

## USING GRAVITY ASSISTS IN THE EARTH-MOON SYSTEM AS A GATEWAY TO THE SOLAR SYSTEM

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For spacecraft departing the Earth-Moon system, lunar flybys can significantly increase the hyperbolic escape energy (C3, in  $\text{km}^2/\text{sec}^2$ ) for a modest increase in flight time. Within  $\sim 2$  months, lunar flybys can produce a C3 of  $\sim 2$ . Over 4-6 months, lunar flybys alone can increase the C3 to  $\sim 4.5$ , or they can provide for additional periapsis burns to increase the C3 from  $\sim 2-3$  to 10 or more, suitable for planetary missions. A lunar flyby departure can be followed by additional  $\Delta\text{-V}$  (such as that efficiently provided by a low thrust system, *eg.* Solar Electric Propulsion (SEP)) to raise the Earth-relative velocity (at a ratio of more than 2:1) before a subsequent Earth flyby, which redirects that velocity to a more distant target, all within not much more than a year.

This paper describes the applicability of lunar flybys for different flight times and propulsion systems, and illustrates this with instances of past usage and future possibilities. Examples discussed include ISEE-3, Nozomi, STEREO, 2018 Mars studies (which showed an 8% payload increase), and missions to Near Earth Objects (NEOs). In addition, the options for the achieving the initial lunar flyby are systematically discussed, with a view towards their practical use within a compact launch period. In particular, we show that launches to geosynchronous transfer orbit (GTO) as a secondary payload provide a feasible means of obtaining a lunar flyby for an acceptable cost, even for SEP systems that cannot easily deliver large  $\Delta\text{-Vs}$  at periapsis. Taken together, these results comprise a myriad of options for increasing the mission performance, by the efficient use of lunar flybys within an acceptable extension of the flight time.

**I. INTRODUCTION**

Throughout the ages, the Moon has held the fascination of mankind, who often awarded it with mystical significance if not deity. In the modern era, the Moon served as the first destination for human space travel, convenient due to the short flight time and relative ease of surface access. However, the Moon's role in human exploration of the rest of the solar system has been contentious, with the purported benefits having at least some reasonable drawbacks.\* Likewise, the possible benefits of using the Moon for robotic missions departing towards solar system targets have generally not been persuasive, and so it has seldom been utilized. While using the Moon still presents some difficulties, this paper outlines the benefits to doing so, including some advantages that have not been widely recognized in the past.

In terms of C3 (*vis-viva* energy per unit mass with respect to the Earth, in  $\text{km}^2/\text{sec}^2$ ), it is generally known that one to two lunar flybys can raise the C3 from about -2 (the minimum required to reach lunar altitude) to about +2. However, reaching any planetary target requires more energy, which must somehow be supplied, and utilizing a lunar flyby makes this inconvenient. The difference in

perigee velocity between these two C3 values is  $\sim 182$  m/s, representing only a 4% mass fraction<sup>†</sup> with cryogenic propellants, which is likely only significant for larger vehicles. There are still some classes of missions that benefit from this launch energy, and they are discussed in the following section.

Multiple lunar flybys provide additional benefits, both with increased C3 and in setting up powered flybys to provide additional energy. Departure energies of as much as 4.5 are possible with three flybys, and adding a periapsis delta-V to two flybys provides an arbitrarily high C3. The details of both approaches, along with examples of previous use and potential future benefits, are given in Section III below.

All of this presupposes an initial lunar flyby, which imposes constraints on the launch period.<sup>‡</sup> Solar system missions typically require 21 launch

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<sup>†</sup> Admittedly, this includes the empty mass of the upper stage, making it more significant.

<sup>‡</sup> The "launch period" is defined as the set of dates over which a launch may be performed at some particular time of day. The "launch window" is the set of acceptable times for launch on a particular launch date, and may be as short as an instantaneous moment.

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\* This paper takes no position in that argument.

dates, spaced as closely together in time as possible (eg. in a 20-day launch period) to avoid a costly extension of the duration of a launch campaign (due to the personnel deployed to support it). The Moon traverses three-fourths of a revolution during this time, so a launch campaign utilizing the Moon must somehow account for this change in inertial direction during the launch period. In addition, launches to higher-energy Earth orbits (primarily geosynchronous transfer orbit (GTO), but also perhaps to Global Positioning System (GPS) orbits), provide the possibility of adding a secondary payload bound for solar system targets at less than the cost of a dedicated launch. All of these considerations for getting to the Moon initially are discussed in Section IV.

## II. SHORT FLYBY SEQUENCES

The simplest analysis of lunar flybys starts with a circular lunar orbit at the average radius of 384,400 km. For a coplanar transfer from low Earth orbit (LEO) to lunar altitude, the launch C3 (that supplied by the launch vehicle) is  $-2.04$ , and the apogee velocity is 186 m/s. With the Moon's velocity of 1.018 km/sec, that produces a relative velocity of 832 m/s, and a maximum turn angle of 105 degrees with a flyby altitude of 100 km. The resulting orbit is hyperbolic, with an escape C3 of only 0.1, much less than the advertised value of 2. However, if a one-month period is achieved initially, before a final flyby to escape, the C3 is 1.33. Higher launch C3 values of up to  $-1.5$  result in higher  $V_\infty$  values (but with smaller turn angles), and higher escape C3 values of up to 2.23, for intermediate orbit periods of one, two, and 1.5 months (with the last being a 3:2 resonance case), as shown in Figure 1. Note that the apogee radius for the 2-month orbits reach as high as 1.15 million km, clearly to high to ignore solar gravity perturbation.

