Exploring the frequency stability limits of whispering gallery mode resonators for metrological applications

Yanne K. Chembo[1], Lukas Baumgartel[2], Ivan Grudinin[2], Dmitry Strekalov[2], Robert Thompson[2] and Nan Yu[2]

[1] FEMTO-ST Institute (CNRS UMR 6174), Optics Department, 16 Route de Gray, 25030 Besançon cedex, France.

[2] Jet Propulsion Laboratory, California Institute of Technology, 4800 Oak Grove Drive, Pasadena, California 91109, USA.

ABSTRACT

Whispering gallery mode resonators are attracting increasing interest as promising frequency reference cavities. Unlike commonly used Fabry-Pérot cavities, however, they are filled with a bulk medium whose properties have a significant impact on the stability of its resonance frequencies. In this context, thermal and other refraction index fluctuations of the medium induce additional frequency instability that has to be reduced to a minimum. On the other hand, a small monolithic resonator provides opportunity for better stability against vibration and acceleration. This feature is essential when the cavity operates in a non-laboratory environment. In this paper, we report a case study for a crystalline resonator, and discuss the a pathway towards the inhibition of vibration- and acceleration-induced frequency fluctuations.

Keywords: Whispering gallery modes resonators, frequency stability, metrology.

1. INTRODUCTION

Ultra-high Q whispering gallery mode (WGM) disk-resonators are expected to play an ever-increasing role in metrological applications. In these resonators, photons are trapped by total internal reflection into the torus-like eigenmodes, *i.e.*, whispering gallery modes. These optical resonators of millimeter-size are characterized by significantly long photon lifetimes (> 1 microsecond), and accordingly, ultra-narrow resonance frequency linewidths. This feature is indeed highly desirable in a wide range of applications from frequency metrology, spectroscopy, sensing, or ultra-low phase noise microwave and terahertz generation.

In many of these applications, the frequency stability of the resonator is critical. Fluctuation in the refraction index and the cavity dimensions will result in jitters and drifts in the resonance frequencies. Both internal and environmentally induced fluctuations will ultimately limit the noise floor for the frequency stability. Thermal noise limits in WGM resonators have been discussed before.[1,2] While the small size of typical WGM resonators make them less sensitive to mechanical disturbances, vibrations and accelerations can still sufficiently deform the cavity and shift the eigenmode frequency.

Effectively, the resonance frequency ω_0 of a cavity generally obeys a simple relationship of the form $\omega_0 \propto c/na$, where c is the velocity of light, n the refraction index at the frequency ω_0, and a is a characteristic dimension of the resonator. As a consequence, any fluctuation of refraction index or cavity geometry induces a frequency fluctuation $\delta\omega_0/\omega_0 = -\delta n/n - \delta a/a$. Hence, metrological applications require the unavoidable fluctuations δa and δn to be reduced to a minimum.

To illustrate the mechanical disturbance impact, consider the mere action of gravity on a WGM calcium fluoride disk-resonator with 5 mm diameter and 0.5 mm thickness (see Fig. 1). It can be shown that the disk resonator in this typical stem-mounting configuration undergoes a radius change of few pm (this is ten times smaller than the size of the hydrogen atom), and the corresponding resonance frequency shift is already as large

Further author information: (Send correspondence to Y. K. C.)
E-mail: yanne.chembo@femto-st.fr ; Telephone: +33 381 666 401.

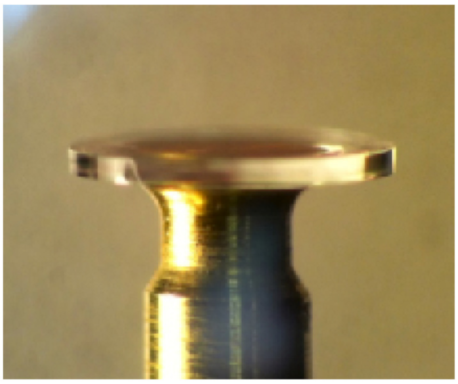

Figure 1. An ultra-high-Q magnesium fluoride disk resonator.

as ~ 200 kHz (for light with a wavelength of 1550 nm). Metrology applications require a frequency precision of the order of 1 Hz, equivalent of cavity radius stability better than 1 fm, which is the size of the proton.

