IRS2012 – International Radiation Symposium 2012
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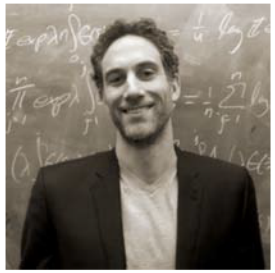
Session 2: Radiative Transfer Theory & Modeling

Parametric 3D Atmospheric Reconstruction in Highly Variable Terrain with Recycled Monte Carlo Paths & an Adapted Bayesian Inference Engine

Ian Langmore, Opera Solutions, New York, NY
Anthony B. Davis, JPL/Caltech, Pasadena, Ca
Guillaume Bal, Columbia University, New York, NY
Youssef M. Marzouk, MIT, Cambridge, Mass

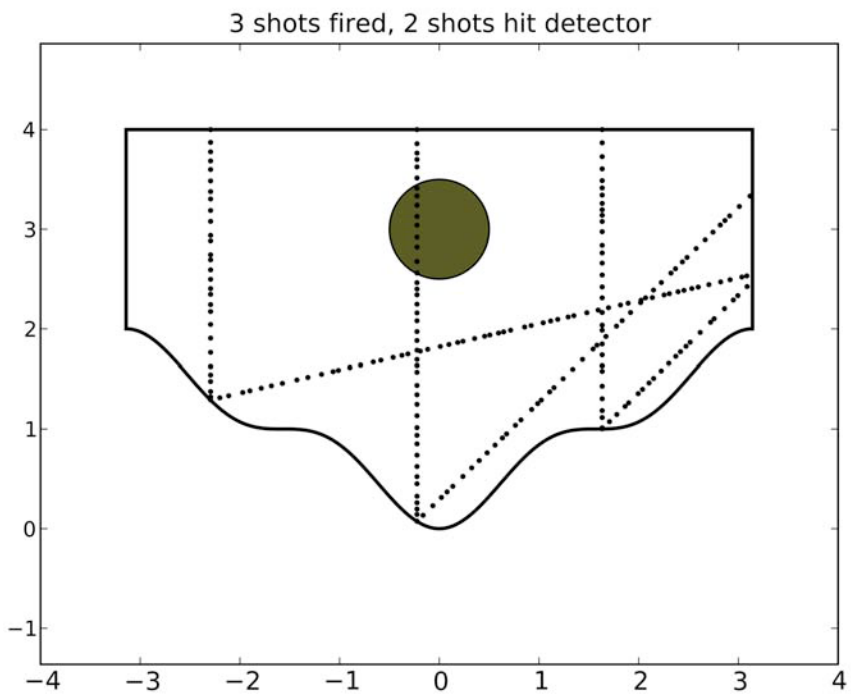


Outline

- **Context**
  - Nuclear treaty verification using remote sensing
- **Remote sensing challenge … and resource!**
  - 2D scene geometry, representative of 3D case
- **Advances in forward modeling**
  - Signal = whole image, not just a pixel
- **Advances in inverse problem**
  - Multi-pixel technique
  - Bayesian posterior inference, not parameter retrieval per se
- **Summary & Outlook**

Context & Challenge

- **Weaponization processes of special nuclear materials (SNM) impact, in particular, the atmospheric environment**
  - Use remote sensing to detect and quantify signature impact
  - In this case, a chemical effluent in a downwind plume
- **Assume a target facility in a deep valley:**



Let there be:

1) a "$\cos^3 x$" terrain;

2) a circular plume;

3) a partially-known/partially-unknown background aerosol;

4) a single crude monochromatic imaging sensor at finite range. (E.g., an airborne device where single spectral band is "informed" by a near-IR hyperspectral gas absorption database.)

Fixed 2D horizontal terrain profile

$z(x) = 1 - \cos^3 2\pi x/L$, $L = 2\pi$ km

Aerosol vertical profile & phase function (log-scale)

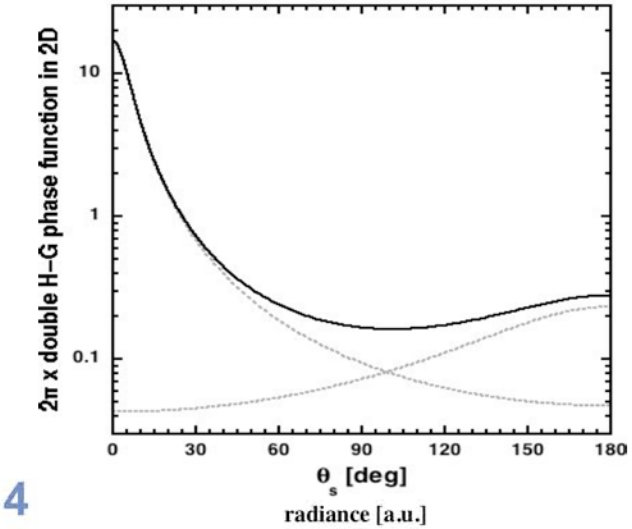
$\sigma(z) = \sigma_0 \exp(-z/H)$, $H = 1/c$, $c = c_0 = 0.5$ km^{-1} and $\delta c = +0.15$ km^{-1} in γ-world

