Model-Based Wavefront Control for CCAT

August 15, 2011

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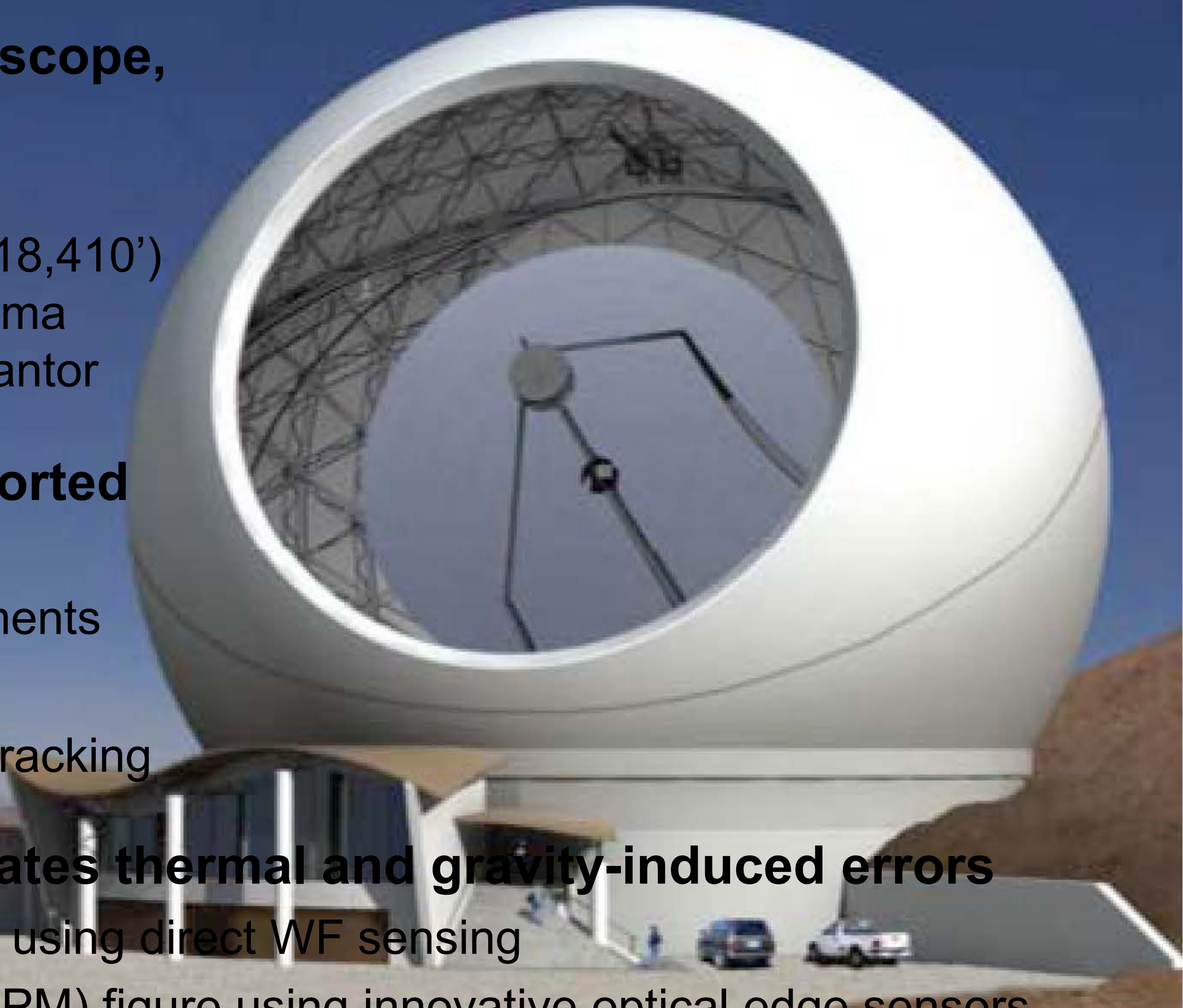
CCAT Telescope



- **Submillimeter-wave telescope, covering 200um to 2mm wavelength band**
  - Will be sited at 5,612m (18,410') altitude in the high Atacama desert near Cerro Chajnantor
- **162 PM segments, supported on composite structure**
  - 3DOF actuation for segments
  - 5DOF actuation for SM
  - Telescope pointing and tracking
- **Active control compensates thermal and gravity-induced errors**
  - Initialize telescope figure using direct WF sensing
  - Maintain Primary Mirror (PM) figure using innovative optical edge sensors
  - Compensate for gravity sag effects using feed-forward, look-up table control of Secondary Mirror (SM) position and pointing

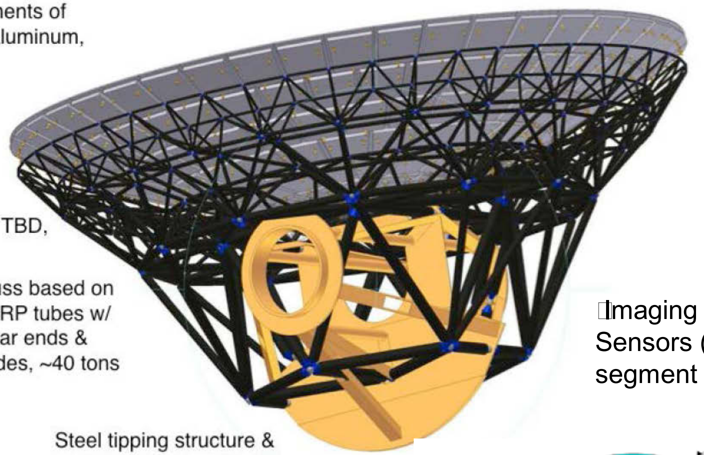
CCAT Control Elements

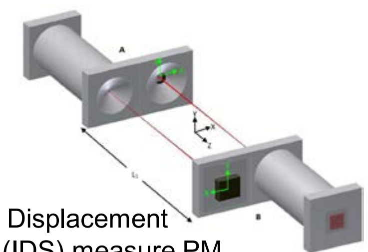
162 Segments of CFRP + aluminum, ~8.5 tons

Actuators, TBD, 1.8 tons

Truss based on CFRP tubes w/ Invar ends & nodes, ~40 tons

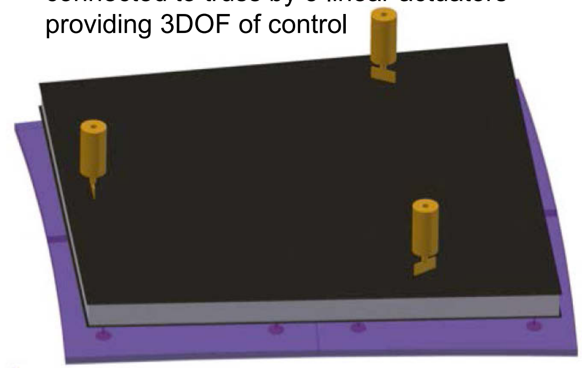
Steel tipping structure & counterweight ~96 tons

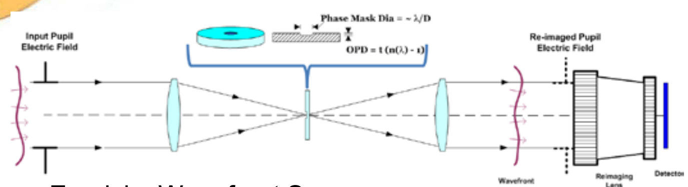




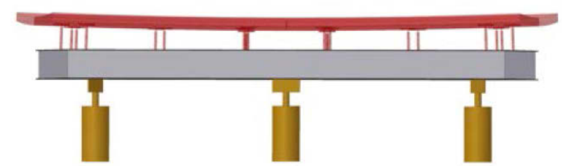
Imaging Displacement Sensors (IDS) measure PM segment displacements

CFRP / Al honeycomb plate 'raft' is connected to truss by 3 linear actuators providing 3DOF of control





Zernicke Wavefront Sensor measures total system WF error



Machined aluminum tiles form reflecting surfaces, 4 per raft, mounted with five invar screw adjustors.

- **CCAT telescope design utilizes CFRP truss structure supporting 162 segments, each about 2 meters in size**
- **Each segment is a CFRP/Al "raft" structure supporting 4 machined Al reflecting panels**
- **Three actuators per segment provide tip/tilt/piston control**
- **Secondary Mirror has 6DOF rigid body control as well**

Imaging Displacement Sensors

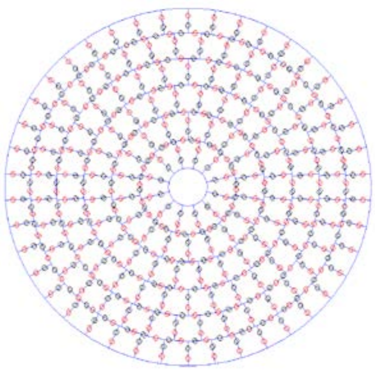
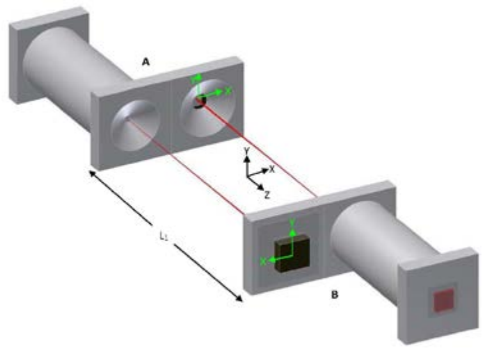


Figure 17. CCAT OES locations. The black circles indicate sensor pairs oriented at -45° to the gap axis, and the red circles those that are oriented at +45° .



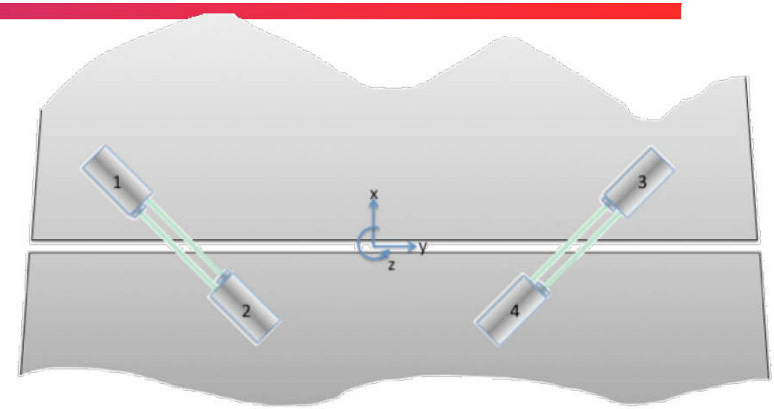


Figure 15. Two OES sensor pairs along a segment edge.