The in the real world, the relationship between launch C3, lunar  $V_\infty$ , and maximum C3 is much more complicated. The inclination of the launch orbit (taken to be 28.5 degrees for launch from the Florida launch complex) and the equatorial inclination of the Moon (varying in its 18-year cycle) produce a  $\sim 100$  m/s change in lunar  $V_\infty$  between the two launch opportunities per day. The eccentricity of the Moon's orbit produces a  $\sim 200$  m/s  $V_\infty$  variation for different true anomaly values at lunar arrival, and the corresponding the lunar velocity and range differences at the final lunar flyby make a significant change in the escape C3, as discussed in [Landau]. Despite these effects, there are ways to optimize the lunar  $V_\infty$  and departure location (primarily by using additional flybys with non-resonant transfers, as will be discussed in section IV), so as to maximize the

escape C3. The same techniques can also be applied to keep the launch C3 nearly constant across a fairly short (but not necessarily continuous) launch period. All of these sequences would be less than  $\sim 3$  months (thus staying within the current section's title), since otherwise solar gravity perturbation becomes a more powerful tool in adjusting lunar  $V_\infty$  and escape C3. Even so, the maximum C3 can reach as high as  $\sim 2.4$  [Landau], somewhat beyond the canonical escape C3 value of 2. In addition to being relatively short in duration, all the sequences considered here are computed with patch-conic methods, without including the effect of solar gravity perturbation.

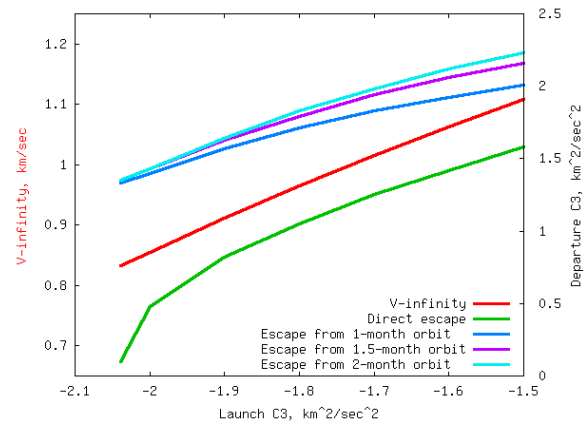


Figure 1: Lunar  $V_\infty$  and escape C3 vs. launch C3, for a circular lunar orbit and coplanar transfer, without solar gravity

What sorts of missions can benefit from C3s of 2 or less? Any vehicle that is targeting a relatively near-Earth orbit, either as a final destination, or as a starting point for beginning a low thrust arc (eg. due to SEP), can do so. To date, near-Earth final orbits have been used primarily for observatory missions, such as Spitzer and Kepler. While those missions utilized a direct launch to a low positive C3 of less than 1, STEREO (another observatory mission, with two spacecraft, "Ahead" and "Behind") used the Moon to send one spacecraft to a lower period heliocentric orbit, and the other to a higher period heliocentric orbit, such that they receded from the Earth in opposite directions, and provided a 3-D view of the Sun, all from the same launch, which would be impossible otherwise [STEREO reference].

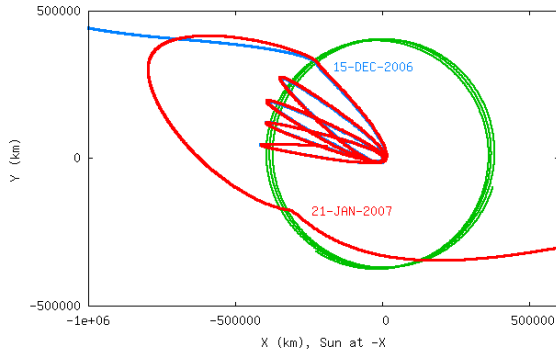


Figure 2: STEREO-Ahead/Behind departure trajectory, in Sun-Earth rotating frame. ‘Ahead’ trajectory is in blue, ‘Behind’ trajectory is in red, and Moon is in green. Lunar flyby dates are given. Note that the Sun is to the left.

The STEREO-A/B departure trajectory is illustrated in Figure 2. From an initial phasing orbit with an apogee barely beyond the Moon, the first flyby of the Moon was targeted such that STEREO-Ahead achieved an Earth-relative C3 of -0.4 with a lower flyby, and STEREO-Behind reached an orbit period of 39 days with a higher one (values?, or in figure). This set up a non-resonant encounter with the Moon 37 days later, where STEREO-Behind then achieved an Earth-relative C3 of -0.3 in the opposite direction from its companion at launch. In both cases, the actual Earth escape was due to solar perturbation, as can be seen in the trend for STEREO-Ahead’s C3 in Figure 3.