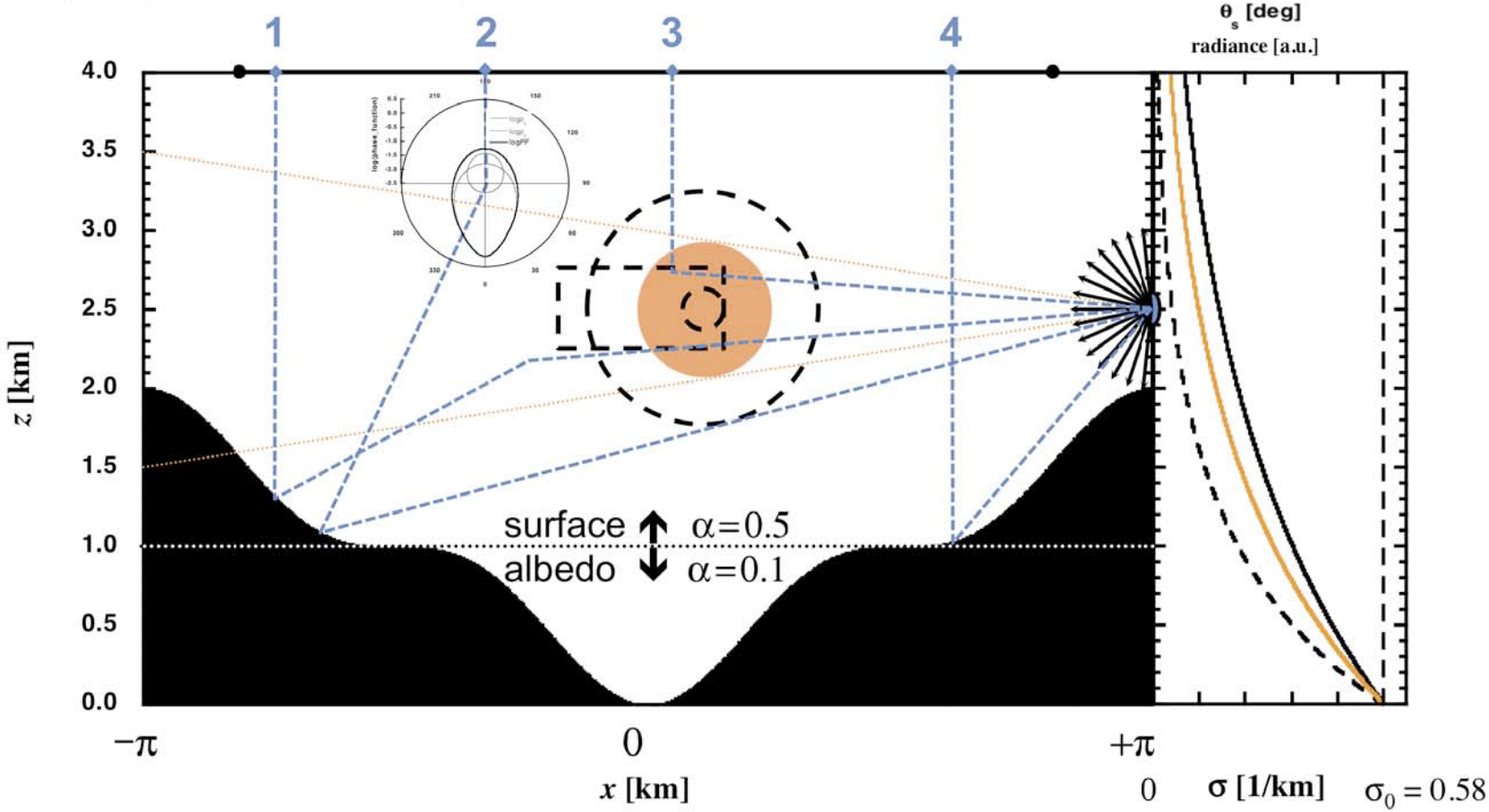
Plume ("true" position and size)

$x_p = +0.35$ km, $z_p = 2.5$ km, $\rho_p = 0.5$ km, and $k_p = 0.5$ km^{-1} (hence $\tau_p = 1$)

Images of reference world &"γ" world (with plume)

