ABSTRACT

A time-series approach is used to estimate the moisture content based on polarimetric SAR data. It is found that under the assumption of constant soil moisture, empirically observed relationships between radar backscatter and moisture are only half as sensitive to moisture as compared to actual radar data. A numerical finite element method is used to calculate the radar backscatter for rough soils with arbitrarily varying soil moisture as a function of depth. Several instances of drying and wetting moisture profiles are considered and the radar backscatter is calculated in each case. Radar backscatter is found to crucially depend on the soil moisture variation in the top half wavelength of soil.

Index Terms—Finite element methods, electromagnetic scattering by rough surfaces, Monte Carlo simulations, subsurface sensing

1. INTRODUCTION

A time-series solution can be used to estimate soil moisture using polarimetric radar data. Polarimetric measurements are required to segment radar data based on the amount of vegetation presented in each pixel. The time-series approach is effective since the backscattering cross section from a natural object changes mainly due to variations in soil moisture over short timescales. It was observed that the measured backscattering sensitivity to soil moisture is much larger than the theoretical predictions for a bare surface [1]. There are at least two possible explanations for this difference. First, the backscattering cross section will increase more than the theoretical predictions if the surface roughness increases during a precipitation event. Although this explanation is feasible, careful roughness measurements are required to support this explanation. Second, the variation of the subsurface soil moisture profile can be an important factor in the change of the backscattering cross section as a soil surface becomes dry after a precipitation event. In this paper, we will examine the effect of the varying soil moisture profile on the backscattering cross section using a fully numerical finite element based method. If radar measurements depend strongly on the subsurface profile, the interpretation of the retrieved soil moisture information from radar data must consider the effect of the varying soil moisture profile.

2. TIME-SERIES OBSERVATION OF BARE SURFACES

For bare surfaces, the backscattering cross section depends on the surface roughness and soil moisture. The backscatter cross section of a given pixel becomes larger as soil moisture increases. Their relationship can be derived empirically as

\[
10 \log_{10}(\sigma_{pp}) = C m_v + D
\]

The polarimetric backscatter cross section is denoted by \(\sigma_{pp}\) where \(p = h\) or \(v\) depending on the radar polarization. The two parameters \((C\) and \(D\)) determine the relationship between the backscattering cross section and volumetric soil moisture \((m_v)\). Based on the actual radar measurements, \(C\) is approximately twice as large as one derived from the theoretical predictions using the small perturbation method (SPM) and the integral equation method (IEM) [1]. This higher value of \(C\) derived from the actual measurements can be caused by the varying subsurface moisture profile since the soil moisture profile changes significantly as a soil surface dries after a precipitation event.

3. EFFECT OF SUBSURFACE PROFILE ON BACKSCATTERING CROSS SECTION

Soil moisture varies with depth depending on the temperature profile, soil and vegetation type, and surface evaporation. The soil moisture profile can be retrieved from L-band radiometric observations using sequential data assimilation techniques [2]. When soil moisture is retrieved from radar data, it is assumed that the soil moisture value is an average value of soil moisture in the top few centimeters. For most semi-analytical
algorithms, retrieved soil moisture is assumed to be the average value in the top few centimeters assuming soil moisture is uniform. Other models assume a piecewise constant soil moisture profile [3].

In this paper, we evaluate the validity of this uniform soil moisture assumption. Experimental observations of soil moisture reveal that moisture is a non-monotonic function of depth [4]. Further, these moisture profiles can not be described by means of simple functional forms. Thus, only fully numerical methods such as finite element methods (FEM) or finite-difference time-domain methods are capable of solving the electromagnetic scattering problem for rough surfaces in all generality.

### 3.1. Overview of the finite element method

The numerical method of choice in this paper is a two dimensional vector-based finite element method. The entire computational domain is fractured into triangles and for each polarization, the vector wave equation is enforced in a weak sense over each of these triangles. ‘Weak’ refers to the fact that the wave equation is not enforced at each point, rather it is enforced in an integral sense over each element [5]. A dielectric mixing model [6] is used to convert volumetric soil moisture to a complex dielectric constant.

The computational domain is terminated using the first order absorbing boundary condition [7], which physically corresponds to the scattered field being purely outgoing at the boundaries. To reduce edge diffraction from the corners of the air-soil interface, a tapered form of the incident field (having a Gaussian amplitude) is considered [8]. Such a tapered incident field simulates to a high degree the incidence of a plane wave on an infinitely long air-soil interface. Finally, the radar scattering cross section is found by evaluating the electromagnetic far field from an (incomplete) contour a small distance above the air-soil interface. The entire setup is shown schematically in Fig. 1.

The full details of the method are described in an upcoming publication [9].

### 3.2. Backscatter cross section for constant moisture soil

To validate the above developed method, comparisons are made with a semi-analytical method, the small perturbation method (SPM) [10, 11]. Methods such as the SPM or the Kirchhoff approximation work well within a very restricted range of surface parameters that correspond to smooth surfaces.

