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Phase-sensitive coherence and the classical-quantum boundary in ghost imaging

by

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Phase-sensitive light

- Unified theory of optical imaging using Gaussian-state light
- Phase-insensitive and phase-sensitive coherence
  - monochromatic versus bichromatic nature
  - quasimonoplanatic versus quasibiplanatic nature
- Ghost Imaging for standoff sensing
  - classical and quantum image signatures
  - signal-to-noise ratio
- Ghost imaging with classical phase-sensitive light

Unified theory via Gaussian states

- Paraxial, z-propagating, photon-units operator $\hat{E}_z(\boldsymbol{\rho}, t)e^{ik_0 z - i\omega_0 t}$
  - Non-zero commutator is $[\hat{E}_z(\boldsymbol{\rho}_1, t_1), \hat{E}_z^\dagger(\boldsymbol{\rho}_2, t_2)] = \delta(\boldsymbol{\rho}_2 - \boldsymbol{\rho}_1)\delta(t_2 - t_1)$
  - Plane-wave decomposition is

$$\hat{E}(\boldsymbol{\rho}, t) = \int \frac{\mathrm{d}\boldsymbol{k}}{2\pi} \int \frac{\mathrm{d}\Omega}{\sqrt{2\pi}} \hat{A}(\boldsymbol{k}, \Omega) e^{i\boldsymbol{k}\cdot\boldsymbol{\rho} - i\Omega t}$$

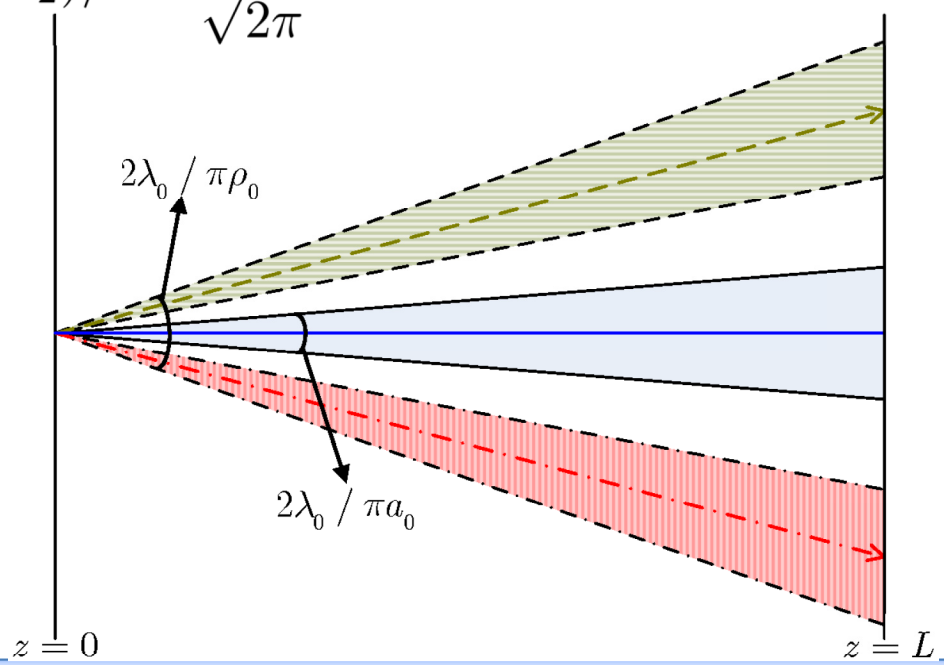
- Zero-mean Gaussian states are completely characterized by
  - Phase-insensitive correlation function $\langle \hat{E}_z^\dagger(\boldsymbol{\rho}_1, t_1)\hat{E}_z(\boldsymbol{\rho}_2, t_2)\rangle$
  - Phase-sensitive correlation function $\langle \hat{E}_z(\boldsymbol{\rho}_1, t_1)\hat{E}_z(\boldsymbol{\rho}_2, t_2)\rangle$

$\langle \hat{E}_z(\boldsymbol{\rho}_1, t_1)\hat{E}_z(\boldsymbol{\rho}_2, t_2)\rangle = 0$	$\langle \hat{E}_z(\boldsymbol{\rho}_1, t_1)\hat{E}_z(\boldsymbol{\rho}_2, t_2)\rangle \neq 0$
Always classical e.g. Lasers, LED's, pseudothermal light	Can be classical *or* quantum e.g. classical phase-sensitive light, squeezed states, output of SPDC, biphoton