15 pixels (centers marked with arrows), 180° Field-of-View





2D scene, representative of 3D case

n **= 1,2,3**

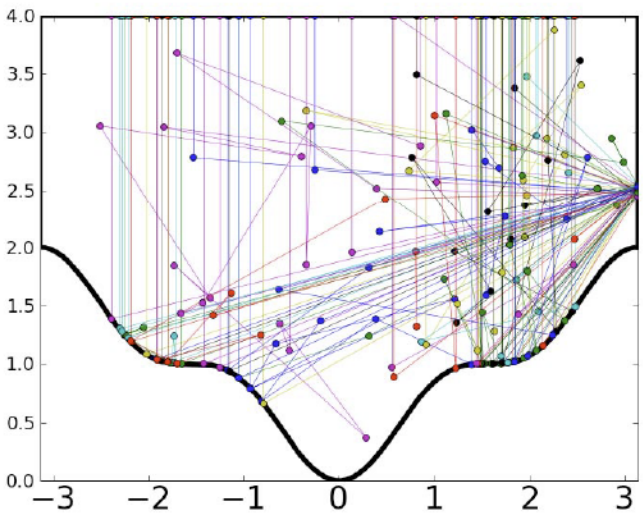
***n*D RTE:** $\left[\vec{\Omega}\cdot\nabla+\sigma(\vec{r})\right]I=\sigma_{\rm s}(\vec{r})\int p_{\rm v}(\vec{r},\vec{\Omega}'\to\vec{\Omega})I(\vec{r},\vec{\Omega}')\mathrm{d}\vec{\Omega}'+Q_{\rm v}(\vec{r},\vec{\Omega})$

***n*D BCs:** $|\vec{\Omega}\cdot\vec{n}(\vec{r})|I(\vec{r},\vec{\Omega})=\alpha(\vec{r})\int\limits_{\vec{\Omega}'\cdot\vec{n}(\vec{r})>0} p_{\rm s}(\vec{r},\vec{\Omega}'\to\vec{\Omega})\,I(\vec{r},\vec{\Omega}')\,\vec{\Omega}'\cdot\vec{n}(\vec{r})\,\mathrm{d}\vec{\Omega}'+Q_{\rm s}(\vec{r},\vec{\Omega})$

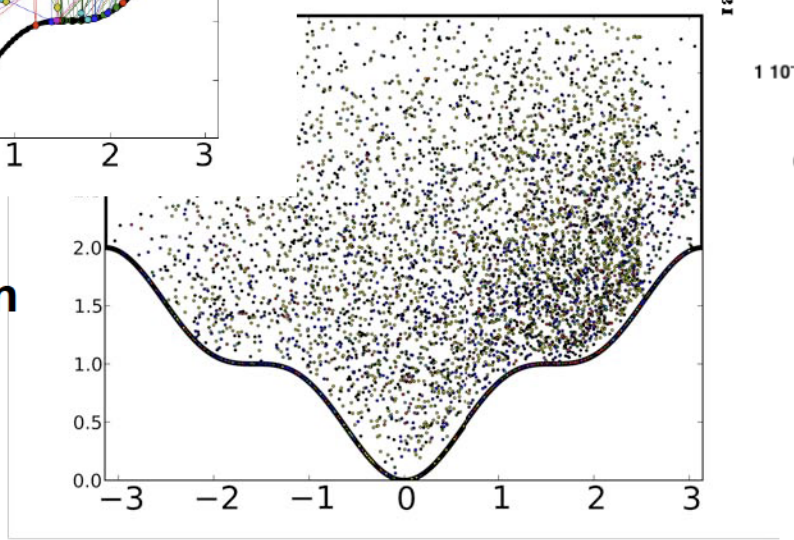
- **Formally, 2D "flatland" RT looks like 3D "real world" RT**
  - Henyey-Greenstein phase function scattering details are different in 2D Monte Carlo (MC)
  - Lambertian surface reflection details are different in 2D MC
- **Path-recycling MC implementation**
  - Save (to file) all paths that end in detection
  - Recompute path weight as needed: Gaseous plume? Aerosols?

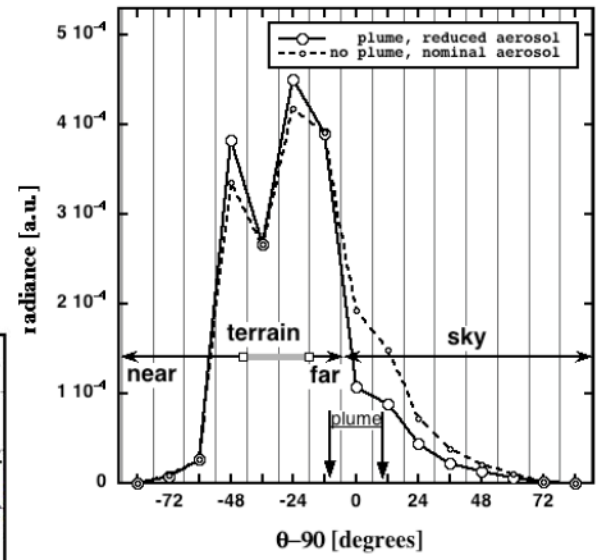
Signal = whole image, not just a pixel

- **Need fast 2D RT model to address inverse problem of plume detection/characterization**
  - Most things in scene don't change: use Monte Carlo path recycling



