Micropixel-level Image Position Sensing Testbed

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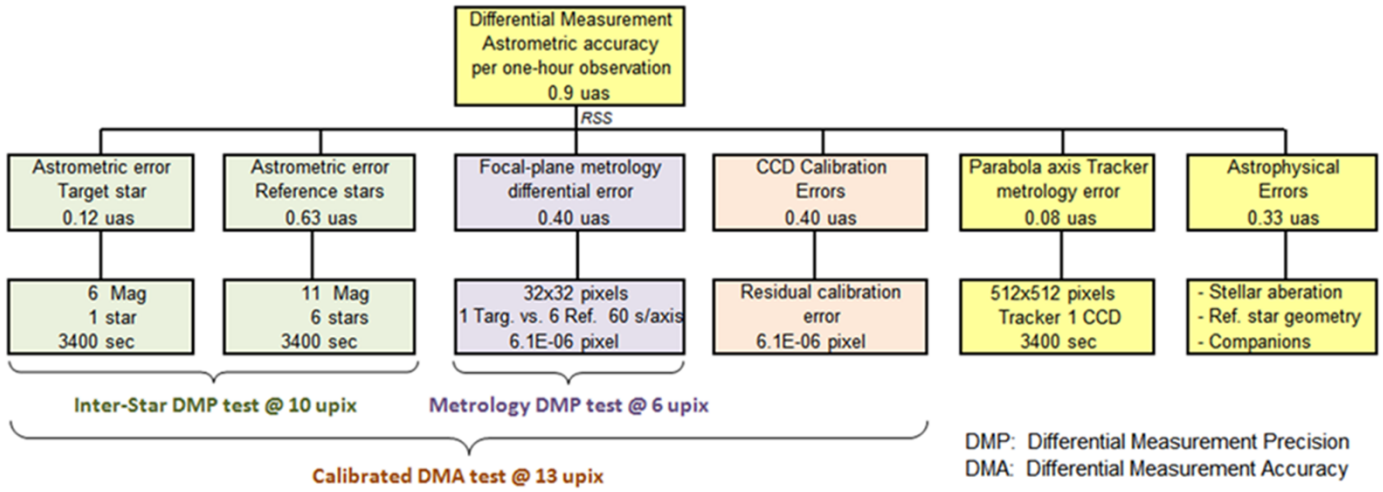
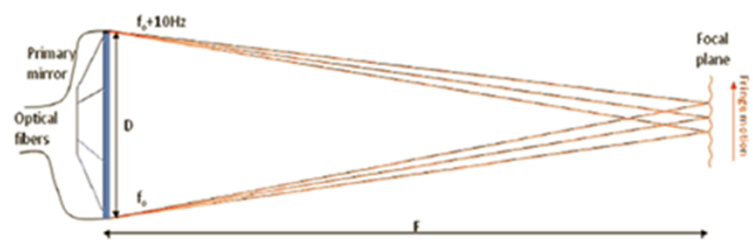
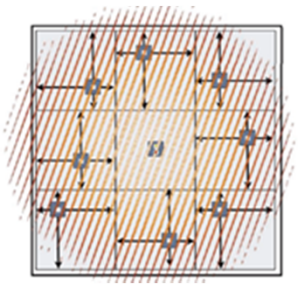
Introduction

- The Micropixel Centroid Testbed (MCT) was designed to demonstrate key aspecs of the technology for Staring Astrometry at 1 uas.
- High-precision centroiding requires Nyquist sampling in the focal plane (> 2 pixels per $f\lambda / D$) .
  - For D = 1 m, f = 20 m focal, λ = 640 nm, 1 pixel ~ 66 mas on the sky.
  - 1 uas accuracy on sky → 15 upix accuracy on the CCD



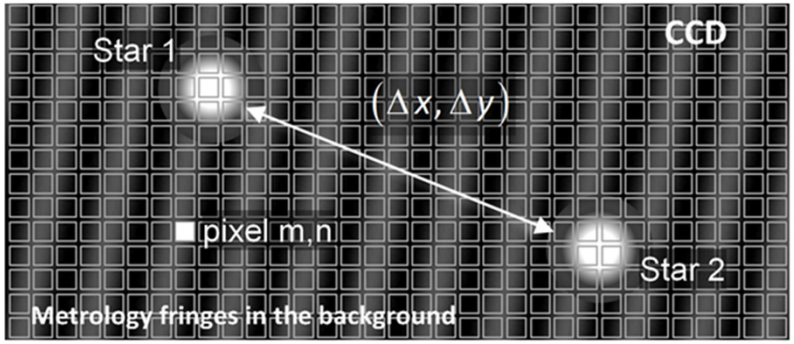
- What is needed:
  1. Inter-pixel metrology
  2. Intra-pixel QE calibration
  3. Image position determination