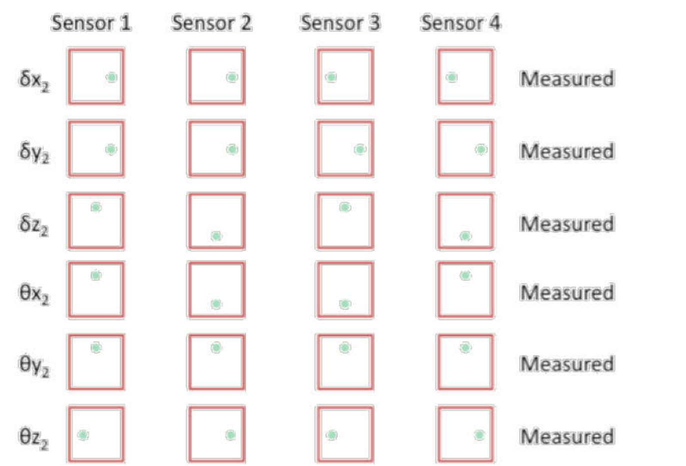
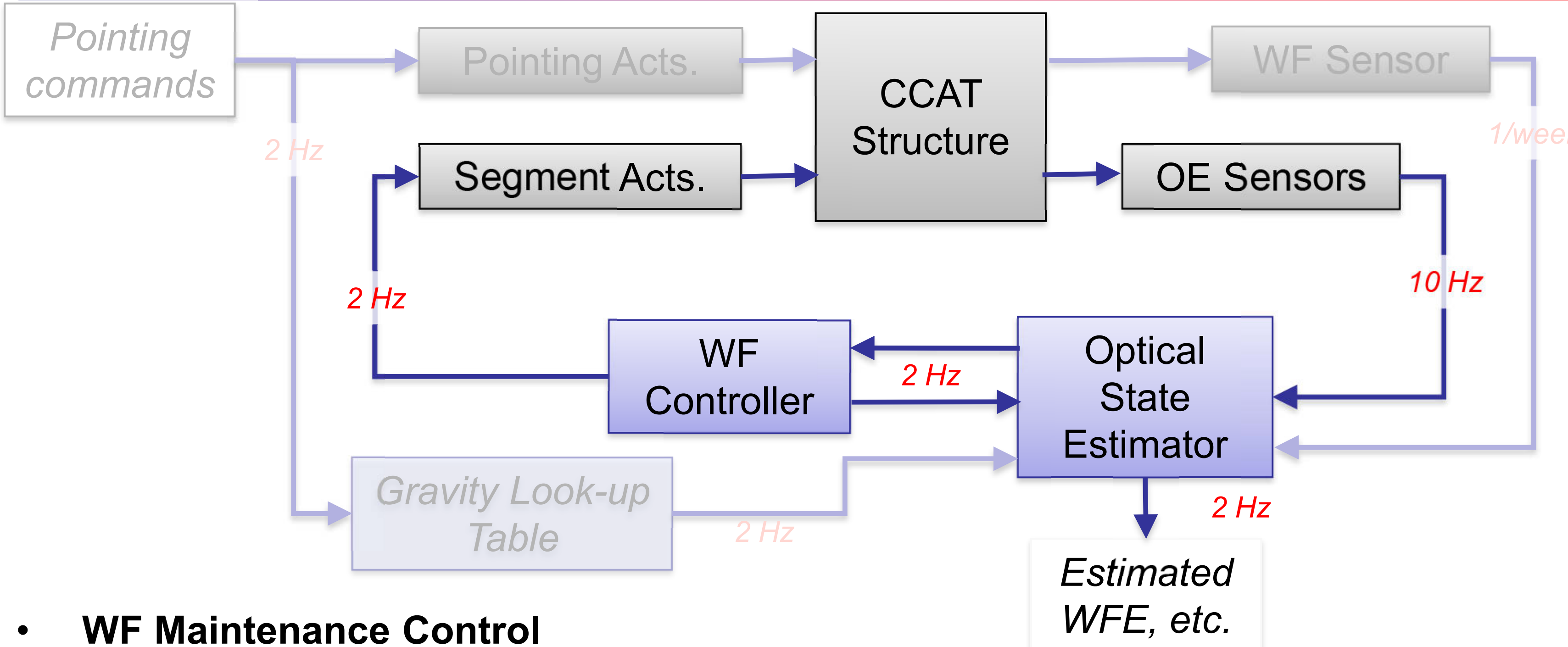


Figure 16. Measurements made by 2 OES sensor pairs fully observe the 6 relative motions between the 2 adjacent segments (the lower segment moves relative to the upper).

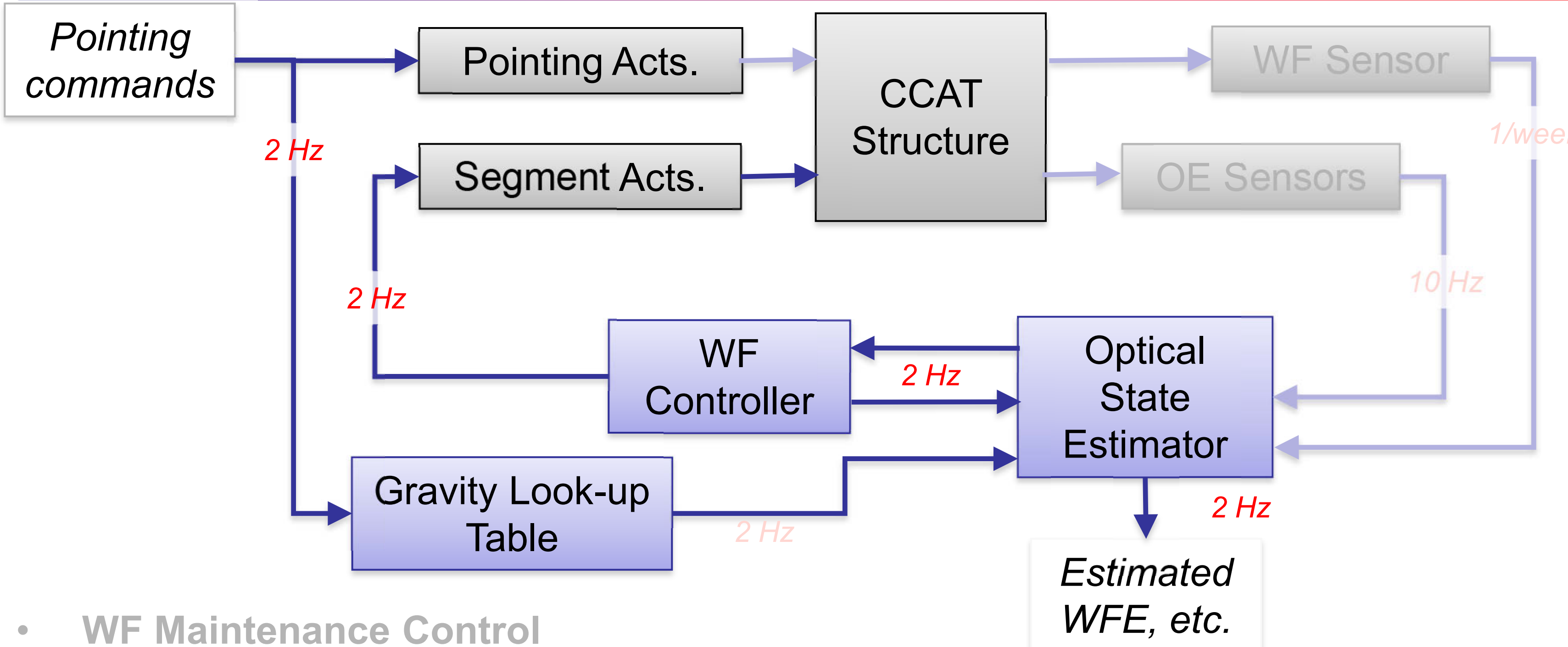
- **Imaging Displacement Sensors (IDS): optical edge sensors**
  - Light source on 1 segment sends beam across the gap to 2nd segment
  - Motions of segments relative to each other move beam centroid
  - Centroiding averages many pixels, produces highly accurate results
- **Angled configuration allows observation of all 6 relative DOFs between 2 segments**

Control Loops



- **WF Maintenance Control**
  - Optical Edge Sensors are used to continuously estimate the WF error
  - WF Controller moves the PM segments to minimize WF error

Control Loops



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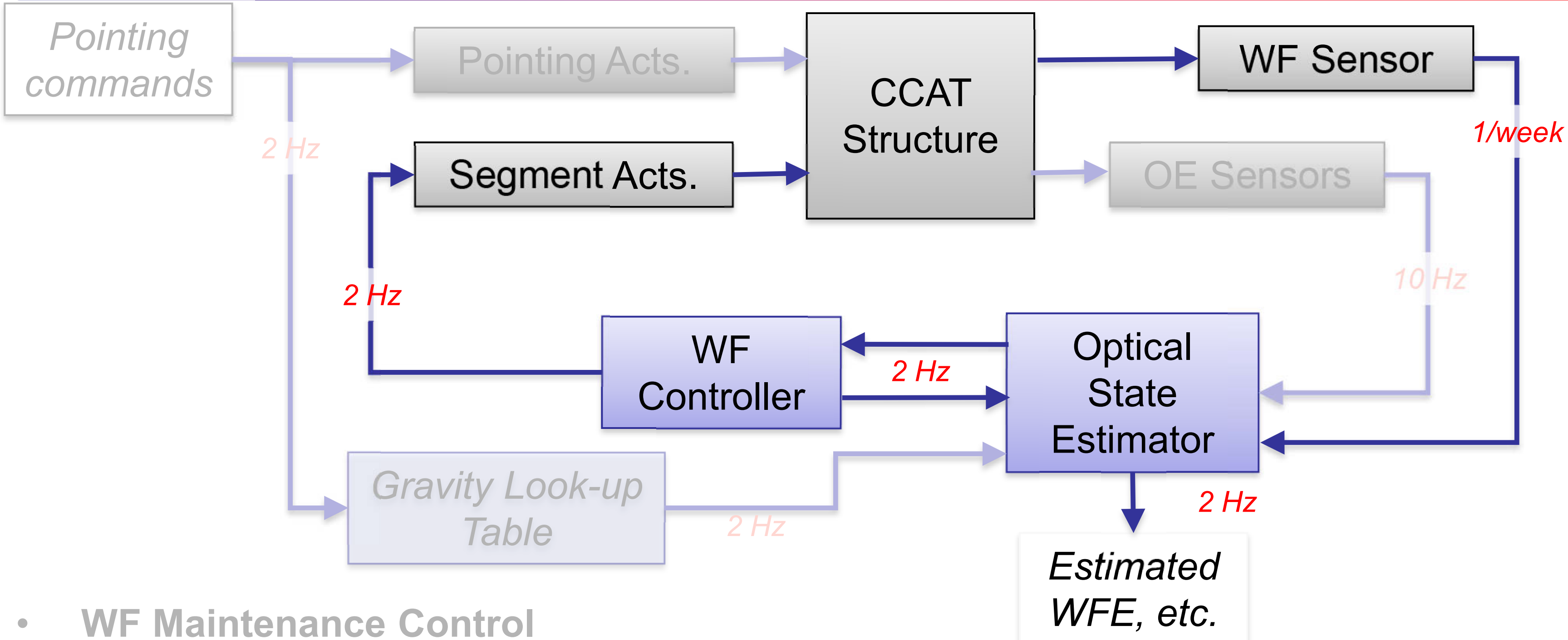
- **Gravity Look-up Table Feed-forward Control**
  - Gravity direction wrt the telescope changes as the telescope pointing changes
  - Secondary Mirror position and angle are controlled open-loop to counter the effect of pointing on the WF