75 paths,
all ending in detection





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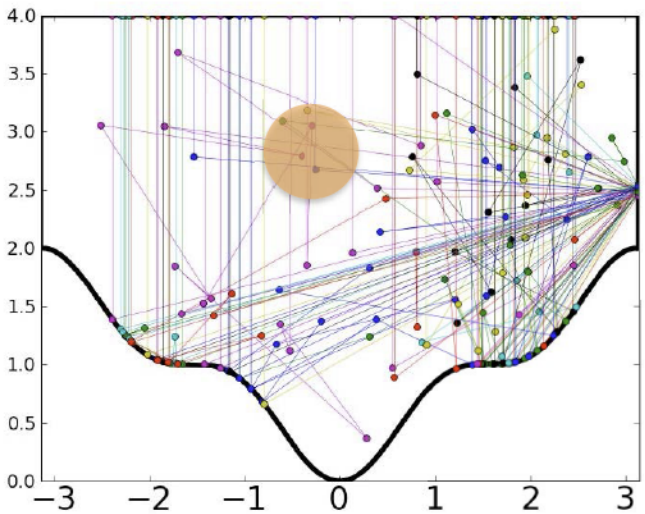

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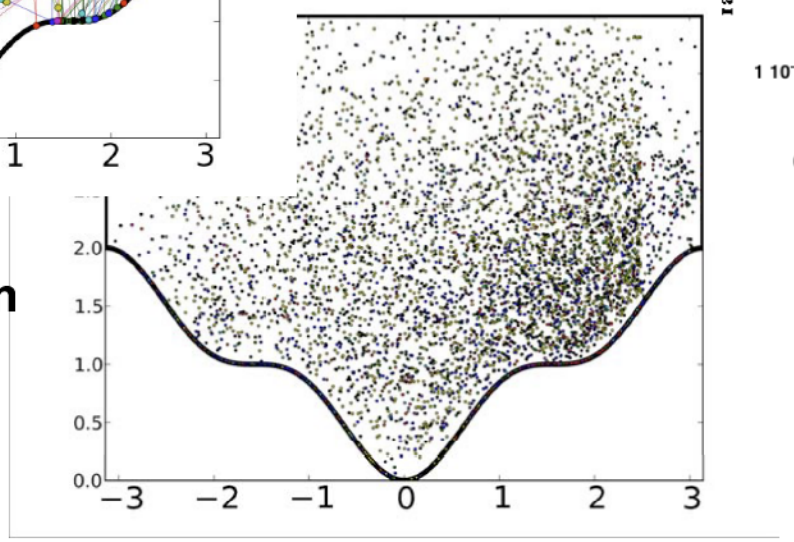

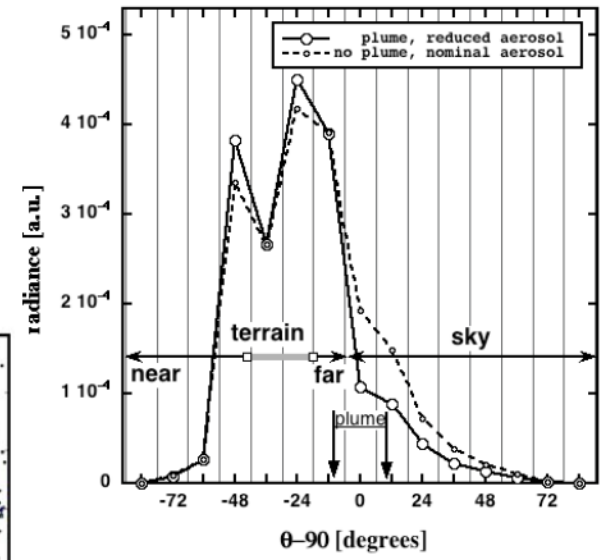

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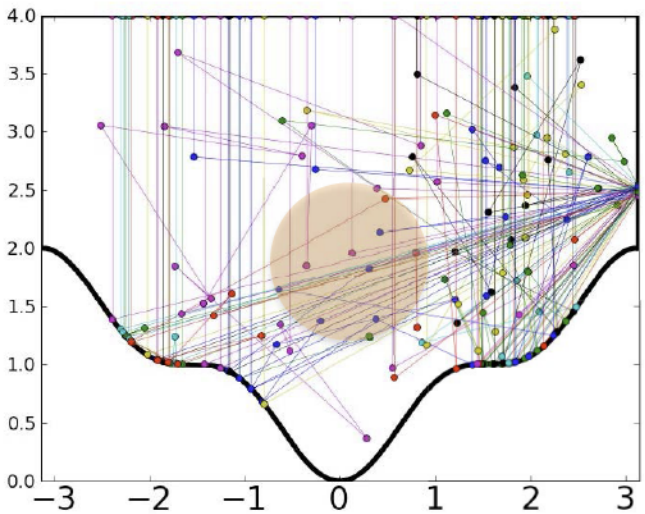