Inter-Pixel Metrology Concept

- Think of the measurement as measuring the distance between the stars using laser fringes as a highly stable yardstick, with the CCD as an intermediary.
- Create traveling fringes using fiber tips illuminated with laser light with a frequency offset



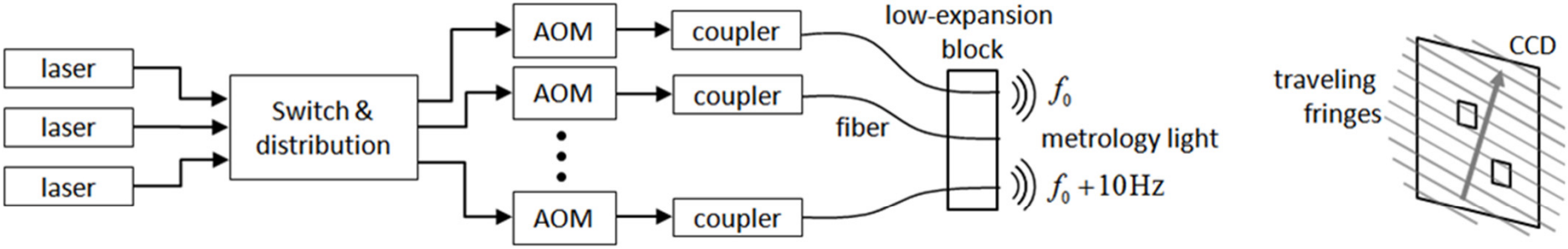


Figure 4: Our approach uses heterodyne metrology beams to create moving fringes on the CCD. The low expansion block and stable laser sources make the fringe spacing highly stable, creating a perfect yardstick that allows unprecedented performance in focal plane characterization.

High-Precision Centroiding

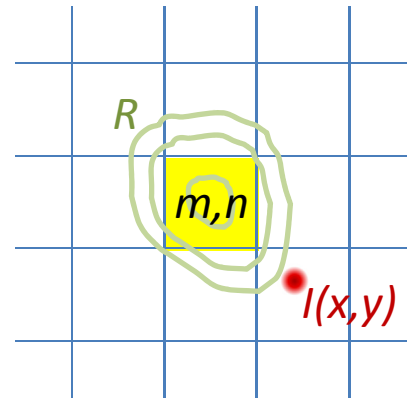
- Define a pixel response function, and determine it using Fourier interpolation to greatly enhance image centroiding accuracy.

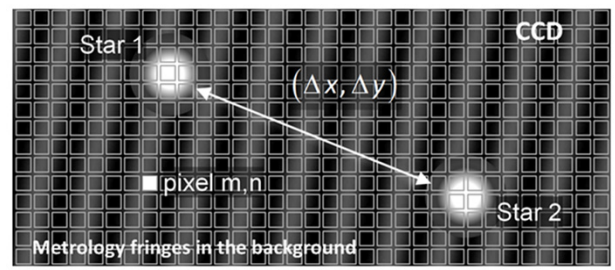
$$I_{mn} = \iint dx\, dy\, R_{mn}(x,y)\, I(x,y)$$

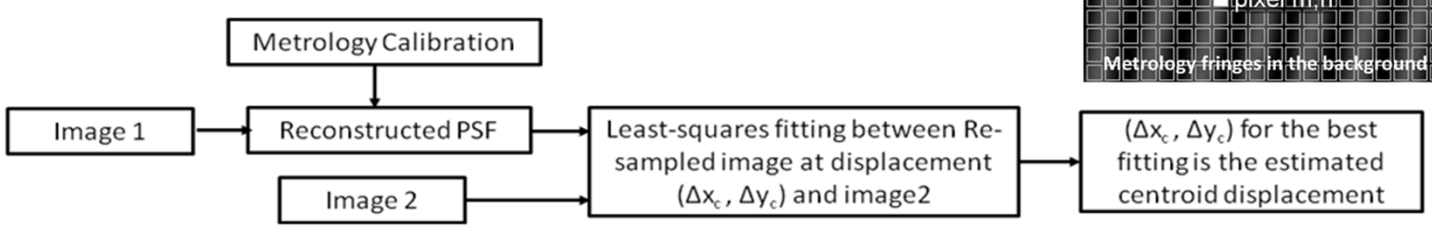
$$I(x,y) = I_0 + A\cos(k_x x + k_y y + \varphi)$$

$$I_{mn} = I_0 \tilde{R}_{mn}(0,0) + A \cdot \mathrm{Re}\left\{\tilde{R}_{mn}(k_x,k_y) e^{i\varphi}\right\}$$

$$\tilde{R}_{mn}(k_x,k_y) = \tilde{R}_{mn}(0,0) \cdot \exp\left\{i\left[k_x(x_{mn} + \Delta x_{mn}) + k_y(y_{mn} + \Delta y_{mn})\right]\right\}$$
$$\times \left[1 + \alpha_{mn} k_x^2 + \beta_{mn} k_y^2 + \gamma_{mn} k_x k_y + \cdots\right]$$

