



Effects of Correlated Errors on the Analysis of Space Geodetic Data

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Motivation



- VLBI error budget (becoming less Gaussian dominated).
- Reduction of thermal errors will require modeling of correlated noise sources.
- Troposphere errors have correlations in both space and time.
 - Kolmogorov frozen flow model.
 - Modeling delay errors Treuhaft & Lanyi (Radio Sci. 1987)
 - Water vapor radiometer (WVR) measurements.
- Instrumental errors introduce station specific temporal correlations.



Thermal Errors

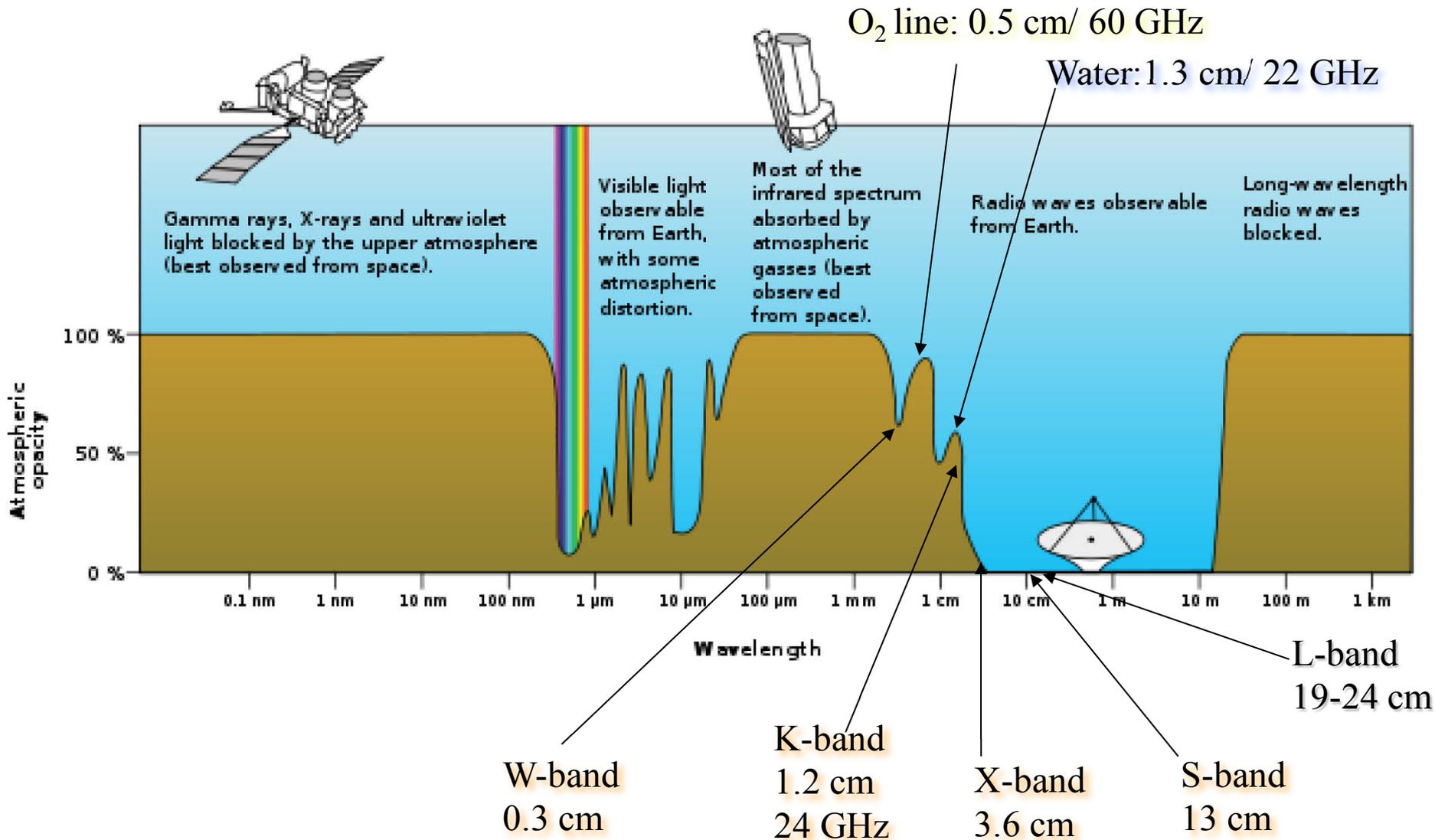


Recording Rate	Improve	
128 Mb/sec	-	Last 10 years
256 Mb/sec	1.4	Recent RDV runs
512 Mb/sec	2	VLBA continuum
2 Gb/sec	4	Mark 5C (R&D fringes)
4 Gb/sec	5.7	Mark 5C dual bank
16-32 Gb/sec	11.3-16	Haystack Mark 6

Data rates are sky-rocketing.
Factors of 10 improvement in the near future.



The Radio Window of the Atmosphere





Future: Trop. Turbulence Dominates

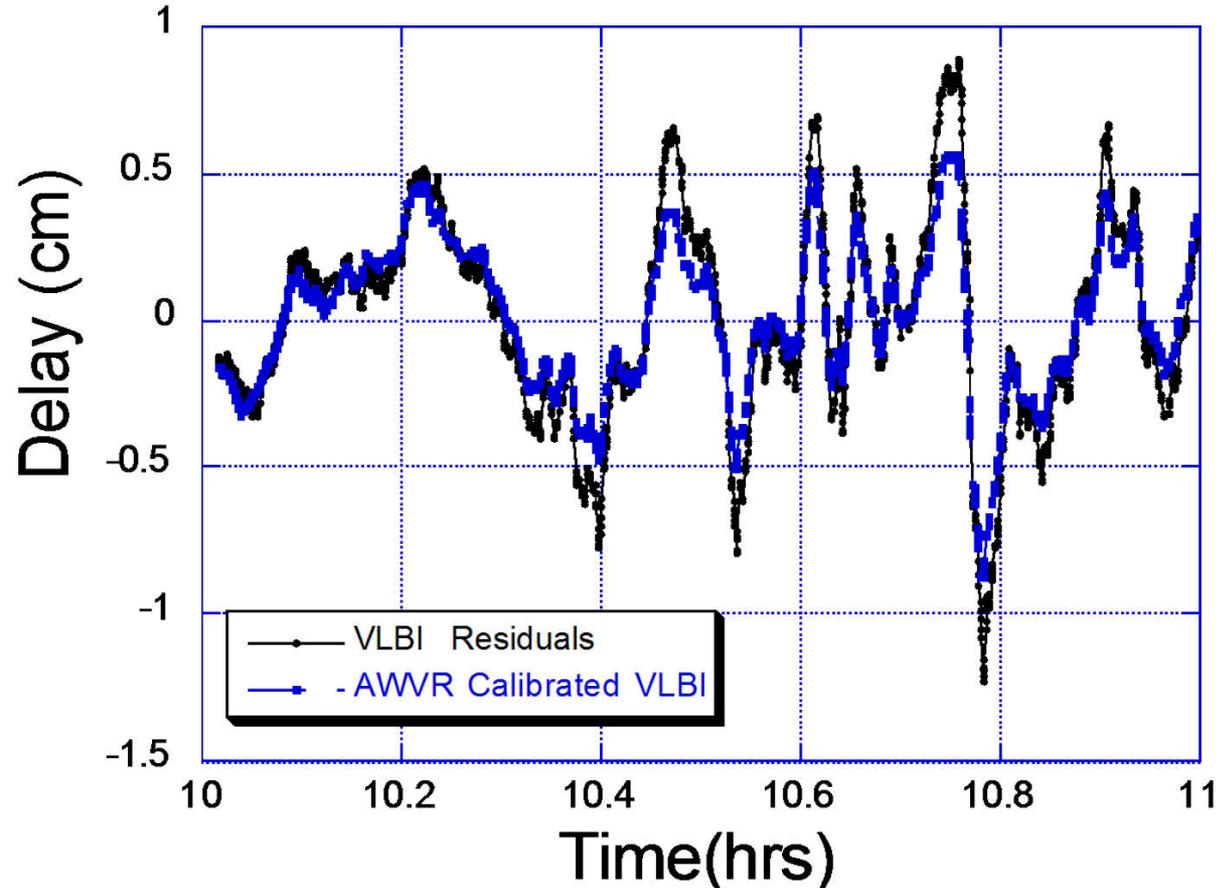


- Monitor 22 GHz/1.3cm water (rotational) line brightness temperature along line-of-sight.
- 3mm scatter reduced to 1mm Goldstone-Madrid 8000 km baseline using X/Ka phase delays.
- As thermal errors average down, the troposphere fluctuations dominate the residuals.