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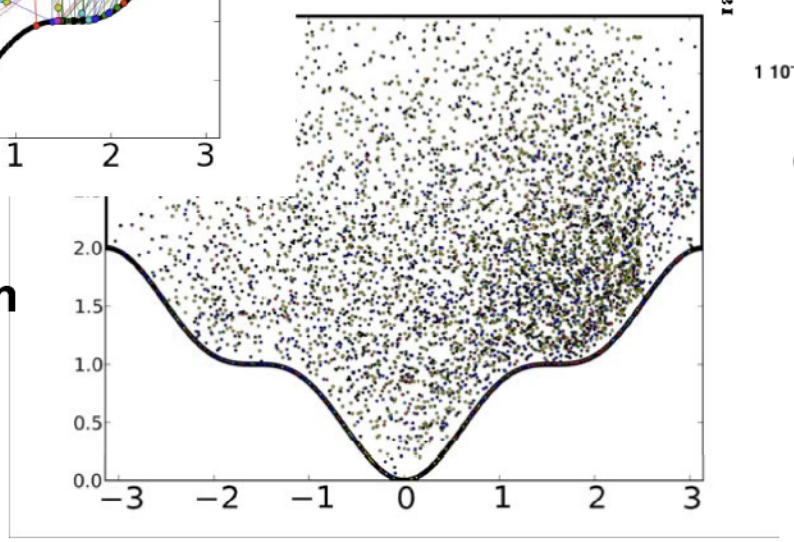
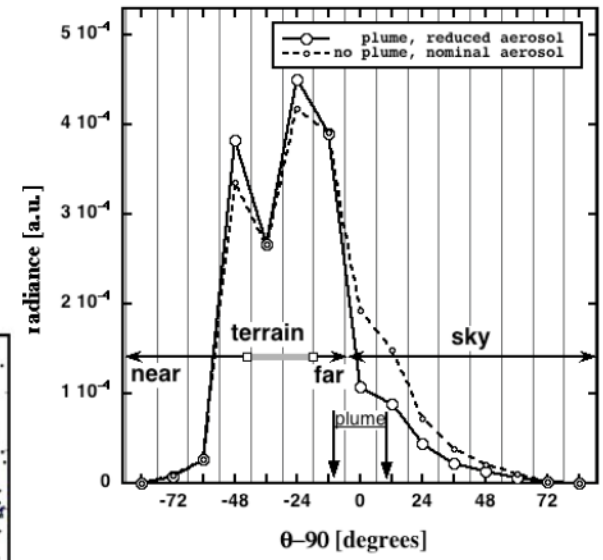




Inverse problem solution

- **5 parameters to infer**
  - 4 for purely absorbing gaseous plume
  - 1 for scattering by background aerosol

SUMMARY OF VARIABLE ATMOSPHERE-SURFACE PARAMETERS

Parameter	Symbol	Min	Max	Unit	Prior
Plume's x-position	x_{p}	-0.5	$+0.5$	km	uniform
Plume's z-position	z_{p}	2.2	2.7	km	uniform
Plume's radius	ρ_{p}	0.15	0.85	km	uniform
Plume's absorption coef.	k_{p}	0	∞	1/km	Gamma
Aerosol perturbation	δc	$-c_0$	$+c_0$	1/km	uniform

x

Inverse problem solution

- **Bayesian posterior inference, rather than parameter (value and error) estimation**

Standard Markov chain Monte Carlo (MCMC):

data

↓ ideal (infinite-precision) forward model

$$\mathbf{d} = \mathbf{F}_\infty(\mathbf{x}) + \mathbf{E}, \qquad \mathbf{E} \sim \mathcal{N}(0, \Sigma_\mathbf{E}), \qquad \mathbf{E} \perp\!\!\!\perp \mathbf{x}.$$

SNR control ↑

$$\pi_\infty(\mathbf{x} \,|\, \mathbf{d}) \propto p_{\text{prior}}(\mathbf{x}) \frac{1}{|\Sigma_\mathbf{E}|^{1/2}} \exp\left(-\frac{1}{2}\|\mathbf{d} - \mathbf{F}_\infty(\mathbf{x})\|^2_{\Sigma_\mathbf{E}^{-1}}\right)$$

Usual cost function to minimize (maximum likelihood approaches)

Inverse problem solution

- **Need to accelerate the standard (Metropolis-Hastings) algorithm!**

Enhanced multi-level (error-contoled) MCMC:

$$\mathbf{F}_j(\mathbf{x}) \sim \mathcal{N}(\mathbf{F}_\infty(\mathbf{x}), \Sigma_j(\mathbf{x})),$$