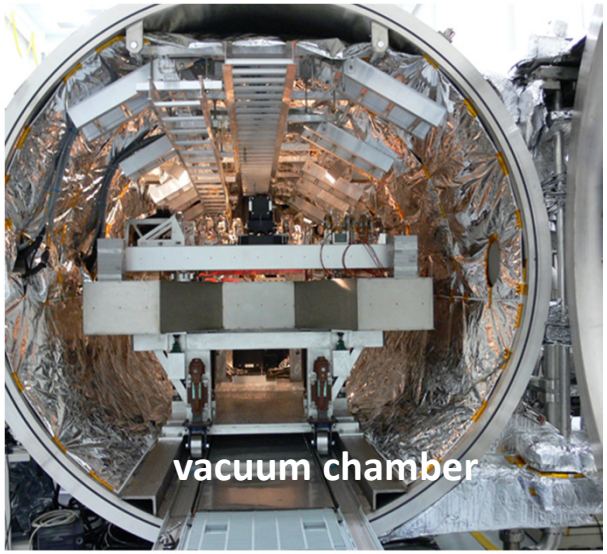


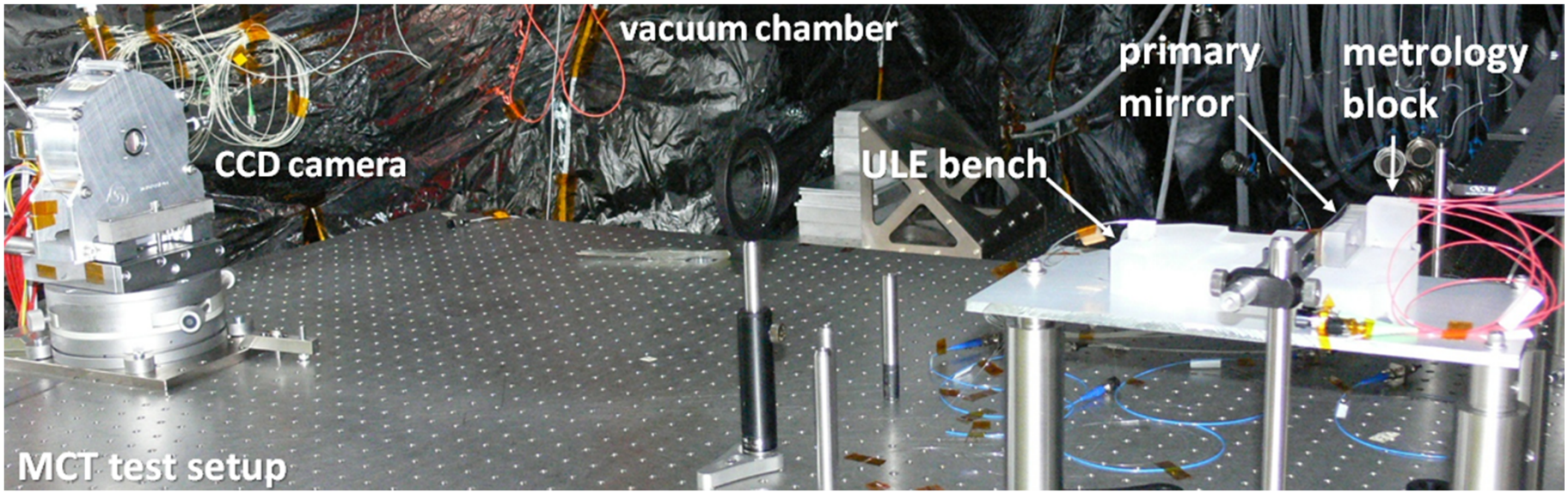




MCT Testbed Setup

- MCT was built in the SIM vacuum chamber with left over SIM parts
- Key components
  - RF electronics and AOM's feeding metrology
  - ULE bench for star simulator and metrology block
  - Scientific grade back-illuminated CCD





Metrology Block and Fringes

- A total of six fiber tips, mounted with non-reduntant spacings (2, 4, 7 mm), give 15 different fringe patterns
- Need a minimum of 2 ~ orthogonal patterns – the rest add redundancy and checking capability