Control Loops



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  - Gravity direction wrt the telescope changes as the telescope pointing changes
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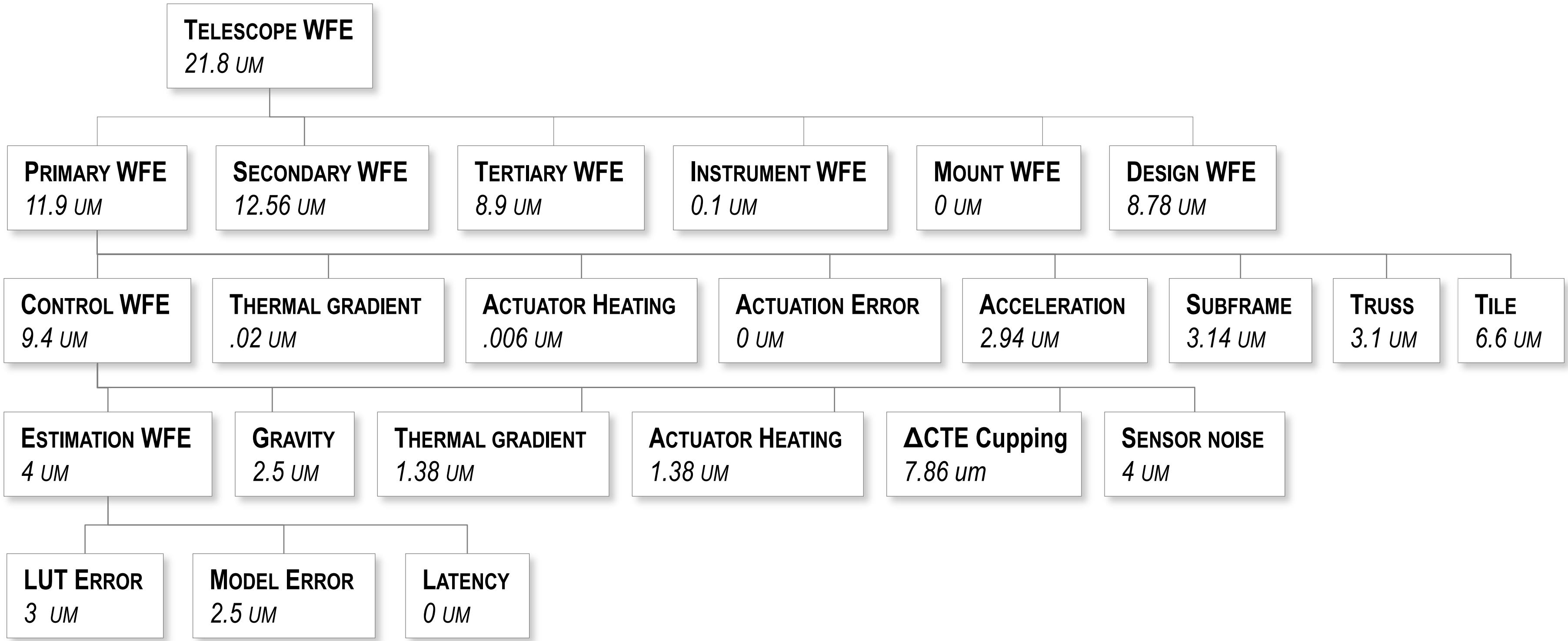
- **WF Initialization Control**
  - Occasional WF Sensing is conducted to directly measure the telescope WF and reset the Maintenance control

Error Tree: a Bottoms-Up Prediction



- **Error tree takes the form of an error budget, but contains predicted values as opposed to allocations**
- **Entries are based on component performance, projected to WF error via model or analysis**
- **No WFC correction of non-PM errors is assumed**

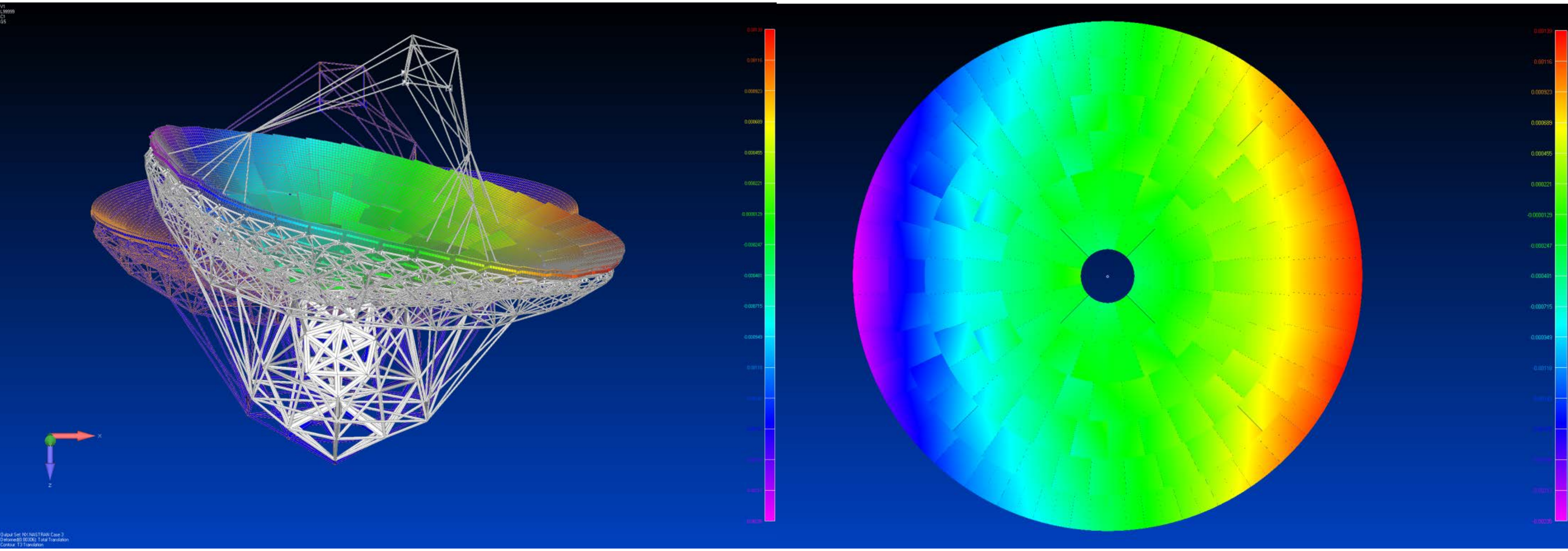
Error Tree Provides WFC Targets

- **Error Tree in spreadsheet form, sets targets for later simulations**
- **Nominal** uses predictions from CCAT Error Budget
- **Model Only** rolls-up just the modeled errors to set targets for model results
  - **With and without Thermal Gradient Cupping**
  - Matches conditions used in later simulations

CCAT WFE Tree

System / Component	Nominal Component WFE (um RMS)	Nominal System WFE (um RMS)	Model only, no cupping Component WFE (um RMS)	Model only, no cupping System WFE (um RMS)	Model, with cupping Component WFE (um RMS)	Model, with cupping System WFE (um RMS)
1 Telescope Totals		**21.55**		**5.14**		**5.32**
Secondary	12.56		0		0	
Tertiary	8.9		0		0	
Instrument	0.1		0		0	
Mount	0		0		0	
Design	8.78		0		0	
2 Primary		12.27		5.14		5.32
Thermal Gradient	0.02		0.02		0.02	
Actuator heating	0.006		0		0	
Actuation error	0		0		0	
Acceleration	2.94		0		0	
Subframe	0.314		0.314		0.314	
Truss	3.1		3.1		3.1	
Tile	6.6		0		0	
3 Control		9.41		4.09		4.32
Gravity	2.5		2.5		2.5	
Thermal Gradient	1.38		0		1.38	
Actuator heating	1.38		0		0	
dCTE cupping	7.86		0		0	
Sensor noise	0.7		0.7		0.7	
4 Estimation		4.03		3.16		3.16
LUT error	3		3		3	
Model error	2.5		0		0	
Latency / Drift	1		1		1	

1G Disturbance Example



- **Gravity sag deforms the PM and SM support structures as the telescope elevation axis changes**
- **This example shows the change incurred as the elevation axis moves from zenith to 45° pointing**

Linearized Model

- **The optical state x_i: perturbations to the nominal position/orientation/figure of each optic**
  - Subscript i indicates time step
- **State x_i changes in time, driven by process noise $\langle_i$, quasi-static modes T_i, actuations u_i**
- **Wavefront output w_i**
  - Wavefront w_i is affected by state changes x_i and actuation u_i
- **IDS optical edge sensor measurements l_i**
  - Edge sensor measurements l_i are affected by state changes x_i and noise ™l_i
- **Wavefront sensor measurements w_{mi}**

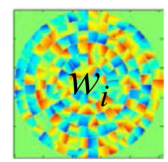
x is the CCAT state vector: 6 rigid-body DOFs per optic

$$x_{Opticj} = \left[\delta_x \delta_y \delta_z \theta_x \theta_y \theta_z\right]^T \qquad x = \begin{bmatrix} x_{Optic1} \\ \vdots \\ x_{Opticn} \end{bmatrix}$$

State transition equation governs the evolution of the state with time

$$x_i = x_{i-1} + \frac{\partial x}{\partial u} u_i + \frac{\partial x}{\partial T} T_i + \xi_i$$