← finite-precision model (well-known for MC RT)

$$\mathbf{d} = \mathbf{F}_j(\mathbf{x}) + \mathbf{E}_j(\mathbf{x}) + \mathbf{E}, \quad \mathbf{E} \sim \mathcal{N}(0, \Sigma_\mathbf{E}), \quad \mathbf{E} \perp\!\!\!\perp \mathbf{x},$$

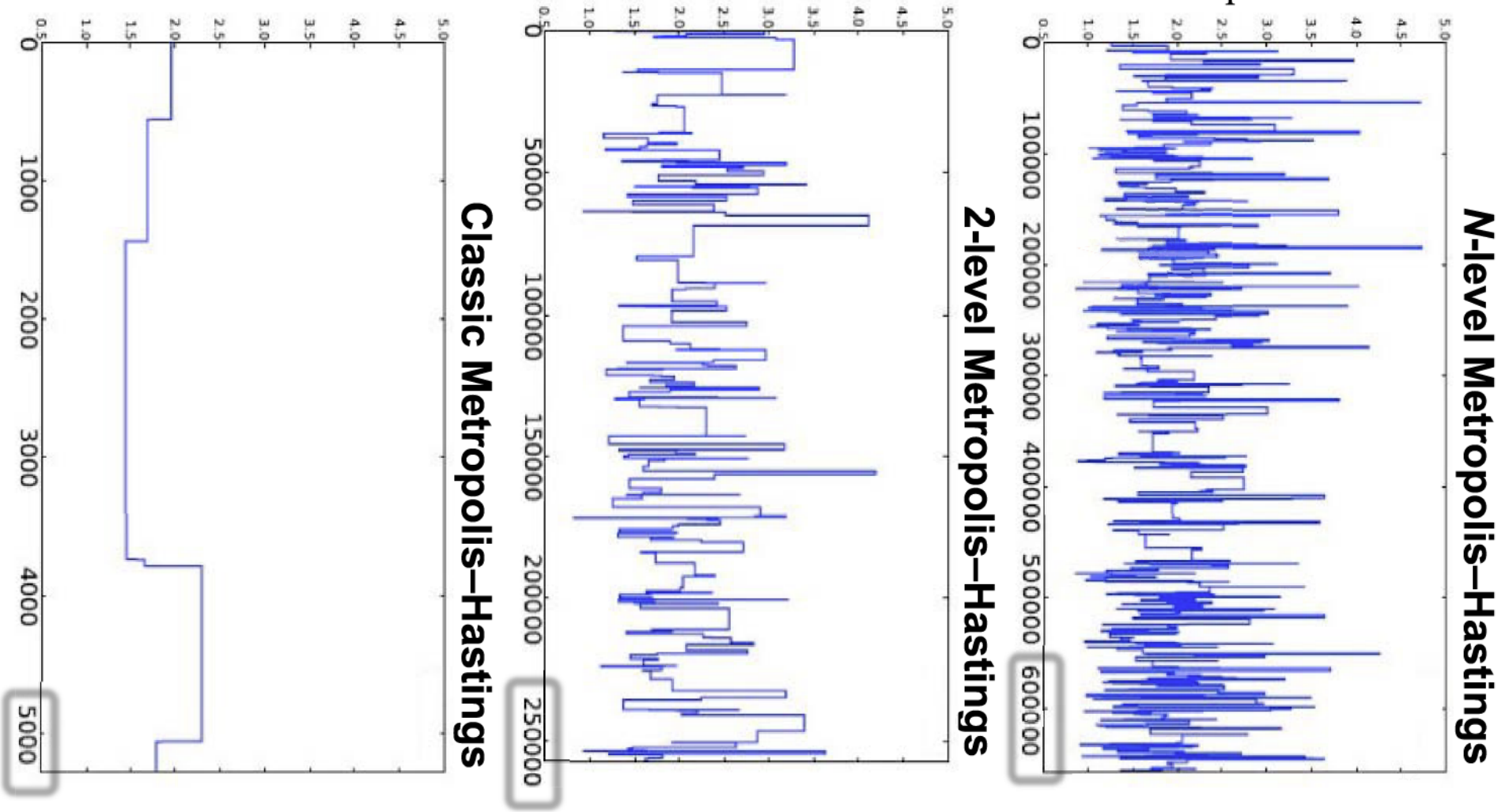
$$\mathbf{E}_j(\mathbf{x}) \sim \mathcal{N}(0, \Sigma_j(\mathbf{x})), \quad \mathbf{E}_j(\mathbf{x}) \perp\!\!\!\perp \mathbf{E}$$

$$\pi_j(\mathbf{x} \,|\, \mathbf{d}) \propto p_{\text{prior}}(\mathbf{x}) \pi_j(\mathbf{d} \,|\, \mathbf{x})$$

$$\pi_j(\mathbf{d} \,|\, \mathbf{x}) = \frac{1}{\sqrt{(2\pi)^m |\Sigma_\mathbf{E} + \Sigma_j(\mathbf{x})|}} \times \exp\left(-\frac{1}{2} \|\mathbf{d} - \mathbf{F}_j(\mathbf{x})\|^2_{(\Sigma_\mathbf{E} + \Sigma_j(\mathbf{x}))^{-1}} \right)$$