Measured with the JPL Advanced Water Vapor Radiometer

VLBI Delay Residuals DOY 200 Ka-Band DSS26-DSS55

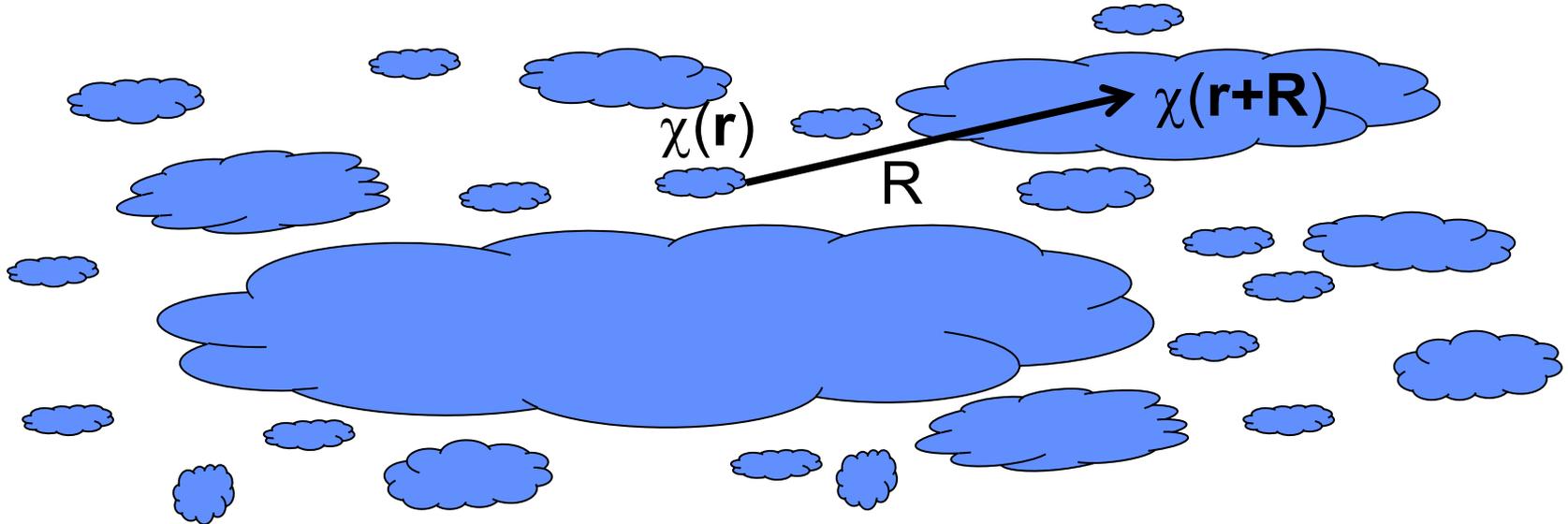


Jacobs et al, AAS Winter 2005.

Bar Sever et al, IEEE, 2007.



Kolmogorov Theory of Turbulence



Structure function of refractivity (Tatarskii, 1961)
Based on Kolmogorov turbulence.

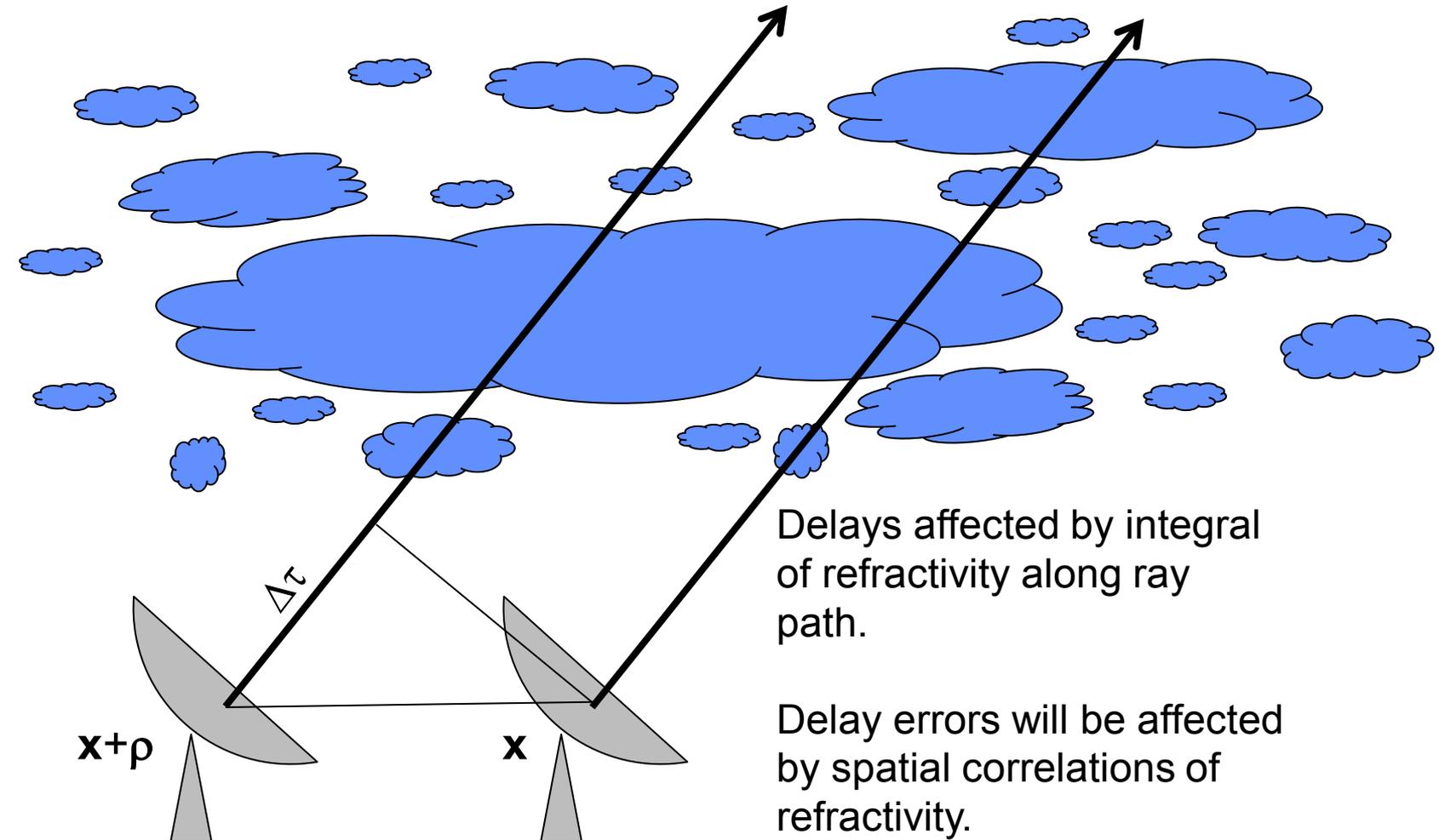
$$D_{\chi}(R) = C^2 R^{2/3}$$

Structure function
contains information
statistical scatter
and covariance.

Constant related
to rate of energy
dissipation and
viscosity.