Wavefront is a function of the state



$$w_i = \frac{\partial w}{\partial x} x_i + w_0$$

Optical edge sensor measurement

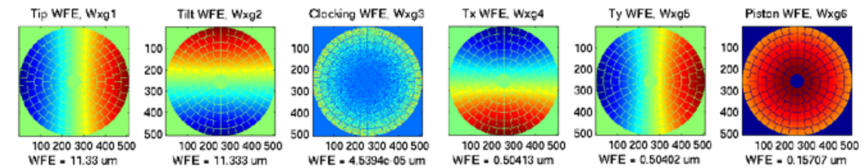
$$l_i = \frac{\partial l}{\partial x} x_i + l_0 + \delta l_i$$

Wavefront sensor measurement

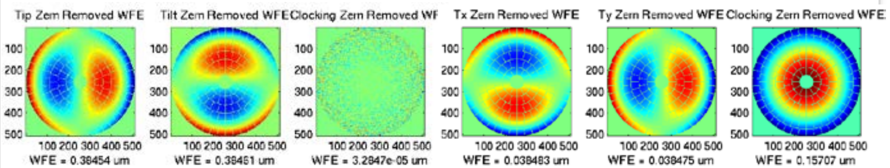
$$w_{mi} = w_i + \delta w_i$$

Global and Singular Modes

- ***Global modes: coordinated RB motions of the PM segments***

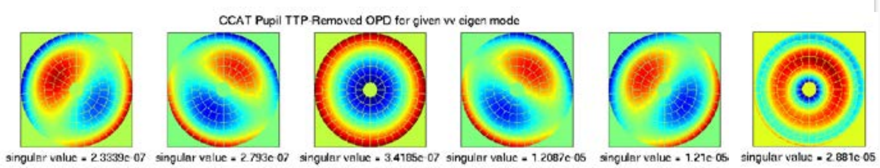


  - *Global modes, TTP removed:*

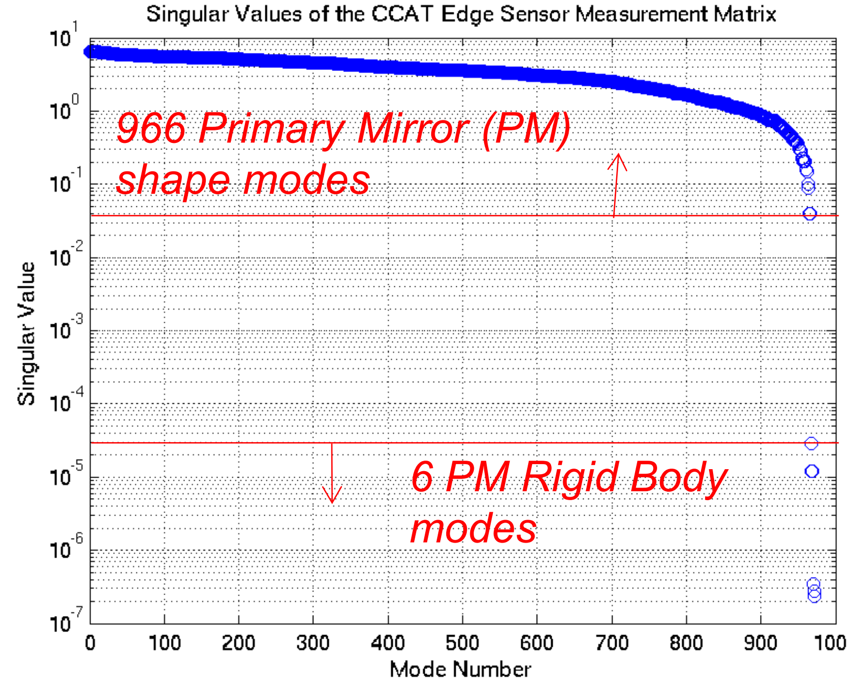


- ***Singular modes: linearly related to the global modes***

$$[U,S,V] = svd\left(\frac{\partial l}{\partial x}\right) \qquad w_{Gj} = \frac{\partial w}{\partial x} U_j$$



- ***The coma WFE in these modes occurs because PM RB motions decollimate the telescope***



- The ratio of highest to lowest singular value is only 161 for the 966 PM shape modes, well within controller dynamic range
- Rigid body modes are not measurable – only relative segment motions are seen by the edge sensors

Control Approaches

- **One approach is to control so that edge sensor measurements are driven to null**
  – This approach nulls errors in the controlled DOFs only

- ***Real objective is to* *minimize WF error*, while keeping control effort in balance**
  – This can be done if WF is measured directly
  – It can also be accomplished using an "*Optical State Estimator*" to estimate WF from edge sensor measurements

- **WF-based control...**
  – *Compensates optical effects of all errors* by actuation of the controlled DOFs

A state-restoring control:

$$u_i = -pinv\left(\frac{\partial l}{\partial x}\frac{\partial x}{\partial u}, ptol\right) l_i$$

WF control cost function

$$\min_u \; J = w^T w + c_u u^T u$$

WF control control law and gain matrix

$$u_i = -G w_i$$

$$G = \left[c_u I + \left(\frac{\partial w}{\partial x}\frac{\partial x}{\partial u}\right)^T \frac{\partial w}{\partial x}\frac{\partial x}{\partial u} \right]^{-1} \left(\frac{\partial w}{\partial x}\frac{\partial x}{\partial u}\right)^T$$

WF control can be used with WF measurement w or WF estimate $\overline{w}$

$$u_i = -G\overline{w}_i$$

Optical State Estimator

- **Kalman filter recursively estimates full state $\hat{x}_i$ from optical edge sensor measurement l_i , previous control u_{i-1}, and prior estimates $\hat{x}_{i-1}$, $\hat{x}_{i-2}$, $\hat{x}_{i-3}$, ...**
  - Balances measurement noise against error in prior estimates to produce optimal estimate
- **Estimated state $\hat{x}_i$ used to estimate WF $\bar{w}_i$ for WF control**
- **Full-state controller feeds back $\bar{w}_i$ to minimize WFE**
  - Observes 6DOF per segment, minus unobservable global tilt modes
  - Additional term c_u weights control effort, allows damping of response to avoid exciting structure and reduce noise sensitivity

Current state estimate computed from current measurement, prior state estimate, and prior actuation

$$\hat{x}_i = \left(I - K_i \frac{\partial l}{\partial x}\right)\hat{x}_{i-1} + \left(I - K_i \frac{\partial l}{\partial x}\right)\frac{\partial x}{\partial u}u_{i-1} + K_i l_i$$

Kalman gain weights contributions of data sources by their expected error

$$K_i = \mathrm{cov}\left(\hat{x}_i - x_i\right)\left(\frac{\partial l}{\partial x}\right)^T \mathrm{cov}\left(\frac{\partial l}{\partial x}\frac{\partial x}{\partial u}u_{i-1}^T + l_i\right)^{-1}$$

Kalman gain changes with time, as the state covariance evolves

$$\mathrm{cov}(\bar{x}_{i+1}) = \mathrm{cov}(\hat{x}_i - x_i) + \frac{\partial x}{\partial u}\mathrm{cov}(u_i)\left(\frac{\partial x}{\partial u}\right)^T + \mathrm{cov}(\delta x_i)$$

Estimated wavefront

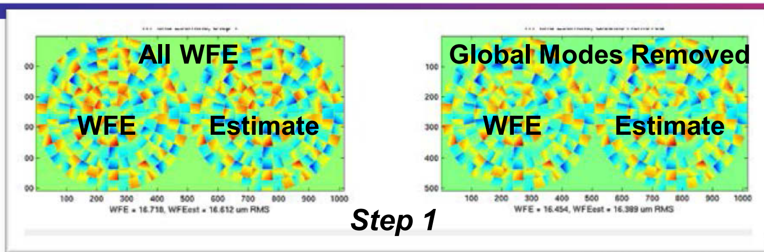
$$\bar{w}_i = \frac{\partial w}{\partial x}\bar{x}_i$$

Wavefront Control

$$u_i = -G\bar{w}_i = -G\frac{\partial w}{\partial x}\hat{x}_i$$

$$G = \left[c_u I + \left(\frac{\partial w}{\partial x}\frac{\partial x}{\partial u}\right)^T \frac{\partial w}{\partial x}\frac{\partial x}{\partial u}\right]^{-1}\left(\frac{\partial w}{\partial x}\frac{\partial x}{\partial u}\right)^T$$

