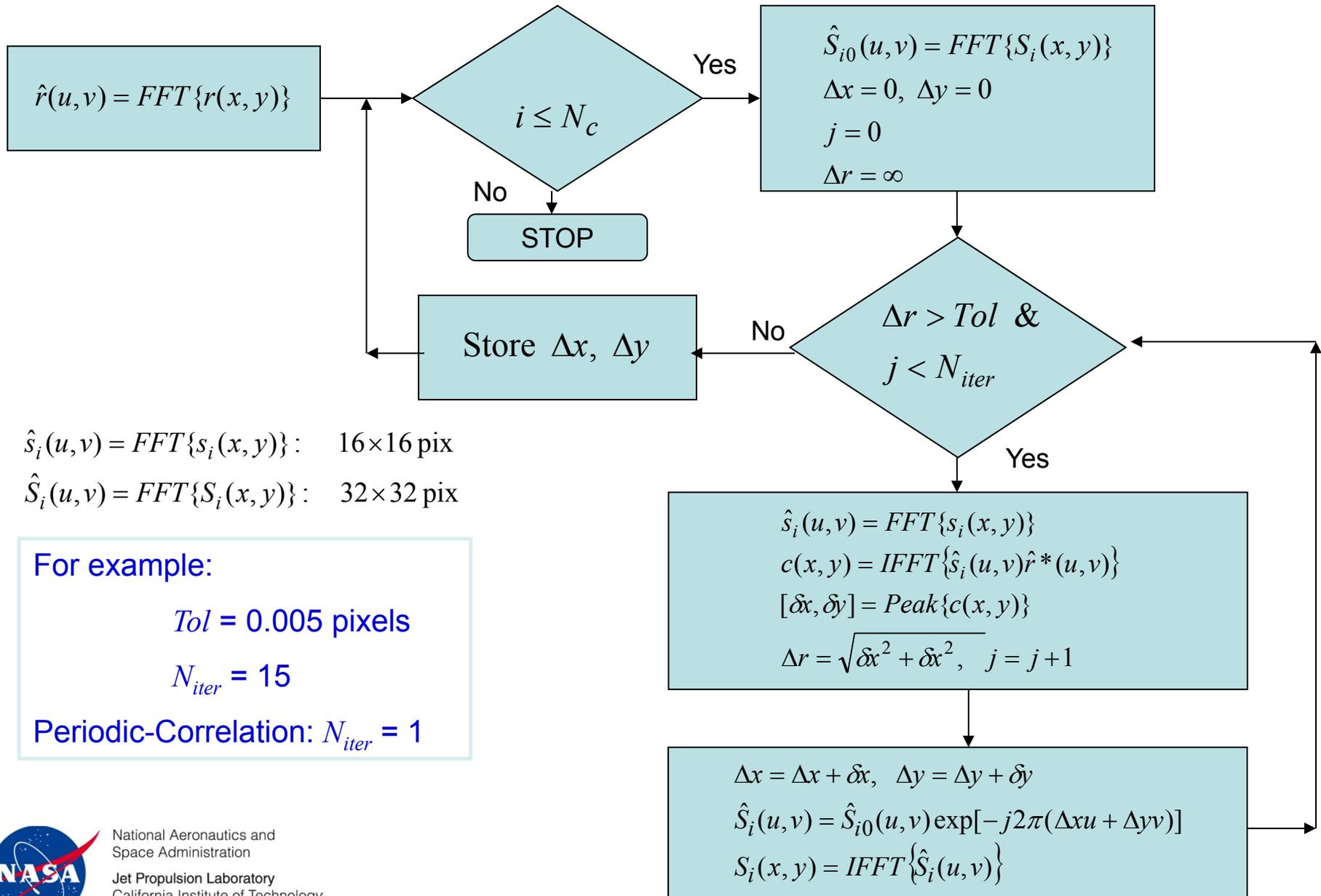
APC Algorithm — How it Works (cont.)