Phase-insensitive correlation

- Gaussian-Schell model phase-insensitive correlation function
  - photon flux P, intensity radius a_0, coherence radius $\rho_0\,(\ll a_0)$, coherence time T_0
- Phase-insensitive correlation spectrum is *monochromatic* and *quasimonoplanatic*

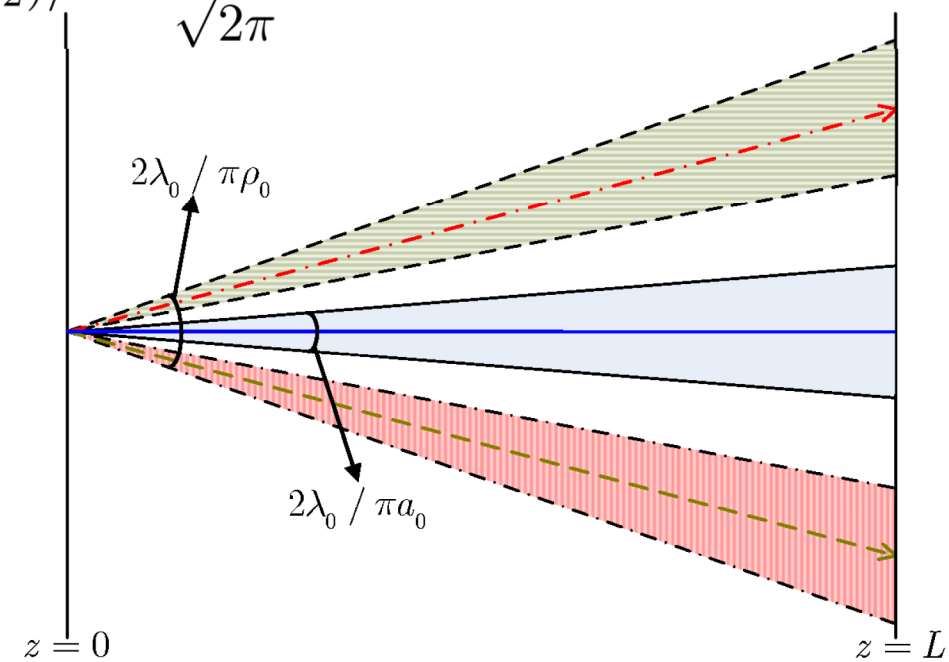
$$\langle \hat{A}^\dagger(\boldsymbol{k}_1,\Omega_1)\hat{A}(\boldsymbol{k}_2,\Omega_2)\rangle = \frac{PT_0\rho_0^2}{\sqrt{2\pi}} e^{-a_0^2|\boldsymbol{k}_2-\boldsymbol{k}_1|^2/8-\rho_0^2|\boldsymbol{k}_1+\boldsymbol{k}_2|^2/8} e^{-T_0^2\Omega_2^2/2}\delta(\Omega_2-\Omega_1)$$



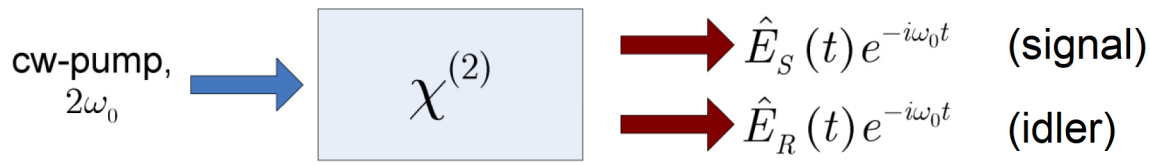
Phase-sensitive correlation

- Gaussian-Schell model phase-sensitive correlation function
  - mean-square flux P_s, intensity radius a_0, coherence radius $\rho_0\ (\ll a_0)$, coherence time T_0
- Phase-insensitive correlation spectrum is *bichromatic* and *quasibiplanatic*

$$\langle \hat{A}(\boldsymbol{k}_1,\Omega_1)\hat{A}(\boldsymbol{k}_2,\Omega_2)\rangle = \frac{P_s T_0 \rho_0^2}{\sqrt{2\pi}} e^{-a_0^2|\boldsymbol{k}_1+\boldsymbol{k}_2|^2/8} e^{-\rho_0^2|\boldsymbol{k}_2-\boldsymbol{k}_1|^2/8} e^{-T_0^2\Omega_2^2/2}\delta(\Omega_2+\Omega_1)$$