The rough soil surface is treated as a Gaussian random process, and as a result it is not enough to consider scattering from a single rough surface. Instead, a Monte Carlo averaging is done over 100 different instances in order to capture the statistical properties of the rough surface (satisfying this requirement turns out to be the biggest computational drawback of fully numerical methods). Fig. 2 graphically shows good agreement between the FEM and SPM for the case of rough soil that is homogeneous in moisture content. Here, the surface properties are characterized in terms of a dimensionless number $kh$, and an average slope, $s = \frac{2h}{l}$, where $k, h, l$ represent the free space wavevector, r.m.s. soil height, and correlation length, respectively. The two polarizations considered correspond to transverse magnetic, or horizontal, and transverse electric, or vertical.

Next, the ensemble averaged backscatter is calculated as a function of soil moisture. The results are shown in Figure 3. It is found that logarithmic fits of the empirical type in (1) fit the FEM derived data very well, and that the fit parameters depend on the surface statistics for a given incidence angle.

### 3.3. Backscatter cross section for wetting/drying soils

We now consider the case where soil moisture monotonically increases or decreases in the top $d$ cm of soil (and constant thereafter). The exact form of the moisture variation is shown in Fig. 4.

In the case of wetting soils, moisture varies from a value of 45% at the top, to 25% at depth $d$, while in the case of drying soils, the moisture increases from 5% at the top, to 25% at depth $d$. The FEM is applied to such a scattering geometry, and the results are presented in Fig. 5, where backscatter is evaluated as a function of the distance $d$ to constant soil moisture.

At very small or very large values of $d$, the asymptotic behavior is along expected lines. Assuming an inversion model based on constant soil moisture, at very small values of $d$ the
Fig. 2. Bi-static radar scattering cross-sections for TM (a) and TE (b) polarizations for $\lambda = 0.24$ m, incidence angle $\theta_i = 40^\circ$, and 15% soil moisture ($E_f = 6.940 - 1.814j$). The rough Gaussian soil has $kh = 0.1$, $s = 5^\circ$, while rougher soil has $kh = 0.5$, $s = 20^\circ$, where $k$, $h$ represent the wavevector in free space, and r.m.s height, respectively. Incoherent first order SPM predictions for rough soil are also shown for comparison.

Fig. 5. Impact on backscatter of soil moisture variation in top $d$ cm of soil that has roughness parameters $kh = 0.1$, $s = 5^\circ$ for horizontal (a) and vertical (b) polarizations (incidence angle is $\theta_i = 40^\circ$). In the legend, ‘Exp:X%-Y%’, and ‘Lin:X%-Y%’, represent exponential and linear moisture profiles, varying in that functional form from X% at the top, to Y% at depth $d$, and constant at Y% below $d$. ‘Const-Z%’ refers to soil having constant Z% moisture everywhere.

backscatter approaches that of sub-soil moisture, whereas for very large values of $d$ the backscatter approaches that of the top-soil moisture.

At intermediate values of $d$, the backscatter displays an oscillating behavior as a function of $d$, on account of an interference effect between the soil layers having different moisture content. It is observed that this interference effect is lost when the soil is made several times rougher [9], and the backscatter is contained within the two asymptotes of sub-soil and top-soil moisture content. For reference, the backscatter corresponding to the average soil moisture in the top $d$ cm is also shown in the case of linear moisture profiles. In the wetting case, this corresponds to 35% moisture, while in the drying case this is 15%.

In closing it must be mentioned that for ease of computation, the results presented are for a single instance of rough soil. Monte Carlo simulations were performed for a select few cases to verify that the observed behavior is indeed also observed in the ensemble.
Fig. 3. Ensemble averaged radar backscatter as a function of soil moisture (mv, expressed as a percentage) for rough Gaussian soil with $kh = 0.5$, $s = 20'$. The fitted curves are of the form $(a \log(mv) + b)$ with $a = 5.63$, $b = -18.75$ for TM and $a = 7.02$, $b = -18.19$ for TE. The fitting parameters for rough soil with $kh = 0.1$, $s = 5'$ (not plotted here) are $a = 5.52$, $b = -31.79$ for TM and $a = 6.99$, $b = -30.10$ for TE.

Fig. 4. Graphical representation of soil moisture variation (in percentage) as a function of soil depth, $z$, in cm, when the depth to constant soil moisture, $d$, is chosen to be 12 cm.

4. CONCLUSIONS

For calculating radar scattering from rough soil surfaces with varying subsurface moisture profiles, we have developed a numerical finite element based method. Our results show that such moisture profiles have a big impact on the radar backscatter, and the accuracy of moisture retrieval depends critically on the knowledge of this moisture profile. By observing the backscatter for rough soils with moisture non-homogeneity in the top half wavelength of soil, it is evident that making the assumption of constant soil moisture can lead to severe errors in moisture retrieval, particularly in the case of drying soils.

5. REFERENCES