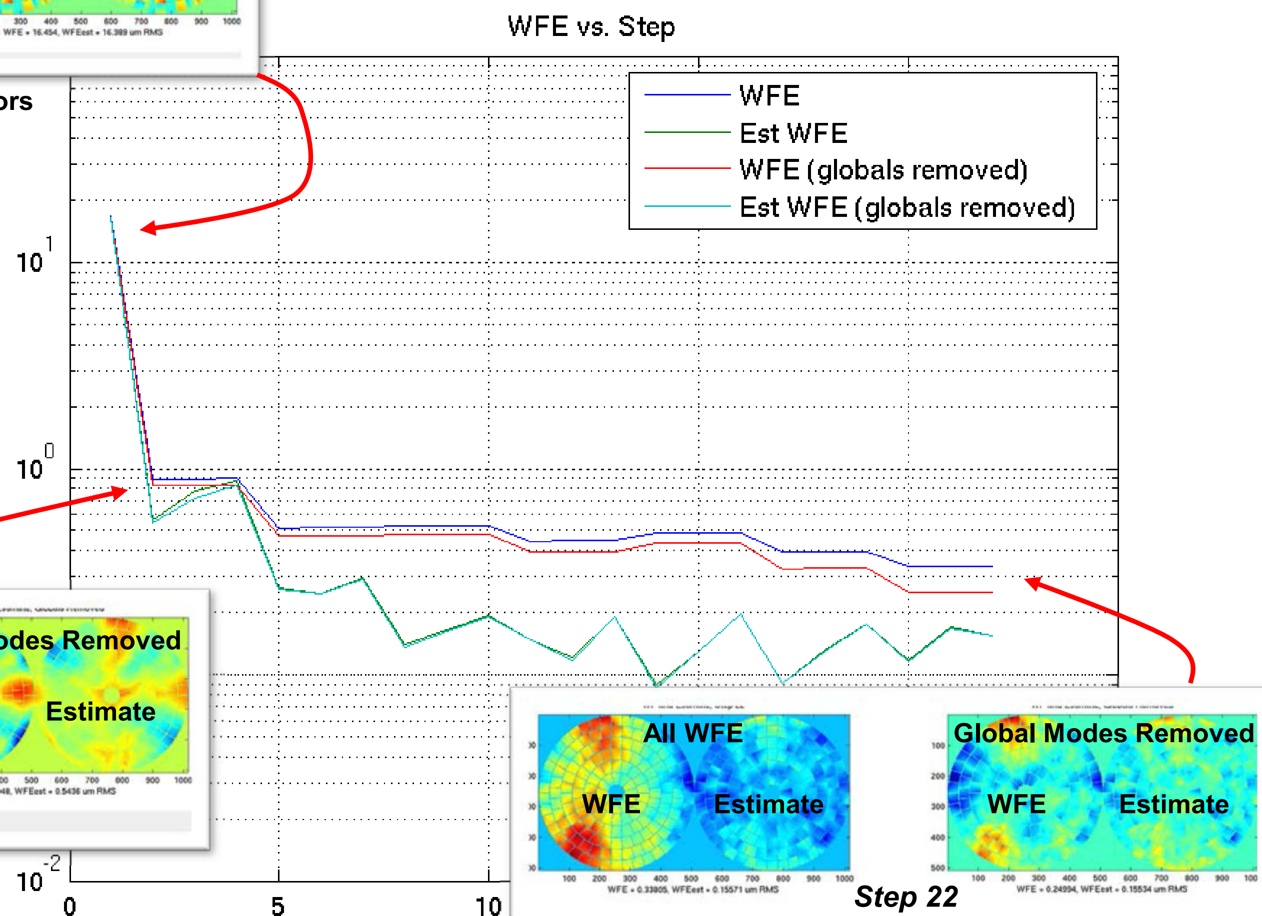
Random IC Example

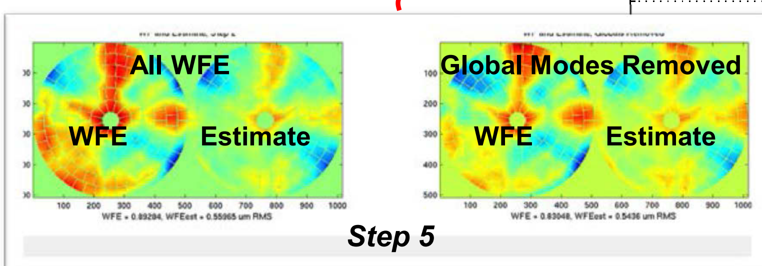


•Initial WFE due to random state errors

Random IC Response

- •Time-step control sim
- •No feed-forward
- •IDS at all steps
- •0.1 um IDS accuracy
- •WFC every third step
- •Full OSE Kalman filter
- •No cupping



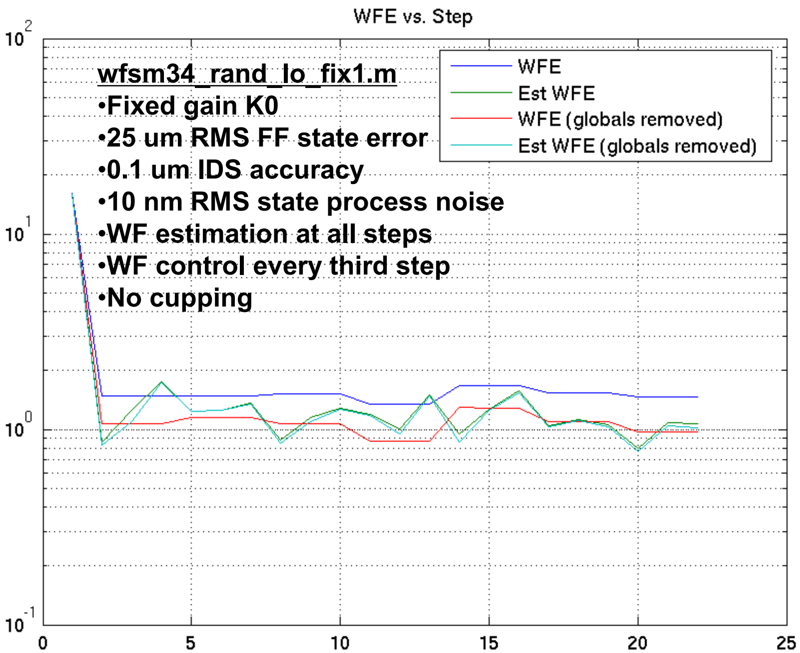


•WFE below 1 um after first step
•Estimates do not see global mode WFE

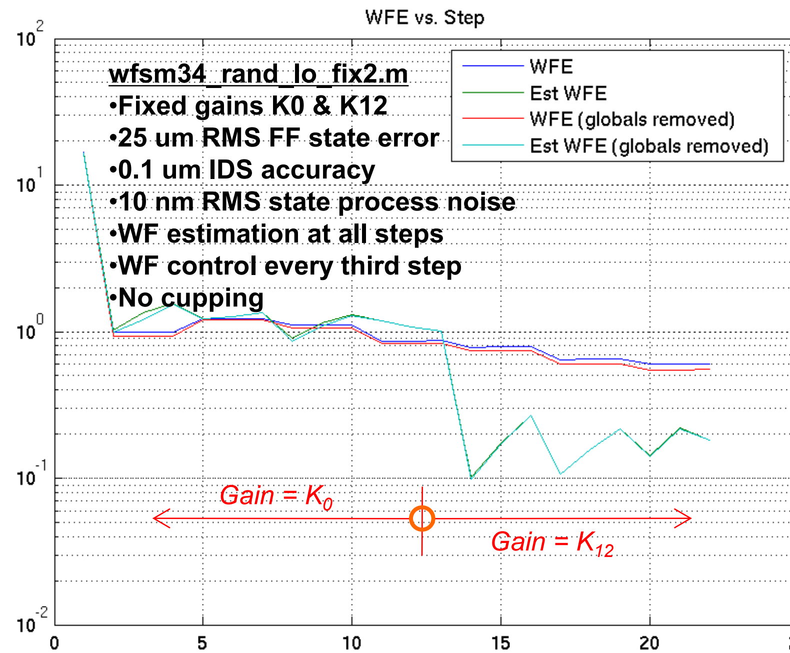
•WFE below 1/2 um, continuing to drop

Fixed-Gain Control

- ***Fixed-gain control with K_0:***



- ***Gain-scheduled control:***



- **Optimal OSE recomputes K_i every step**
- **Fixed-gain OSE is simpler**
  - Performance can still be tailored to conditions by switching gain matrices
- **CCAT performance analyses use conservative K_0 gains**

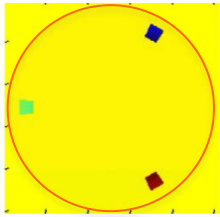
Fixed-gain OSE does not recompute gains or covariances

$$\hat{x}_i = \left(I - K_0 \frac{\partial l}{\partial x}\right)\hat{x}_{i-1} + \left(I - K_0 \frac{\partial l}{\partial x}\right)\frac{\partial x}{\partial u}u_{i-1} + K_0 l_i$$

"Three Fixed Points" Control

- **Three piston DOFs are removed from the control, for segments 264, 300 & 336**

$$\frac{\partial x}{\partial u}(:,k)=[]$$



- **Estimator remains unchanged**

- **Control objective is to *minimize PM modes only*: global modes are stripped**

$$w_{PM}=\left[I-w_g w_g^T\right]w\equiv P_{wg}w=w-w_g\left(w_g^T w\right)$$

$$\frac{\partial w_{PM}}{\partial x}=P_{wg}\frac{\partial w}{\partial x}=\frac{\partial w}{\partial x}-w_g\left(w_g^T\frac{\partial w}{\partial x}\right)$$

- **Resulting control law does not respond to global errors**

$$\min_u \quad J=w_{PM}^T w_{PM}+c_{wu}u^T u$$

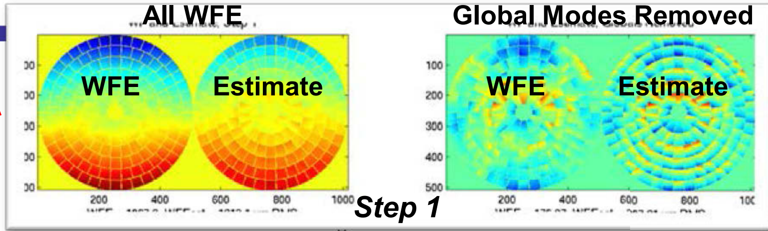
$$u_i=-G\frac{\partial w_{PM}}{\partial x}\hat{x}_i$$

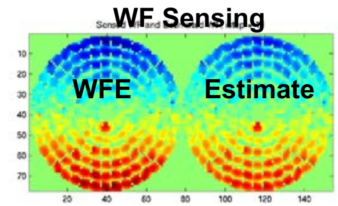
- **Use *all* segment controls (and normal *K*)when information on global modes is available**
  - Feed-forward
  - WF sensing

$$G=\left[c_u I+\left(\frac{\partial w_{PM}}{\partial x}\frac{\partial x}{\partial u}\right)^T\frac{\partial w_{PM}}{\partial x}\frac{\partial x}{\partial u}\right]^{-1}\left(\frac{\partial w_{PM}}{\partial x}\frac{\partial x}{\partial u}\right)^T$$

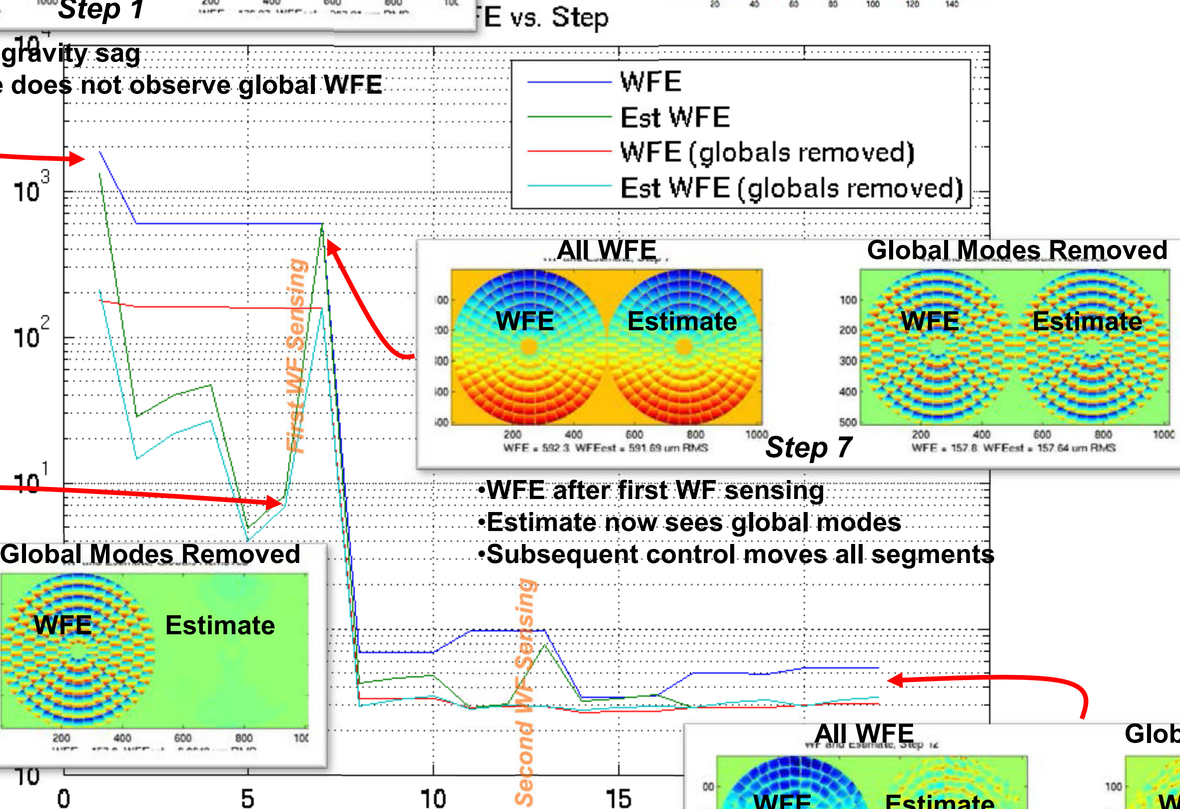
$$x_i=x_{i-1}+\frac{\partial x}{\partial u}u_i+\frac{\partial x}{\partial u_{SM}}u_{SMi}+\frac{\partial x}{\partial T}T_i+\xi_i$$

