Nondimensional representations for occulter design and performance evaluation

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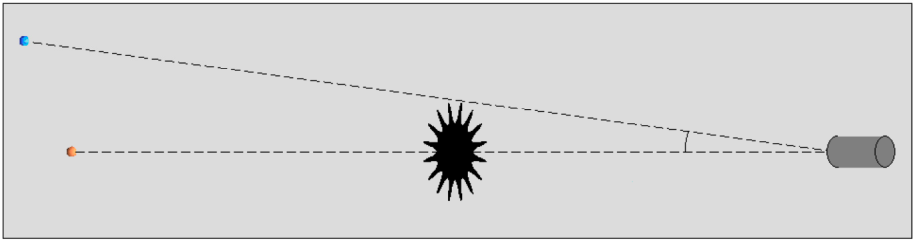
What is an occulter?

An *occulter* or *starshade* is an optical element which is placed in front of the telescope to block most of the light from a star before it reaches the optics inside, without blocking the planet.



In our case, we use two spacecraft flying in formation:

- First has its edge shaped to cancel the starlight
- Second is the telescope which images the star and planet

Optimization for occulter design

We design these occulters by optimizing the function $A(r)$, which determines edge shape, to minimize electric field amplitude:

$$E_{occ}(\rho) = E_0 e^{\frac{2\pi i z}{\lambda}} \left(1 - \frac{2\pi}{i\lambda z}\int_0^R A(r) J_0\left(\frac{2\pi r\rho}{\lambda z}\right) e^{\frac{\pi i}{\lambda z}(r^2+\rho^2)} r dr\right)$$

This minimization is done for points across the plane of the telescope aperture

$$0 \leq \rho \leq \rho_{\max}$$

and across a range of wavelengths

$$\lambda_{\min} \leq \lambda \leq \lambda_{\max}$$

and generally includes additional constraints, such as a solid central region for the spacecraft bus:

$$A(r) = 1 \quad \forall \quad 0 \leq r \leq a \tag{1}$$

Scaling properties of the system

Making an occulter work at two distances doesn't require any changes to shape.

$$z \to z c \qquad\qquad \lambda \to \lambda / c$$

$$E_{occ}(\rho) = E_0 e^{\frac{2\pi i z}{\lambda}} \left(1 - \frac{2\pi}{i\lambda z} \int_0^R A(r) J_0 \left(\frac{2\pi r \rho}{\lambda z} \right) e^{\frac{\pi i}{\lambda z}(r^2+\rho^2)} r dr \right)$$

$$\Downarrow$$

$$E_{occ'}(\rho) = E_0 e^{\frac{2\pi i z c^2}{\lambda}} \left(1 - \frac{2\pi}{i\lambda z} \int_0^R A(r) J_0 \left(\frac{2\pi r \rho}{\lambda z} \right) e^{\frac{\pi i}{\lambda z}(r^2+\rho^2)} r dr \right)$$

Result: same electric field, within a constant phase factor; band goes from $[\lambda_L, \lambda_H]$ to $[\lambda_L/c, \lambda_H/c]$.

Other scaling properties ($r \to r/c, \rho \to \rho/c, z \to z/c^2$) used to reduce systems to laboratory scale.

Spanning the parameter space (I)

The fact the same occulter works at two distances hints that not all of the input parameters to the optimization are independent. We can do a change of variables and rewrite the propagation integral in terms of independent nondimensional parameters:

$$r' \equiv \frac{r}{R}, 0 \leq r' \leq 1$$

$$\rho' \equiv \frac{\rho}{\rho_{\max}}, 0 \leq \rho' \leq 1$$

$$N_o \equiv \frac{R^2}{\lambda z}, N_t \equiv \frac{\rho_{\max}^2}{\lambda z}$$

$$A'(r') = A(r)$$

$$E_{occ}(\rho') = 1 + 2\pi i N_o \int_0^1 A'(r') J_0\left(2\pi\sqrt{N_o N_t} r' \rho'\right) e^{\pi i\left(N_o r'^2 + N_t \rho'^2\right)} r' dr'$$

Spanning the parameter space (II)