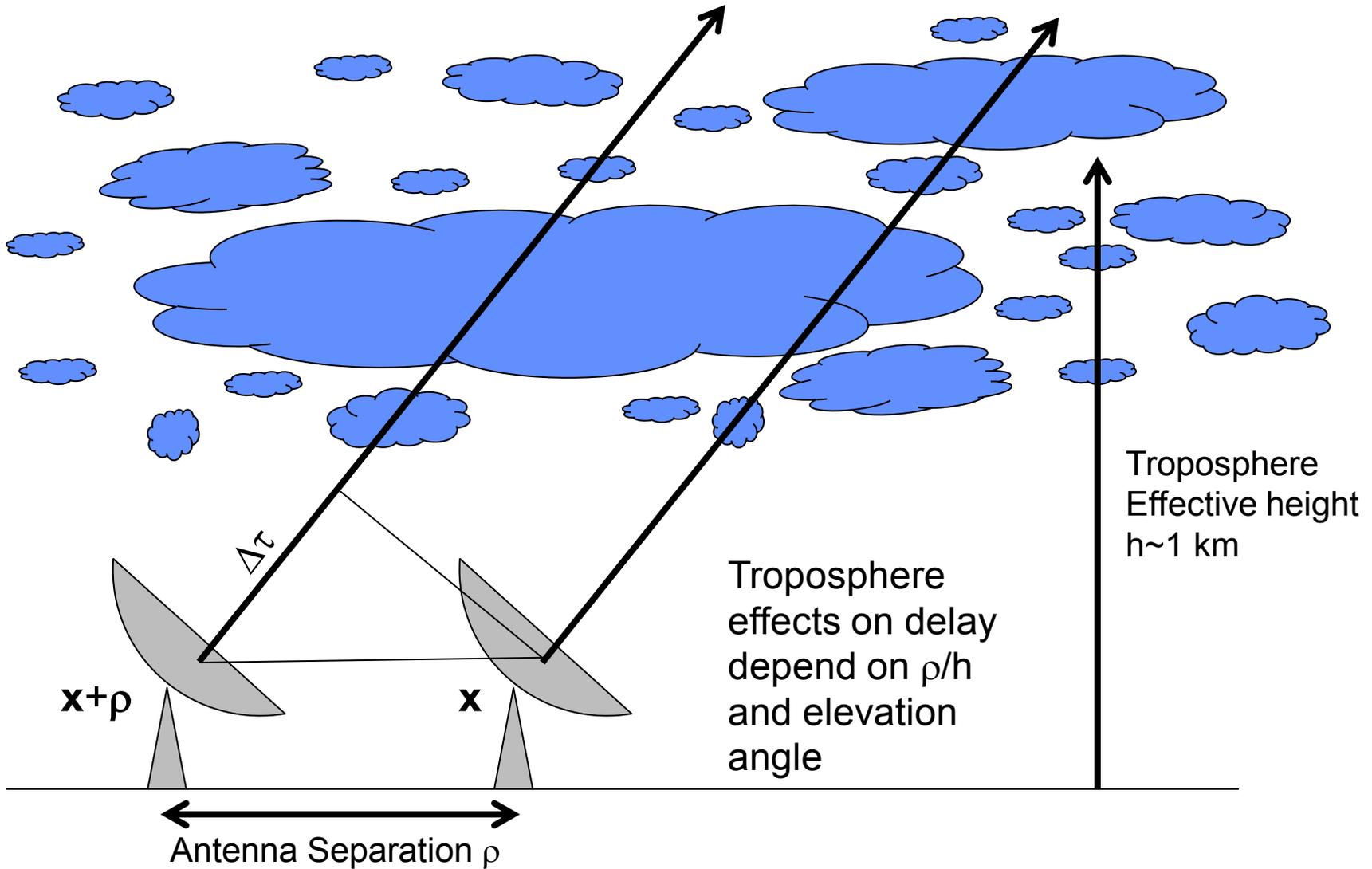


Spatial Correlations on Delay Errors



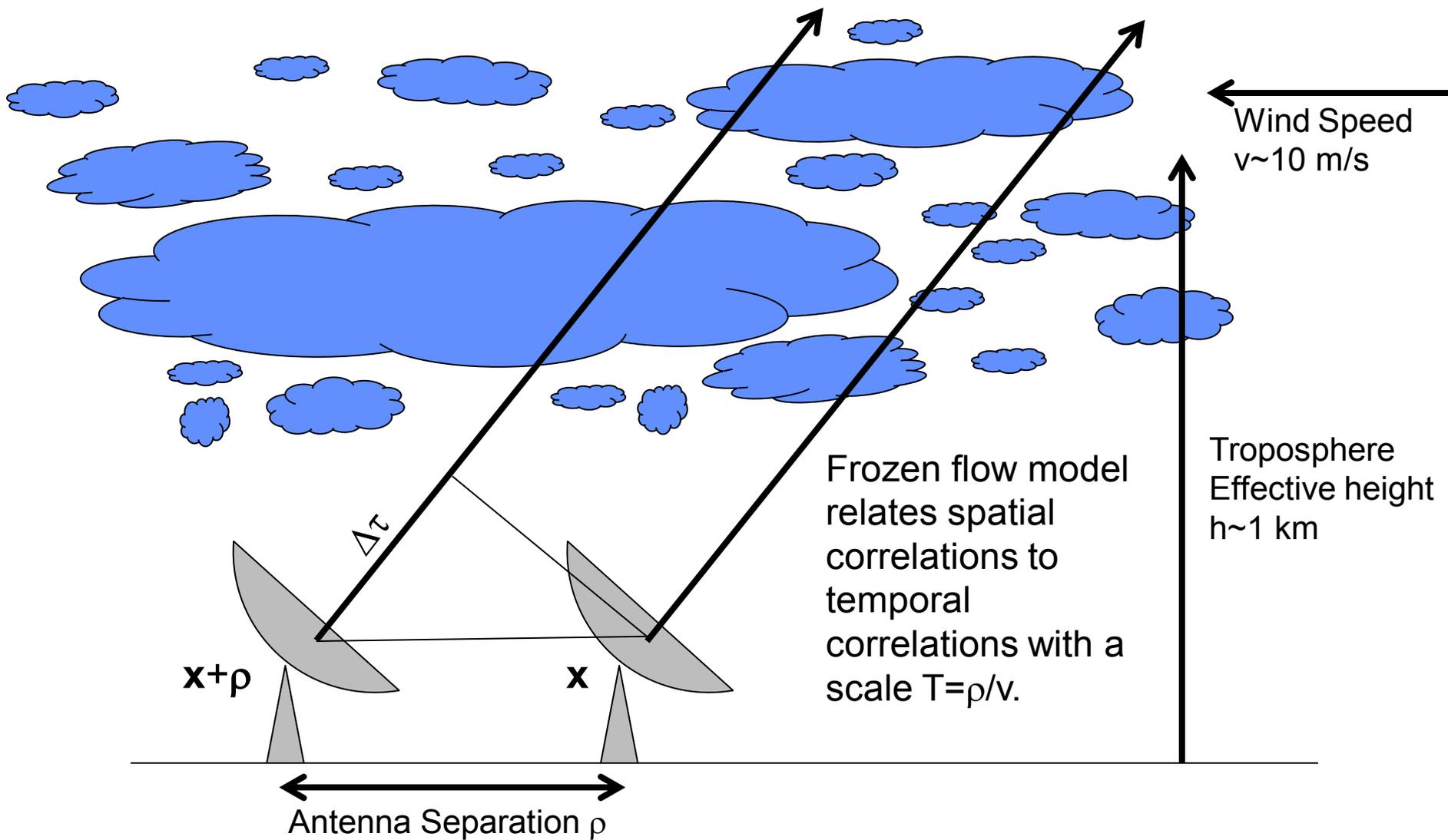


Tropospheric Effects on Delays





Temporal Correlations on Delay



Delay structure function

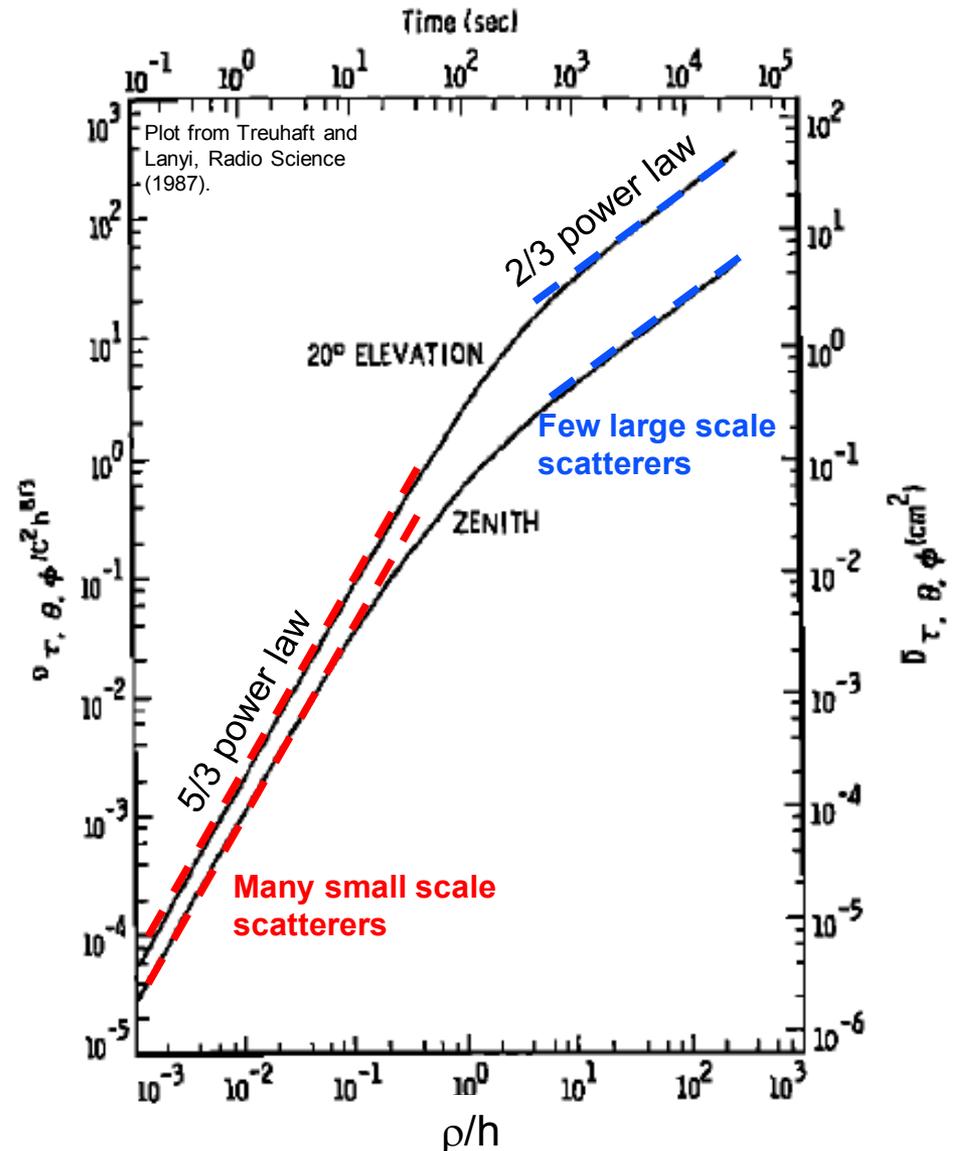
$$D_{\tau}(\rho) = \langle (\tau(\mathbf{x}+\rho) - \tau(\mathbf{x}))^2 \rangle$$

takes in refractivity variations via the media effects on the delay.

Delay error correlations depend on the ratio of antenna distance to troposphere height (ρ/h) and elevation angle.

Broken power law with smooth switch over at $\rho/h \sim 1$.

Correlations in space and time are derived from structure functions.