- Advantages of using smaller dimensions for $r(x,y)$ & $s(x,y)$:
 - Avoids wrap-around error when performing sub-image multi-pixels shifting
 - Makes the APC calculations much faster
 - Increases the WFS dynamic range
- To shift $S(x,y)$ by $(-\Delta x, -\Delta y)$:
 - Obtain $S(u,v)$ by FFT $\rightarrow S(u,v)\exp[-j2\pi(-\Delta xu - \Delta yv)] \rightarrow$ (by IFFT) $S(x+\Delta x, y+\Delta y)$



APC Algorithm Flow-Chart



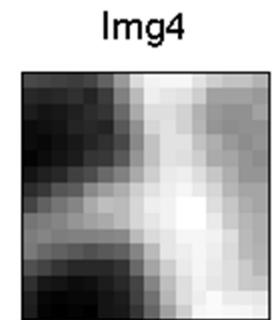
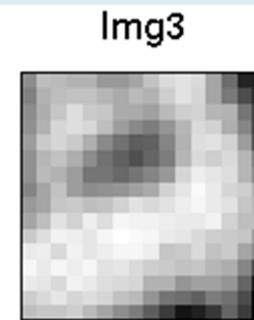
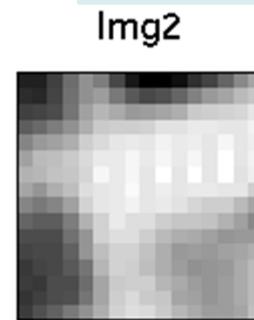
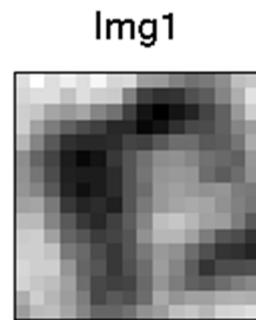
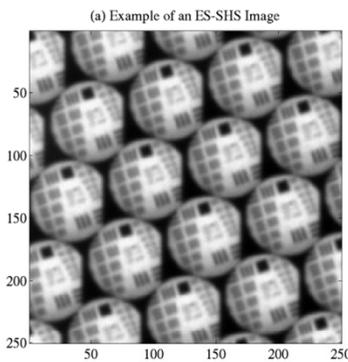
$\hat{s}_i(u, v) = FFT\{s_i(x, y)\} : 16 \times 16 \text{ pix}$
 $\hat{S}_i(u, v) = FFT\{S_i(x, y)\} : 32 \times 32 \text{ pix}$

For example:
 $Tol = 0.005 \text{ pixels}$
 $N_{iter} = 15$
 Periodic-Correlation: $N_{iter} = 1$



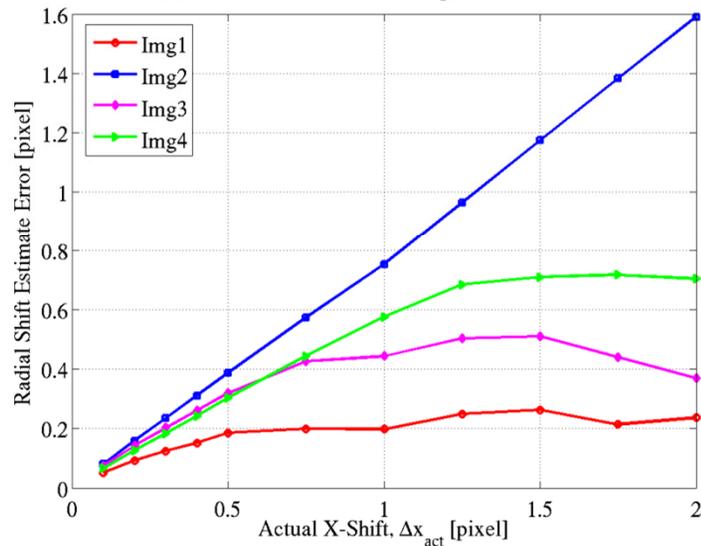
Shift Estimation Error versus Actual Shift — Case of 16x16 pix Ref-Cell

- SHC image was taken on an ES-SHS testbed at JPL
- Original image is used as the reference frame, and the shifted one as the test frame
- Shifted image is obtained as follows: Up-sampled the image to 4x before shifting, shifted the 4x image by convolution, then down-sampled the shifted image back to its original sampling format
- No additional noises are introduced to the images