Classical-quantum boundary



- $\hat{E}_\ell(t)$ in zero-mean jointly-Gaussian state
  - phase-insensitive auto-correlations

$$\langle \hat{E}_\ell^\dagger(t+\tau)\hat{E}_\ell(t)\rangle = \mathcal{F}^{-1}\{S(\Omega)\}$$

$$\begin{cases} \ell = S, R \\ S(\Omega) = S(-\Omega) \end{cases}$$

  - phase-sensitive cross correlation

$$\langle \hat{E}_S(t+\tau)\hat{E}_R(t)\rangle = \mathcal{F}^{-1}\left\{\sqrt{S(\Omega)(1+S(\Omega))}\right\}$$

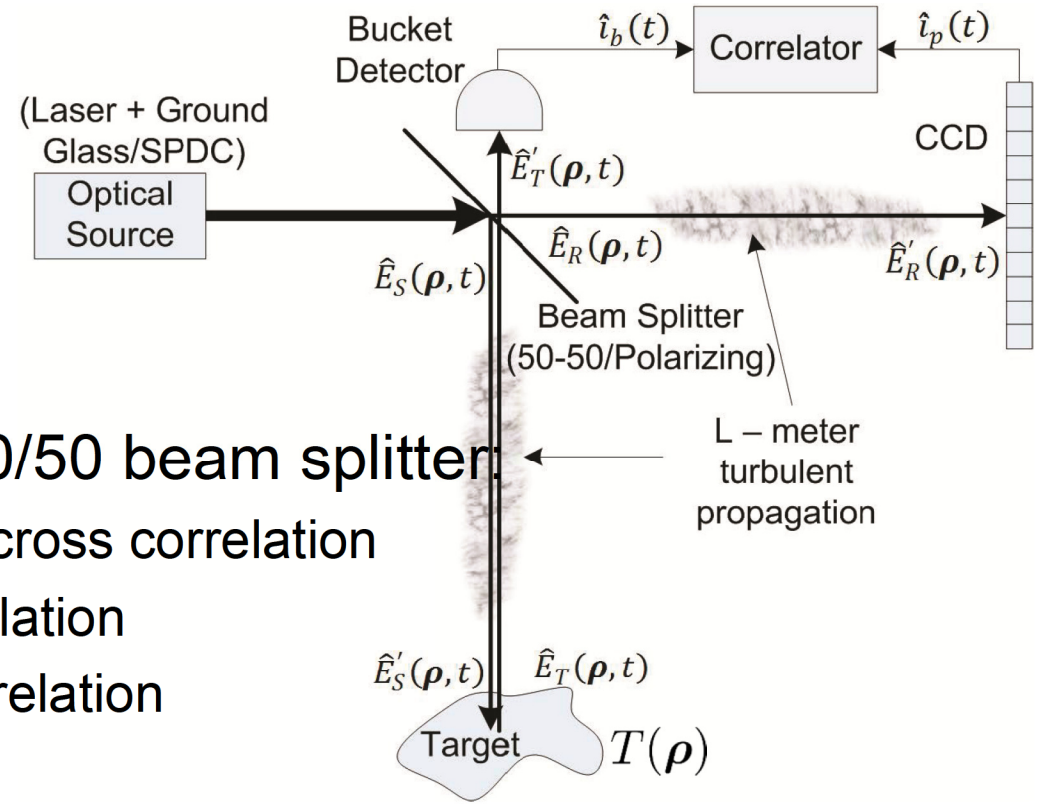
- When $S(\Omega) \ll 1$

$$\sqrt{S(\Omega)\big(1+S(\Omega)\big)} \gg S(\Omega)$$

Maximum phase-sensitive correlation in quantum physics

Maximum phase-sensitive correlation in classical physics

Standoff-sensing via Ghost imaging



- Pseudothermal source + 50/50 beam splitter:
  - maximum phase-insensitive cross correlation
  - no phase-sensitive autocorrelation
  - no phase-sensitive cross correlation

- Type-II SPDC source + polarizing beam splitter:
  - maximum *quantum* phase-sensitive cross-correlation
  - no phase-sensitive autocorrelation
  - no phase-insensitive cross correlation