VLBA K-band Test Data Set



VLBA baselines:
Maximum 8600 km
Mauna Kea to St. Croix

Minimum 240 km
Los Alamos to Pie Town

Tropospheric and instrumental correlated errors begin to affect the ability to average down random errors from 45 baselines.

Catalog runs with 12 sessions of 24 hours each.

Data credit: K/Q VLBI collaboration.
Lanyi et al. AJ 2010
Charlot et al. AJ 2010



Image credit: http://www.nasa.gov/centers/goddard/news/topstory/2007/nrao_agreement.html



VLBA Thermal Errors



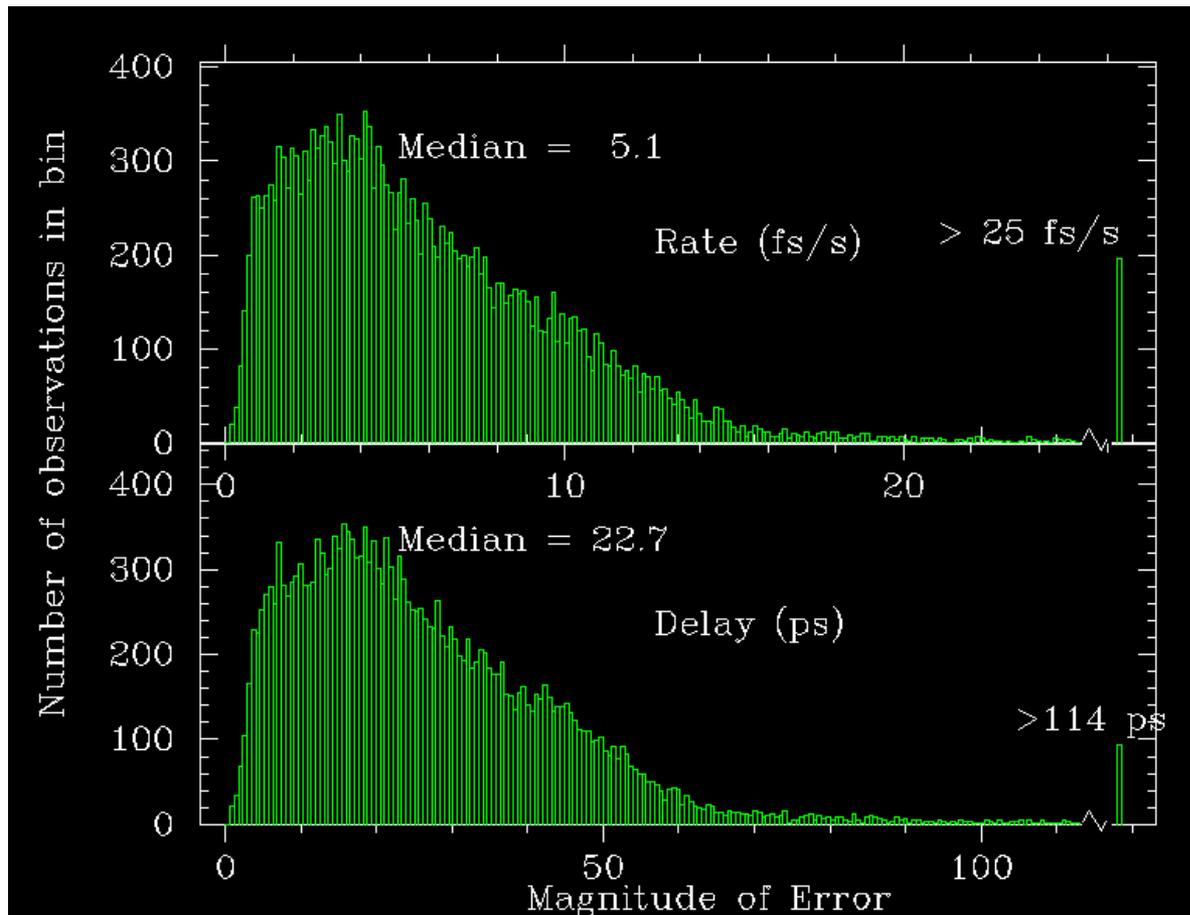
Sampling at 128 Mb/s
Delay rate errors at 5 fs/s
Delay errors at 23 ps

With future recorders this will decrease by a factor of 10 to delay errors of 2 ps.

