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Astrodynamics Research

LiAISON-Supplemented Navigation for Geosynchronous and Lunar L_1 Orbiters

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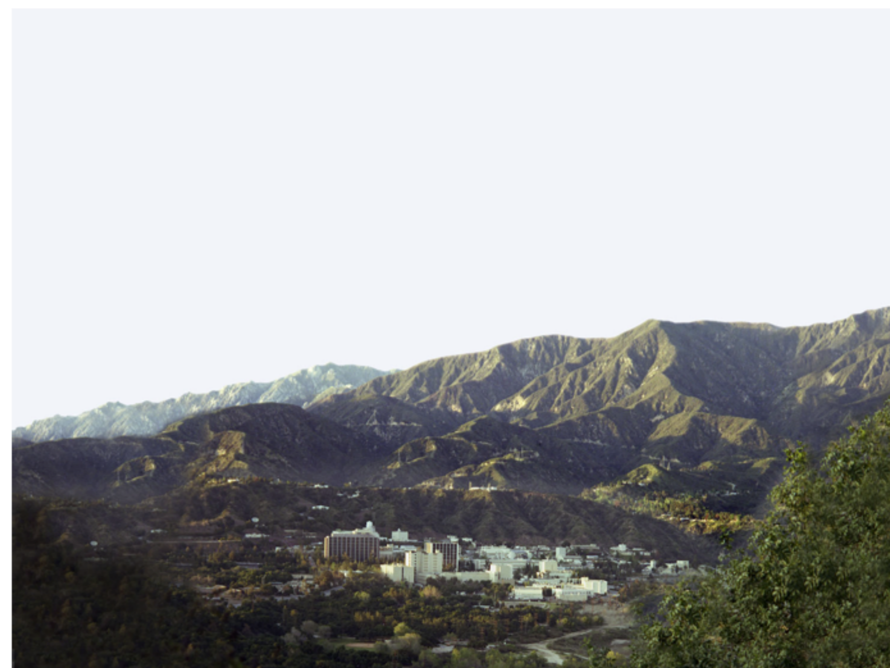

LiAISON Navigation: Introduction

LiAISON: Linked Autonomous Interplanetary Satellite Orbit Navigation

- Extends a GPS-like navigation capability to GEO and the Moon
- A single navigation satellite at the Earth-Moon L_1 point tracks other satellites anywhere near the Earth and Moon.
- This research focuses on tracking GEO satellites, but works for any Earth / Moon orbits.

***Costs*:**

- Requires a navigation satellite, which may double as a communication relay if needed.
- The customer satellite may require additional communication hardware, like a GPS receiver.

Objective:

- Measure the benefit of LiAISON Navigation applied to future GEO satellites.
- Quantify the cost and accuracy of LiAISON compared to DSN-only navigation.
- Determine how many DSN tracks may be removed and achieve the same navigation accuracy.

***Benefits*:**

- The L_1 location provides a huge geometrical advantage for navigation.
- Improves navigation accuracy.
- Reduces the number of DSN/ground tracking passes per mission.
- With ultra-stable clocks currently being demonstrated, one navigation satellite can track any number of customers – just like GPS.
- If the satellite shifts to L_2 (free transfer), then it can communicate with satellites on the far side of the Moon.

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Innovation

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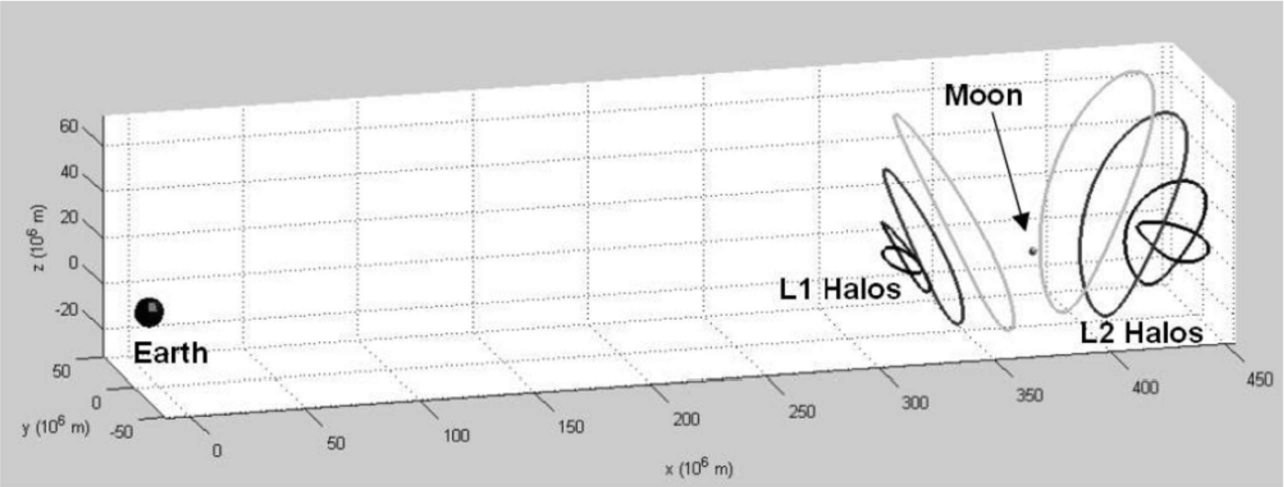
- Three-Body orbits (such as this L_1 orbit) are anchored to the Earth and Moon.
- This permits *relative* range/Doppler measurements to contain information about each spacecraft's *absolute* position and velocity.
- No ground measurements necessary, whatsoever!

