

Whispering Gallery Mode Resonators as Optical Reference Cavities

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Abstract—Highly stabilized lasers are an increasingly valuable tool for metrology. For many applications, however, existing Fabry P erot systems are too bulky and cumbersome. We are investigating the use of miniature monolithic whispering gallery mode resonators as reference cavities for laser stabilization. We seek to exploit the benefit of small size and vibration resistance by suppressing thermally induced frequency fluctuations. We have theoretically investigated the viability of using a thin-film coating to achieve temperature compensation. We have experimentally investigated an active temperature stabilization scheme based on birefringence in a crystalline resonator. We also report progress of laser locking to the resonators.

I. INTRODUCTION

Optical atomic clocks have attracted widespread interest due to their high stability and accuracy. These systems are facilitated by advances in laser technologies that enable optical trapping, cooling, and octave-spanning frequency combs (see, e.g., [1]). Such self-referencing combs enable a direct link from microwave to optical frequencies.

Limiting the short-term stability of optical frequency standards is the local oscillator – a highly stabilized laser capable of producing sub-hertz linewidths. Typical stabilization techniques utilize a Fabry-P erot cavity fabricated from materials with extremely low thermal expansion coefficients. These cavities are further stabilized via operation under vacuum and with extensive vibration isolation. The net result is a large and complex local oscillator package.

Therefore, aiding the development of a portable and compact optical clock is a laser reference cavity that is both small and highly robust to mechanical vibrations and thermal fluctuations. At first glance, whispering gallery mode resonators (WGMRs) might not seem promising for such applications: because the light travels completely within bulk material, they are subject to higher levels of thermorefractive noise than is present in conventional Fabry-P erot cavities. Furthermore, the most thermally stable materials, such as ultra-low expansion (ULE) and zerodur glass, are not suitable for WGMRs due to unacceptably high absorption at optical wavelengths. On the other hand, if the thermorefractive noise can be suppressed, WGMRs have potential to be high-performance cavities owing to their extremely high quality factor (Q) over a wide range of wavelengths, low sensitivity to mechanical noise, and small size, which enables precise and fast temperature control.

The dominant coupling mechanisms between temperature and resonant frequency in a WGMR are thermal expansion and thermorefraction. In case of the former, a change in resonator

temperature leads to a change in its size – i.e., the cavity length – and a corresponding change in frequency. The latter refers to a temperature dependence of the host material’s refractive index n , and is a quantity which varies widely from material to material. For WGMRs to be useful as reference cavities, both effects must be highly suppressed.

II. THERMAL COMPENSATION

We have explored two schemes for suppression of thermally induced frequency fluctuations: (1) a passive scheme whereby a material with negative thermorefractive coefficient is applied as a coating to a resonator to compensate for thermal expansion, and (2) an active scheme whereby a differential in frequency fluctuations between differently polarized modes in a birefringent crystalline resonator is used to stabilize its temperature.

A. Coated WGMR simulations

Coating a whispering gallery mode resonator with a thin-film of negative thermorefractive coefficient material could potentially be used to suppress the effects of thermal expansion. Such a compensation scheme was proposed by Han and Wang [2] and experimentally demonstrated by Jestin et al. [3], albeit with a resonator demonstrating relatively low Q . In such a heterogeneous resonator, the mode volume is shared between two materials so that part of the mode is contained in a medium demonstrating substantial negative thermorefraction (thermal expansion is always positive in the relevant materials).

We have performed simulations to investigate the viability of such a thin-film temperature compensation scheme following the procedure outlined in [2]. The method begins with a solution to Maxwell’s equations over the geometry of the resonator, accounting for the different refractive index of resonator and coating (n_1 and n_2 , respectively). The boundary condition demands continuity of tangential field components at resonator-coating and coating-air interfaces of the heterogeneous structure, yielding

$$\frac{1}{n_2} \frac{\chi_l'(z_4)}{\chi_l(z_4)} - \frac{C_l \psi_l'(z_3) + \chi_l'(z_3)}{C_l \psi_l(z_3) + \chi_l(z_3)} = 0 \quad (1a)$$

where the coefficient C_l is given by

$$C_l = \frac{\frac{n_2}{n_1} \psi_l(z_1) \chi_l'(z_2) - \psi_l'(z_1) \chi_l(z_2)}{\psi_l'(z_1) \psi_l(z_2) - \frac{n_2}{n_1} \psi_l'(z_2) \psi_l(z_1)}. \quad (1b)$$

