Phase-sensitive coherence and the classical-quantum boundary in ghost imaging

by

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Phase-sensitive light

- Unified theory of optical imaging using Gaussian-state light
- Phase-insensitive and phase-sensitive coherence
  - monochromatic versus bichromatic nature
  - quasimonoplanar versus quasibiplanar nature
- Ghost Imaging for standoff sensing
  - classical and quantum image signatures
  - signal-to-noise ratio
- Ghost imaging with classical phase-sensitive light
Unified theory via Gaussian states

- Paraxial, z-propagating, photon-units operator \( \hat{E}_z(\rho, t) e^{i k_0 z - i \omega_0 t} \)
  - Non-zero commutator is \( [\hat{E}_z(\rho_1, t_1), \hat{E}_z^\dagger(\rho_2, t_2)] = \delta(\rho_2 - \rho_1) \delta(t_2 - t_1) \)
  - Plane-wave decomposition is
    \[
    \hat{E}(\rho, t) = \int \frac{dk}{2\pi} \int \frac{d\Omega}{\sqrt{2\pi}} \hat{A}(k, \Omega) e^{ik \cdot \rho - i \Omega t}
    \]

- Zero-mean Gaussian states are completely characterized by
  - Phase-insensitive correlation function \( \langle \hat{E}_z^\dagger(\rho_1, t_1) \hat{E}_z(\rho_2, t_2) \rangle \)
  - Phase-sensitive correlation function \( \langle \hat{E}_z(\rho_1, t_1) \hat{E}_z(\rho_2, t_2) \rangle \)

<table>
<thead>
<tr>
<th>\langle \hat{E}_z(\rho_1, t_1) \hat{E}_z(\rho_2, t_2) \rangle = 0</th>
<th>\langle \hat{E}_z(\rho_1, t_1) \hat{E}_z(\rho_2, t_2) \rangle \neq 0</th>
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</thead>
<tbody>
<tr>
<td>Always classical e.g. Lasers, LED’s, pseudothermal light</td>
<td>Can be classical or quantum e.g. classical phase-sensitive light, squeezed states, output of SPDC, biphoton</td>
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Phase-insensitive correlation

- Gaussian-Schell model phase-insensitive correlation function
  - photon flux $P$, intensity radius $a_0$, coherence radius $\rho_0 \ll a_0$, coherence time $T_0$

- Phase-insensitive correlation spectrum is monochromatic and quasimonoplanar

$$\langle \hat{A}^\dagger(\mathbf{k}_1, \Omega_1) \hat{A}(\mathbf{k}_2, \Omega_2) \rangle = \frac{PT_0\rho_0^2}{\sqrt{2\pi}} e^{-a_0^2|\mathbf{k}_2-\mathbf{k}_1|^2/8-\rho_0^2|\mathbf{k}_1+\mathbf{k}_2|^2/8} e^{-T_0^2\Omega_2^2/2} \delta(\Omega_2 - \Omega_1)$$
Phase-sensitive correlation

- Gaussian-Schell model **phase-sensitive** correlation function
  - mean-square flux $P_s$, intensity radius $a_0$, coherence radius $\rho_0$ ($\ll a_0$), coherence time $T_0$

- Phase-insensitive correlation spectrum is **bichromatic** and **quasibiplanar**

$$\langle \hat{A}(k_1, \Omega_1) \hat{A}(k_2, \Omega_2) \rangle = \frac{P_s T_0 \rho_0^2}{\sqrt{2\pi}} e^{-a_0^2|k_1+k_2|^2/8} e^{-\rho_0^2|k_2-k_1|^2/8} e^{-T_0^2 \Omega_2^2/2} \delta(\Omega_2 + \Omega_1)$$
Classical-quantum boundary

\[ \chi^{(2)} \]

- \( \hat{E}_\ell(t) \) in zero-mean jointly-Gaussian state
  - phase-insensitive auto-correlations
    \[ \langle \hat{E}_\ell^\dagger(t + \tau) \hat{E}_\ell(t) \rangle = \mathcal{F}^{-1} \{ S(\Omega) \} \]
  - phase-sensitive cross correlation
    \[ \langle \hat{E}_S(t + \tau) \hat{E}_R(t) \rangle = \mathcal{F}^{-1} \left\{ \sqrt{S(\Omega)(1 + S(\Omega))} \right\} \]

- When \( S(\Omega) \ll 1 \)
  \[ \sqrt{S(\Omega)(1 + S(\Omega))} \gg S(\Omega) \]