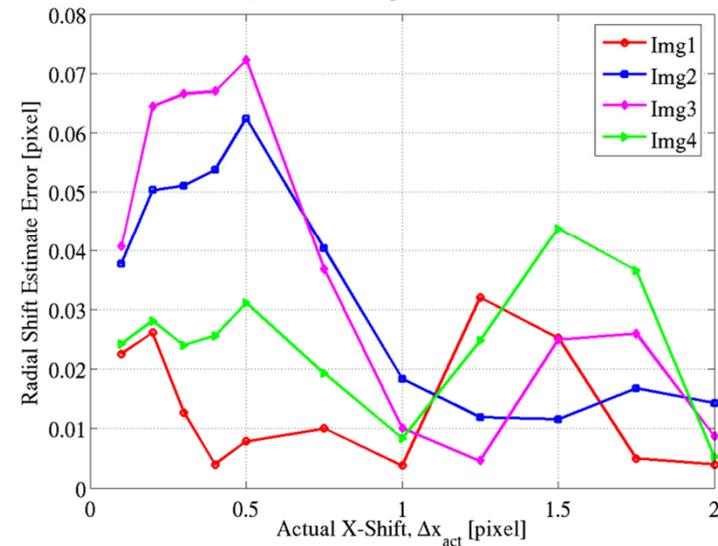


16x16 pix Cells

(a) Periodic-Correlation: 16x16 pix Reference Cell

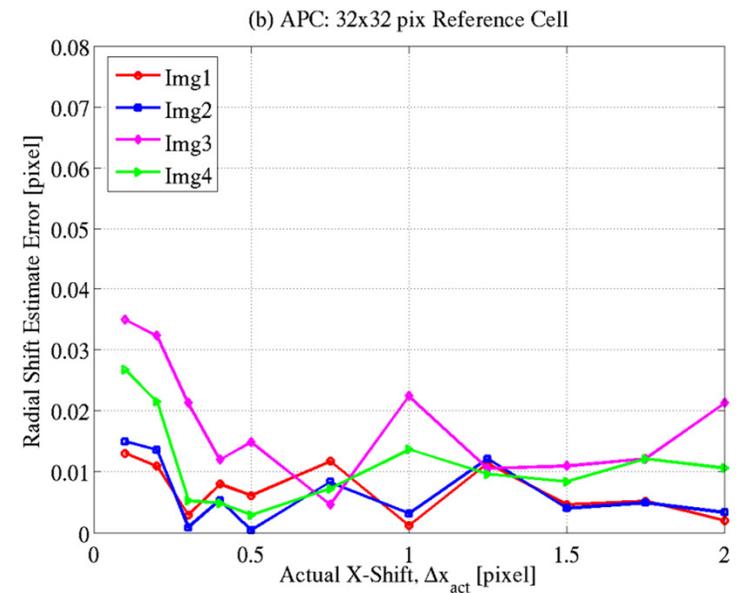
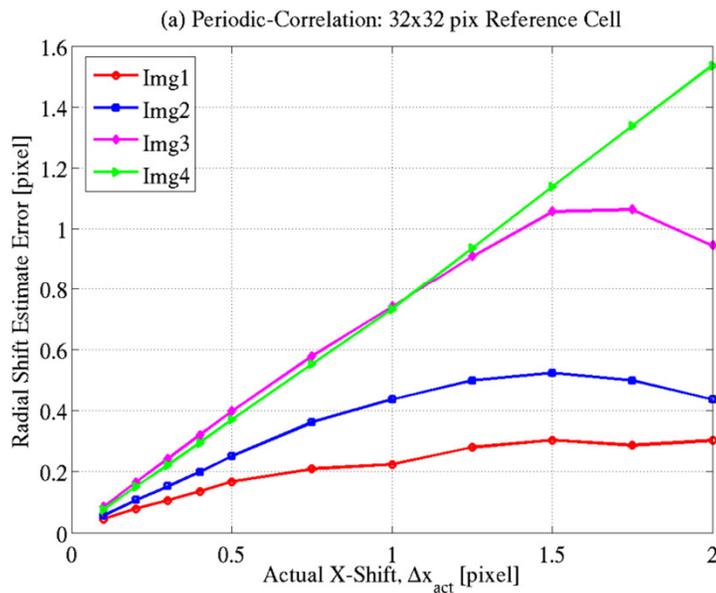
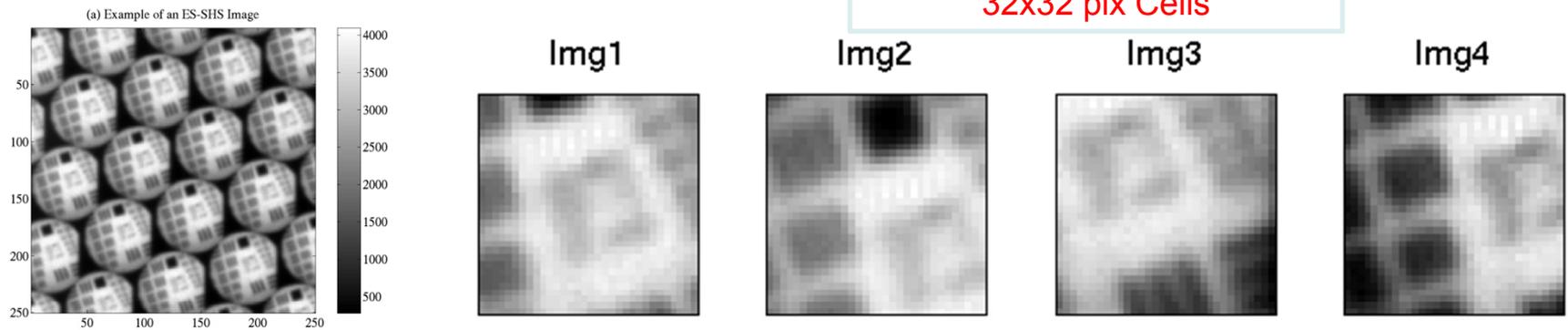