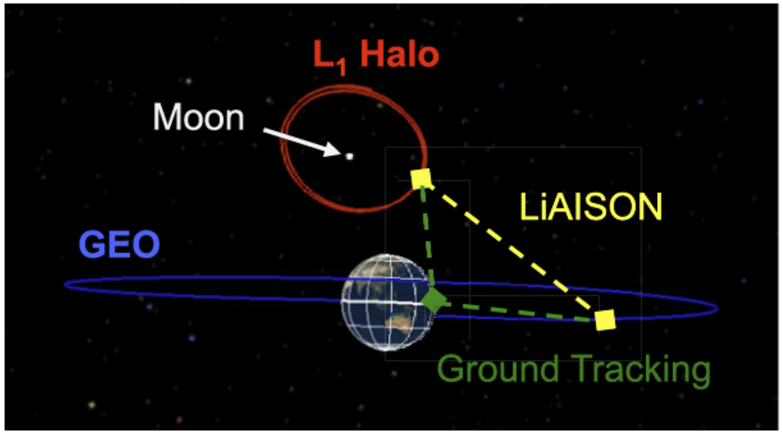
- The result is akin to GPS at the Moon, using only a single dedicated navigation satellite.



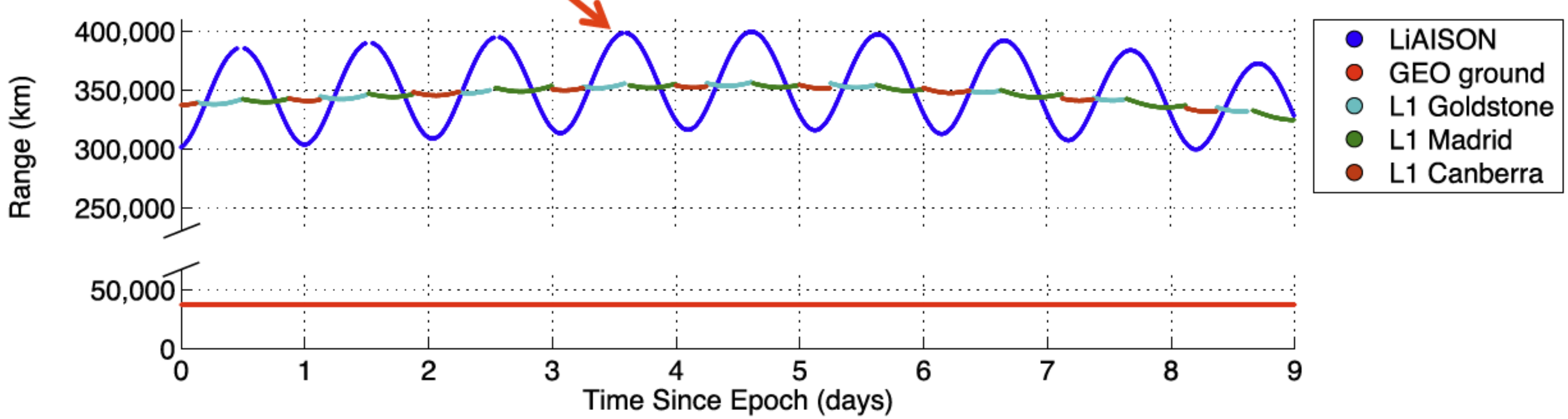
LiAISON Observation Geometry

The geometry

- Ground – GEO: Static observations
- Ground – L_1: Relatively static observations
- L_1 – GEO: Dynamic observations
  - *Observable geometry*



Very clear signal

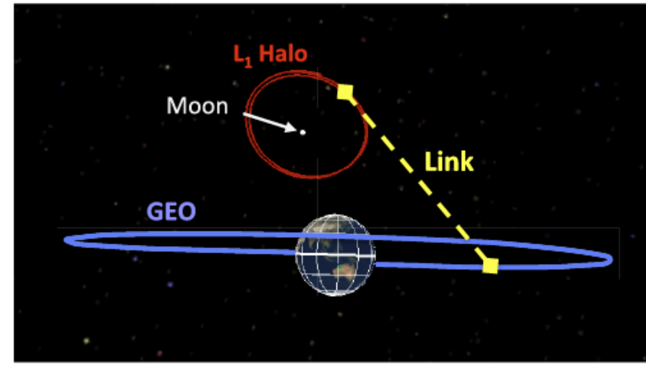


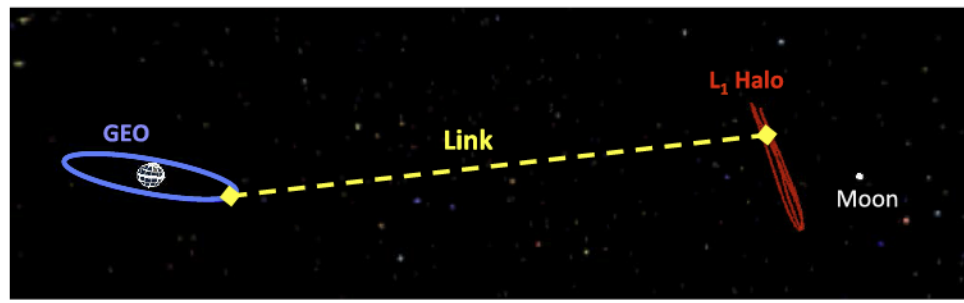
Dynamic Models

- Reference Frame
  - Geocentric Celestial Reference Frame (GCRF) with 1976 IAU Precession and 1980 IAU Nutation with no corrections.
- Planetary Ephemeris
  - DE405
- Force Models
  - Two-Body
  - Geopotential (20x20 GGM02C for GEO)
  - N Body
    - JPL DE405
  - Solar radiation pressure
    - A/m = 0.01 m^2/kg

LiAISON:

Linked
Autonomous
Interplanetary
Satellite
Orbit
Navigation





$$\begin{bmatrix}\dot{\mathbf{r}}_i\\ \dot{\mathbf{v}}_i\end{bmatrix} = \begin{bmatrix}\mathbf{v}_i\\ \mathbf{a}_g(t,\mathbf{r}_i)+\mathbf{a}_{SRP}(t,\mathbf{r}_i)+\mathbf{a}_{n\text{-}body}(\mathbf{r}_i,\mathbf{r}_{\oplus 3})+[\gamma]^T_{RTN}\,\mathbf{w}_i\end{bmatrix}$$

Measurement Models

- Simplified measurement models for range and range-rate.
- No light time is assumed.
- Geometric range plus a constant bias and Gaussian noise.