The characteristic equation (1a) is derived in [4], and uses the following notation: The Riccati-Bessel functions are defined by $\psi_l(z) = zj_l(z)$ and $\chi_l(z) = zn_l(z)$, where l is the angular mode number; $j_l(z)$ and $n_l(z)$ are spherical Bessel and Neumann functions, respectively. The arguments are $z_1 = n_1ka$, $z_2 = n_2ka$, $z_3 = n_2k(a+t)$, and $z_4 = k(a+t)$, and the prime denotes derivative with respect to the argument. Solution of equation (1a) depends on the radius of the disc, a , the thickness of the coating, t , and the refractive index of each material, all of which in turn depend upon temperature via

$$\begin{aligned} a &= a_0(1 + \alpha_a^{(1)} \Delta T) \\ t &= t_0(1 + \alpha_t^{(2)} \Delta T) \\ n &= n_0(1 + \alpha_n^{(i)} \Delta T) \end{aligned} \quad (2)$$

where $\alpha_i^{(i)}$ is the linear thermal expansion coefficient, $\alpha_n = (1/n_i)\partial n_i/\partial T$ is the thermorefractive coefficient, $i = 1, 2$ for the disc and coating, respectively, and the zero subscript indicates the initial ($\Delta T = 0$) value.

Thus, the temperature dependence of eigenfrequencies can be found from roots of (1a). We find these roots using a numerical solver in MATLAB, with an initial guess provided by asymptotic approximation. The derivatives are expanded using standard recursion relations (e.g., [5]). Furthermore, we make the approximation that both materials are non-magnetic and that the refractive index of air is identically 1. The characteristic equation has many roots for a given set of parameters; we choose the first one – corresponding to the “fundamental” whispering gallery mode.

First, we simulated the thermally induced frequency shift of a fused silica (SiO_2) resonator that has been coated with calcium fluoride (CaF_2). As shown in figure 1, we find that the the frequency shifts of the coated resonator have been suppressed by nearly three orders of magnitude over the uncoated device (in agreement with [2]). Furthermore, evidence of a turning point suggests the potential for extreme stability.

Although this improvement is substantial, fused silica is an amorphous material. A crystalline resonator offers several advantages, among them higher quality factor [6], and the possibility of birefringence (the benefit of which is described below). This motivates simulation of a crystalline magnesium fluoride (MgF_2) resonator coated with calcium fluoride. These simulations suggest that the coating provides some improvement, though the gains are much less dramatic than in the case of the amorphous fused silica. Temperature induced frequency shift remains in the gigahertz per degree Celsius range, even for a coating of $5 \mu\text{m}$ (figure 2). The explanation for the decreased improvement is the relatively low thermal expansion coefficient of SiO_2 : $0.55 \times 10^{-6}/^\circ\text{C}$ [2] compared to $9.0 \times 10^{-6}/^\circ\text{C}$ for MgF_2 [7].

The simulation results shown in figures 1 and 2 are performed for spherical resonators with a radius of $50 \mu\text{m}$ with light in the mode having a free-space wavelength of $1.5 \mu\text{m}$. This corresponds to an angular mode number l in the neighborhood of 250. While larger resonators can in principle be treated with the same simulation, the angular number gets