(b) APC: 16x16 pix Reference Cell



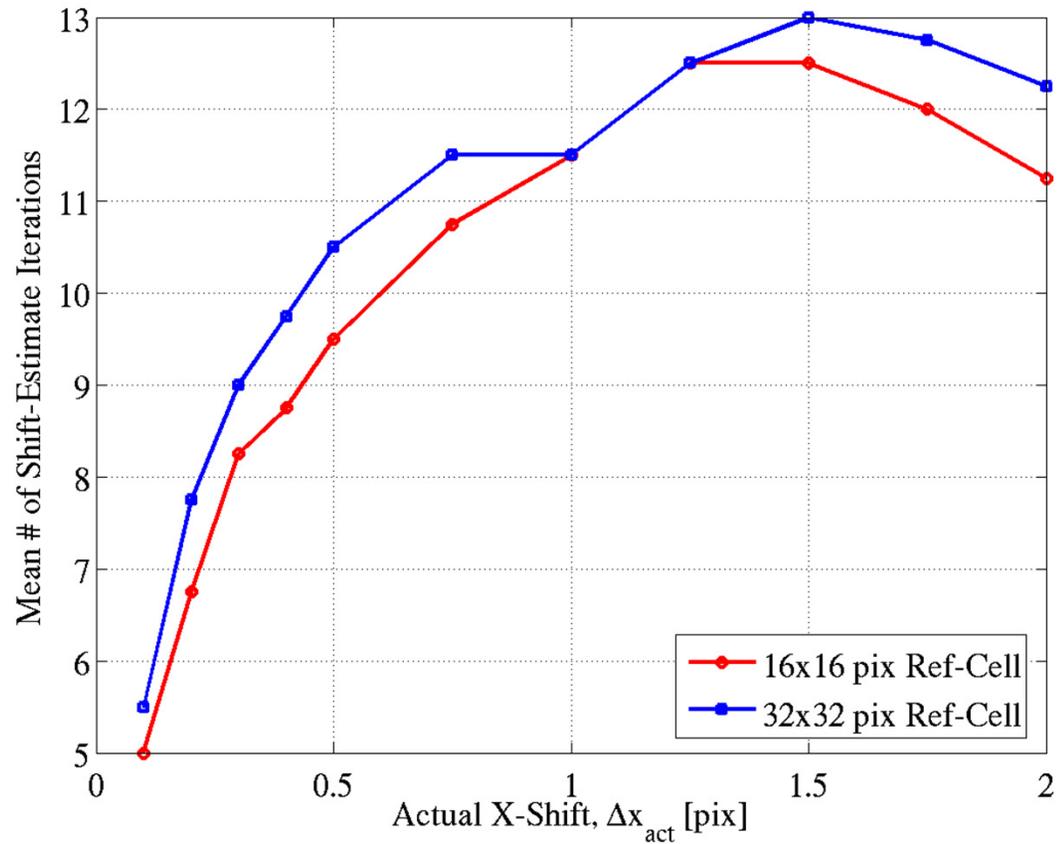
Shift Estimation Error versus Actual Shift — Case of 32x32 pix Ref-Cell

- SHC image was taken on an ES-SHS testbed at JPL
- Original image is used as the reference frame, and the shifted one as the test frame
- Shifted image is obtained as follows: Up-sampled the image to 4x before shifting, shifted the 4x image by convolution, then down-sampled the shifted image back to its original format
- No additional noises are introduced to the images