$$\rho = \sqrt{(\mathbf{r}_1 - \mathbf{r}_2) \cdot (\mathbf{r}_1 - \mathbf{r}_2)} + \rho_{bias} + \rho_{noise}$$

- Idealized range-rate with Gaussian noise.

$$\dot{\rho} = \frac{\rho \cdot \dot{\rho}}{\rho} + \dot{\rho}_{noise}.$$

- DSN stations used to track the L_1 orbiter
  - Goldstone, California
  - Madrid, Spain
  - Canberra, Australia
- Ground station tracking GEO
  - Sky Valley, California

Filter Model

- Utilizing a conventional Kalman filter (CKF).
- Time Update
  - State deviation propagation

$$\bar{x}_{k+1} = \Phi(t_{k+1}, t_k)\hat{x}_k$$

  - Covariance propagation

$$\bar{P}_{k+1} = \Phi(t_{k+1}, t_k) P_k \Phi(t_{k+1}, t_k)^{\mathrm{T}} + \bar{Q}(t),$$

$$\bar{Q}(t) = \Gamma(t_{k+1}, t_k) Q \Gamma(t_{k+1}, t_k)^{\mathrm{T}}$$

- Process noise is added into an RTN frame define to coincide with the SRP direction.

$$\bar{Q}_{GCRF} = \begin{bmatrix} [\gamma]^{\mathrm{T}}_{RTN}\, \bar{Q}_{(\mathbf{r},\mathbf{r})}\, [\gamma]_{RTN} & [\gamma]^{\mathrm{T}}_{RTN}\, \bar{Q}_{(\mathbf{r},\mathbf{v})}\, [\gamma]_{RTN} & [\gamma]^{\mathrm{T}}_{RTN}\, \bar{Q}_{(\mathbf{r},\mathbf{a})} \\ [\gamma]^{\mathrm{T}}_{RTN}\, \bar{Q}_{(\mathbf{v},\mathbf{r})}\, [\gamma]_{RTN} & [\gamma]^{\mathrm{T}}_{RTN}\, \bar{Q}_{(\mathbf{v},\mathbf{v})}\, [\gamma]_{RTN} & [\gamma]^{\mathrm{T}}_{RTN}\, \bar{Q}_{(\mathbf{v},\mathbf{a})} \\ \bar{Q}_{(\mathbf{a},\mathbf{r})}\, [\gamma]_{RTN} & \bar{Q}_{(\mathbf{a},\mathbf{v})}\, [\gamma]_{RTN} & \bar{Q}_{(\mathbf{a},\mathbf{a})} \end{bmatrix}$$

Filter Model

- Utilizing a conventional Kalman filter (CKF).
- Measurement Update
  - Linearized observation

$$\mathbf{y}_{k+1} = \tilde{H}_{k+1}\mathbf{x}_{k+1} + \epsilon_{k+1}.$$

  - State deviation update

$$\hat{\mathbf{x}}_{k+1} = \bar{\mathbf{x}}_{k+1} + K_{k+1}[\mathbf{y}_{k+1} - \tilde{H}_{k+1}\bar{\mathbf{x}}_{k+1}].$$

$$K_{k+1} = \bar{P}_{k+1}\tilde{H}_{k+1}^T \left[\tilde{H}_{k+1}\bar{P}_{k+1}\tilde{H}_{k+1}^T + R_{k+1}\right]^{-1}$$

  - Covariance Update (Joseph formulation)

$$P_{k+1} = (I - K_{k+1}\tilde{H}_{k+1})\bar{P}_{k+1}(I - K_{k+1}\tilde{H}_{k+1})^T + K_{k+1}R_{k+1}K_{k+1}^T.$$

- A posteriori state update

$$\hat{\mathbf{X}}_{k+1} = \mathbf{X}_{k+1}^* + \hat{\mathbf{x}}_{k+1}$$

LiAISON Navigation: Trade Study Overview

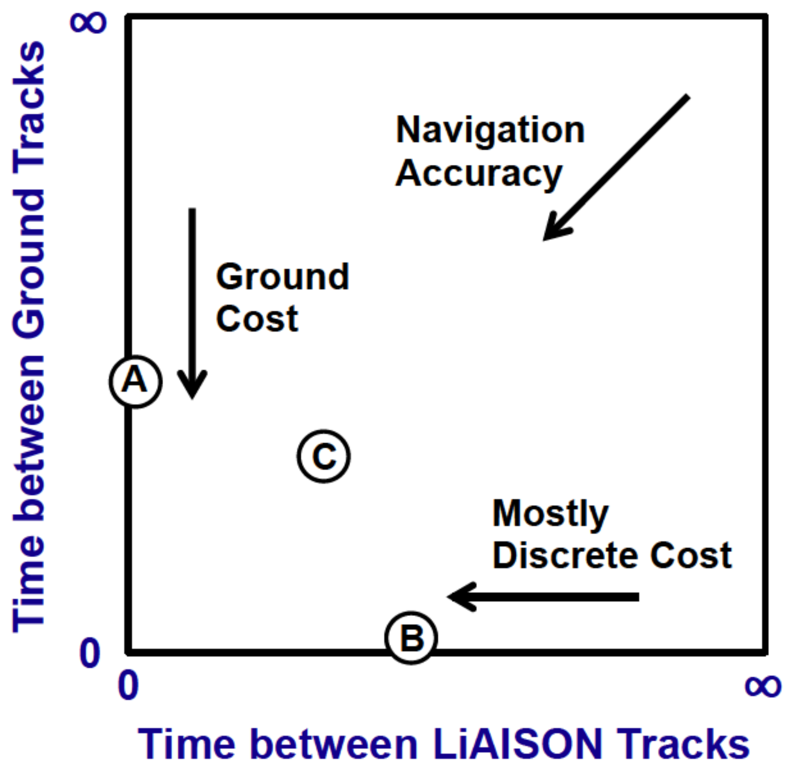
- High fidelity trade study utilizing varying levels of LiAISON tracking and ground tracking. This will answer the questions:
  - "How many ground tracks may we remove and still obtain the same level of accuracy for our mission?" – this quantifies the cost-savings by using LiAISON over ground-only tracking.
  - "How much improvement may we expect by adding LiAISON to our mission?" – this quantifies the improvement in satellite navigation accuracy by adding LiAISON.
  - "How does general observation scheduling influence the expected accuracy and uncertainty?" – this quantifies the necessity as to when observations should be obtained.
  - This research applies directly to GEO missions like TDRSS, but also easily applies to LEO missions, lunar missions, and any mission near the Earth or Moon.