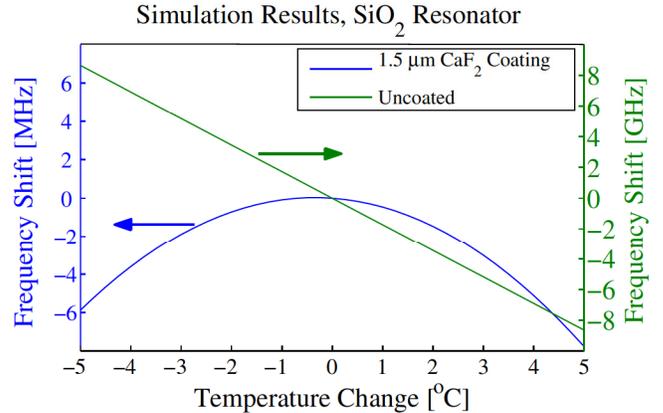


Fig. 1. Simulation results for a SiO_2 resonator with $1.4876 \mu\text{m}$ CaF_2 coating, showing dramatic improvement from the negative thermorefractive coefficient of the latter. Not only is the thermal frequency shift about 1000 times less, a turning point appears for the proper thickness.

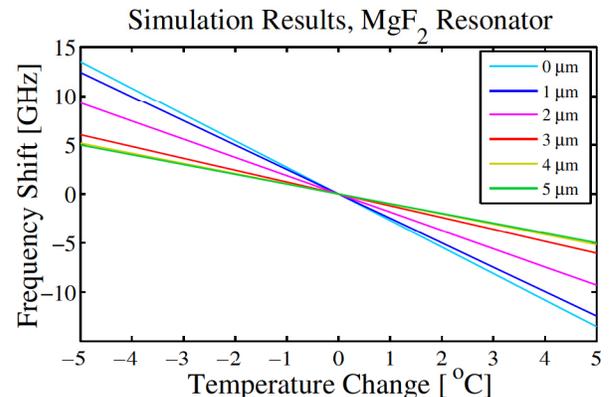


Fig. 2. Simulation results for a crystalline MgF_2 resonator with varying thicknesses of CaF_2 coating. Though some increase in temperature stability is predicted, the frequency shift remains monotonic and in the $\text{GHz}/^\circ\text{C}$ range.

very large if the $1.5 \mu\text{m}$ wavelength is maintained, exhausting computational resources.

B. Active Stabilization

We have developed an active temperature stabilization scheme that utilizes a differential in thermorefractive coefficients of the ordinary and extraordinary polarizations in a birefringent resonator. Since the two polarizations have different thermal dependencies, a change in the frequency spacing between their modes corresponds to a change in the resonator’s temperature. By selecting a TE and TM mode that are close together (in frequency) and measuring the difference between them in real time, one can measure temperature fluctuations. Feeding-back this measurement to a temperature control mechanism on or near the WGMR constitutes closed-loop temperature stabilization.

The experimental setup is as follows. Light from a fiber laser is coupled via an angle-polished fiber into a resonator made from magnesium fluoride. The laser frequency is swept over a few MHz, with a center frequency such that the two

polarizations are simultaneously observable. Upon exiting the coupler, a polarizing beam splitter separates polarizations and each is detected with a separate photodiode (PD). Signals from the two detectors are acquired and processed digitally so that the frequency difference, Δf , between their peak centers can be precisely determined. The resulting error signal is passed through two proportional-integration stages to generate a correction signal. For temperature control, the resonator is placed in a house-made brass mount which contains two resistive heating elements, one above and one below the device, and in good thermal contact with it.

To predict the stabilization performance, we first determine the frequency differential's temperature dependence for a given, typical pair of modes. Such a pair is shown in figure 3. We make simultaneous, independent measurements of Δf and the resonator's temperature (with a precision thermistor) for several values of temperature. A fit to this data yields a value of $d(\Delta f)/dT \approx -90 \text{ MHz}/^\circ\text{C}$, a very sensitive function. Assuming we can make a shot-noise limited measurement of the mode locations (mode linewidth is 100 kHz), we can expect frequency resolution of 0.1 Hz, which corresponds to temperature stabilization at the nK level. This in turn yields a projected frequency stability of approximately one part in 10^{14} [8].

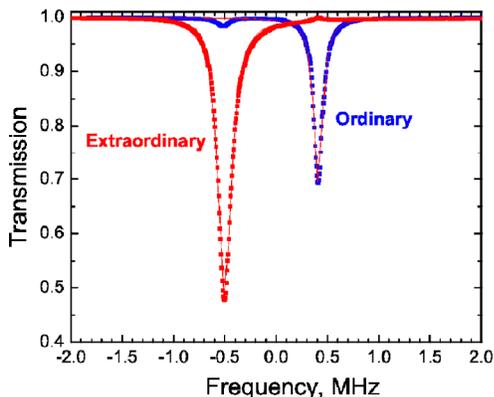


Fig. 3. Frequency spacing between the ordinary and extraordinary modes is a function of temperature and can be used for active temperature stabilization.