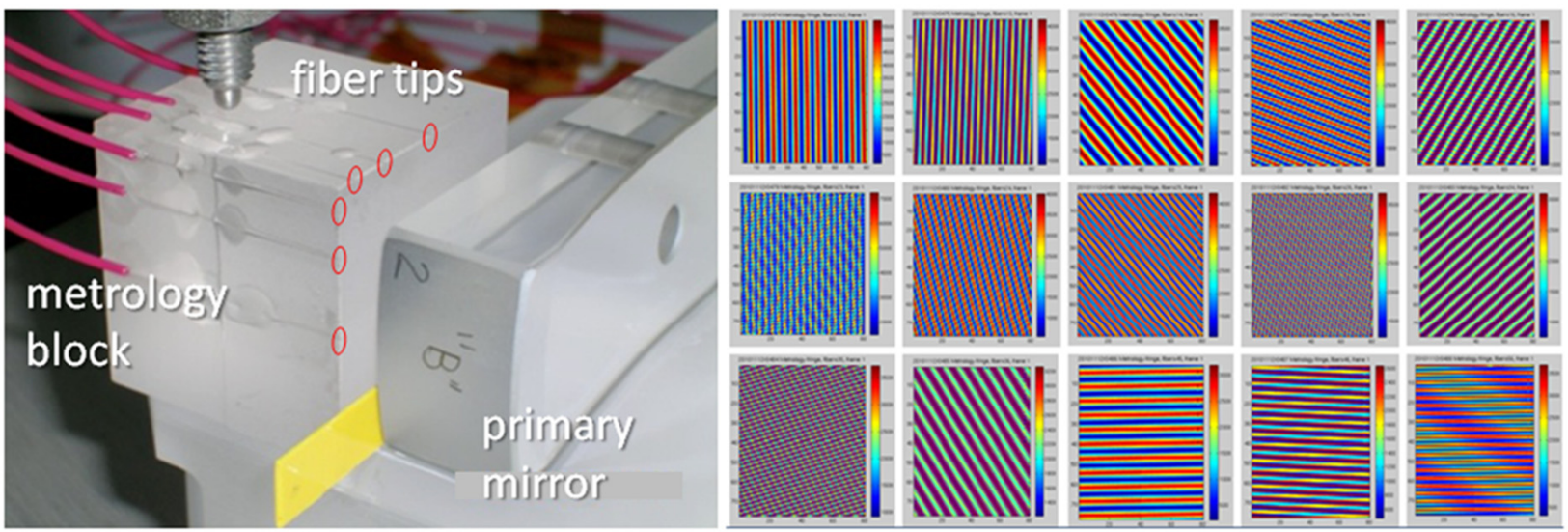
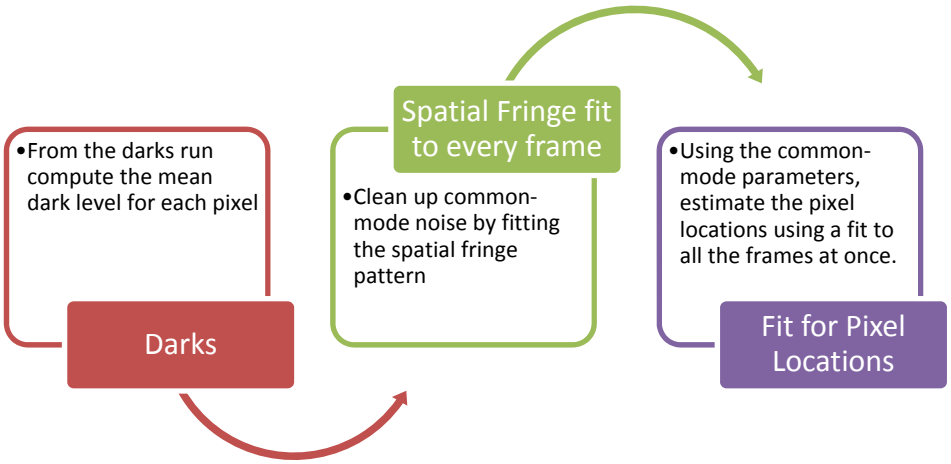
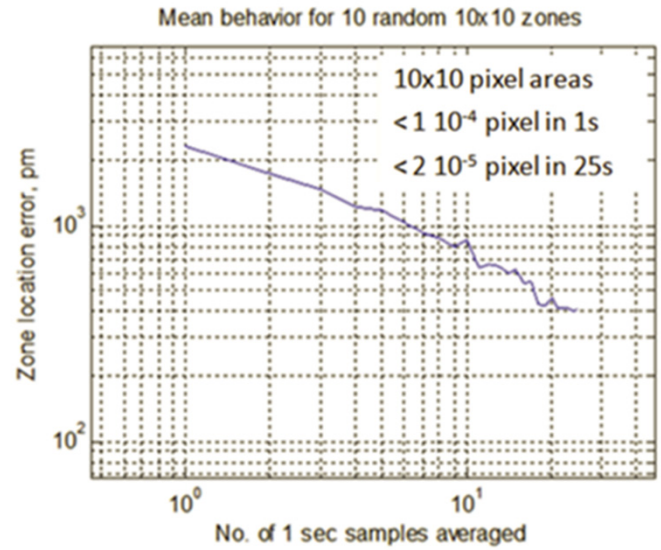


Figure 8: Close-up (left) of the metrology block and primary mirror in MCT, highlighting the metrology fiber tips, and (right) fringe patterns on the CCD for the 15 different lit fiber pairs.