Frequency stabilization of lasers locked to passive Fabry-Pérot optical cavities nowadays routinely reaches at $\sim 10^{-15}/\mathrm{Hz}^{1/2}$ (see refs.[3,4]). Performances so far with WGM disk resonators are typically of the order of $\sim 10^{-12}$ (see ref.[5]), but the thermal limit down to $\sim 10^{-14}$ has already been reached.[6]

To achieve a frequency stability comparable of Fabry-Pérot reference cavities, especially when practical devices are considered, we will have to address the mechanical instability. A key challenge is therefore to design clever mounting architectures capable of reducing and eliminating environmental fluctuations of mechanical nature in WGM disk-resonators. An interesting solution relies on the concept of "neutral mounting", where the mounting forces are distributed in a way such that the deformation of the optical cavity is null in a given direction, regardless of their intensity.[7] As explicitly displayed in Fig. 2, the null force-to-displacement conversion guarantees that mechanical fluctuations are not transferred to a cavity-length deformation, that is, to the resonance frequency.

The aim of this work is to present a neutral mounting architecture for WGM resonators. In particular, we will show that clamping a WGM disk between two coaxial cylinders induces a radial displacement field that is null in a neutral circumference path. This optimal trajectory, that is unaffected by the mounting force, can therefore host the WGMs and preserve their eigenfrequencies from mechanical fluctuations.

The plan of the article is the following. In the next section, we will explain the mounting architecture and discuss its advantageous properties from a theoretical point of view. Section 3 will be devoted to the finite-differences numerical simulations of the stress, strain and displacement fields inside the disk, in order to evidence the validity of this neutral mounting scheme. These simulations will also enable to understand the potential of this architecture for the inhibition of low-frequency vibrations. These simulations will also enable to understand numerically the influence of accelerations on the reference eigen-frequency, and thereby evaluate the typical acceleration sensitivity of the mounted resonator. The last section will conclude the article.

2. CONCEPT OF NEUTRAL MOUNTING CONFIGURATIONS

The design of a neutral mounting architecture is not straightforward in WGM disk-resonators, as mounting such disks is subject to a number of stringent constraints. The most important one is that the rim of the disk (lateral surface) has to be free. Any mechanical contact with the rim could preclude the propagation of whispering gallery modes. The mounting scheme must also allow physical access of a fiber or prism to the edge of the disc for optical coupling.

Therefore, WGM disk resonators can only be loaded either through tangential stresses applied onto the surface of an inner hole (where far from the spatial location of the WGMs), or through normal stresses exerted on the bottom and top surfaces of the disk.

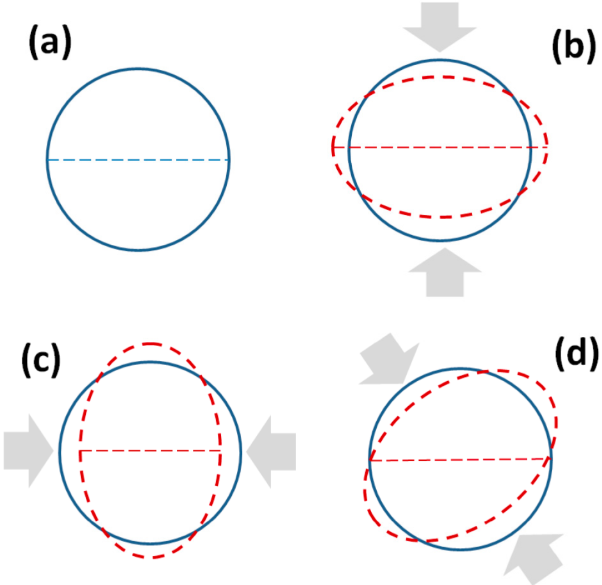

Figure 2. Illustration of the principle of neutral mounting, as explained in ref.[7] The solid (discontinuous) circle (ellipse) stands for the unloaded (loaded) spherical cavity. The arrows indicate the mounting forces. (*a*) The unloaded sphere, with the horizontal axis (diameter) being the reference length of a Fabry-Pérot cavity. (*b*) Vertical mounting. The sphere is elongated horizontally, and the reference cavity length is larger. (*c*) Horizontal mounting. The sphere is elongated vertically, and the reference cavity length is shorter. (*d*) Neutral mounting. The sphere diameter has a zero displacement along the axis of interest. The mounting force may fluctuate without inducing undesirable cavity length fluctuations.