This is going from 1 cm to 1mm errors.

Can we begin to see improvements from correlated error models in these data?

K-band Data



Troposphere turbulence is expected to introduce a 20-40 ps error.



Troposphere Covariance



We look for improvements due to troposphere covariance error modeling by comparing catalog solutions to ICRF2.

As a simple test we use only nine baselines with the Mauna Kea station as reference.

Rather than introducing troposphere breaks every twenty minutes we use only one troposphere parameter estimate for station over a 24 hour period with clock parameters estimate breaks every three hours.

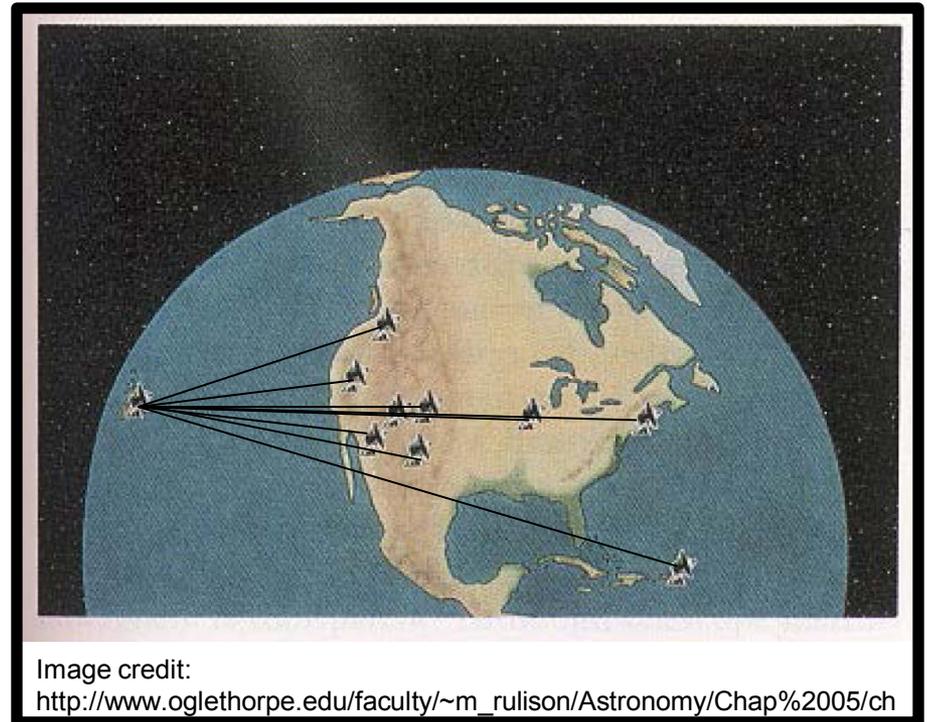


Image credit:
http://www.oglethorpe.edu/faculty/~m_rulison/Astronomy/Chap%2005/chapter_5_lecture_notes.htm

Troposphere covariance re-defines χ^2 such that drifts do not add to χ^2 nearly as much.

Number of free parameters reduced from what would typically be 1200 to 74.



Troposphere Covariance



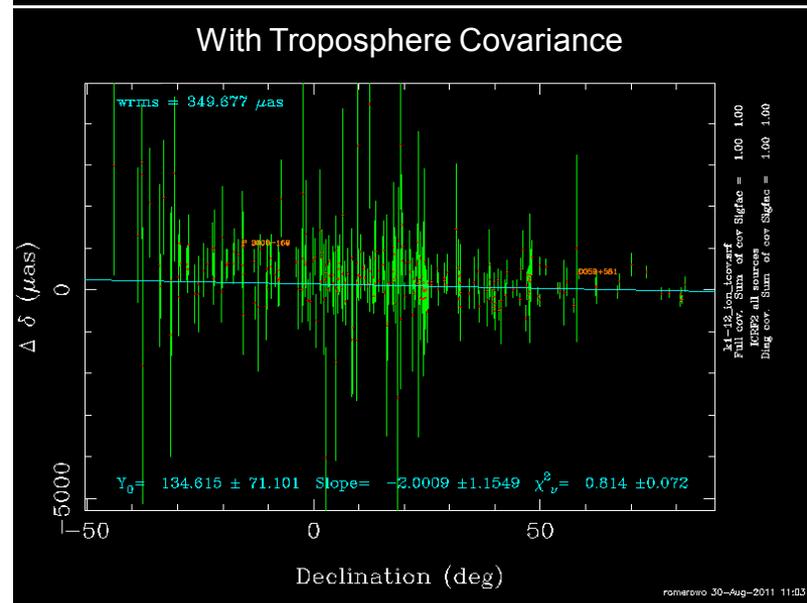
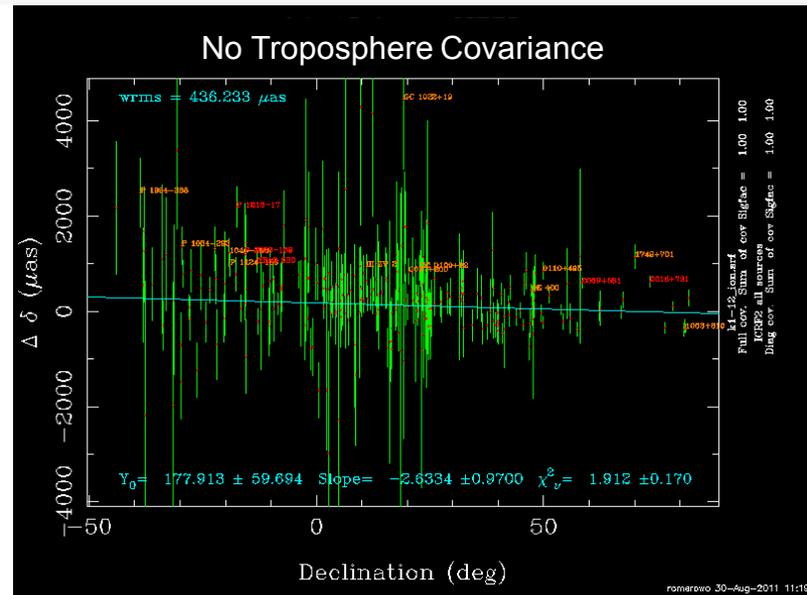
9 baseline MK referenced VLBA K-band catalog solution compared to ICRF2.

Look at δdec vs. dec since it is expected to be more affected by troposphere.

Both solutions have only one troposphere break every 24hrs with clock breaks every 3 hours.

We begin to see some improvement in the results.

$\Delta\delta$ vs. Dec	No TropCov	TropCov
wRMS (μas)	436	350
Y_0 (μas)	178 +/- 57	135 +/- 71
Slope ($\mu\text{as}/\text{deg}$)	-2.6 +/- 1.0	-2.0 +/- 1.2
χ^2	1.91 +/- 0.17	0.81 +/- 0.07





Troposphere Covariance in X/Ka Catalog



Including trop. cov. in X/Ka catalog improves the wRMS by 7% (using the exact same data and exact same modeling).

A 7% improvement would take 14% more data to get the same result by pure averaging.

For X/Ka going into its 6th year that would mean almost another year of data to get the improvement we get from trop cov.

wRMS in RA

w/o Trop. Cov. : 215 μ as

w/ Trop. Cov. : 198 μ as

wRMS in DEC

w/o Trop. Cov. : 300 μ as

w/ Trop. Cov. : 283 μ as

δ Dec vs. Dec slope

w/o Trop. Cov. : 1.6 +/- 1.0 μ as/deg

w/ Trop. Cov. : 1.1 +/- 0.9 μ as/deg



Instrumentation Errors



- Mark 4 analog system being replaced by digital back ends.
 - Better filtering (phase linear FIR filter).
- Sampling right after down converter to eliminate most cable delays.
- Stochastic behavior of hydrogen maser clocks remains.
- Thermal variations in cables and filters.