Preliminary Results: CCD pixel Locations





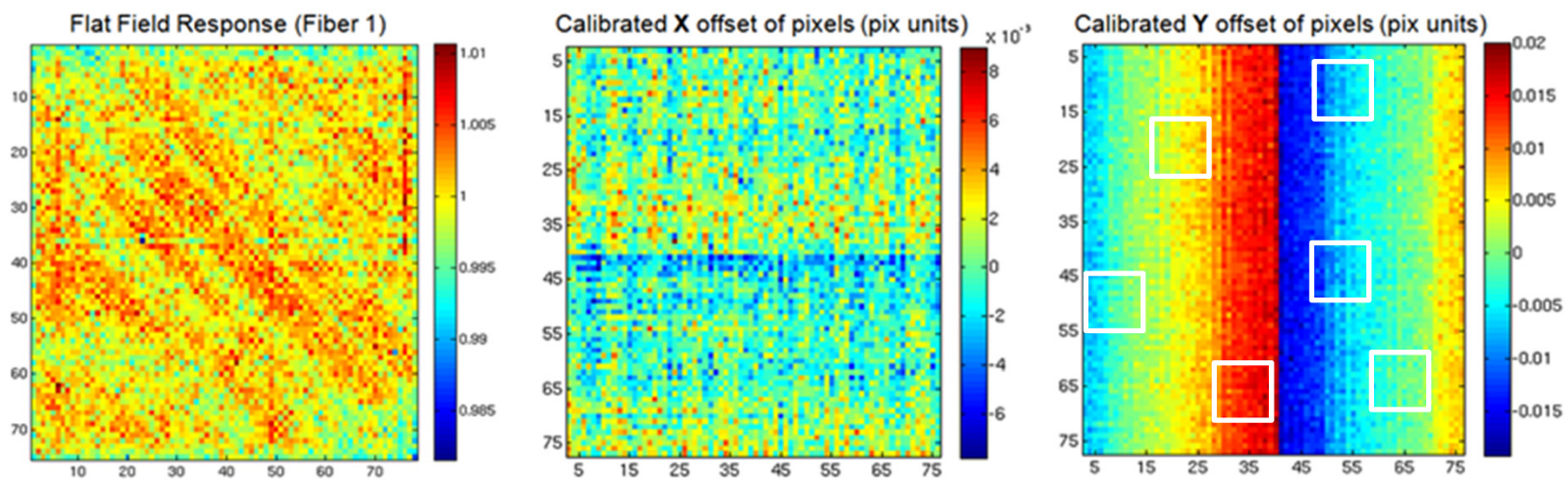


Figure 9: MCT test results showing the flat field response (left) and effective pixel location deviations from a regular grid along row (middle) and column (right) directions.

Optical Bench, the Camera and Image

- The optical bench is a recycled SIM internal beam launcher bench
- On it is mounted a parabola, which is masked down to a 14 mm circle which acts as the primary
- The metrolog block is mounted behind the primary
- A fiber bundle with seven closely packed fibers make up the stars

