

# TOWARDS AN OPTIMAL NOISE VERSUS RESOLUTION TRADE-OFF IN WIND SCATTEROMETRY

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## 1. INTRODUCTION

A scatterometer is a radar that measures the normalized radar cross section ( $\sigma^0$ ) of the Earth's surface. Over the ocean this signal is related to the wind via the geophysical model function (GMF). The objective of wind scatterometry is to estimate the wind vector field from  $\sigma^0$  measurements; however, there are many subtleties that complicate this problem—making it difficult to obtain a unique wind field estimate.

Conventionally, wind estimation is split into two stages: a wind retrieval stage in which several ambiguous solutions are obtained, and an ambiguity removal stage in which ambiguities are chosen to produce an appropriate wind vector field estimate. The most common approach to wind field estimation is to grid the scatterometer swath into wind vector cells and estimate wind vector ambiguities independently for each cell. Then, fieldwise structure is imposed on the solution by an ambiguity selection routine. Although this approach is simple and practical, it neglects fieldwise structure in the retrieval step and does not account for the spatial correlation imposed by the sampling. This makes it difficult to develop a theoretically appropriate noise versus resolution trade-off using pointwise retrieval.

Fieldwise structure may be imposed in the retrieval step using a model-based approach. However, this approach is generally only practical if a low order wind field model is applied, which may discard more information than is desired. Furthermore, model-based approaches do not account for the structure imposed by the sampling.

A more general fieldwise approach is to estimate all the wind vectors for all the WVCs simultaneously from all the measurements. This approach can account for structure of the wind field as well as structure imposed by the sampling in the wind retrieval step. Williams and Long in 2010 [1] developed a fieldwise retrieval method based on maximum a posteriori estimation (MAP). This MAP approach can be extended to perform a noise versus resolution trade-off, and deal with ambiguity selection.

This paper extends the fieldwise MAP estimation approach and investigates both the noise versus resolution trade-off as well as ambiguity removal in the fieldwise wind retrieval step. The method is then applied to the SeaWinds scatterometer and the results are analyzed.

## 2. BACKGROUND

This section presents background on scatterometry. The scatterometer sampling and noise models are also presented.

Wind scatterometers make multiple  $\sigma^0$  measurements from different look directions of the same location on the Earth's surface. For the SeaWinds scatterometer, each scatterometer pulse is partitioned into several 'slices' with

different spatial response functions that are narrow ( $\sim 5$  km) in the range direction and long ( $\sim 25$  km) in the azimuth direction. The slice  $\sigma^0$  measurements from different looks (or ‘flavors’) of measurements sample the same location with response functions that overlap irregularly and with different orientations. Neglecting noise, the  $i$ th slice  $\sigma^0$  measurement  $\sigma_{t,i}^0$  can be expressed as an inner product of the underlying  $\sigma^0$  field  $\sigma_{t,i}^0(x)$  with the spatial response function  $A_i(x)$ , where the  $\sigma^0$  field is a nonlinear function of the wind field  $\vec{U}(x)$ . In practice, the integration is made discrete. The multiple measurements can be stacked into a vector, producing the discrete scatterometer sampling operator [1]

$$\vec{\sigma}_t^0 = \begin{bmatrix} \sum_x A_1(x) \text{gmf}_1(\vec{U}(x)) \\ \vdots \\ \sum_x A_N(x) \text{gmf}_N(\vec{U}(x)) \end{bmatrix} = \mathbf{T}(\vec{U}(x)). \quad (1)$$

where  $\text{gmf}_i(\cdot)$  represents the GMF that relates the wind to  $\sigma^0$  with viewing geometry, polarization, and frequency corresponding to the  $i$ th slice measurement, and  $x$  represents a two-dimensional spatial variable.