The first strategy, which is somehow equivalent to the mounting architecture of Fig. 1. It is easy to foresee that in case of vertical vibration, the free outer edge (where the WGMs are located) would undergo relatively large deformations, resulting in significant frequency shifts.

This second strategy is indeed more robust, particularly when the loading is symmetrical (stresses applied on both the top and the bottom surfaces of the disk). In particular, it can be shown that clamping the disk between two cylinders with different radii can, when properly arranged, create a radial displacement field that is null at the rim of the disk, regardless of the mounting force intensity. This possibility can in fact be foreshadowed from the Euler-Bernoulli beam theory, which states that, when a beam is loaded (for example bent downwards), the top-half is stretched while the bottom-half is compressed. Therefore, the plane of symmetry (or a nearby surface) should not undergo any longitudinal deformation to the first order.

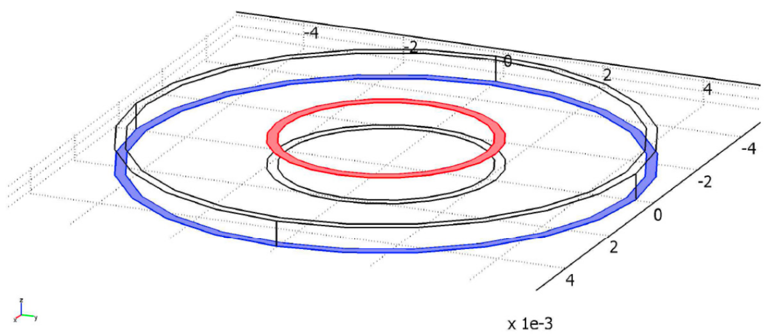

Figure 3. The clamping boundary conditions. The blue band is supported from below while a varying pressure is applied to the top pink band.

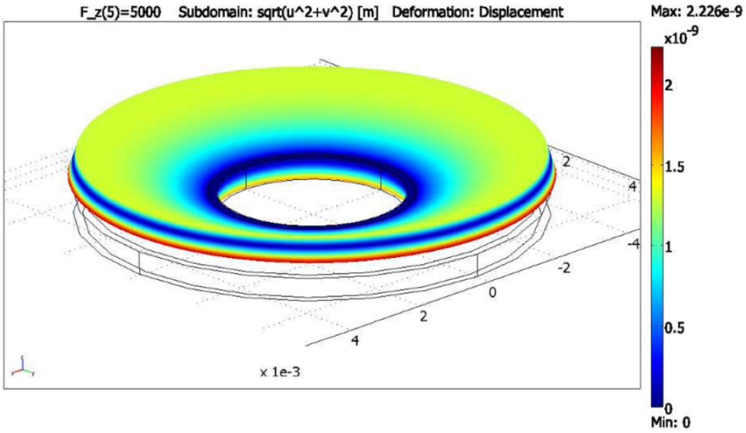

Figure 4. Finite Element numerical results showing the disc-resonator deformed from the co-axial mounting scheme. Colors represent the magnitude of radial deformation, and a band where this deformation is zero can be seen along the perimeter of the disc.

The next section is devoted to finite-difference numerical simulations aiming to evaluate the efficiency of this neutral mounting scheme.

3. NUMERICAL SIMULATIONS

The heuristic analysis presented above provides intuition that a neutral clamping configuration exists for WGM disk resonators. However, to verify this intuition and furthermore provide the quantitative results that would be necessary for designing a physical system for experiment, numerical simulations are necessary.

3.1 Modeling Configuration

We performed finite element method (FEM) simulations using COMSOL. Because our co-axially clamped resonator can be thought of as a hybrid of the sheared and bent disc situations, we worked with a geometry where the disc has a central hole. The dimensions of the disc are, inner radius, $a = 2$ mm, outer radius, $b = 5$ mm, and a thickness of 0.5 mm, with the disc symmetry plane aligned on the z axis (-250 μm< z <250 μm). We assigned material properties to the disc based on those of calcium fluoride, with Young's modulus $E = 75.8$ GPa, Poisson ratio $\nu = 0.26$, and density $\rho = 3180$ kg m^{-3}.

The co-axial clamping boundary conditions are applied as follows. A static force of F_z is applied upwards to a 200 μm wide strip around the lower, outer portion of the disc is. A 200 μm wide strip around the top, inner portion of the disc is held fixed, completing the clamping scheme (see figure 3). The model is then meshed with 70,000 triangular, quadratic Lagrange elements, and varying values of F_z are used in finding static solutions to the geometry deformation, stress, and strain.