Inverse problem solution

- **Trade speed and accuracy in sampling strategy**
  - All 3 runs are for 5 hours of CPU time (only k_p shown)



Inverse problem solution

- **Resulting posterior PDFs**
  - Here: intrinsic plume properties
- **Here: SNR = 5**
- **Done: SNR = 2 (yields degraded precision)**

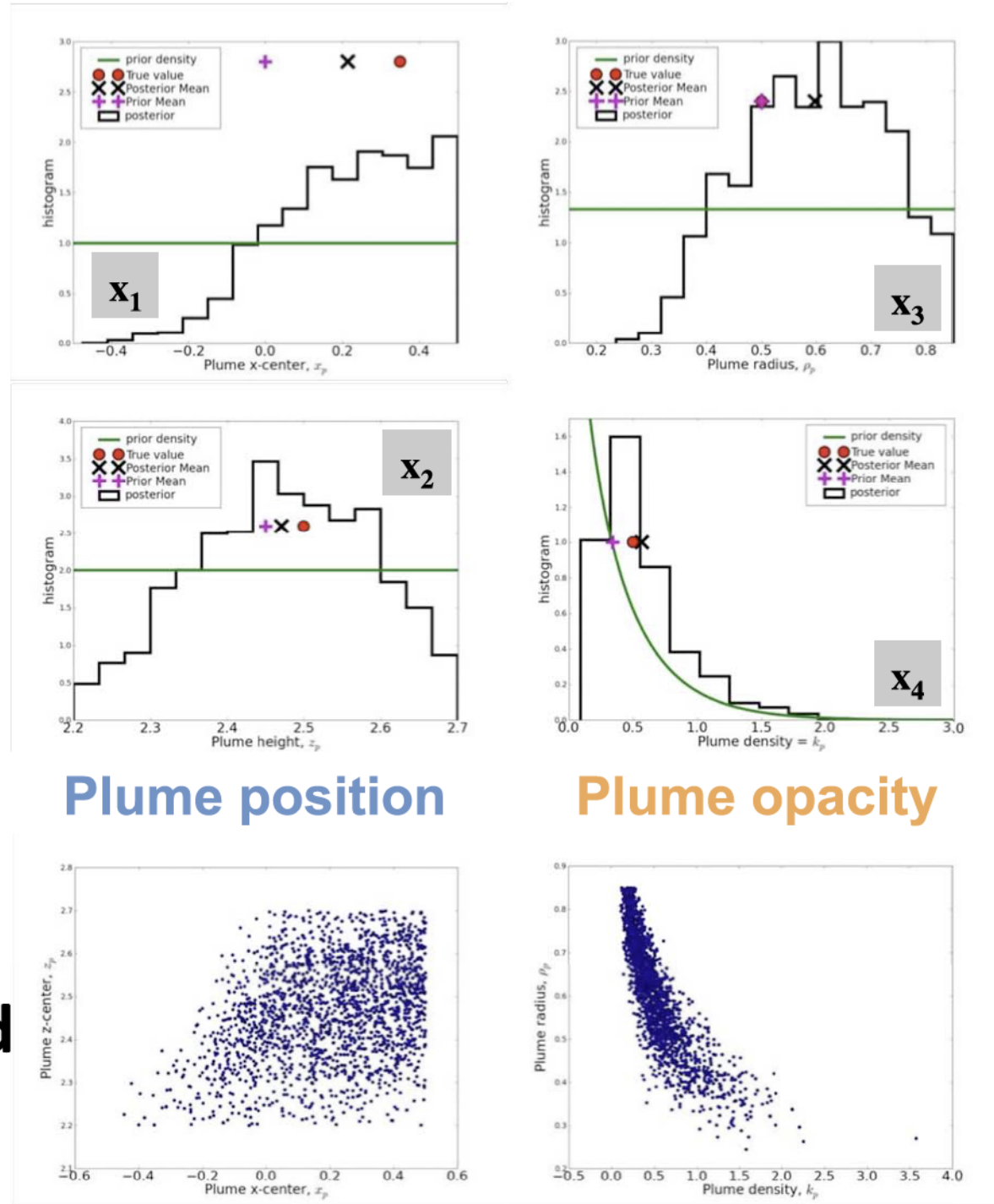
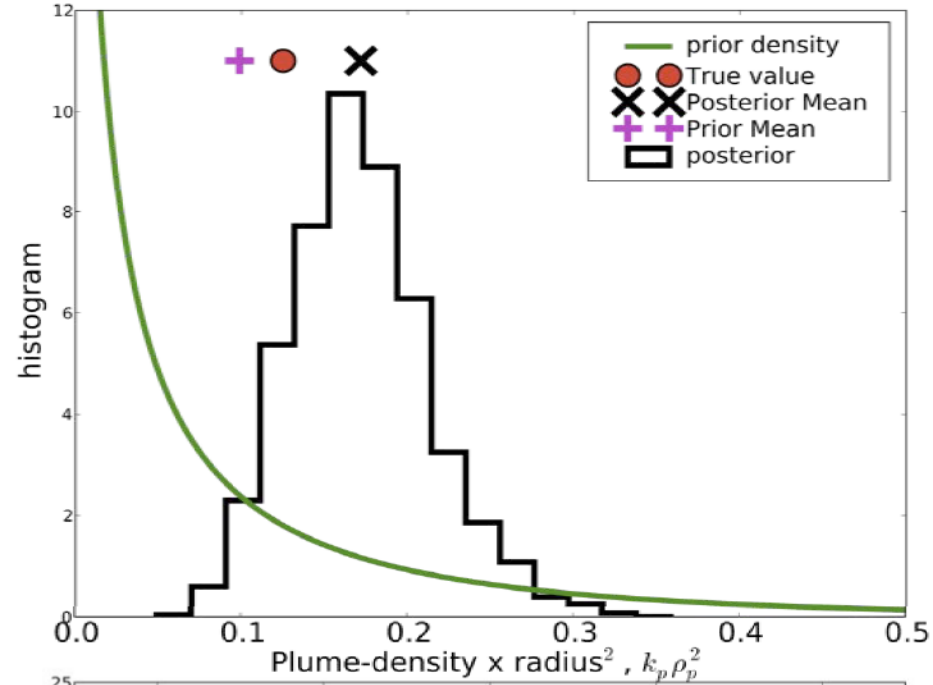
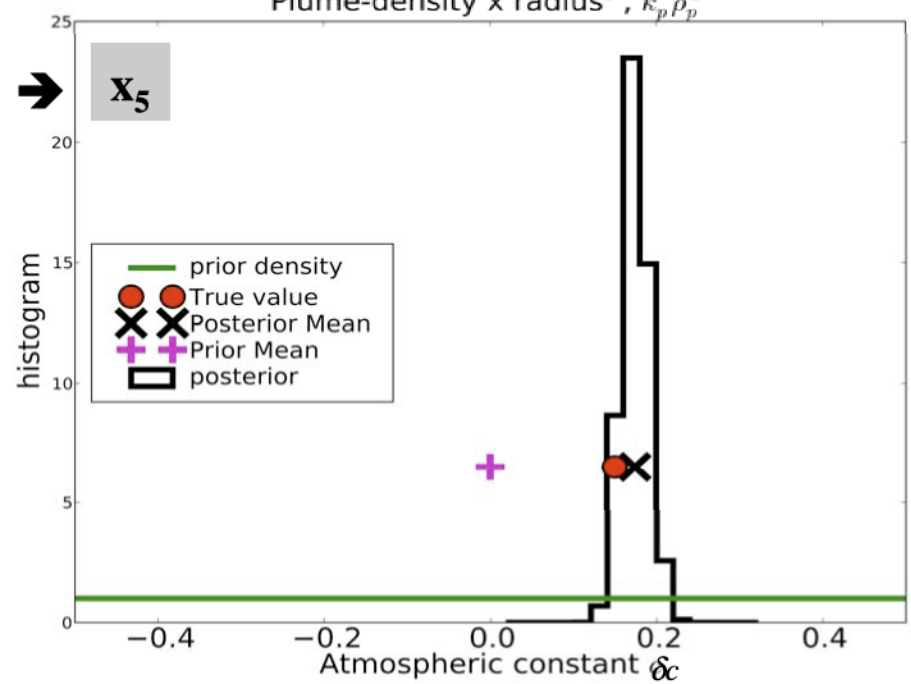


Fig. 3b. *MCMC samples for joint posterior distributions at SNR = 5, without inverse crime, for plume-specific parameters.* Left: (x_p, z_p). Right: (k_p,ρ_p). More discussion in main text.

Inverse problem solution

- **Resulting posterior PDFs**
  - plume "mass"
  - and … **Aerosol amount** ➔ x_5





- **Here: SNR = 5**
- **Done: SNR = 2 (yields degraded precision)**

Summary & Outlook

- **Introducing a new class of (physics-based) "multi-pixel" methods in remote sensing of the environment, formally**
- **Need *fast* forward 2D or 3D RT model**
  - Parameterized structural and optical properties
  - Path-recycling Monte Carlo implementation
- **Introducing the "MC^3" Bayesian inference algorithm**
- **IEEE-TGRS paper is available from presenter for more details (revised version currently under review)**
- **Collaboration** COLUMBIA UNIVERSITY IN THE CITY OF NEW YORK JPL

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