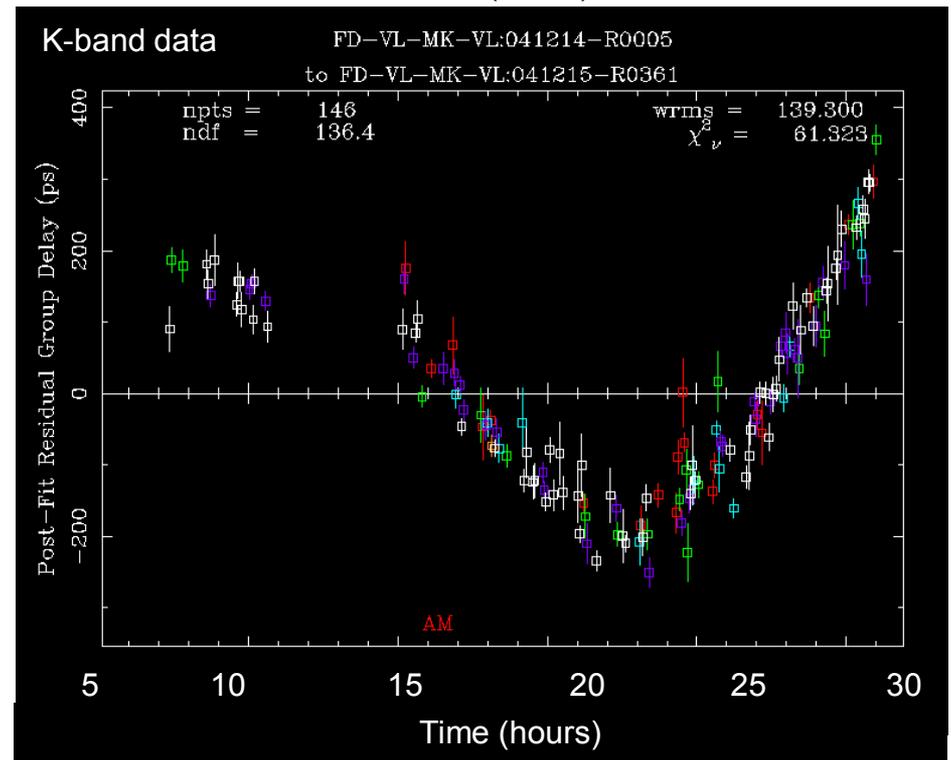
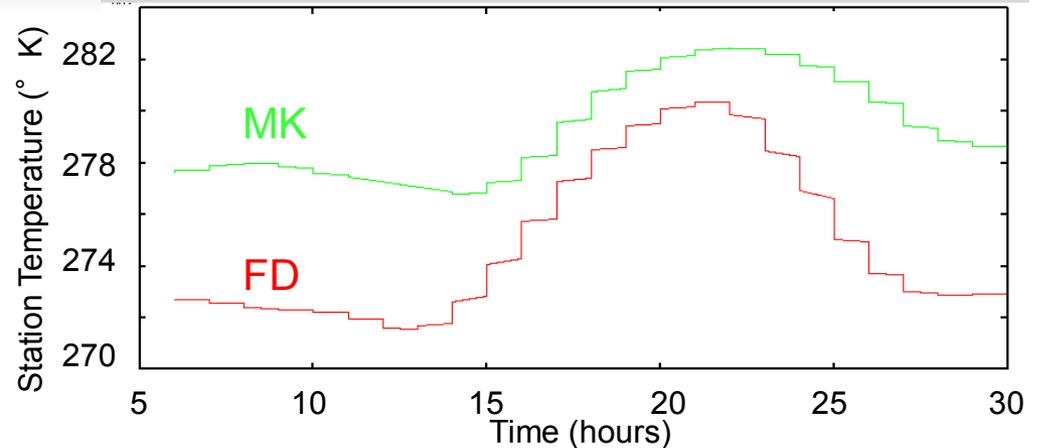
Instrumental Correlations



Some baselines show delay residuals with strong temporal correlations.

This temporal correlation in many cases is correlated with station temperature variations.

Typically these errors are absorbed by adding troposphere parameter breaks every 20 minutes and clock parameter breaks every few hours.





Conclusions



As thermal errors are reduced instrumental and troposphere correlated errors will increasingly become more important.

Work in progress shows that troposphere covariance error models improve data analysis results. We expect to see stronger effects with higher data rates.

Temperature modeling of delay errors may further reduce temporal correlations in the data.



BACK UP SLIDES



Outlook



Include 24 hr scale clock covariance to error modeling (reduce the need for clock breaks).

Implement troposphere covariance error models to VLBA catalog solution with 45 baselines.

Include troposphere covariance error model on TEMPO Earth orientation solution.



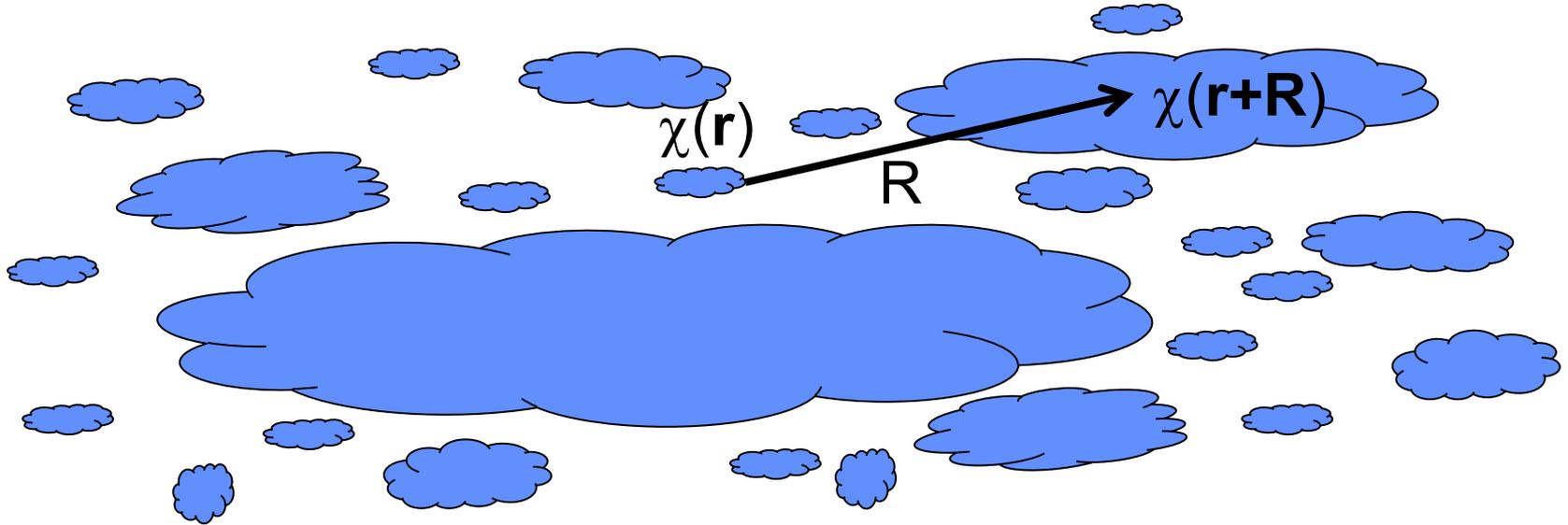
Tropcov Solution Comparison



$\Delta\alpha\cos\delta$ vs. RA	No TropCov	TropCov
wRMS (μas)	283.8	211.9
Y0 (μas)	-47.9 +/- 19.5	-46.6 +/- 25.2
Slope ($\mu\text{as/hr}$)	4.3 +/- 1.7	4.0 +/- 2.2
Chisq	1.88 +/- 0.17	1.00 +/- 0.09
$\Delta\alpha\cos\delta$ vs. Dec	No TropCov	TropCov
wRMS (μas)	222.9	181.1
Y0 (μas)	25.7 +/- 8.4	32.3 +/- 25.2
Slope ($\mu\text{as/deg}$)	-2.2 +/- 0.6	-3.0 +/- 0.7
Chisq	1.86 +/- 0.17	0.95 +/- 0.08
$\Delta\delta$ vs. RA	No TropCov	TropCov
wRMS (μas)	469.7	380.212
Y0 (μas)	75.5 +/- 27.8	44.8 +/- 32.3
Slope ($\mu\text{as/hr}$)	-4.6 +/- 1.4	-2.3 +/- 1.5
Chisq	1.90 +/- 0.17	0.82 +/- 0.07
$\Delta\delta$ vs. Dec	No TropCov	TropCov
wRMS (μas)	436.2	349.7
Y0 (μas)	178 +/- 57	135 +/- 71
Slope ($\mu\text{as/deg}$)	-2.6 +/- 1.0	-2.0 +/- 1.2
Chisq	1.91 +/- 0.17	0.81 +/- 0.07