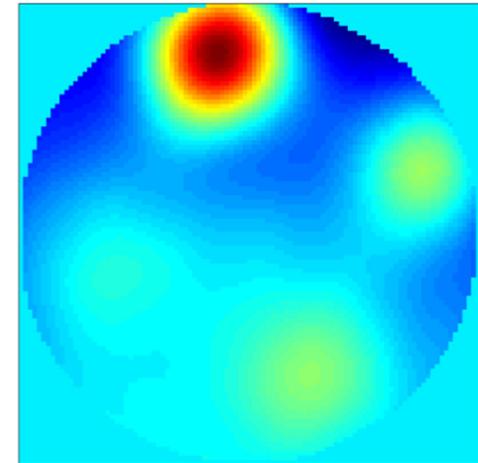
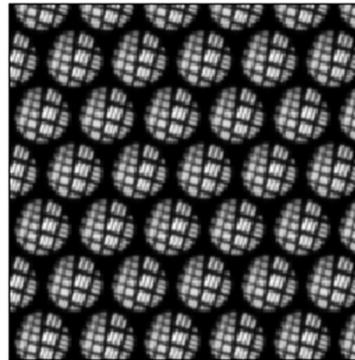
Mean Number of Shift-Estimate Iterations Used

- Corresponds to the cases in the previous 2 pages

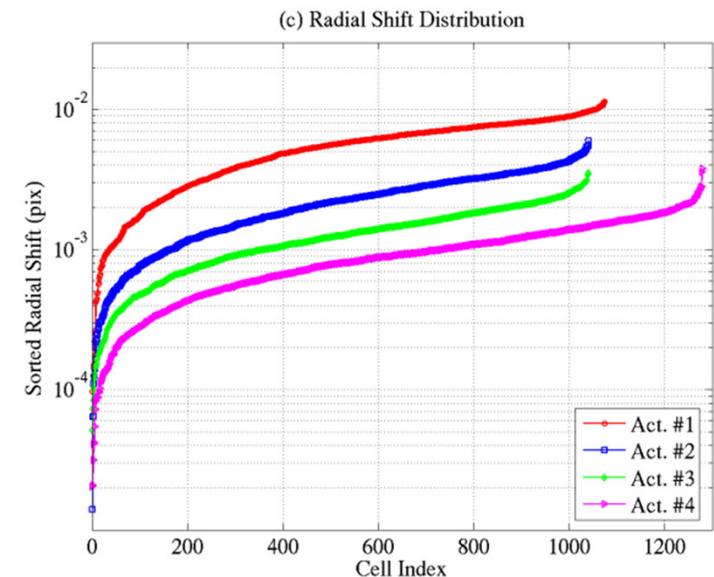
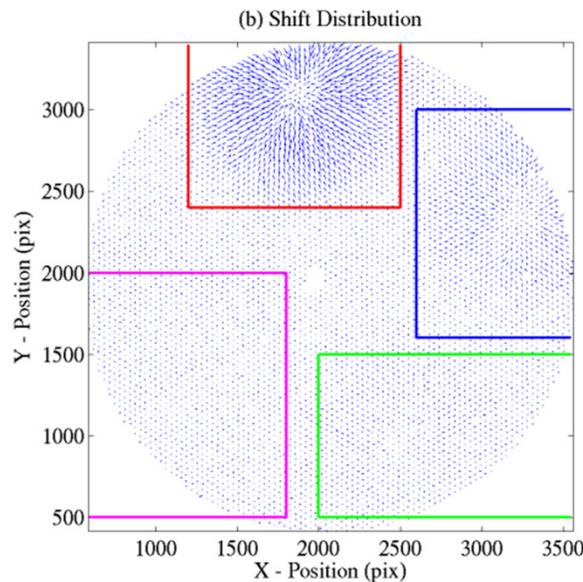


OPD Obtained from Extended-Scene Images—An Example

- Reference image is obtained from a "flat-state" of the system
- Test image is obtained by actuating 4 actuators of the DM by different command voltages
- Same extended-scene is used for both reference and test images—OPD of a differential mode
- Reference cell is 16x16 pix
- 4 curves in part (c) are obtained from the offsets of the same-color 4 areas in part (b)—Correspond to the 4 different actuators
- Measured maximum radial sub-image shift is <0.02 pix in this case



RMS = 6.3, PV = 45.0 nm



Summary

- Extended-scene SH sensor is useful when point-source is not available but SH-WFS is needed
- APC is much less sensitive to sub-image non-uniformity—much more reliable than ACC
- APC works well with both point-source spot-images and extended-scene images, just as ACC
- **Future work:**
 - Examine and eliminate the scene-content dependent shift-estimation errors
 - Establish an acceptance criteria for extended-scene images—Failsafe criteria for extended scenes
- **Acknowledgement**
 - The extended-scene SHC images used in this presentation were measured by Fang Shi, Paul Best, Rhonda M. Morgan and Zhou Hanying at JPL



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