Pseudothermal ghost imaging

- Assume Gaussian-Schell model source
  - intensity radius a_0, coherence radius $\rho_0\,(\ll a_0)$
- Assume rough-surface target $\langle T^*(\boldsymbol{\rho}_1)T(\boldsymbol{\rho}_2)\rangle = \lambda_0^2\mathcal{T}(\boldsymbol{\rho}_1)\delta(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)$
- Assume independent uniform turbulence along each path
  - square-law approximation to wave structure function
- Assume far field operation $k_0 a_0 \rho_0/2L \ll 1$
- Assume object lies within field of view $\lambda_0 L/\pi\rho_0$
- Photocurrent cross correlation is

free-space far-field coherence radius $\rho_L = \lambda_0 L/\pi a_0$

$$\langle \hat{C}(\boldsymbol{\rho}_1)\rangle \propto \int d\boldsymbol{\rho}_2\,\mathcal{T}(\boldsymbol{\rho}_2) + \frac{1}{\alpha}\int_{A_2} \mathrm{d}\boldsymbol{\rho}_2\mathcal{T}(\boldsymbol{\rho}_2)e^{-|\boldsymbol{\rho}_2-\boldsymbol{\rho}_1|^2/\alpha\rho_L^2}$$

turbulence degradation factor $(\alpha \geq 1)$

Featureless background $\sim$ Image

SPDC ghost imaging

- Assume Gaussian-Schell model source:
  - intensity radius a_0, coherence radius $\rho_0 \, (\ll a_0)$
- Assume rough-surface target $\langle T^*(\boldsymbol{\rho}_1) T(\boldsymbol{\rho}_2) \rangle = \lambda_0^2 \mathcal{T}(\boldsymbol{\rho}_1) \delta(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)$
- Assume independent uniform turbulence along each path
  - square-law approximation to wave structure function
- Assume far field operation $k_0 a_0^2 / 2L \ll 1$
- Assume object lies within field of view $\lambda_0 L / \pi \rho_0$
- Photocurrent cross correlation is

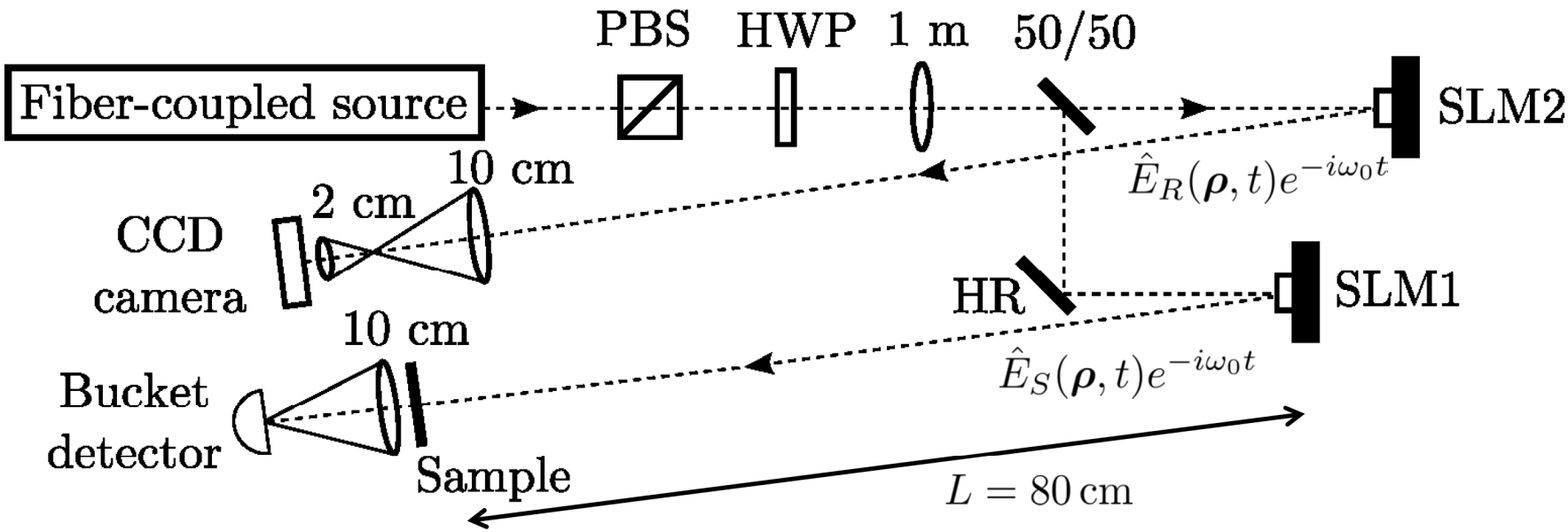
$$\langle \hat{C}(\boldsymbol{\rho}_1) \rangle \propto \int d\boldsymbol{\rho}_2 \, \mathcal{T}(\boldsymbol{\rho}_2) + \frac{1}{\alpha} \left(1 + \frac{1}{\sqrt{8\pi \mathcal{I}}} \right) \int_{A_2} \mathrm{d}\boldsymbol{\rho}_2 \mathcal{T}(\boldsymbol{\rho}_2) e^{-|\boldsymbol{\rho}_2 + \boldsymbol{\rho}_1|^2 / \alpha \rho_L^2}$$