III. WGMR AS A REFERENCE CAVITY

The ultimate goal of this work is to lock a laser to the temperature-stabilized WGMR. We have begun development of the locking system. Using the Pound-Drever-Hall (PDH) method [9], we have achieved preliminary stable locking. The magnesium fluoride cavity demonstrated $Q=1.9 \times 10^9$ (corresponding to $\mathcal{F} \approx 100,000$), and is temperature stabilized by a conventional resistive heater and foam insulation.

The experimental setup is shown in figure 4 and described as follows. Light from a 1550 nm laser is coupled into the resonator via an angle-polished optical fiber, while a photodiode placed near the end of the fiber collects light from a single mode family. Our electronics employ a 1 MHz modulation frequency, which is split into two, with one portion being

amplified and fed to an electro-optic phase modulator, while the other portion is used to demodulate the photodiode signal in an rf mixer. The resulting error signal (figure 5) is then fed to proportional-integrator-differentiator stages to generate the correction signal for the laser's piezoelectric tuning. So far, we have measured 3.2 kHz rms residuals over the loop bandwidth.

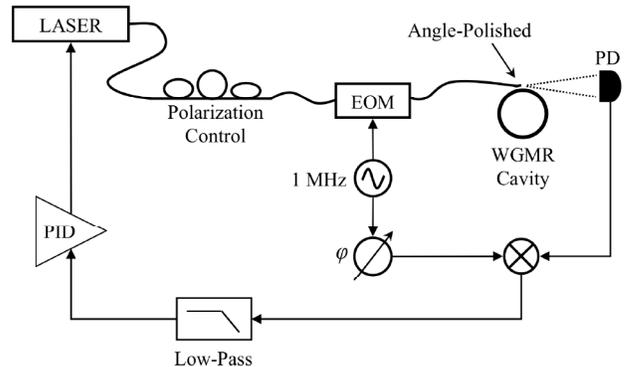


Fig. 4. Schematic of the experimental setup used for locking the Koheras fiber laser to the WGMR cavity. Locking has been achieved to a non-temperature stabilized resonator.

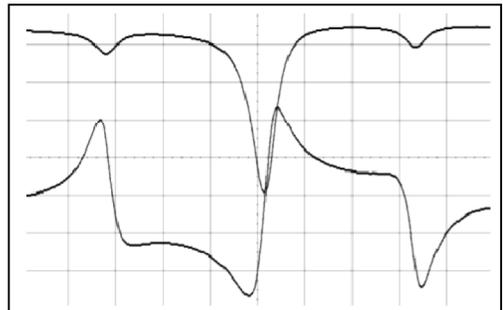


Fig. 5. Oscilloscope capture showing the photodiode signal (upper trace) and PDH error signal (lower trace). Sidebands are 1 MHz from the carrier.

IV. CONCLUSION

We have investigated temperature compensation schemes for whispering gallery mode resonators applied as frequency reference cavities. Thermal compensation from a thin-film coating of material with negative thermorefractive coefficient is predicted to yield substantial improvements to an amorphous silica resonator, but insufficient improvement for a crystalline MgF_2 device. We note, however, that other material combinations may facilitate successful implementation with a crystalline resonator. A scheme utilizing the differential in thermorefractivity of differently polarized modes in a birefringent WGMR yields temperature resolution in the nK range and can be used for frequency stabilization. A cavity stabilized in this way has many advantages, chief amongst them being that it eliminates the need for thermal shielding, dramatically reducing the size of the apparatus. Our data support potential

stabilization to a few parts in 10^{14} at 1 second. Furthermore, we believe that this technique is among the best performance of an active temperature control system operating near room temperature, and that it will have applications beyond laser stabilization.

V. ACKNOWLEDGMENT

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