Stochastic Convection Parameterizations

João Teixeira, Carolyn Reynolds (*), Kay Suselj & Georgios Matheou

Jet Propulsion Laboratory
California Institute of Technology
Pasadena, California

(*) Naval Research Laboratory
Monterey, California

Small-scale Processes in Atmospheric Models

It is not a simple computational fluid dynamics problem ....

.... We need to represent radiation, clouds, turbulence, convection, gravity waves, surface interaction.
### Physical Parameterizations

1) **Small-scale fluid dynamics:**
   - Turbulence and Convection (dry and moist)
   - Gravity waves
   - Clouds (phase transition / latent heat)
   - Would be mostly solved if NWP models had resolutions of around 1-10m

2) **Small-scale physical processes (NOT just fluid dynamics):**
   - Radiation Interaction
   - Cloud and aerosol microphysics
   - Some of these equations are well-known but others not at all

3) **Surface interaction can be (1) and (2) but also includes additional complexity (vegetation, biogeochemistry)**
Physical Parameterizations:
Example of radiation vs turbulence/convection

Turbulence and convection parameterization:

\[ \frac{\partial \phi}{\partial t} = -\frac{\partial}{\partial z} (\bar{w}\phi) \]

Reynolds decomposition and averaging

\[ \frac{\partial \phi}{\partial t} = -\frac{\partial}{\partial z} (\bar{w}\phi) - \frac{\partial}{\partial z} (\bar{w}'\phi') \]

needs to be parameterized due to non-linearity \( w\phi \)

Radiation parameterization:

Theory leads to

\[ C_p \frac{\partial \bar{T}}{\partial t} = -\frac{\partial}{\partial z} \left[ F_R (\bar{T}, q, l) \right] \]

This is incomplete

Complex non-linearities

But models have

\[ C_p \frac{\partial \overline{T}}{\partial t} = -\frac{\partial}{\partial z} \left[ F_R (\overline{T}, \overline{q}, \overline{l}) \right] \]

Stochastic approach
Need for stochastic approaches

Non-linearities in cloud-radiation interaction, cloud microphysics are too complex \( \rightarrow \) need stochastic approaches.

To sample these different states

We need to sample (Monte Carlo) these PDFs

Kawai & Teixeira, JCLI, 2010
Large-Eddy Simulation (LES) models

- High-resolutions (~ 10-50m) in all 3 dimensions
- Resolutions good enough to represent key dynamics in convection
- Closures still needed for scales < 10m (but simpler to do)

Matheou et al., MWR, 2011
Representing moist convection with stochastic plumes leads to more realistic results.

LES provides detailed statistics about cloud structure.
Mass Flux Model for Cumulus Mixing

\[
\left( \frac{\partial \bar{\phi}}{\partial t} \right)_{\text{conv}} = - \frac{\partial \bar{w}' \phi'}{\partial z} \approx - \frac{\partial M (\phi_u - \bar{\phi})}{\partial z}
\]

Originally proposed by Arakawa 1969, Betts 1973

\[
\frac{\partial \phi_u}{\partial z} = -\epsilon (\phi_u - \bar{\phi}) \text{ for } \phi \in \{\theta_1, q_t\}
\]

\[
M = \sigma_u w_u
\]

\[
\frac{1}{2} \frac{\partial w_u^2}{\partial z} = -b \epsilon w_u^2 + a \frac{g}{\theta_0} \left( \theta_{v,u} - \bar{\theta}_v \right)
\]

Lateral entrainment rate: \( \epsilon = \frac{1}{w_u \tau} \approx \frac{1}{h_c} \)

Constant \( \tau \): Neggers et al 01; Chein et al and Teixeira, 03.