free-space far-field coherence radius $\rho_L = \lambda_0 L / \pi a_0$

Featureless background $\ll$ Image, when $\mathcal{I} \ll 1$

turbulence degradation factor $(\alpha \geq 1)$

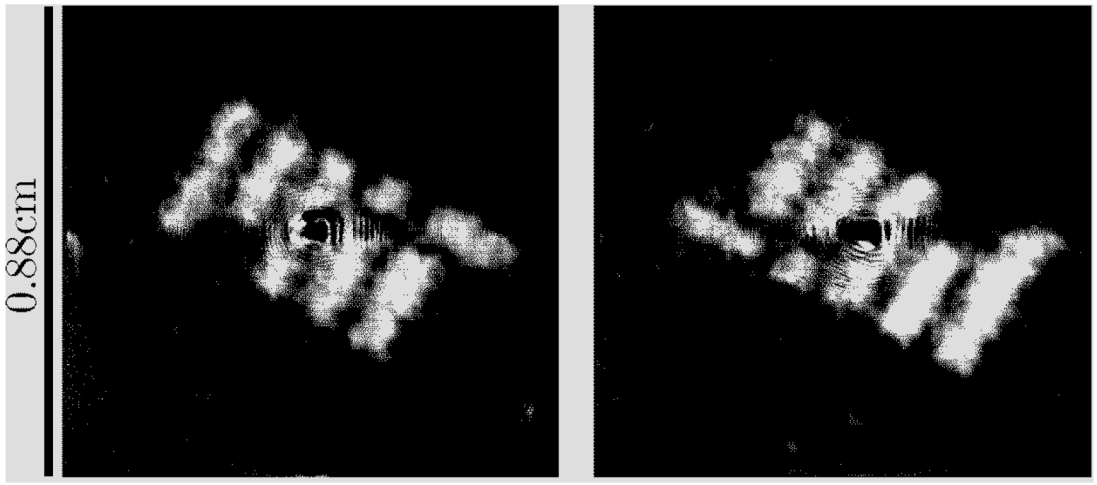
Imaging with classical phase-sensitive light



- $10\,\mathrm{mW}$, $\lambda_0 = 795\,\mathrm{nm}$ cw laser, phase-modulated by SLMs
- Far-field operation $k_0 a_0^2/2L \approx 0.2$
- Phase-conjugate SLM modulations generate *classical phase-sensitive* light, i.e., $\langle \hat{E}_S(\boldsymbol{\rho}_1,t_1)\hat{E}_R(\boldsymbol{\rho}_2,t_2)\rangle \neq 0$
- Identical SLM modulations generate *phase-insensitive* light, i.e., $\langle \hat{E}_S^\dagger(\boldsymbol{\rho}_1,t_1)\hat{E}_R(\boldsymbol{\rho}_2,t_2)\rangle \neq 0$

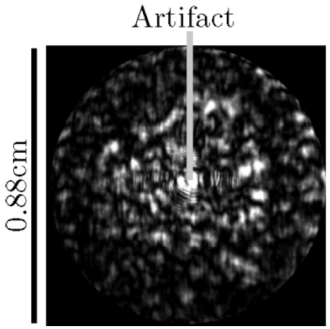
Ghost images with classical phase-sensitive light

- Ghost images averaged over 7000 realizations at $2\,\mathrm{Hz}$



phase-insensitive light phase-sensitive light

- Image inversion, as predicted by theory, seen in ghost images with phase-sensitive light



- Resolution and SNR for phase-sensitive and phase-insensitive images are comparable

- Finite fill-factor of SLMs responsible for imaging artifact

Conclusions

- Phase-sensitive coherence theory unifies Gaussian-state classical and quantum imaging
  - strength of phase-sensitive cross correlation defines classical-quantum boundary
- Phase-sensitive light is biplanatic and quasibichromatic
- Standoff ghost imaging gives equal resolution in classical and quantum cases
  - low-flux, low-brightness SPDC has significant contrast advantage
  - turbulence in either path degrades resolution
- Classical phase-sensitive light yields ghost images
  - same resolution and SNR as phase-insensitive light
  - image is reversed due to phase-sensitive coherence propagation