WF Initialization Example





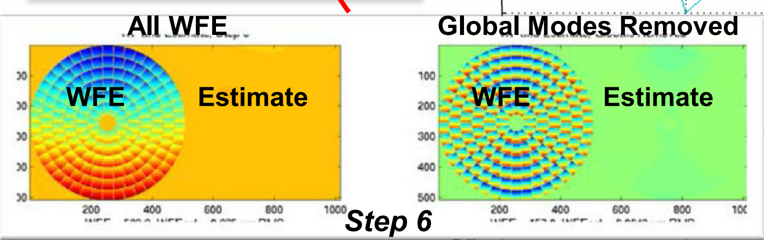
- Initial WFE due to gravity sag
- Initial WF estimate does not observe global WFE



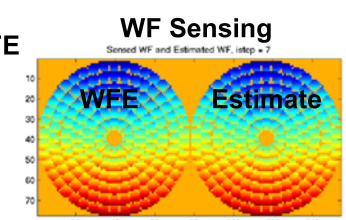
- WFE after first WF sensing
- Estimate now sees global modes
- Subsequent control moves all segments

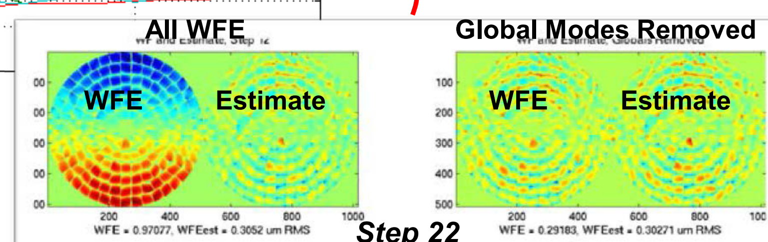
CCAT 1G Response

- Time-step control sim
- No feed-forward
- WFS at steps 7 and 13
- 80x80 WFS sampling
- 0.8 um WFS accuracy
- IDS at all steps
- 3 Fixed Points control
- 0.5 um IDS accuracy
- WFC every third step



- WFE after OES control only
- Estimates do not see global mode WFE
- OES control only does not correct global mode WFE





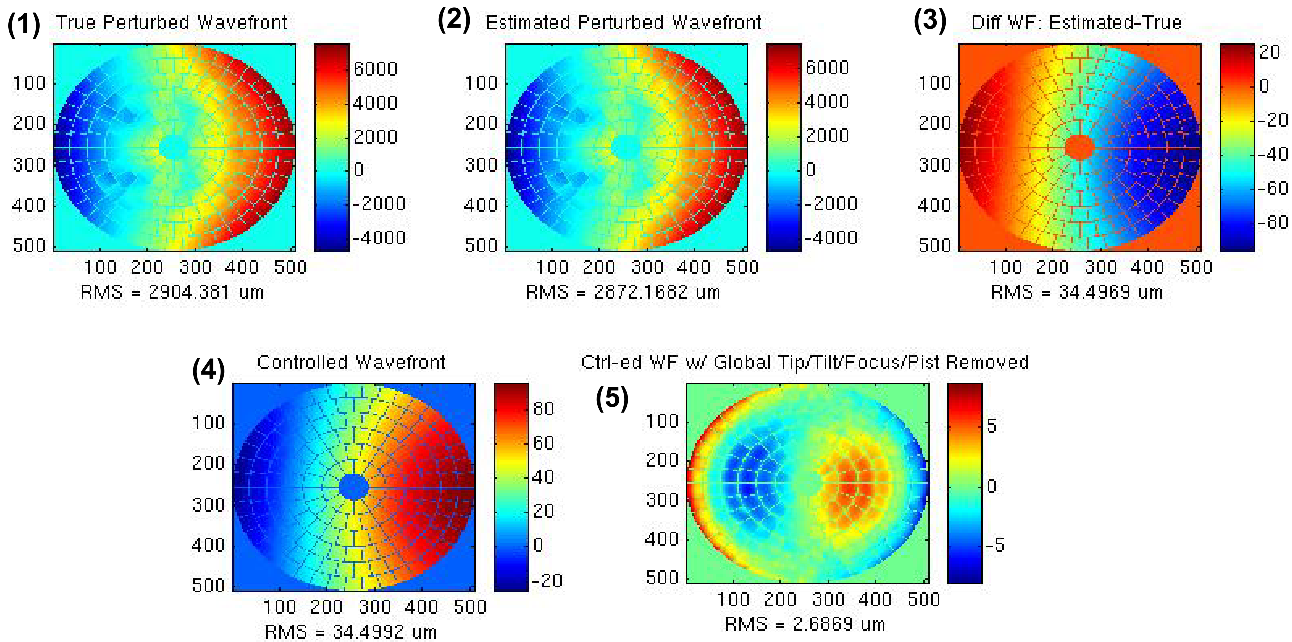
- Final WFE below 1 um, with long-term WFE growth driven by process, actuator and edge sensor noise

Feed-forward control options

- **Determining change with gravity (etc.)**
  - Model-based LUT
  - Model updated by: WF sensing; IDS; laser range-finder metrology
- **Feed-forward control implementation options**
  1. Move PM in global mode only
     - Optical axis varies with sag, variation accounted for in pointing
  2. Move SM only
  3. Move both to minimize WFE and keep axis at nominal
- **LUT-based PM commanding mode options**
  - Send global motion commands only, which are invisible to the IDS
  - Send WF error signals (including global WFE) to be added to the ES-derived WF estimate, and let controller pull out the global correction

Segment State Estimate & Wavefront Control, *1G* on *segments only*

Sensors tilted 45 deg, no sensor noise



(1) 1G load on all PM segments computed from structural FEM model are applied to segments in the optical model, SM and TM are in their design positions (no load). Pupil wavefront is shown in this telescope state.

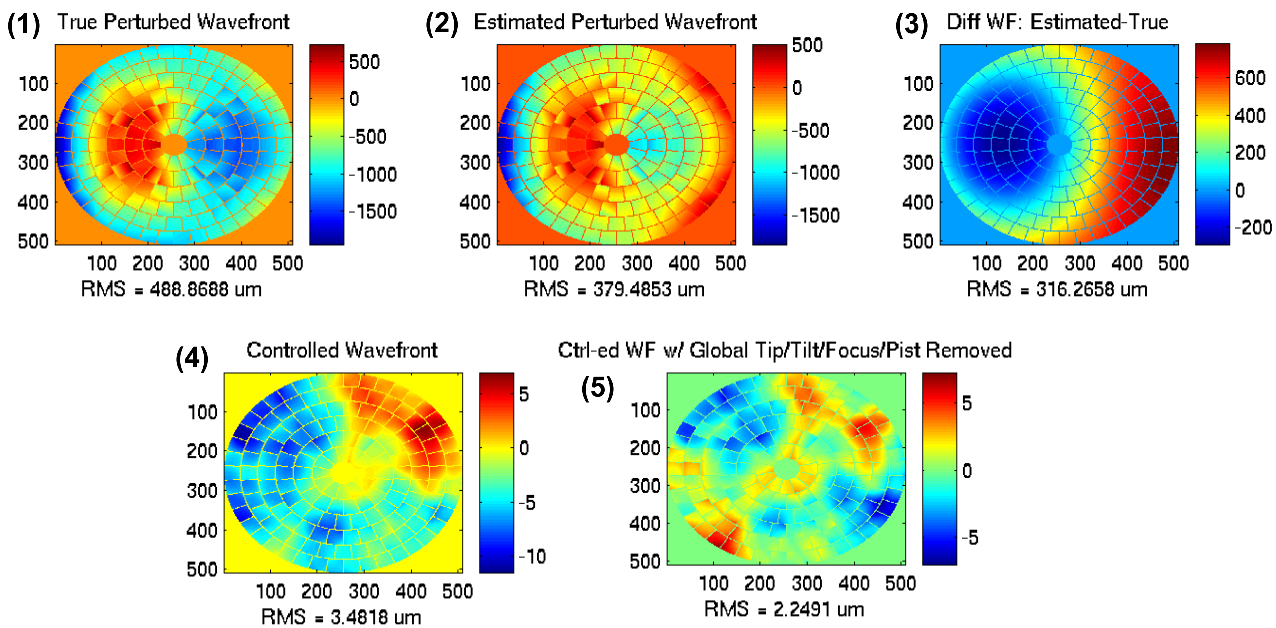
(2) Pupil wavefront is evaluated with estimated segment state, SM and PM are still in design state.

(3) is the difference of (2) and (1).

(4) True wavefront error *after* segment 3DOF control.

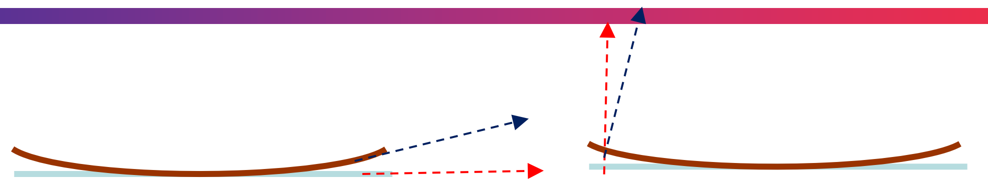
(5) With global tip/tilt/focus/piston removed from (4).