LiAISON Trade Study

- **Quantify the cost and accuracy of LiAISON compared to ground-only navigation.**
  - We'll illustrate this objective using:
    (A) a continuous LiAISON with sparse Ground tracking solution, and
    (B) a continuous Ground with sparse LiAISON tracking solution

- **Determine how many ground tracks may be removed and achieve the same navigation accuracy.**
  - We'll illustrate this objective using:
    (B) a continuous Ground with sparse LiAISON tracking solution, and
    (C) a LiAISON-supplemented solution.
  - such that both result in similar tracking uncertainty.

Trade Study



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LiAISON Navigation: Trade Study Example Results

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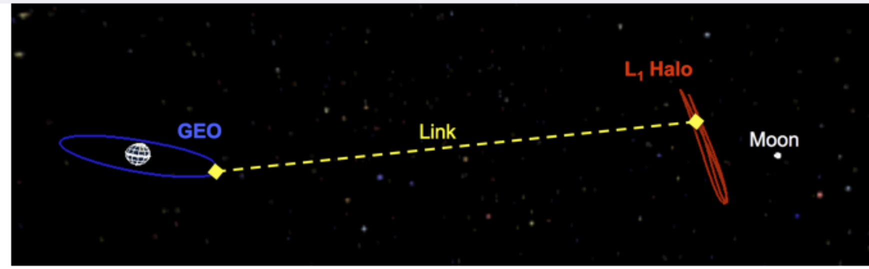
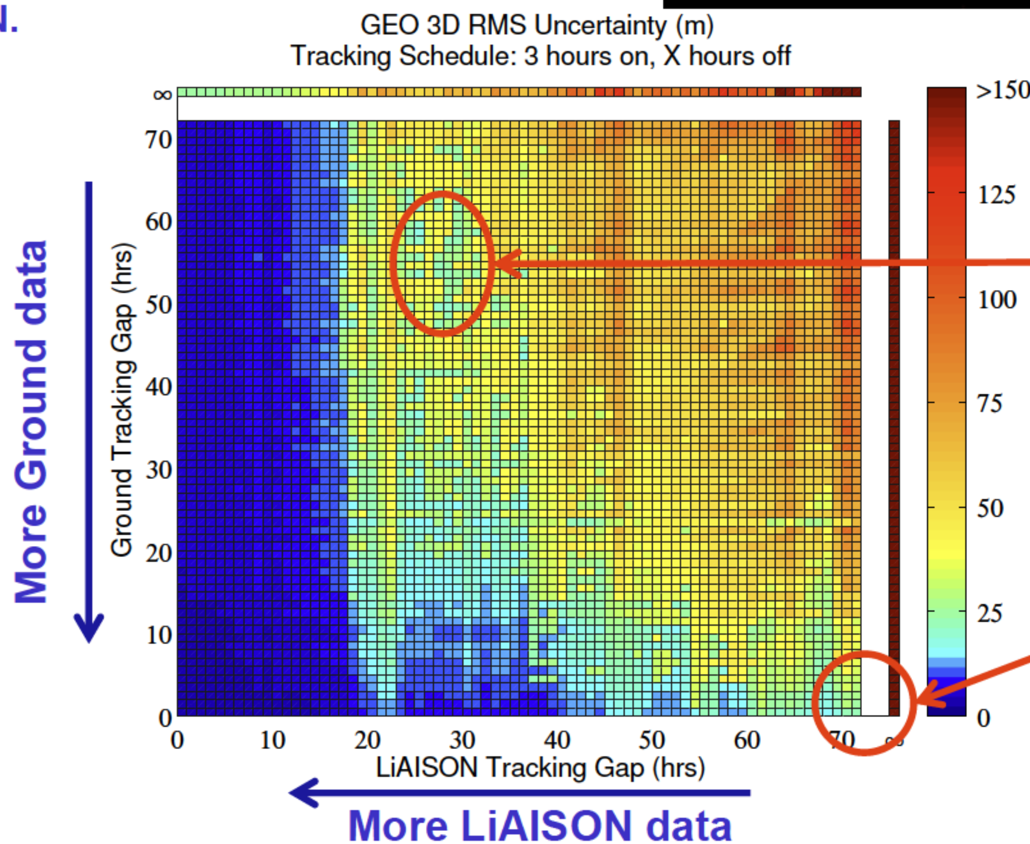
- Example results for tracking a GEO satellite using ground stations and LiAISON
- We've shown that you can reduce the number of ground tracks *tremendously* and achieve even better navigation accuracy by using LiAISON.





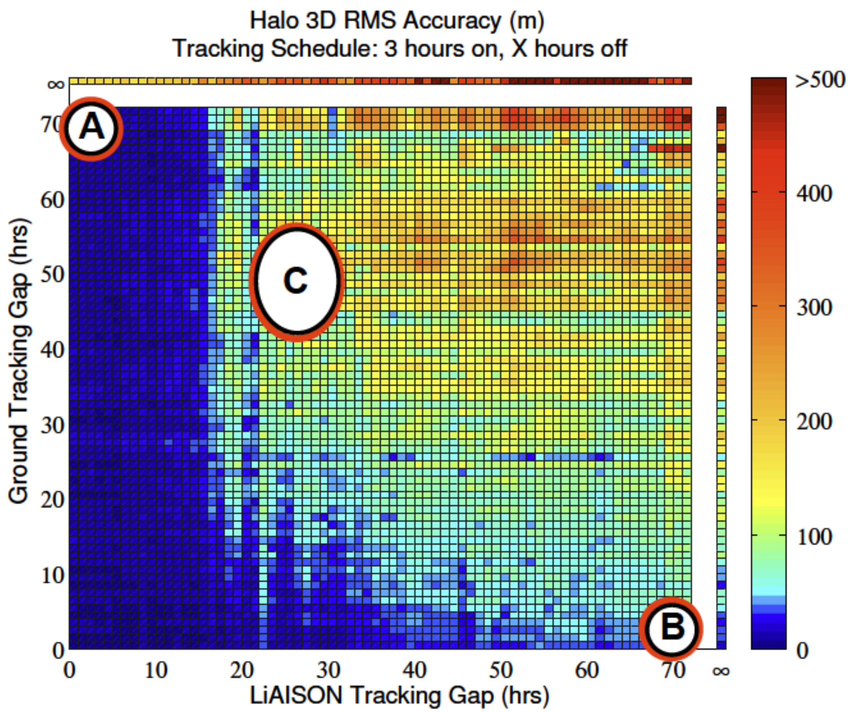
<u>Cheaper GEO nav:</u>
1 ground track every 2-3 days,
1 LiAISON track every 1-2 days:
~30 meter uncertainty