Scatterometers make noisy  $\sigma^0$  measurements due to several sources. A standard scatterometer noise model is that the measurements are Gaussian random variables, where the mean is the ‘true’ or noise-free measurement and the variance is a quadratic function of the mean [2]. This form accounts for fading as well as other noise sources. The distribution for noisy scatterometer measurements can be expressed as

$$f(\vec{\sigma}_m^0 | \vec{U}(x)) = \frac{(2\pi)^{-\frac{N}{2}}}{|\mathbf{R}|} \exp\{(\vec{\sigma}_m^0 - \vec{\sigma}_t^0)^T \mathbf{R}^{-1} (\vec{\sigma}_m^0 - \vec{\sigma}_t^0)\} \quad (2)$$

where  $\vec{\sigma}_t^0$  is a function of the wind field as expressed in Eq. 1, and  $\mathbf{R}$  is a diagonal covariance matrix with diagonal elements  $R_{i,i} = \alpha_i (\sigma_{t,i}^0)^2 + \beta_i \sigma_{t,i}^0 + \gamma_i$ , where  $\alpha_i$ ,  $\beta_i$ , and  $\gamma_i$  are functions of the scatterometer design.

The problem of wind scatterometry is to invert the noisy sampling model and estimate the wind field given the noisy scatterometer measurements. [1] develops a MAP estimation approach for wind field retrieval. The MAP estimator can be expressed as  $\text{argmax}_{\vec{U}(x)} \{\log f(\vec{\sigma}_m^0 | \vec{U}(x)) + \log f(\vec{U}(x))\}$ , where  $f(\vec{U}(x))$  is a prior distribution of the wind field. The maxima can be found using a gradient search approach. The method developed in [1] employs a simplistic prior designed to regularize the problem in a practical manner in order to reconstruct a high resolution wind field. We desire a more useful prior that appropriately handles the noise versus resolution trade-off.

### 3. IMPLEMENTATION

In general, we desire a prior that imposes as little information as possible to obtain the desired variability of the estimates. Thus, we take a conservative approach employing a maximum entropy prior under certain constraints. We wish to impose structure on the power spectrum or correlation of the signal, which uniquely determines the covariance of the prior distribution. The maximum entropy distribution with a given covariance is the Gaussian distribution. Also, we desire the energy in the unobservable components of the signal to go to zero—suggesting a zero mean Gaussian prior distribution. Thus, we employ independent, zero-mean Gaussian priors on the U and V components of the wind fields with a Toeplitz covariance (which implies a wide-sense stationary process with a particular power spectrum). Note that for a diagonal covariance (i.e., white process), this results in a Rayleigh distributed wind speed prior and a uniform direction prior.

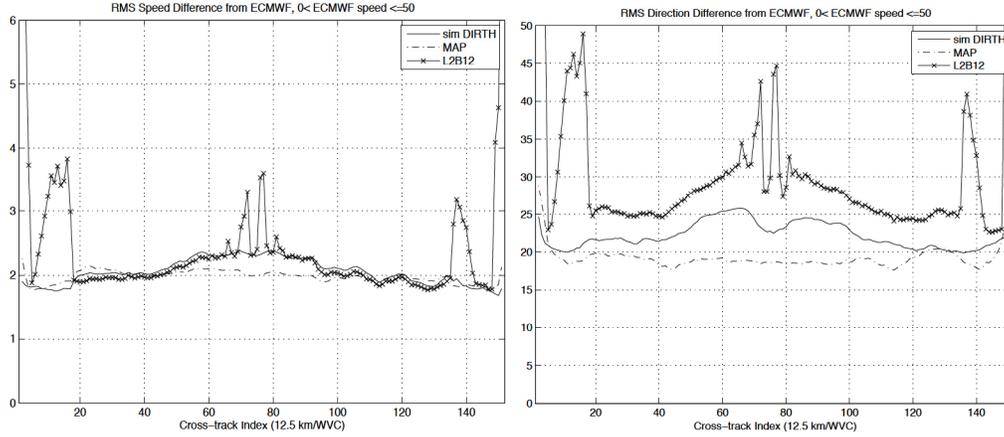
The power spectrum of wind components over the ocean tends to fall off approximately as one over the wave number squared [3]. We impose this structure on the high frequency components of the wind field by assuming an exponential correlation function, which has a flat spectrum up until the correlation length where it begins to fall off as one over the wave number squared. We use a correlation length of 6 km in order to significantly impose the prior only on the finest scales.