WFC with combined 1G errors on segments / SM, no cupping
Segment 3DOF Controlled with 500 nm CCD XY noise
SM Lookup Position (*no SM Y-motion from 1G*) Computed with Segment RB Errors



(1) 1G load on all PM segments and SM, computed from structural FEM model, are applied to segments and SM in the optical model, TM is in its design position (no load). Pupil wavefront is shown in this telescope state.
(2) Pupil wavefront is evaluated with estimated segment state and SM in a pre-computed lookup table (LUT) position; TM is in its no-load design position. SM LUT position compensates for PM 1G errors.
(3) is the difference of (2) and (1); show big difference because SM is in difference positions in (1) and (2).
(4) True wavefront error *after* segment 3DOF control.
(5) With global tip/tilt/focus/piston removed from (4).

Ideal Segment Cupping, Sensor/Detector State Change

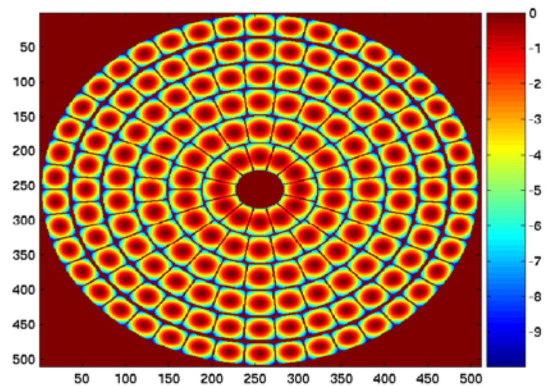


Sensor beam positions/directions before & after segment cupping

Detector plane positions/directions before & after segment cupping

Beam/detector rotations due to cupping are computed with differential displacements along nominal beam/detector directions.

True WF (um) at 3 um cupping

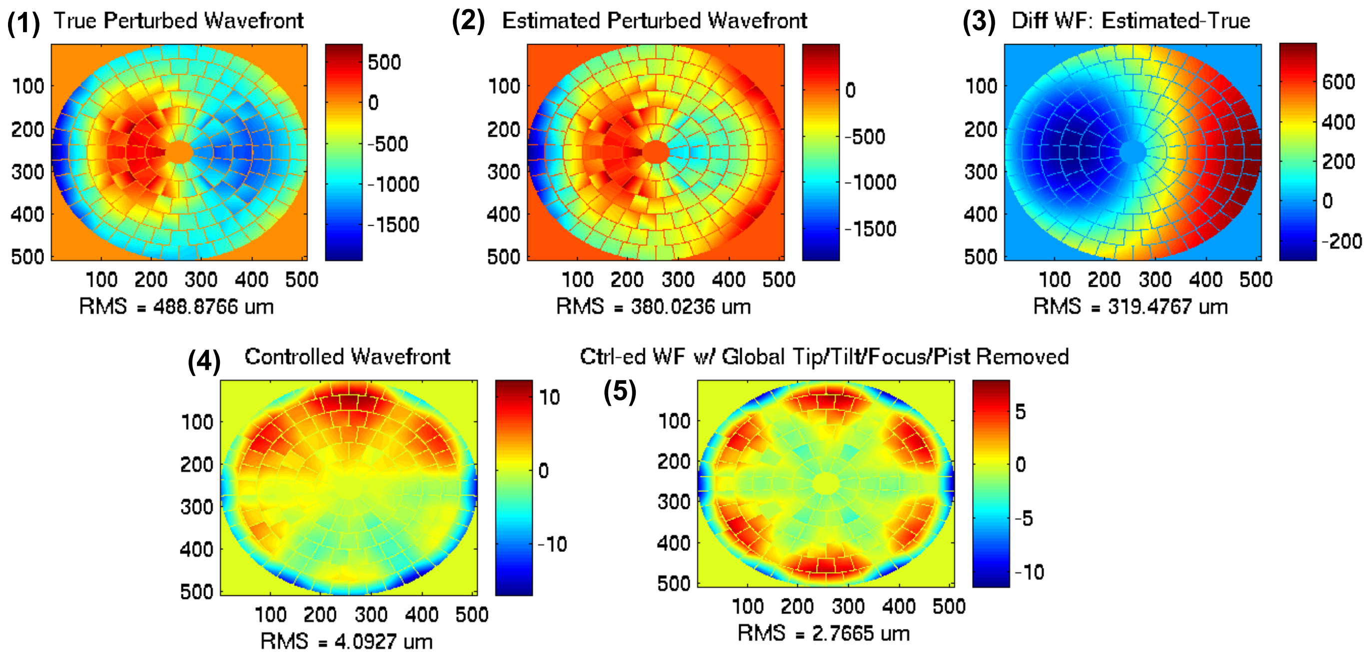


RMS = 2.087 um

Adding cupping deformation to PM segments

1) An ideal segment cupping on *k-th* segment is described by the model: $z_k = c_k*(r/r_k)$^2 in the segment frame. Currently the amplitude c_k is constant for all segments, *r* is distance to segment center and r_k is segment linear size.
2) Cupping deformation is added to each segment through a grided surface on the optical surface of each segment.

WFC with combined 1G errors on segments / SM and 1 um cupping on all segments
Cupping not estimated, Segment 3DOF Controlled with 100 nm CCD XY noise
SM at Lookup Table Position



(1) 1G load on all PM segments and SM, computed from structural FEM model, are applied to segments and SM in the optical model, *and a 1 um uniform cupping is applied to all segments*, TM is in its design position (no load). Pupil wavefront is shown in this telescope state.

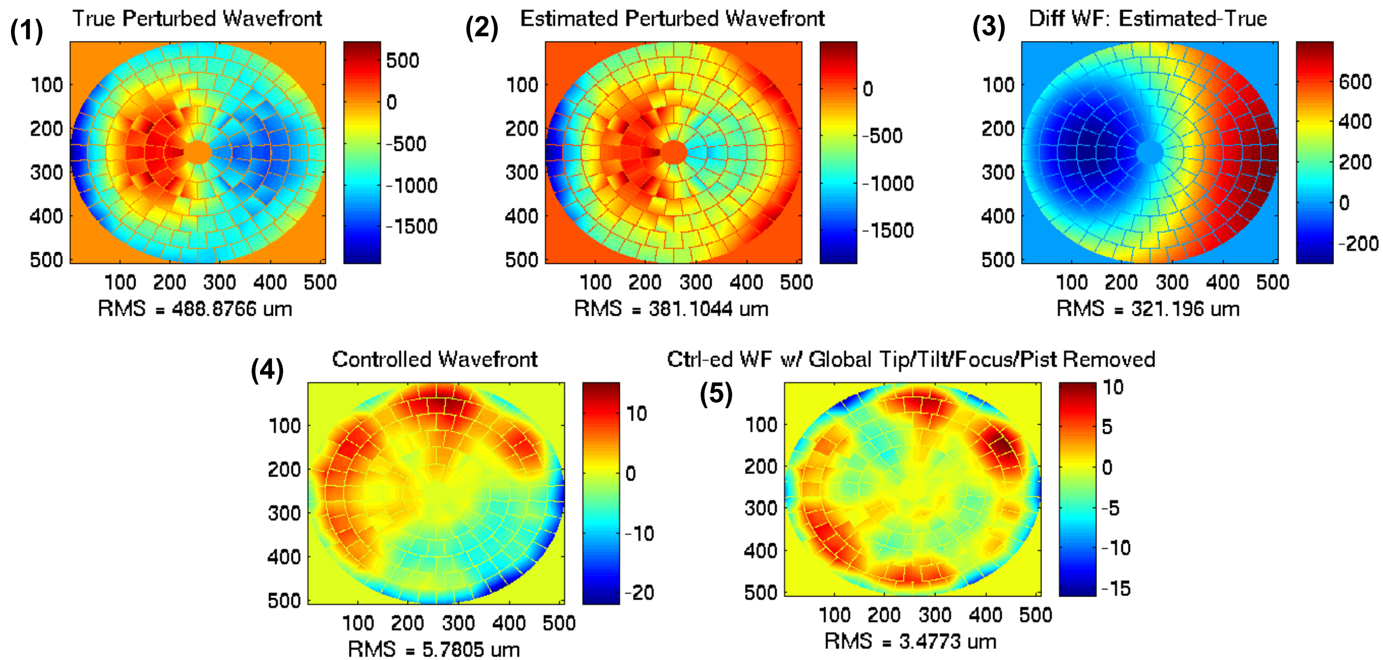
(2) Pupil wavefront is evaluated with estimated segment state and SM in a pre-computed lookup table (LUT) position; TM is in its no-load design position. SM LUT position compensates for PM 1G errors. Cupping is not estimated.

(3) is the difference of (2) and (1); show big difference because SM is in difference positions in (1) and (2).

(4) True wavefront error *after* segment 3DOF control.

(5) With global tip/tilt/focus/piston removed from (4).

WFC with combined 1G errors on segments/SM and 1 um cupping on all segments
Cupping not estimated, Segment 3DOF Controlled with 500 nm CCD XY noise
SM at Lookup Table Position



(1) 1G load on all PM segments and SM, computed from structural FEM model, are applied to segments and SM in the optical model, *and a 1 um uniform cupping is applied to all segments*, TM is in its design position (no load). Pupil wavefront is shown in this telescope state.

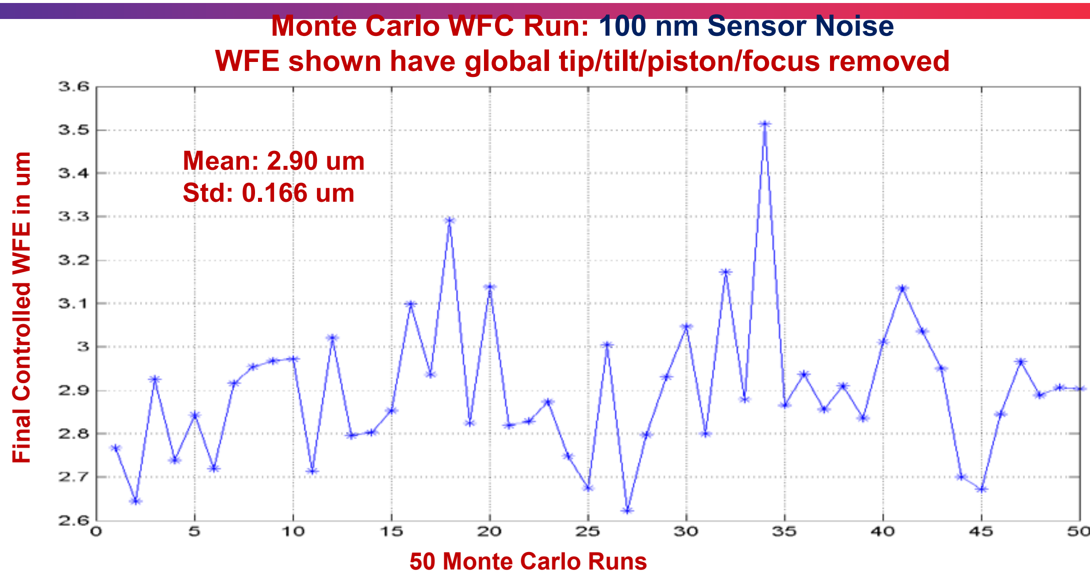
(2) Pupil wavefront is evaluated with estimated segment state and SM in a pre-computed lookup table (LUT) position; TM is in its no-load design position. SM LUT position compensates for PM 1G errors. Cupping is not estimated.