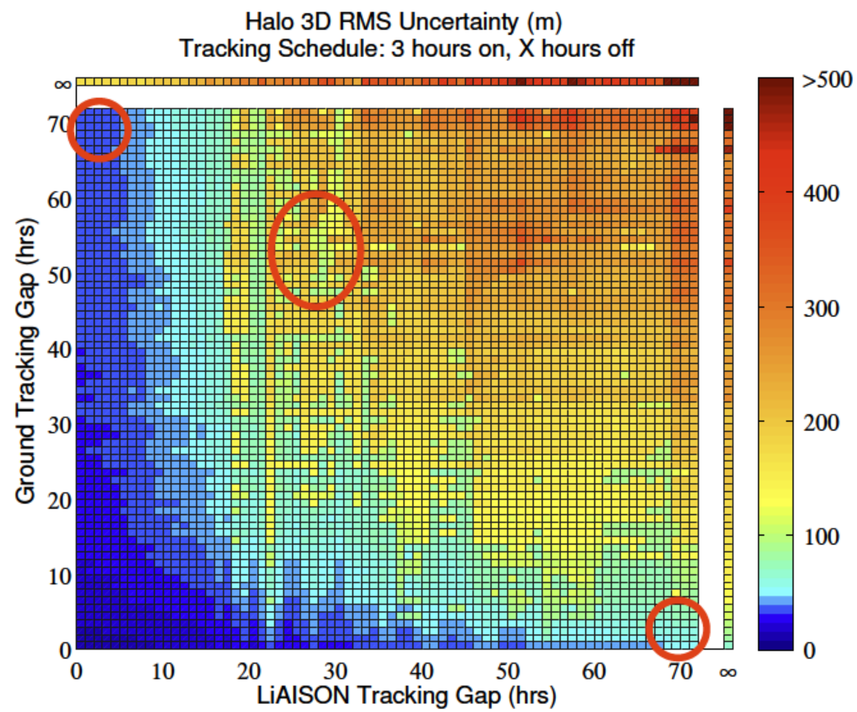
Fewer ground tracks, better accuracy!

<u>Ground-only GEO nav:</u>
Near-continuous ground observations with little (or no) LiAISON:
>150 meter uncertainty w/o LiAISON
~25 meter uncertainty w/ LiASION

LiAISON Navigation: L_1 Results

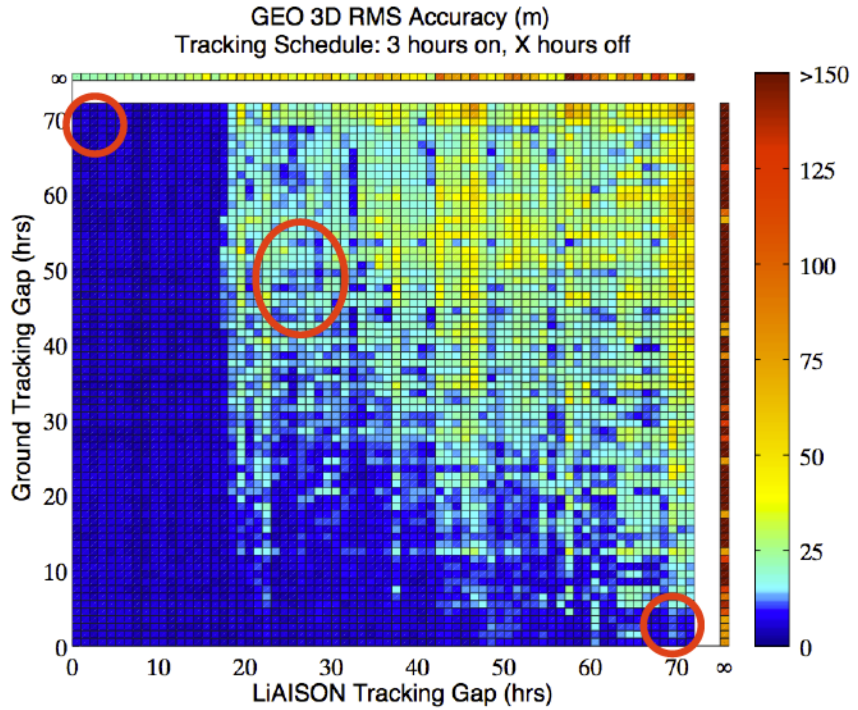
- L_1 3D RMS position accuracies (left) and uncertainties (right).
- Tracking schedule ranges from continuous, to 72 hour tracking gaps.
- Infinity corresponds to LiAISON or ground tracking only (no mixed schedule).
- Continuous ground tracking only gives an estimate with an uncertainty of 70 meters.
- Continuous LiAISON tracking only gives an estimate with an uncertainty of 150 meters.