This MAP approach is implemented using a gradient search, which produces a reconstructed wind field estimate near the initialization wind field. The method can be considered either as a resolution enhancement procedure or a smoothing procedure on the initialization field, depending on the structure of the initialization field and the prior. There are many possible approaches to come up with an initialization field, which can be considered as ambiguity selection. We could use a numerical weather prediction (NWP) field; however, the NWP direction fields tend to smooth over fronts and mislocate storms. Alternatively, the result of a standard ambiguity selection routine may be used as the initialization. For example, the direction interval retrieval with thresholded nudging (DIRTH) method uses the relative likelihoods of the direction ambiguities to exclude the less likely solutions from the nudging process [4]. However, DIRTH cannot account for spatial structure imposed by the sampling. We develop a similar approach to obtain an initialization field, but search for fieldwise ‘ambiguities’.

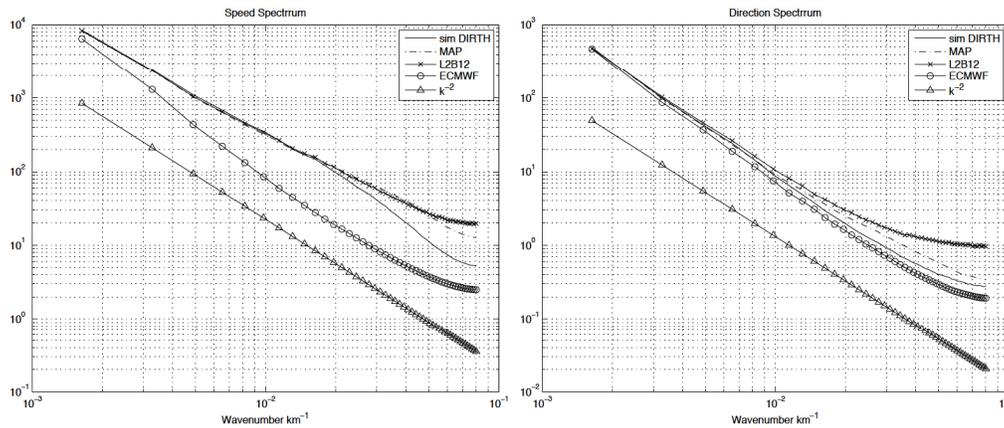
We begin with the thresholded nudge field (which requires that conventional pointwise retrieval also be performed). Then, we apply fieldwise MAP estimation using a gradient search. For practical purposes, we use the prior developed in [1] for this phase of the processing. Next, the field is low pass filtered and the fieldwise MAP approach is applied again. This filtering followed by MAP estimation process is then repeated until convergence or a maximum number of iterations is achieved. This field is then used to initialize fieldwise MAP estimation using the prior described above that assumes a  $k^{-2}$  spectrum. This final MAP procedure is similar to the final DIRTH step in that it smooths the wind field where there are wide intervals in the direction with similar high likelihoods. Furthermore, this final step smooths regions where data is more noisy or the sensitivity is low (i.e., regions that are expected to produce noisier estimates), while reconstructing fine details that are observable.

#### 4. ANALYSIS

Here, we take a statistical approach to analyse the results of the MAP method applied to real data. We process several revolutions worth of SeaWinds data and compare the results to the standard 12.5km DIRTH product (L2B12), ECMWF, and the DIRTH approach using 100 percent overlap gridding (sim DIRTH). Figure 1 shows the RMS speed and direction difference with respect to ECMWF of the various approaches. The MAP speed and direction RMS differences are the most flat across the swath. Also the MAP result significantly reduces the RMS direction error. Figure 2 displays the along-track one-sided power spectra of the speed and direction of the various winds. The direction spectra are taken as the magnitude squared of the Fourier transform of  $\exp\{-id(x)\}$  where  $d(x)$  represents the direction field expressed in radians. The MAP speed spectrum follows the L2B12 closely until the L2B12 hits the noise floor and levels out, while the MAP result continues to follow the  $k^{-2}$  fall off. This suggests that the resolution of the retrieved speed is at least as high as the L2B12 product but is less noisy. The MAP direction spectrum has more information in the high frequencies than the sim DIRTH, and does not level out in noise like the L2B12 direction spectrum. Although the MAP approach can be applied at a higher resolution, the MAP speed and direction spectra are beginning to level out at the last bin, suggesting that 12.5km is an appropriate WVC resolution.



**Fig. 1.** Speed and direction RMS difference from ECMWF.



**Fig. 2.** Speed and direction power spectra.

## 5. REFERENCES

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