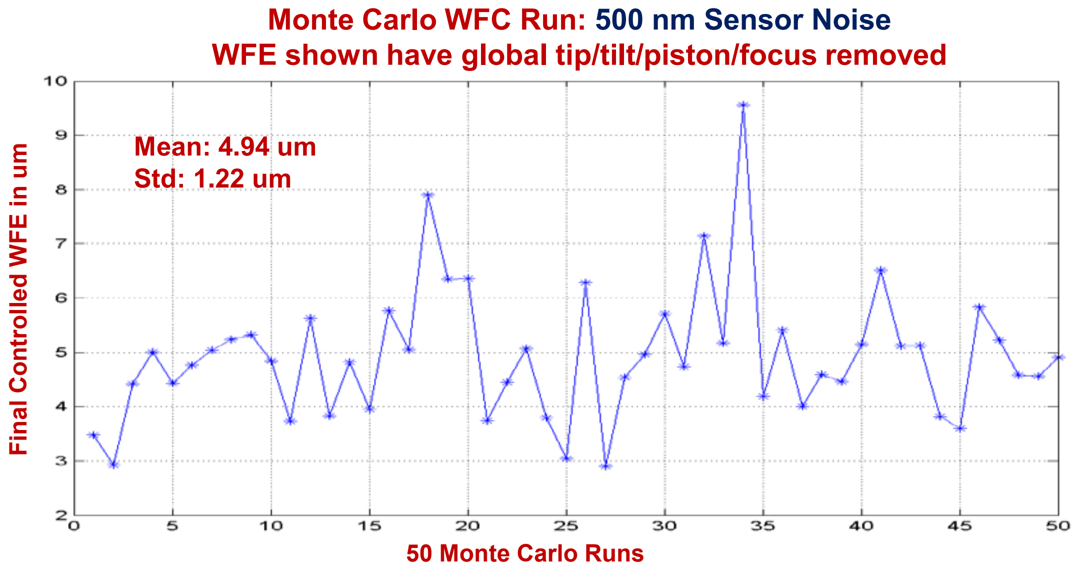
(3) is the difference of (2) and (1); show big difference because SM is in difference positions in (1) and (2).

(4) True wavefront error *after* segment 3DOF control.

(5) With global tip/tilt/focus/piston removed from (4).

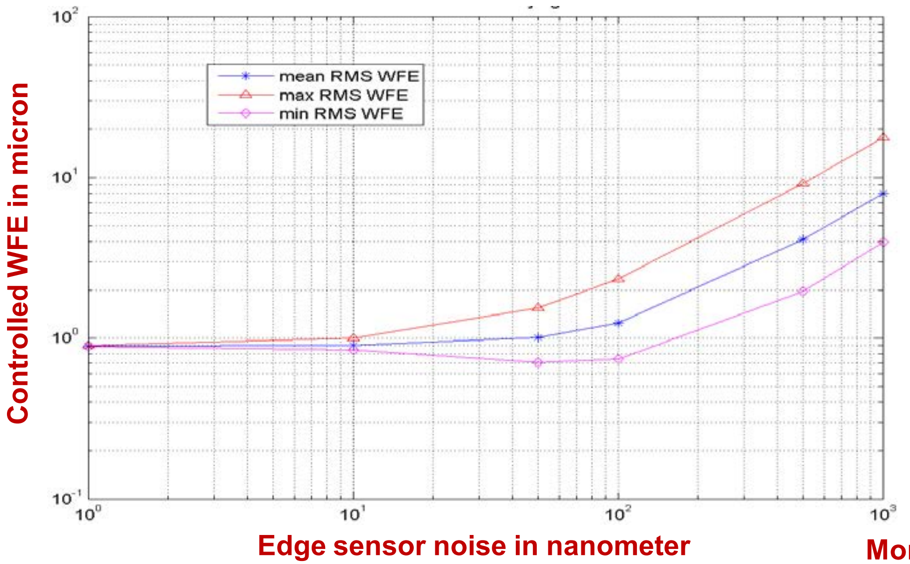
WFC with combined 1G errors on segments / SM and 1 um cupping on all segments
Segment cupping deformation added but not estimated
SM Lookup Position (*no SM Y-motion from 1G*)





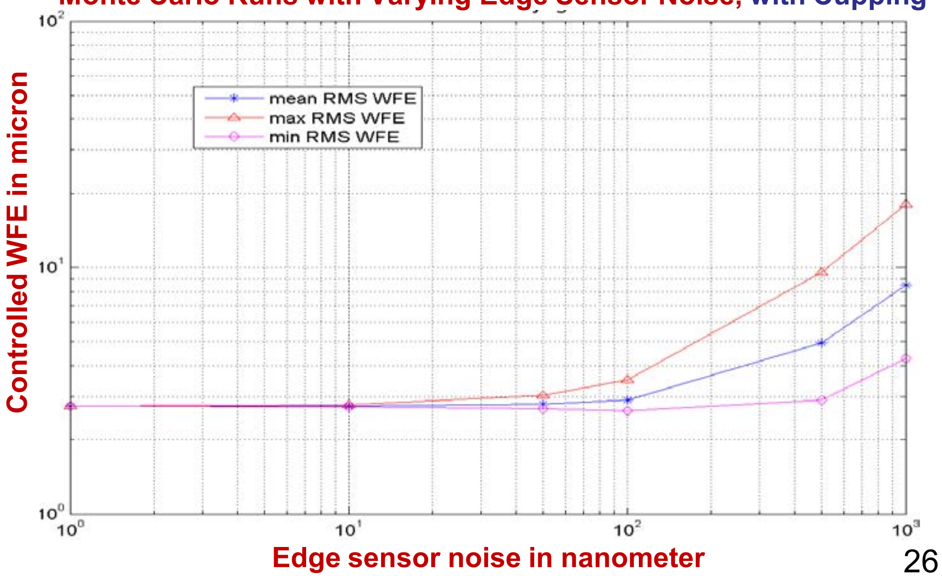
WFC with 1G errors on segments and SM, SM at LUT Position Sensor noise varied, Segment 3DOF Controlled

Monte Carlo Runs with Varying Edge Sensor Noise, No Cupping



With each sensor noise value as standard deviation, 50 Monte Carlo runs were performed.

Monte Carlo Runs with Varying Edge Sensor Noise, with Cupping



Preliminary Performance Summary

CCAT WFE Tree					
		Nominal		Model only, with cupping	
System			System WFE (um RMS)		System WFE (um RMS)
	Component	Component WFE (um RMS)		Component WFE (um RMS)	
1	**Telescope Totals**		**21.55**		**5.32**
	Secondary	12.56		0	
	Tertiary	8.9		0	
	Instrument	0.1		0	
	Mount	0		0	
	Design	8.78		0	
2	Primary		12.27		5.32
	Thermal Gradient	0.02		0.02	
	Actuator heating	0.006		0	
	Actuation error	0		0	
	Acceleration	2.94		0	
	Subframe	0.314		0.314	
	Truss	3.1		3.1	
	Tile	6.6		0	
3	Control		9.41		4.32
	Gravity	2.5		2.5	
	Thermal Gradient	1.38		1.38	
	Actuator heating	1.38		0	
	dCTE cupping	7.86		0	
	Sensor noise	0.7		0.7	
4	Estimation		4.03		3.16
	LUT error	3		3	
	Model error	2.5		0	
	Latency / Drift	1		1	

Performance Snapshot

- **Modeled performance exceeds the predictions of the CCAT WF Error Tree for the included component errors**
  - 2.9 um vs. 5.3 um allocated
- **This performance requires no heroic measures**
  - Fixed-gain OSE
  - No estimation of thermal modes
  - No exotic components
  - No continuous measurement of telescope collimation
- **As design matures, we will update models to track**
- **We also hope to demonstrate performance with full-sized hardware in the next 1-2 years**

CCAT WFE Tree

System		Component	Model only, with cupping: Component WFE (um RMS)	System WFE (um RMS)
1	**Telescope Totals**			**5.32**
		Secondary	0	
		Tertiary	0	
		Instrument	0	
		Mount	0	
		Design	0	
2	Primary			5.32
		Thermal Gradient	0.02	
		Actuator heating	0	
		Actuation error	0	
		Acceleration	0	
		Subframe	0.314	
		Truss	3.1	
		Tile	0	
3	Control			4.32
		Gravity	2.5	
		Thermal Gradient	1.38	
		Actuator heating	0	
		dCTE cupping	0	
		Sensor noise	0.7	
4	Estimation			3.16
		LUT error	3	
		Model error	0	
		Latency / Drift	1	

Conclusion

- **CCAT IDS optical edge sensors provide high dynamic range, excellent accuracy, full observability of segment DOFs, and are linear**
- **Optical State Estimator with full Kalman filter maintains WFE < 1 um vs. 5 um target with included errors**
- **Simpler OSE with fixed gain filter maintains WFE < 3 um: "good enough"**
- **Submillimeter WF Sensing can meet performance needs when a suitable calibration source is in view (Mars, e.g.)**
- **Plan forward**
  - Update Modeling and Control analyses to keep pace with evolving design, improving fidelity
  - Near-term POC demonstration at CSO on Mauna Kea is being planned
  - Also planning WF Maintenance control demonstration for next year, using full-sized CCAT hardware

WFE Performance Checklist

- **WFE of modeled contributors exceeds Error Tree performance targets**
- **WFE contributors**
  - OES sensing errors
  - Thermal cupping
    - WF impacts
    - OSE impacts
  - Design error
  - FoV
  - WF sensing
  - FF LUT mismatch
  - Estimation model mismatch
  - Atmosphere
  - Actuator error
  - Process noise/dynamics
  - Fitting error
  - Segment panel figure
  - Segment panel alignment
  - BW effects
- **IDS control is effective in maintaining PM WF quality**
  - Error multiplier ~3
- **WF performance meets targets without heroic measures**
  - Fixed gains
  - No estimation of thermal cupping