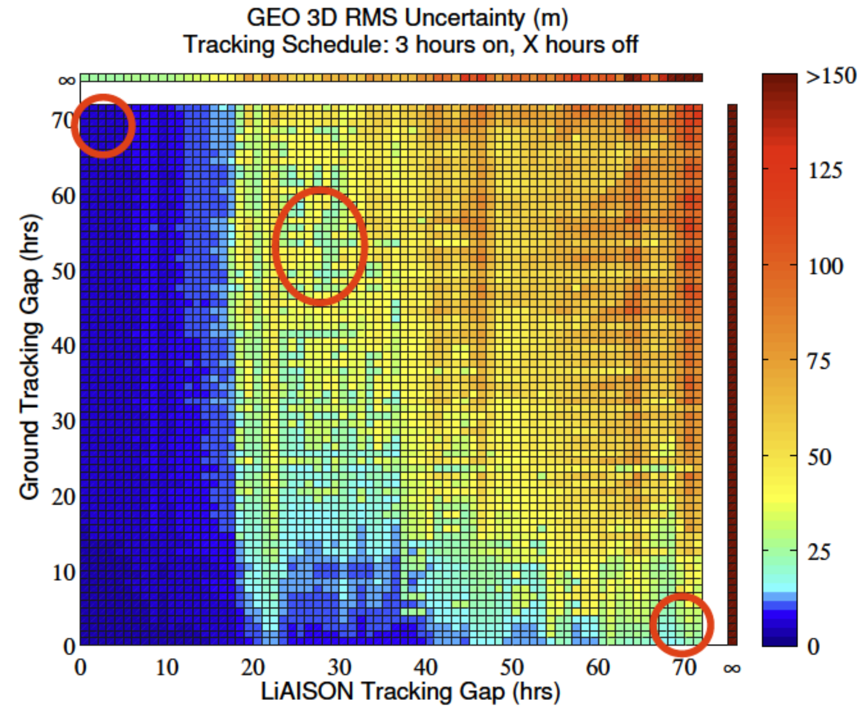




LiAISON Navigation: GEO Results

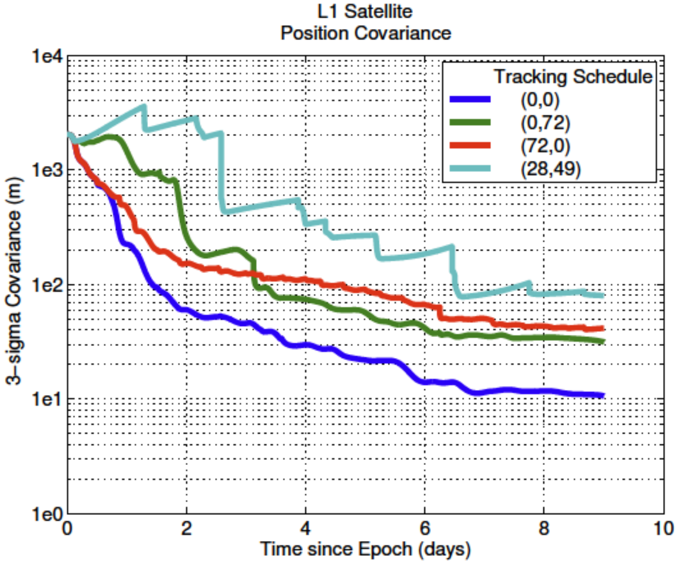
- GEO 3D RMS position accuracies (left) and uncertainties (right).
- Tracking schedule ranges from continuous, to 72 hour tracking gaps.
- Infinity corresponds to LiAISON or ground tracking only (no mixed schedule).
- Continuous ground tracking only gives an estimate with an uncertainty greater than 150 meters.
- Continuous LiAISON tracking only gives an estimate with an uncertainty of 30 meters.

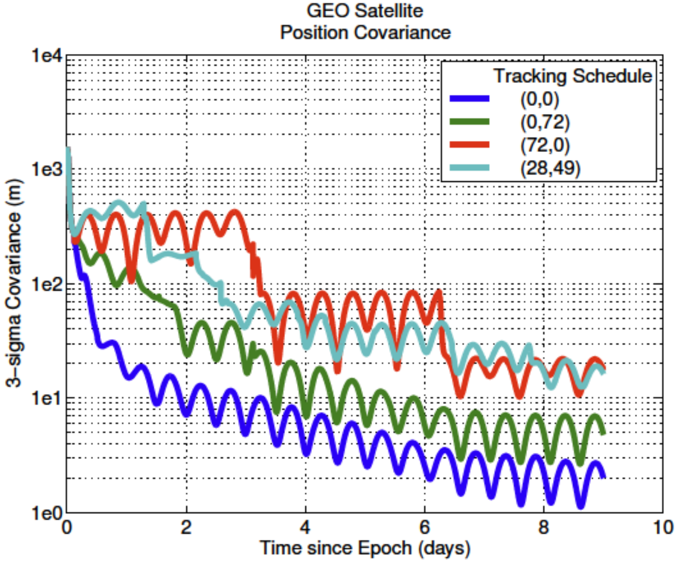




Tracking Schedule Analysis

- **Trade study was analyzed for four different tracking schedules.**
  - **Continuous (0,0)**
  - **Sparse LiAISON (72,0)**
  - **Sparse ground (0,72)**
  - **Mixed Tracking (28,49)**

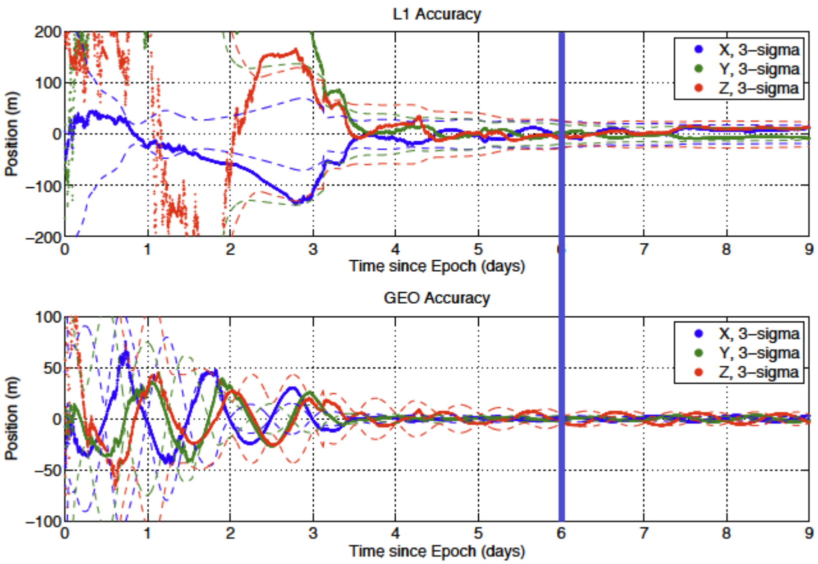




- **Continuous tracking (0,0) of both data types is the best solution.**
- **Mixed schedule (28,49) gives about the same uncertainty after 6 days as continuous ground with sparse LiAISON (72,0).**