3.2 Numerical Results

Results of the FEM simulation confirm the existence of a location at the edge of the disc where radial deformation is zero for a wide range of mounting-force values. Figure 4 shows the deformed disc and magnitude of radial displacement. Figure 5 shows radial deformation $u_r(z)$ plotted along the edge of the disc for mounting-forces spanning three orders of magnitude. In all cases, the disc does not deform radially at a position 64.8 μm above the mid-plane of the disc. The fact that this location does not coincide with the geometric symmetry plane is a result of the asymmetric boundary conditions, i.e., that the inner mounting surface is fixed (and the material cannot deform radially), but the outer mounting surface is allowed to deform radially. However, the results are consistent with the intuition from the Euler-Bernoulli theory, which predicts a neutral plane between regions of compression and expansion.

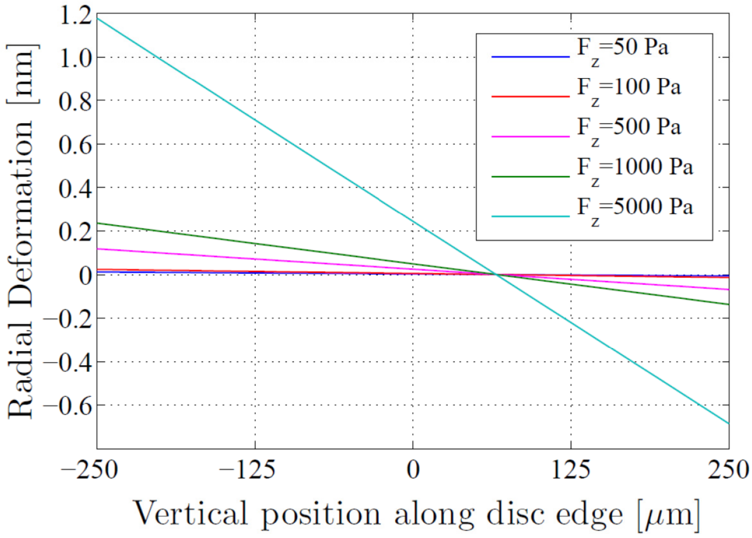

Figure 5. Plot of $u_r(z)$ along the disc edge for five different values of mounting pressure. The fact that $u_r(z) = 0$ for $z = 68.4$ μm in all cases suggests that the frequency of a WGM propagating in this region would be independent of mounting pressure.

The fact that the turning point remains the same even when the loading force spans across 3 orders of magnitude is a reliable indicator of the robustness of this mounting scheme. It indicates that if the WGMs are travelling at the height of this turning point, they would not undergo any eigenfrequency shift. In particular, it proves that the system would be very insensitive to low-frequency vibrations (which induce only small load variations), but also to large accelerations (which may induce significantly large load variations). Asymmetric WGM resonators as presented in Fig. 6 would therefore enable to improve the frequency stability of the cavity, particularly as far as vibrations and linear accelerations are concerned.

4. CONCLUSION

In this work, we have investigated a neutral mounting architecture for WGM disk resonators. Numerical simulations have shown that optimal configurations enable frequency stability for a very wide range of loading forces.

This theoretical analysis, which will have to be confirmed experimentally, indicates that the reduction of vibration- and acceleration-induced frequency fluctuations in WGM disk-resonators can be achieved with a simple mounting strategy. Successful implementation of such device would offer a small and robust frequency reference for portable and practical devices in a non-laboratory environment.

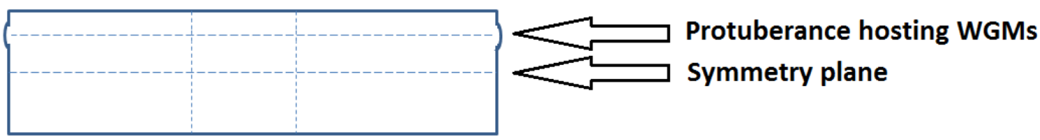

Figure 6. Lateral view of an optimally designed WGM resonator. A protuberance in the circumference of the disk is located at the height of the load-insensitive turning point ($z = 68.4$ μm in or example of Fig. 5). Such protuberance-like WGM resonators have already been demonstrated in the literature (see for example refs.[8,9]).

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