Parameter	Definition	Description
N_1	$\frac{R^2}{\lambda_{\max} z}$	maximum-wavelength occulter Fresnel number
N_2	$\frac{R^2}{\lambda_{\min} z}$	minimum-wavelength occulter Fresnel number
N_3	$\frac{\rho_{\max}^2}{\lambda_{\max} z}$	maximum-wavelength shadow Fresnel number
N_4	$\frac{a}{R}$	ratio of central disk radius to full radius

We can define four independent parameters to bound N_o and N_t for broadband optimization

$$N_1 \leq N_o \leq N_2 \tag{2}$$

$$N_3 \leq N_t \leq \frac{N_3 N_2}{N_1} \tag{3}$$

$$A'(r') = 1 \text{ for } 0 \leq r' \leq N_4 \tag{4}$$

$$0 \leq \rho' \leq 1 \tag{5}$$

and let us examine the parameter space of possible optimized occulters.

Spanning the parameter space (III)

We create a 4D grid in the four variables and run an optimization at each grid point, giving the worst-case suppression level for each (N_1, N_2, N_3, N_4) at any point in the occulter shadow and any wavelengths in the occulter bandpass.

Our grid consists of 27000 points, with the values of N_1 through N_4 given at left.

Such a grid can be mined for information about the performance and limitations of occulters under various requirements.

N_1	N_2	N_3	N_4
26 pts.	12 pts.	9 pts.	10 pts.
7.5	15.0	0.1	0.250
8.0	17.5	0.15	0.306
8.5	20.0	0.2	0.361
9.0	22.5	0.25	0.417
9.5	25.0	0.3	0.472
10.0	27.5	0.35	0.527
10.5	30.0	0.4	0.583
11.0	32.5	0.45	0.638
11.5	35.0	0.5	0.694
12.0	37.5	-	0.750
12.5	40.0	-	-
13.0	42.5	-	-
13.5	-	-	-
14.0	-	-	-
14.5	-	-	-
15.0	-	-	-
15.5	-	-	-
16.0	-	-	-
16.5	-	-	-
17.0	-	-	-
17.5	-	-	-
18.0	-	-	-
18.5	-	-	-
19.0	-	-	-
19.5	-	-	-
20.0	-	-	-

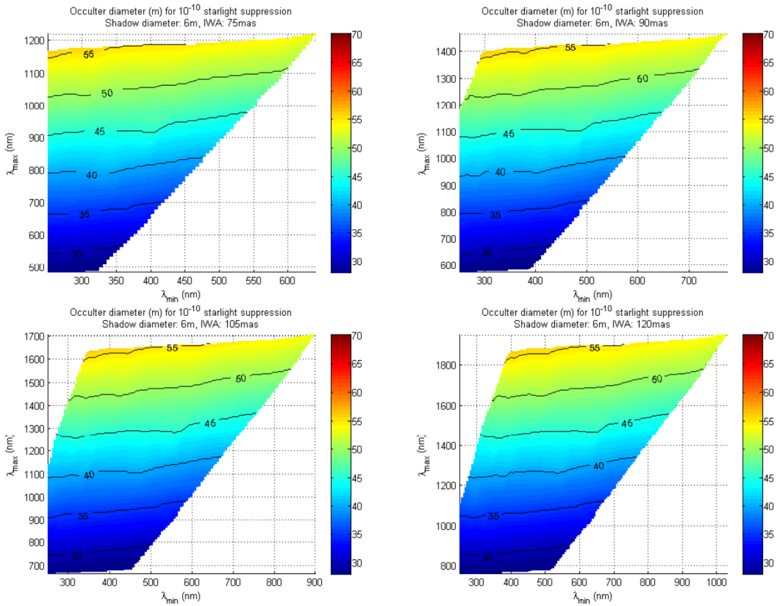
Existing designs

This grid covers the parameter space of existing optimized designs:

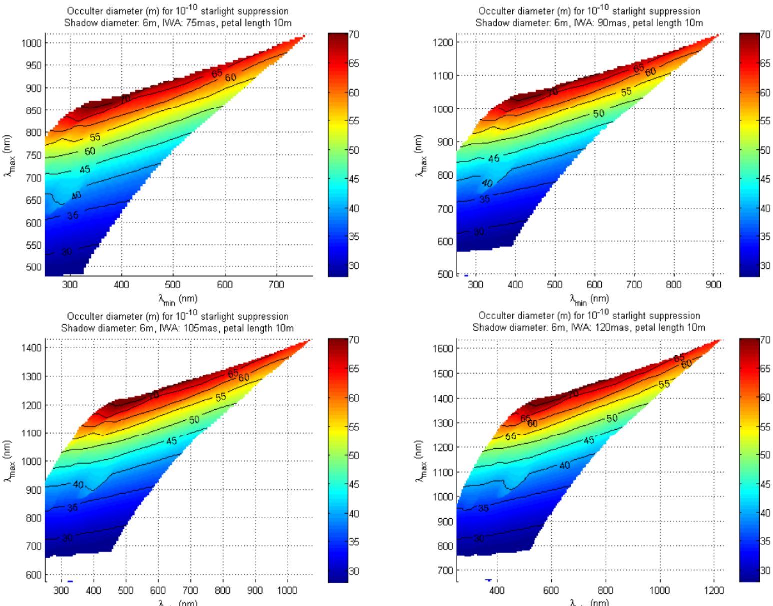
	N_1	N_2	N_3	N_4
Lower grid bound	7.5	15.0	0.10	0.25
Upper grid bound	20.0	42.5	0.50	0.75
THEIA (Kasdin *et al.* 2009)	10.39	29.09	0.23	0.50
1.1m O_3 (Kasdin *et al.* 2010)	9.92	21.82	0.10	0.52
1.5m O_3 variant (Shaklan *et al.* 2011)	12.69	27.92	0.20	0.63
New Worlds Probe (Soummer *et al.* 2010)	8.48	21.21	0.10	0.26

Additional constraints (for tip size, red leak, etc.) may cause their performance to differ slightly from subsequent plots.

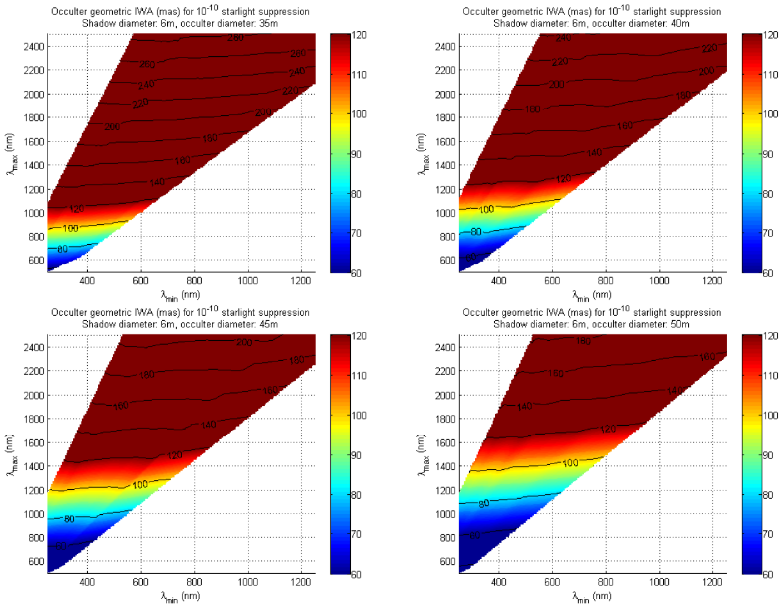
Occulter diameter vs. bandpass



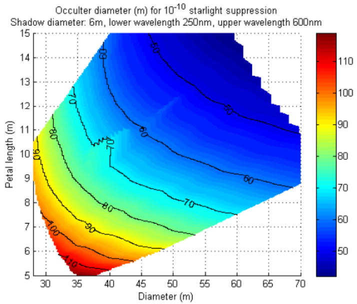
Occulter diameter vs. bandpass, fixed 10m petals