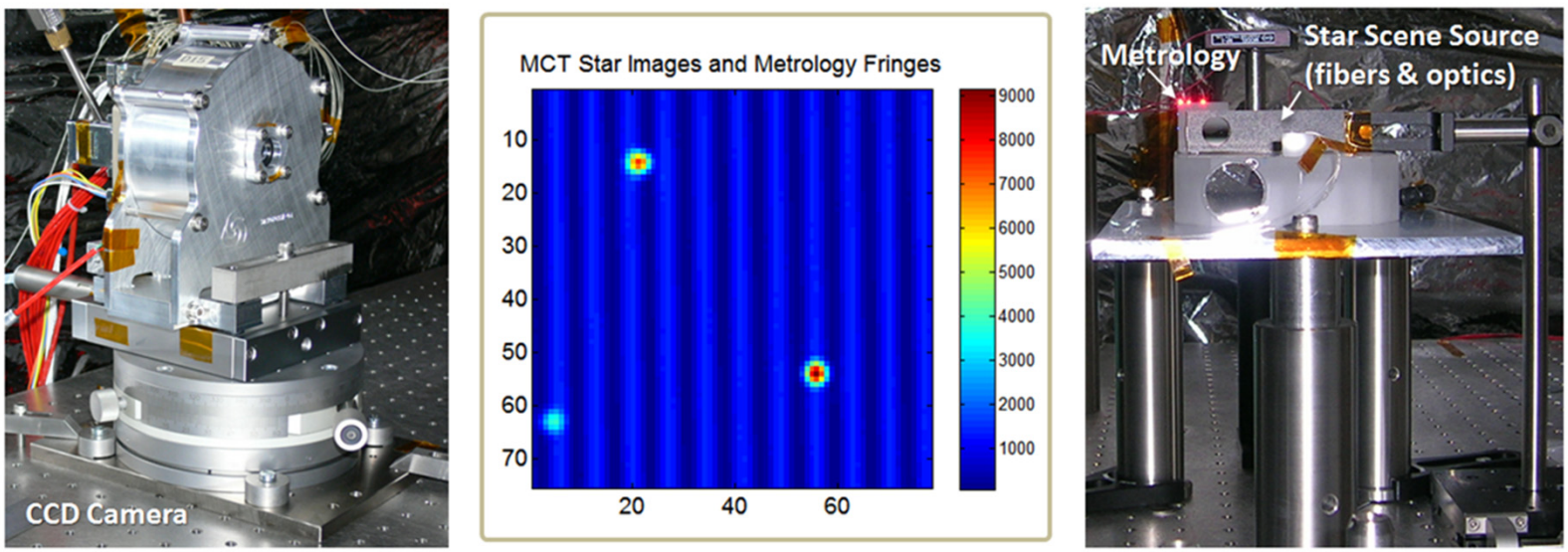


Figure 7: MCT test setup in the SIM vacuum chamber at JPL prior to shut-down. Shown are the test camera (left) a single frame with star images and metrology fringes (center), and the optical bench (right) with the primary mirror and the metrology block, showing some of the metrology fiber tips lit.

Check Centroid Displacement Estimation Algorithm

- Using simulated image pairs and simulated pixel metrology calibration errors, keeping terms up to 3rd order in the expansion
- Repeat the estimation for simulated centroid displacements from -0.5 → +0.5 pix in 0.1 pix steps along *x* (row) and *y* (column)
- Find that RMS estimation error is less than 4 µpix, with the error being small near the center (smallest displacements) and increasing towards the edges.