Kolmogorov Theory of Turbulence



Structure function of refractivity (Tatarskii, 1961)

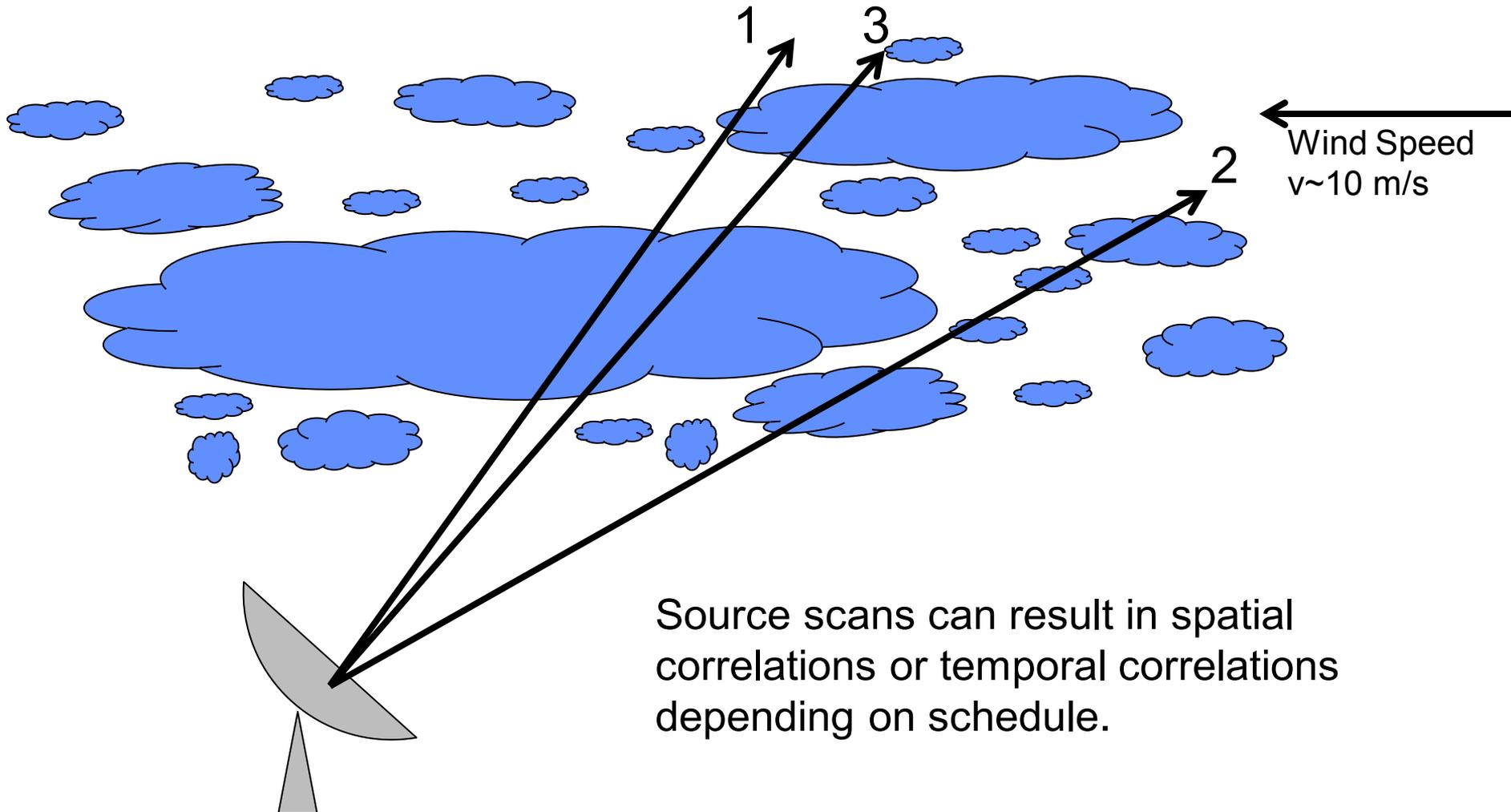
$$D_{\chi} = \langle (\chi(\mathbf{r}+\mathbf{R}) - \chi(\mathbf{r}))^2 \rangle$$

$$D_{\chi} = 2\sigma_{\chi}^2 - 2\text{cov}(\chi(\mathbf{r}+\mathbf{R}), \chi(\mathbf{r}))$$

$$D_{\chi} = C^2 R^{2/3} \text{ (Kolmogorov)}$$



Single Site Spatio-Temporal Correlations

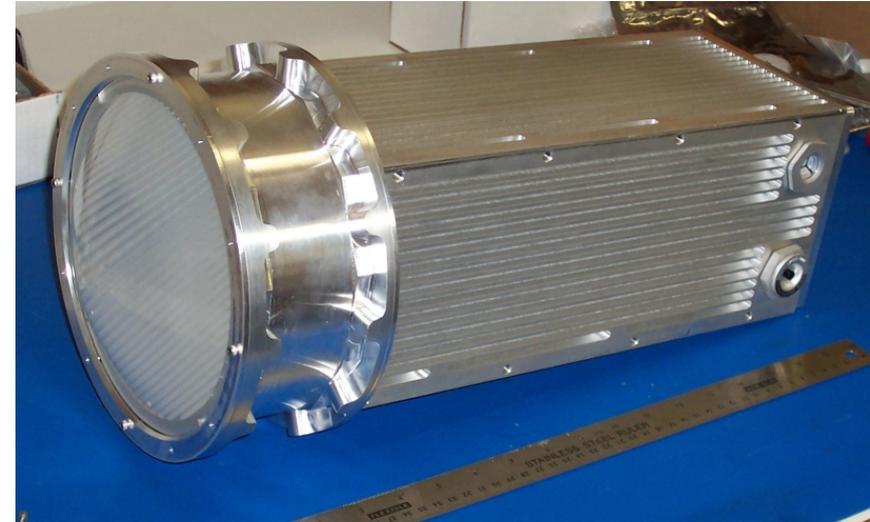




Troposphere Solution 3: Cheaper WVR?



- Advanced R&D WVR is very accurate but expensive on order ~\$500K
Mounted 50-100m from VLBI antenna
- ~~Investigating “Radiometrics” WVR~~
 - ~~21 channels cover 20-30 GHz~~
 - ~~“Inexpensive” at \$120K (U.S.)~~
 - ~~Light weight: ~7 kg~~
- Subreflector mounting
 - enabled by low weight
 - ideal co-pointing, no offset
 - better match with VLBI beam volume
 - enables calibration of doppler/rate
 - improves performance at high freq.
- Gain stability needs investigation