(A) LiAISON vs. (B) Ground Trade

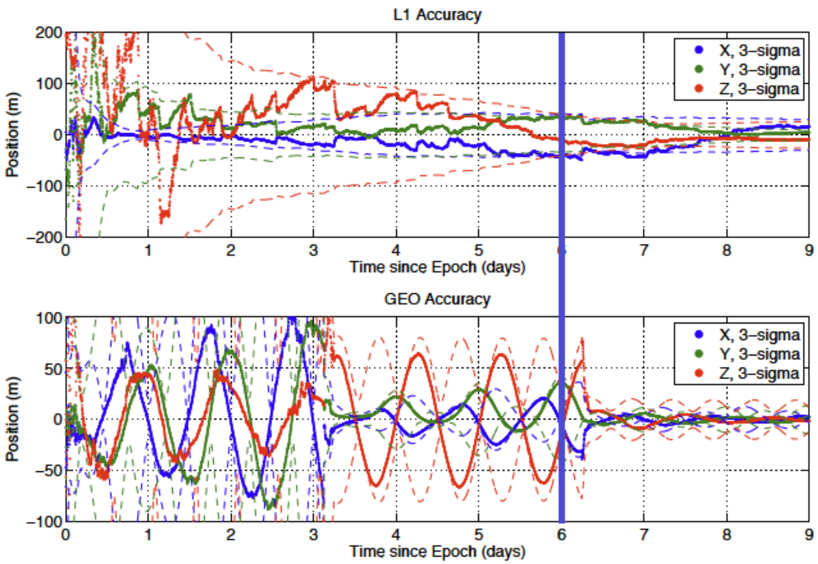
(A) Continuous LiAISON schedule with some ground: 3 hours on, 72 hours off



L1 Satellite: 13.52 m ± 35.03 m (3σ)

GEO Satellite: 4.96 m ± 5.65 m

(B) Continuous Ground schedule with some LiAISON: 3 hours on, 72 hours off



L1 Satellite: 35.01 m ± 47.17 m

GEO Satellite: 17.11 m ± 26.40 m

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(C) Mixed vs. (B) Ground Trade

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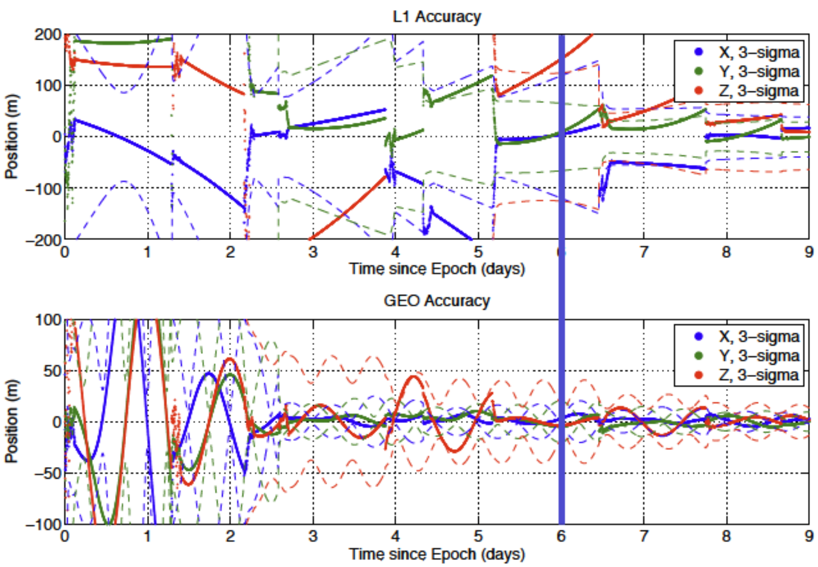
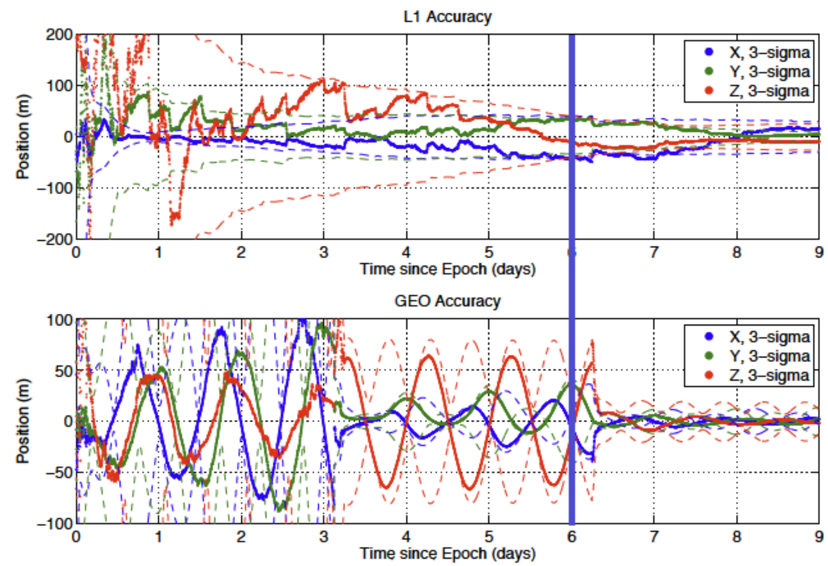
(C) Mixed schedule. LiAISON: 3 hours on, 28 hours off. Ground: 3 hours on, 49 hours off.