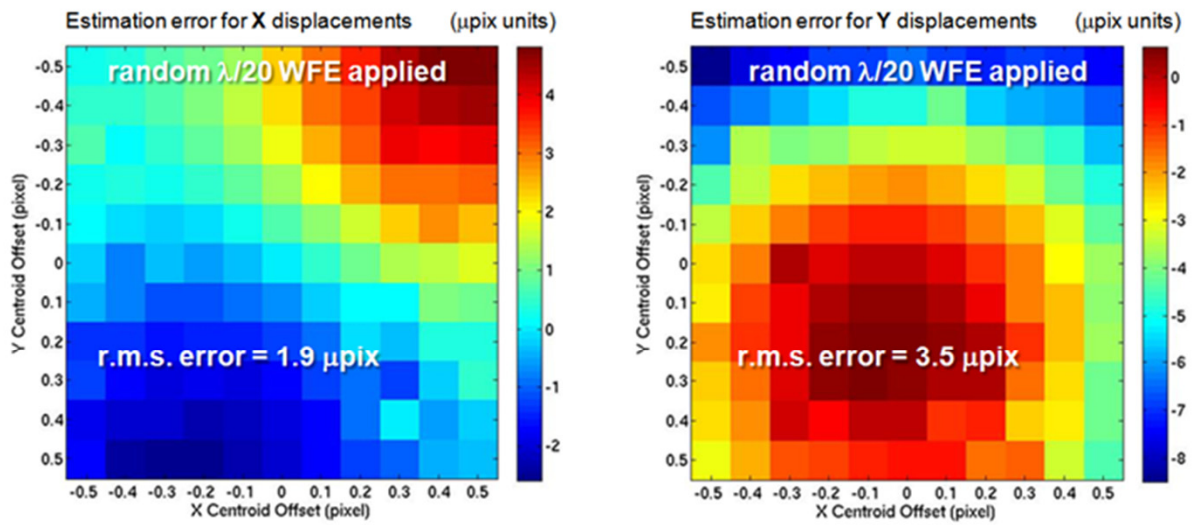
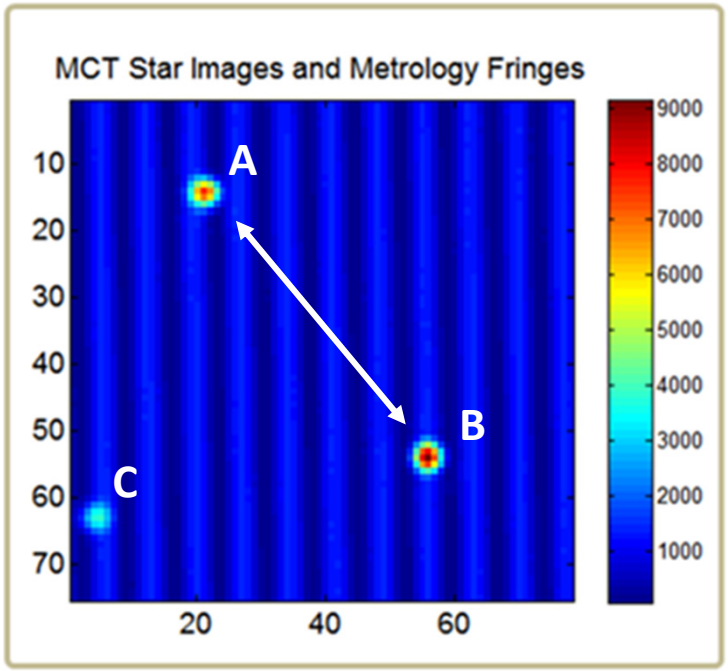
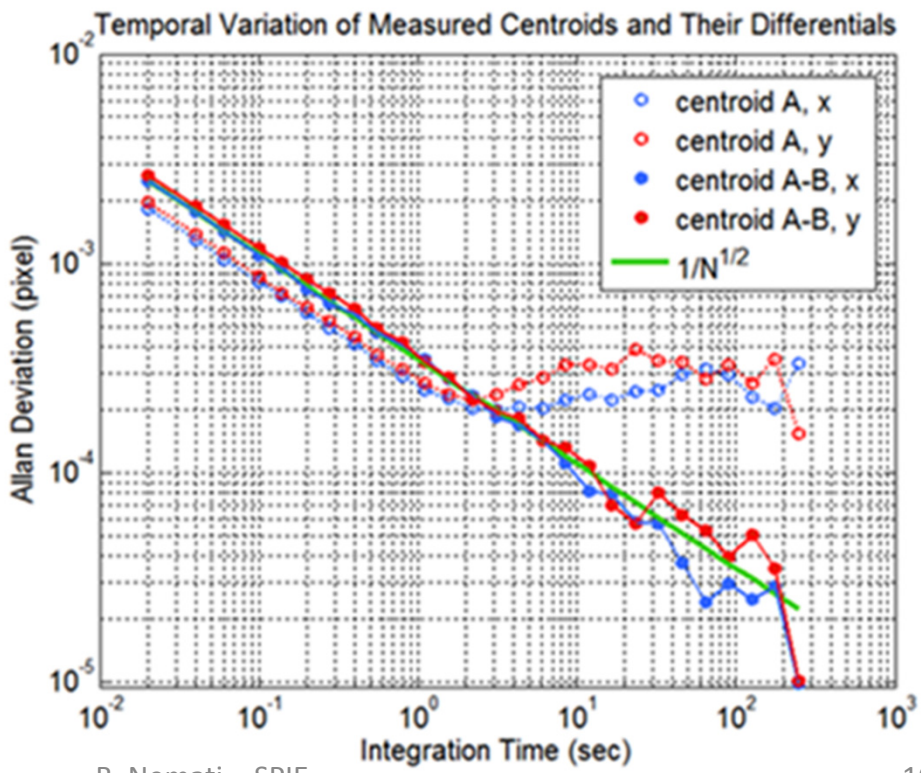


Figure 8: Simulation results showing the expected systematic errors in our estimation of image displacements for various applied x and y displacements. A wavefront error of λ/20 was simulated. The grids correspond to relative two-dimensional centroid displacements along x and y within the range [-0.5, 0.5] pix. The error is expressed in µpix units.

Stability of Centroid Differentials

- The centroids of star A or B vary at 100 µpix level at time scale of a few seconds
- It is the difference that is of interest in astrometry.
- In the lab, the centroids of A & B each by about 250 µpix in ~ 200 sec
- The random error in the separation of the two images averages down to 30 µpix after integrating for 200 sec





Summary and Future Work

- We have made a preliminary study of micro-pixel centroid estimation feasibility for use in narrow angle astrometry at 1 uas
- Simulation results show that, if the images are Nyquist (or better) sampled, the systematic error in centroid displacement estimation is at a few micro pixels in the presence of $\lambda/20$ wavefront errors
- We have demonstrated metrology of a back-illuminated CCD down to 20 upix
  - The study seems to indicate a surprising 4% shift in pixel locations in one direction for an E2V CCD39 at the midpoint boundary
- First lab results of centroid estimation and differencing show that we have the precision to do the tests (30 upix after 200 s)
- We expect further funding this year to complete the demonstration

BACKUP

Metrology Processing Algorithm – I

1. Spatial fringe fit

$$I_n^{i,j} = I_n^0 + A_n^0 \sin\left(\phi_n^0 + \vec{\kappa}_n^0 \cdot \vec{r}_{i,j}\right) \qquad \text{where:} \quad \vec{\kappa}_n^0 \cdot \vec{r}_{i,j} = \kappa_{n,x}^0 \cdot j + \kappa_{n,y}^0 \cdot i$$