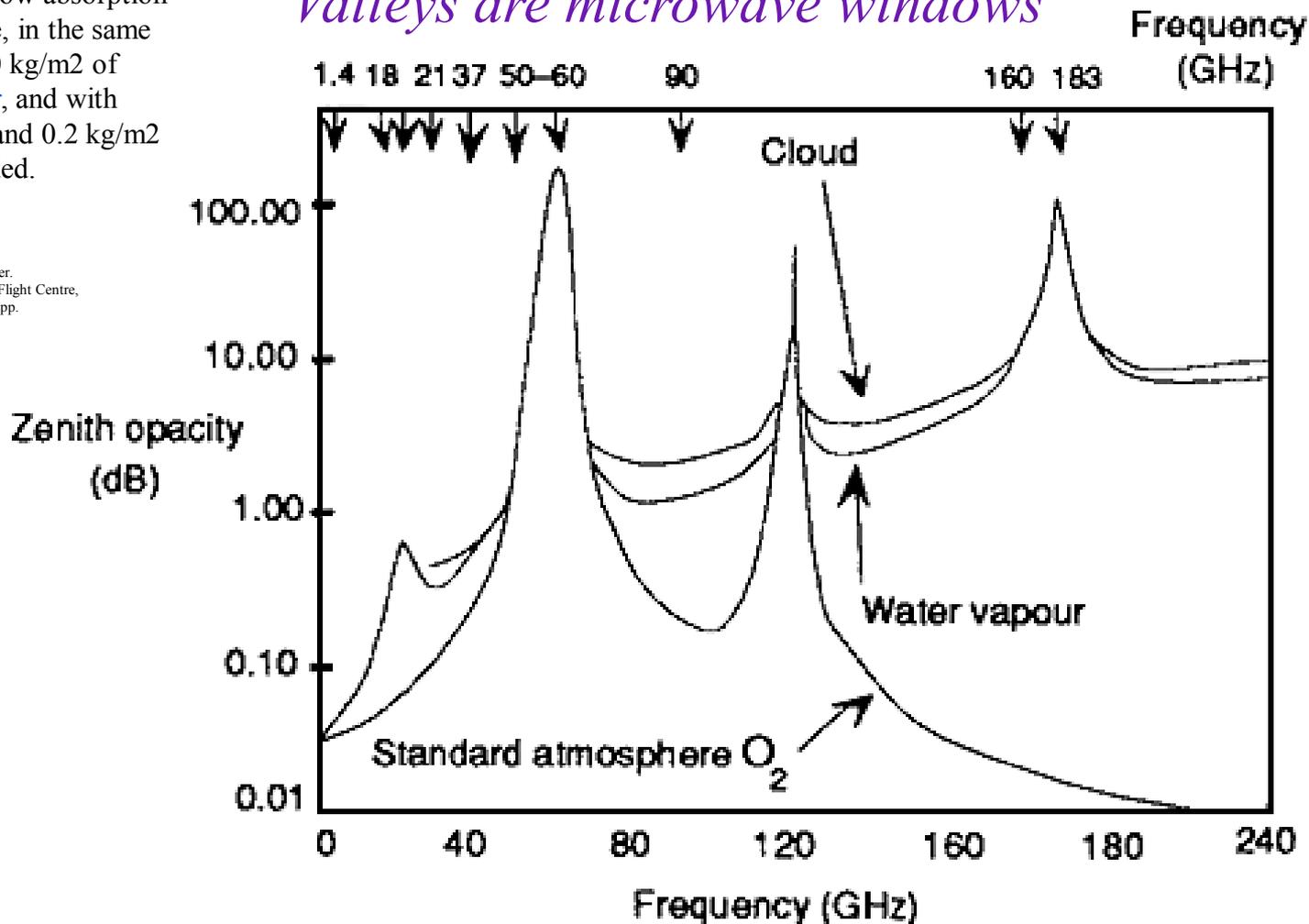
WVR looks at 22 GHz water line



The three curves show absorption in a dry atmosphere, in the same atmosphere with 20 kg/m² of added water vapour, and with both water vapour and 0.2 kg/m² of stratus cloud added.

Murphy, R. et al., 1987, Earth Observing System Volume II: HMRR High-Resolution Multifrequency Microwave Radiometer. Published by NASA, Goddard Space Flight Centre, Greenbelt, Maryland 20771, USA, 59pp.

Valleys are microwave windows



Murphy, R. et al., 1987, Earth Observing System Volume II: HMRR High-Resolution Multifrequency Microwave Radiometer. Published by NASA, Goddard Space Flight Centre, Greenbelt, Maryland 20771, USA, 59pp.



Need instrumental Phase calibrations



Problem:

180 psec
~diurnal
effect

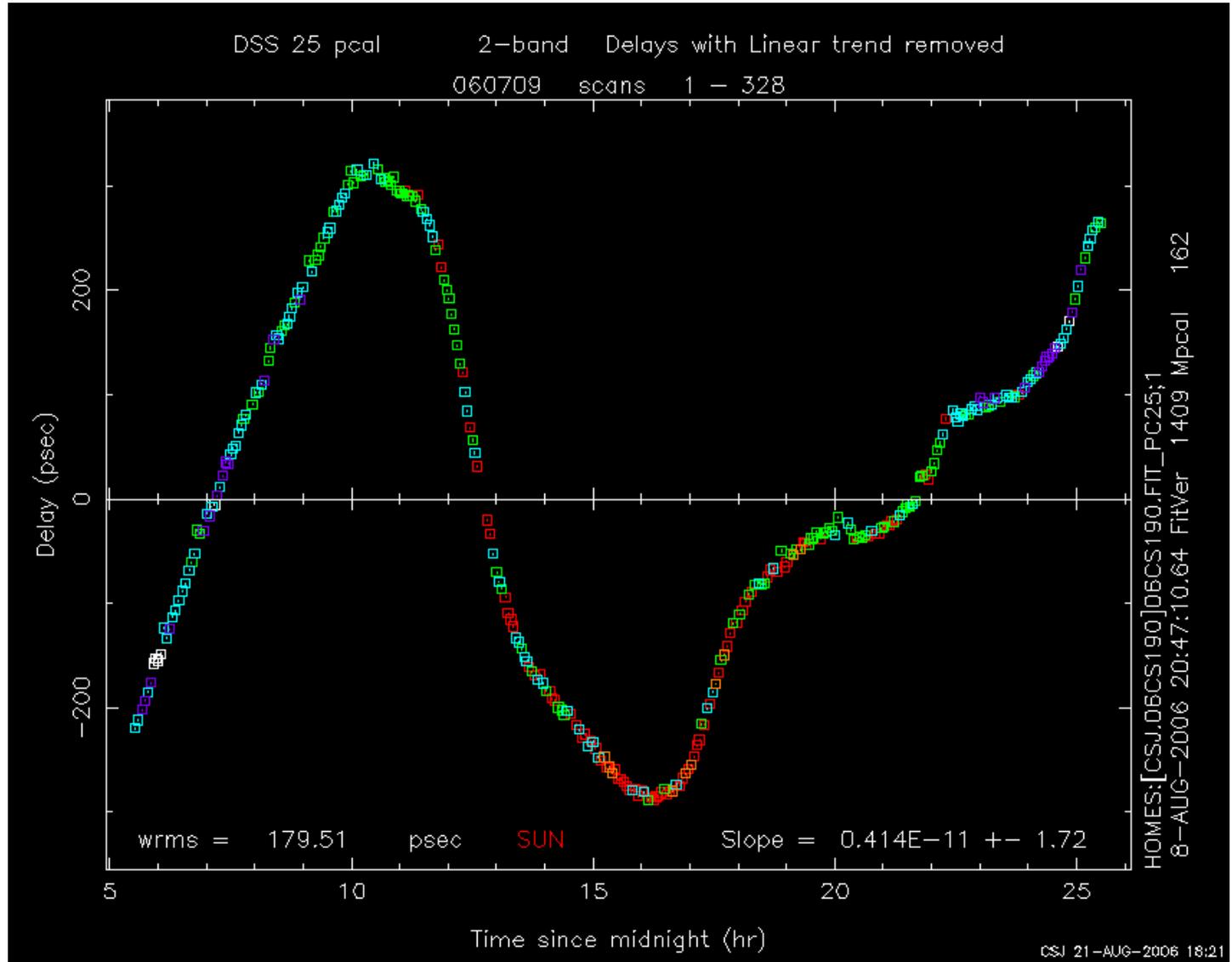
Solution:

Ka-band
Phasecal
Prototype
Demo' d

-- >

Units being
Built.

Operations
in ~1 year



Proto-type shows need for calibrations. *Credit: C. Jacobs, B. Tucker, L. Skjerve*