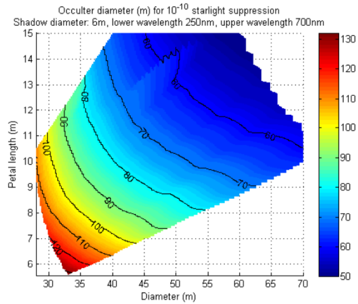


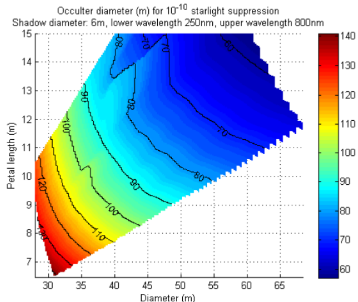
Bandpass versus inner working angle



IWA vs. occulter diameter and petal length







Summary

Nondimensionalization of propagation integrals provides a way to manageably investigate the limits of optimized occulter performance. The gathered data provides a framework for looking at design sensitivities and performing trade studies for occulter missions.

The data shown is representative but not exhaustive; if there's a particular slice of data you'd like to see, please feel free to contact me.

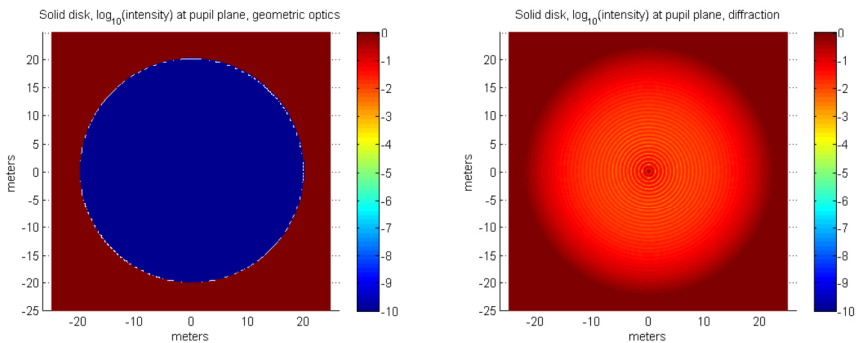
Thank you for your attention; I'd be happy to take any questions.

Additional slides

Why not a disk for planets?

Problem with a simple disk: while geometric optics predicts complete suppression, wave optics predicts diffraction around the edges.

1. On-axis, creates Poisson's spot in the center of the shadow, where the intensity is not attenuated at all
2. Off-axis, full diffraction calculation shows we can only suppress the starlight by $\sim 10^3$



Designing an occulter (I)

Following (Vanderbei, Cady, and Kasdin 2007), we start by thinking about Fresnel propagation from a plane wave incident on an apodized aperture with circular symmetry :

If the apodization is given by a function $A(r)$, then after a distance z, the electric field is:

$$E_{\mathrm{ap}}(\rho) = E_0 e^{ikz}\left(\frac{2\pi}{i\lambda z}\int_0^R e^{\frac{\pi i}{\lambda z}(r^2+\rho^2)} J_0\left(\frac{2\pi r\rho}{\lambda z}\right) A(r) r \,\mathrm{d}r\right)$$

Designing an occulter (II)

An apodized occulter is the complement of this aperture:

$\Rightarrow$

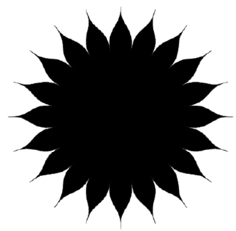
We can write the equation for this using Babinet's Principle:

$$
\begin{aligned}
E_{\mathrm{occ}}(\rho) &= E_0 e^{ikz}\left(1 - \frac{2\pi}{i\lambda z}\int_0^R e^{\frac{\pi i}{\lambda z}(r^2+\rho^2)} J_0\left(\frac{2\pi r\rho}{\lambda z}\right) A(r) r \,\mathrm{d}r\right) \\
&= E_0 e^{ikz} - E_{\mathrm{ap}}(\rho)
\end{aligned}
$$

Designing an occulter (III)

Can't build an apodized occulter from real materials, so we convert it to a binary occulter with N petals:

$\Rightarrow$



Choose petal width so electric field mostly unchanged for small ρ:

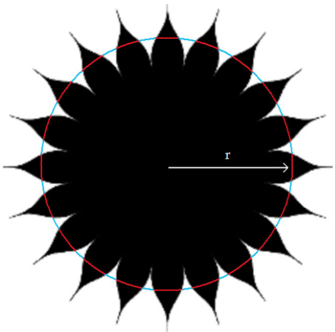
$$E_{\mathrm{bin}}(\rho,\phi) = E_{\mathrm{occ}}(\rho) - E_0 e^{ikz} \sum_{j=1}^{\infty} \frac{4\pi(-1)^j}{i\lambda z} \cos\left[jN(\phi - \pi/2)\right]$$

$$\times \int_0^R e^{\frac{\pi i}{\lambda z}(r^2+\rho^2)} J_{jN}\left(\frac{2\pi r\rho}{\lambda z}\right) \frac{\sin\left(j\pi A(r)\right)}{j\pi} r \,\mathrm{d}r$$

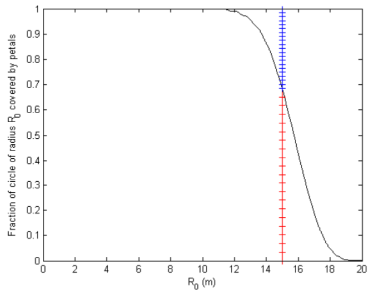
Designing an occulter (IV)

How do we choose the width of the petals?

1. For profile $A(r)$, we design the petals so that a circle of radius r has a fraction of the circle equal to $A(r)$ blocked by a petal
2. We repeat the petals N times to place the scattered light outside the aperture



$\Rightarrow$



Designing an occulter (V)

Lastly, need to choose $A(r)$ and N. We choose $A(r)$ by setting up a linear optimization on the real and imaginary parts of $E_{\rm occ}$ (Vanderbei *et al.* 2007, Cady *et al.* 2008) to constrain the intensity at the telescope pupil:

$$E_{\rm occ}(\rho;\lambda) = E_0 e^{ikz}\left(1 - \frac{2\pi}{i\lambda z}\int_0^R e^{\frac{\pi i}{\lambda z}(r^2+\rho^2)} J_0\left(\frac{2\pi r\rho}{\lambda z}\right) A(r) r \,\mathrm{d}r\right)$$

- Strict bounding should be $Re(E_{\rm occ}(\rho;\lambda))^2 + Im(E_{\rm occ}(\rho;\lambda))^2 \leq c$, with c an upper bound on the intensity; these are quadratic constraints on $A(r)$
- Bounding the real and imaginary parts independently introduces some slightly conservative assumptions, but assures the optimization remains linear.
- Linear optimizations have globally optimal solutions, so we get good apodization profiles with each run of the optimization rather than getting caught in local minima.
- Even better, we can use c as a variable and put it in the cost function, so we minimize the upper bound on the intensity.

Designing an occulter (VI)

We also add some constraints to ensure center of the occulter is solid (to ensure there is a place for the spacecraft bus) and the petal edges are smooth (as the optimization will tend to produce spiky bang-bang solutions otherwise). Can tweak further if desired.

The full problem:

$$\begin{aligned}
\text{Minimize}: \quad & c \\
\text{subject to}: \quad & Re(E_{\text{occ}}(\rho;\lambda)) - c/\sqrt(2) \leq 0 \\
& -Re(E_{\text{occ}}(\rho;\lambda)) - c/\sqrt(2) \leq 0 \\
& Im(E_{\text{occ}}(\rho;\lambda)) - c/\sqrt(2) \leq 0 \\
& -Im(E_{\text{occ}}(\rho;\lambda)) - c/\sqrt(2) \leq 0 \\
& \forall \quad \rho \leq \rho_{\max}, \quad \lambda \in [\lambda_{\min}, \lambda_{\max}] \\
& A(r) = 1 \quad \forall \quad 0 \leq r \leq a \\
& A'(r) \leq 0, \quad |A''(r)| \leq \sigma \quad \forall \quad 0 \leq r \leq R
\end{aligned}$$

Lastly, we choose N so $|E_{\text{bin}} - E_{\text{occ}}| \ll c$ for $\rho \leq \rho_{\max}$.

Optimized shadow

Result: a dark shadow at the telescope aperture.