L1 Satellite: 66.98 m ± 88.89 m

GEO Satellite: 8.09 m ± 19.13 m

(B) Continuous Ground schedule with some LiAISON: 3 hours on, 72 hours off

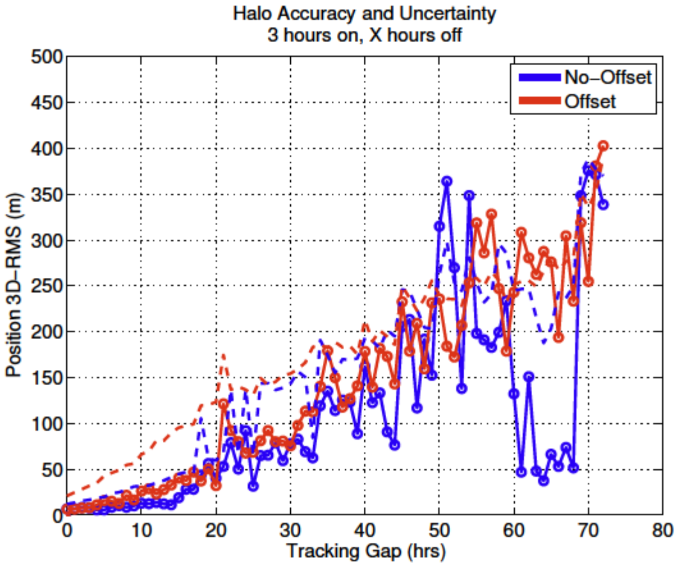


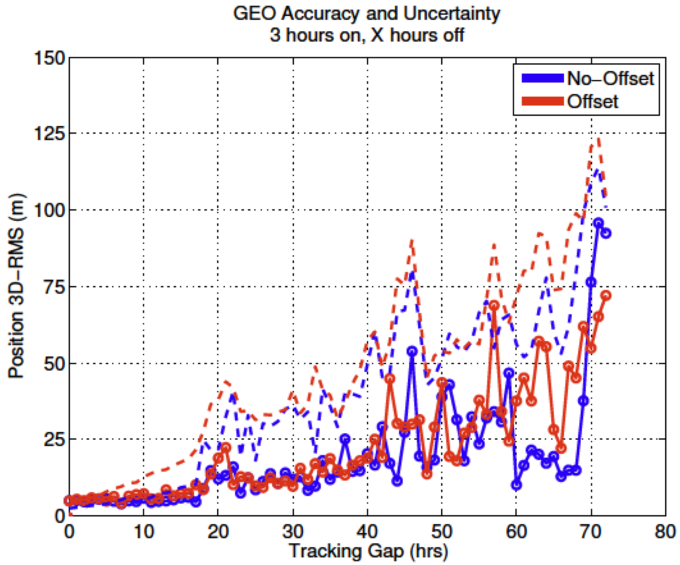
L1 Satellite: 35.01 m ± 47.17 m

GEO Satellite: 17.11 m ± 26.40 m

Measurement Offset Analysis

- Investigating the diagonal of the Trade Study for measurement offset sensitivity.





- No-offset has less uncertainty when Tracking Gap < 22 hours
- Resonance at 22 hours is seen in both GEO and Halo uncertainties due to observation geometry.

- No-offset approaches Offset uncertainty for Tracking Gaps > 22 hours
- Offset analysis for Tracking Gaps > 22 hours has little effect on the accuracy and uncertainty.

Conclusions

- **Relative and absolute positioning of the L_1 and GEO satellite is possible using only LiAISON.**
- **LiAISON is a valuable measurement type to improve radiometric measurements from DSN.**
- **High fidelity simulations and filter show that supplementing radiometric DSN with LiAISON can improve the solution greatly.**
- **There exists regions of mixed tracking that produces the same uncertainties as continuous tracking.**
- **We've shown that you can reduce the number of ground tracks *tremendously* and achieve even better navigation accuracy by using LiAISON.**
- **A simple measurement offset analysis showed that for large tracking gaps one could expect the same filter performance.**
- **However, for tracking gaps of less than a day, a no-offset in the measurement**

Thank You

Questions?

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Backup Slides

Table 2. Summary of models used in the LiAISON simulations.

Model	LiAISON Standard			Reference
	Reference Frame			
Conventional inertial system	GCRF			–
Precession	1976 IAU			–
Nutation	1980 IAU			–
Planetary ephemerides	JPL DE405			Ref. 21, 22
Polar motion	IERS			–
UT1-TAI	IERS			–
JED/TDB/TT	IERS			–
Station coordinates	Lat. (deg)	Lon. (deg)	Alt. (km)	–
Goldstone	35.247	243.205	1.0711	–
Madrid	40.427	4.251	0.834	–
Canberra	-35.398	148.982	0.692	–
Sky Valley	33.930	243.610	0.445	–
Reference ellipsoid	$a_e = 6378136.3$ m			–
	$1/f = 298.257$			–
	Force Models			
GM	398600.4415 km^3/s^2			Ref. 30
Geopotential	GGM02C & 3–sigma clone 20×20			Ref. 30
N body	JPL DE405			Ref. 21, 22
Solar radiation	solar constant $= 4.5298 \times 10^{-6} Pa$ at 1 AU, conical shadow for Earth.			–
	Measurement Models			
Range	Instantaneous			Eq. 18
Doppler	Instantaneous			Eq. 19

Table 3. Orbit determination filter uncertainties.

Estimation Parameters	*a priori* uncertainty (1-sigma)	Number of Parameters
Spacecraft position	1,000 m	6
Spacecraft velocity	1 mm/s	6
SRP Coefficient	20%	2
Empirical Accelerations		6
Radial	5e-10 m/s^2	–
Transverse	1e-12 m/s^2	–
Normal	1e-12 m/s^2	–
SST range bias	3 m	1
DSN range bias	6 m	3
GEO-supporting ground station range bias	6 m	1
SST measurements		
range	1 m	–
range-rate	1 mm/s	–
DSN measurements		
range	100 m	–
range-rate	0.1 mm/s	–
GEO-supporting ground station measurements		
range	100 m	–
range-rate	1 mm/s	–