Constant \( h \): Siebesma 97

\( \sigma_u \) is the udraft/core area fraction - not to be confused with cloud fraction
LES models and Eddy-Diffusivity/Mass-Flux (EDMF) Parameterization

Large Eddy Simulation (LES) model - BOMEX shallow cumulus case

Well mixed sub-cloud layer: Eddy-Diffusivity (ED) mixing

Cloud core updrafts: Mass-Flux (MF) transport

Bimodal joint pdf of $w$ and $q_t$
1) Estimate PDF of plume/updraft properties ($T, q, w$)
2) Sample PDF to generate a variety of plumes (diff. properties)
3) Integrate different plumes in the vertical

Produces more realistic results than purely deterministic parameterization
Stochastic Nature of Parameterizations in Ensemble Prediction

For parameterizations:

Ensemble and deterministic prediction are essentially different

In ensemble prediction systems:
- Parameterizations should be viewed as stochastic
- But within the context of current parameterizations (without imposing artificial stochastic terms)

Parameterizations:
- Typically used to predict the evolution of grid-mean quantities
- Can also provide estimates of higher moments (can be used to constrain random sampling)
Stochastic parameterizations in ensemble systems: a methodology

Methodology for stochastic parameterizations

A variable after being updated by a parameterization (e.g. moist convection) can be written:

$$\Phi^{stoch} = \Phi^{conv} + \mathcal{E}$$

- $\Phi^{conv}$ - mean value of the variable after convection
- $\Phi^{stoch}$ - stochastic value after convection
- $\mathcal{E}$ - normally distributed stochastic variable with mean $\mu(\mathcal{E}) = 0$
- standard deviation $\sigma(\mathcal{E}) = \sigma_{\phi, conv}$
- $\sigma_{\phi, conv}$ - standard deviation due to moist convective processes

After discretizing the first term on the rhs, the following equation is obtained

$$\Phi^{stoch} = \Phi + \Delta t \left( \frac{\Delta \Phi}{\Delta t} \right)^{conv} + \mathcal{E}$$

- $\Phi$ - mean value before the moist convection parameterization

Teixeira & Reynolds, MWR, 2008
Stochastic convection: a simple approach

Assuming standard deviation proportional to convection tendency leads to:

\[
\frac{\phi_{\text{conv}}^{\text{stoch}} - \bar{\phi}}{\Delta t} = (1 + \eta \beta) \left( \frac{\Delta \bar{\phi}}{\Delta t} \right)_{\text{conv}}
\]

\(\beta\) - constant of proportionality

\(\eta\) - normally distributed stochastic variable with mean \(\mu(\eta) = 0\) and standard deviation \(\sigma(\eta) = 1\)

(Teixeira & Reynolds, MWR, 2008)

Simple vertical correlation: single random number per column

No horizontal or temporal correlations:
- Simpler
- Perturbations assumed much smaller than grid-size
- Variance already possesses a certain degree of correlation
- Physically unclear how to construct correlations
US Navy NOGAPS ensemble spread due to stochastic physics only: 850 hPa Temperature

- Perturbations grow in time
- At 24 h: mostly in Tropics/Sub-tropics
- At 144 h: mostly in Mid-latitudes
- Similar for U at 250 and 850 hPa, Z at 500 hPa

(Teixeira & Reynolds, MWR, 2008)
Simple stochastic convection approach in Navy’s ensemble prediction system

NOGAPS stochastic convection after 5 to 10 days:
- Saturation in Tropics
- Synoptic (sub-synoptic) peak in NH Extra-tropics

Stochastic Convection:
- Is able to produce substantial ensemble spread in the Tropics
- Produces sizeable impact in ensemble spread in the extratropics

Initial-condition + stochastic convection show promising increase in ensemble spread and decrease in number of outliers in the Tropics
Stochastic Convection significantly improves NOGAPS ET performance.

- ET: Stoc. Conv.
- Multi-model Consensus
- Official Forecast

Reynolds, Goerss, Mc Lay

Number of Forecasts:
- 24: 359
- 48: 293
- 72: 239
- 96: 183
- 120: 139

Atlantic 2005 - TC Forecast Error (nm)
NOGAPS ET with stochastic convection better than high-res deterministic model, and competitive with official forecast at 96 and 120 hours.
Summary

- Sub-grid scale physical processes possess complex non-linearities
- Some parameterizations need to be stochastic (e.g. radiation-cloud interaction, cloud microphysics) EVEN in deterministic models
- Moist convection parameterizations using plumes (mass-flux) are more realistic if stochastic
- Simple stochastic convection parameterizations produce improvements in hurricane forecasts with NOGAPS ensemble system
- Stochastic physics and resolution independent parameterizations