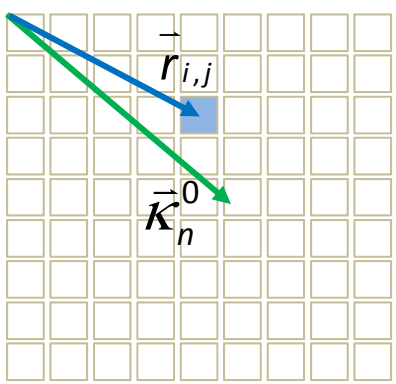
$$\xrightarrow[\text{fit all pixels for each frame}]{\text{non-linear least squares (lsqnonlin)}} \left\{ I_n^0, A_n^0, \kappa_{n,x}^0, \kappa_{n,y}^0, \phi_n^0 \right\}$$

2. Pixel locations fit

$$I_n^{i,j} = I_n^0 \iota_{i,j} + A_n^0 \alpha_{i,j} \sin\left(\phi_n^0 + \vec{\kappa} \cdot \vec{r}'_{i,j}\right)$$

$$= I_n^0 \iota_{i,j} + A_n^0 \sin\phi_n^0 \cdot \underbrace{\left(\alpha_{i,j} \cos\left(\vec{\kappa} \cdot \vec{r}'_{i,j}\right)\right)}_{C_{i,j}} + A_n^0 \cos\phi_n^0 \cdot \underbrace{\left(\alpha_{i,j} \sin\left(\vec{\kappa} \cdot \vec{r}'_{i,j}\right)\right)}_{S_{i,j}}$$

where: $\vec{\kappa} \equiv \left\langle \vec{\kappa}_n^0 \right\rangle \qquad \vec{r}'_{i,j} = (j,i) + \hat{\kappa}\,\delta r_{i,j}$ and $\hat{\kappa} \equiv \vec{\kappa} / |\vec{\kappa}|$



$$\xrightarrow[\text{fit all frames for each pixel}]{\text{psuedo inverse}} \begin{bmatrix} \iota_{i,j} \\ C_{i,j} \\ S_{i,j} \end{bmatrix} = \begin{bmatrix} I_1^0 & A_1^0 \sin\phi_1^0 & A_1^0 \cos\phi_1^0 \\ I_2^0 & A_2^0 \sin\phi_2^0 & A_2^0 \cos\phi_2^0 \\ \vdots & \vdots & \vdots \\ I_N^0 & A_n^0 \sin\phi_N^0 & A_N^0 \cos\phi_N^0 \end{bmatrix}^{\dagger} \begin{bmatrix} I_1^{i,j} \\ I_2^{i,j} \\ \vdots \\ I_N^{i,j} \end{bmatrix}$$

$$\alpha_{i,j} = \sqrt{C_{i,j}^2 + S_{i,j}^2}$$

$$\vec{\kappa} \cdot \vec{r}'_{i,j} = \tan^{-1} \frac{S_{i,j}}{C_{i,j}} + m2\pi \quad \rightarrow \quad \delta r_{i,j} = \frac{1}{|\vec{\kappa}|}\left(\tan^{-1} \frac{S_{i,j}}{C_{i,j}} + m2\pi - \vec{\kappa} \cdot (j,i)\right) \text{ along } \hat{\kappa}$$

Note: *m* obtained by unwrapping the phase

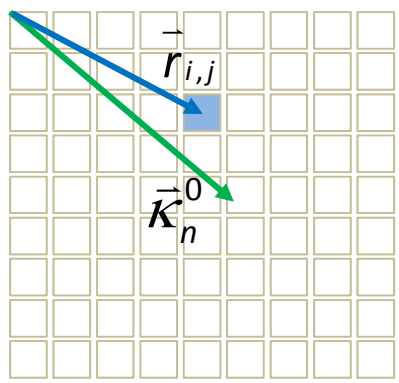
K can also be gotten from a fit of K.r to r, rather than an average of all the Ks from the frames

Metrology Processing Algorithm – II

3. Iterated spatial fit

$$I_n^{i,j} = I_n^0 \iota_{i,j} + A_n^0 \alpha_{i,j} \sin\left(\phi_n^0 + \vec{\kappa}_n^0 \cdot \vec{r}'_{i,j}\right)$$

$$\xrightarrow[\text{fit all pixels for each frame}]{} \left\{ I_n^0, A_n^0, \kappa_{n,x}^0, \kappa_{n,y}^0, \phi_n^0 \right\}$$ revised values

Single fiber data can provide estimates of iota and alpha
Iota is ver close to QE

4. Final pixel locations fit – Same as step 2, except using the revised values from step 3