Maximum phase-sensitive correlation in quantum physics

Maximum phase-sensitive correlation in classical physics
Standoff-sensing via Ghost imaging

- **Pseudo-thermal source + 50/50 beam splitter:**
  - maximum phase-insensitive cross correlation
  - no phase-sensitive autocorrelation
  - no phase-sensitive cross correlation

- **Type-II SPDC source + polarizing beam splitter:**
  - maximum *quantum* phase-sensitive cross-correlation
  - no phase-sensitive autocorrelation
  - no phase-insensitive cross correlation
Pseudo-thermal ghost imaging

- Assume Gaussian-Schell model source
  - intensity radius $a_0$, coherence radius $\rho_0 (\ll a_0)$
- Assume rough-surface target $\langle T^*(\rho_1)T(\rho_2) \rangle = \lambda_0^2 T(\rho_1) \delta(\rho_1 - \rho_2)$
- Assume independent uniform turbulence along each path
  - square-law approximation to wave structure function
- Assume far field operation $k_0 a_0 \rho_0 / 2L \ll 1$
- Assume object lies within field of view $\lambda_0 L / \pi \rho_0$
- Photocurrent cross correlation is

$$\langle \hat{C}(\rho_1) \rangle \propto \int d\rho_2 T(\rho_2) + \frac{1}{\alpha} \int_{A_2} d\rho_2 T(\rho_2) e^{-|\rho_2 - \rho_1|^2 / \alpha \rho_L^2}$$

Featureless background $\sim$ Image

Free-space far-field coherence radius $\rho_L = \lambda_0 L / \pi a_0$

Turbulence degradation factor $(\alpha \geq 1)$
SPDC ghost imaging

- Assume Gaussian-Schell model source:
  - intensity radius \( a_0 \), coherence radius \( \rho_0 \) (\( \ll a_0 \))
- Assume rough-surface target
  \[ \langle T^*(\rho_1)T(\rho_2) \rangle = \lambda_0^2 T(\rho_1)\delta(\rho_1 - \rho_2) \]
- Assume independent uniform turbulence along each path
  - square-law approximation to wave structure function
- Assume far field operation
  \[ k_0 a_0^2 / 2L \ll 1 \]
- Assume object lies within field of view
  \[ \lambda_0 L / \pi \rho_0 \]
- Photocurrent cross correlation is
  \[ \langle \hat{C}(\rho_1) \rangle \propto \int \mathcal{D}\rho_2 \mathcal{T}(\rho_2) \]
  \[ + \frac{1}{\alpha} \left(1 + \frac{1}{\sqrt{8\pi\mathcal{I}}}\right) \int_{A_2} d\rho_2 \mathcal{T}(\rho_2) e^{-|\rho_2 + \rho_1|^2 / \alpha \rho_L^2} \]

Free-space far-field coherence radius \( \rho_L = \lambda_0 L / \pi a_0 \)

Featureless background \( \ll \) Image, when \( \mathcal{I} \ll 1 \)

Turbulence degradation factor \( (\alpha \geq 1) \)
10 mW, $\lambda_0 = 795$ nm cw laser, phase-modulated by SLMs

Far-field operation $k_0 a_0^2 / 2L \approx 0.2$

Phase-conjugate SLM modulations generate classical phase-sensitive light, i.e., $\langle \hat{E}_S(\rho_1, t_1) \hat{E}_R(\rho_2, t_2) \rangle \neq 0$

Identical SLM modulations generate phase-insensitive light, i.e., $\langle \hat{E}^*_S(\rho_1, t_1) \hat{E}_R(\rho_2, t_2) \rangle \neq 0$
Ghost images with classical phase-sensitive light

- Ghost images averaged over 7000 realizations at 2 Hz

- Image inversion, as predicted by theory, seen in ghost images with phase-sensitive light

- Resolution and SNR for phase-sensitive and phase-insensitive images are comparable

- Finite fill-factor of SLMs responsible for imaging artifact
Conclusions

- Phase-sensitive coherence theory unifies Gaussian-state classical and quantum imaging
  - strength of phase-sensitive cross correlation defines classical-quantum boundary

- Phase-sensitive light is biplanatic and quasibichromatic

- Standoff ghost imaging gives equal resolution in classical and quantum cases
  - low-flux, low-brightness SPDC has significant contrast advantage
  - turbulence in either path degrades resolution

- Classical phase-sensitive light yields ghost images
  - same resolution and SNR as phase-insensitive light
  - image is reversed due to phase-sensitive coherence propagation