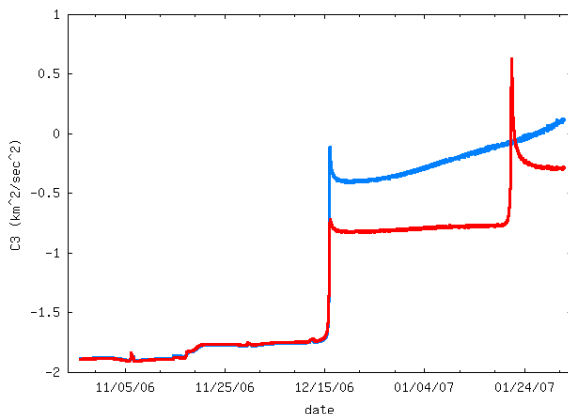


Figure 3: C3 vs. time for STEREO-A (blue) and STEREO-B (red). Note that the final Earth escape is due to solar perturbation, rather than the lunar flybys.

In addition to this actual example, many proposed SEP missions have an optimal initial launch C3 of  $\sim 2$ , followed by thrusting to increase the Earth relative  $V_\infty$  at a subsequent flyby to 4-6 km/sec, with a delta-V cost of less than half the increase (eg. 1.5 km/sec of thrusting from a C3 of 2 produces an Earth flyby at 5 km/sec) [Landau]. An Earth flyby is much more effective at producing higher inclinations than direct thrusting, and a  $\sim 1$ -year increase in flight time is an acceptable penalty for the greatly increased launch mass and range of possible outbound declinations. Furthermore, SEP trajectories bound for Near Earth Objects (NEOs, almost all of them asteroids), often can’t make use of a C3 of more than  $\sim 4$ , due to phasing issues. For many of these missions, a pair of lunar flybys, initially analyzed with patched-conic methods, are sufficient.

### III. LONGER FLYBY SEQUENCES

#### Ballistic Trajectories

While the play of the tides on the Earth is attributed primarily to the Moon, one third of their size is non-lunar, which reminds us that the Earth-Moon system has another major dynamic element: the Sun. By using the solar gravitational perturbation at the edges of the Earth’s sphere of influence, the lunar  $V_\infty$  can be significantly changed, and the orbit direction itself can be reversed if desired. In the case of GRAIL, the solar perturbation is used to reduce the lunar  $V_\infty$  to allow a barely sub-parabolic arrival, but in this case we generally want to increase the lunar  $V_\infty$ , balanced against the decrease in turn angle that higher velocities require. The solar perturbation is capable of producing an Earth escape by itself, but not with a higher energy than that obtainable by a lunar flyby. So the goal is to use the solar perturbation to set up a final series of one or two lunar flybys that maximize the Earth departure energy, after an initial flyby that raises apogee enough to receive a significant solar perturbation.

Since the trajectory segment with the solar perturbation (hereafter the “solar loop”) is necessarily bounded by lunar flybys, every useful solar loop has a discrete set of defined durations between crossings of the Moon’s orbit, such that the Moon meets the trajectory at these points. The changes in the trajectory are such that it is meaningless to talk about resonant flybys, but the durations can still be characterized by the number of months between lunar encounters. In addition, the initial and final flyby can be characterized as inbound or outbound, producing four trajectory families, each with a set of feasible durations. Note that there is always a lunar flyby associated with each perigee surrounding a solar loop, with no additional revolutions between flybys.

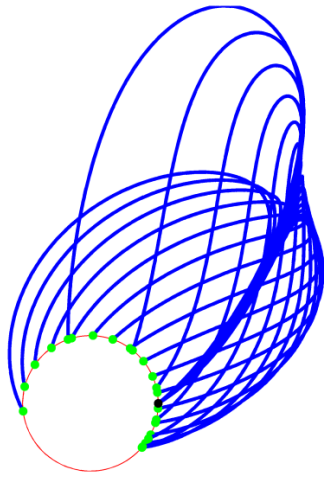


Figure 4: ‘Doi’ family of trajectories, for  $V_\infty$  of 1.1 km/sec. Solutions exist for a solar phase angle of about 0 to 90.

In addition to the initial apogee height (or equivalently period), the solar loops are strongly affected by the angle between the first lunar flyby location and the solar direction (hereafter the “solar phase angle”), and much less strongly affected by the lunar  $V_\infty$  (since all reasonable  $V_\infty$  values permit arbitrarily high apogees). Once a combination of period and solar phase angle for an initial flyby has been found to result in a lunar approach at the end of the solar loop, a differential corrector is used to target an exact patched-conic flyby through the center of the Moon. Then all possible solar phase angles and lunar  $V_\infty$  values for this trajectory type are found by continuation. All of this is performed in a simplified model with patched-conic lunar flybys using a circular orbit for the Moon, and solar perturbation added only outside the Moon’s orbit. For convenience, the families of solutions have an initial letter corresponding to the number of complete lunar revolutions between flybys, followed by ‘o’ or ‘i’ depending on whether that flyby is inbound or outbound. So a ‘Doi’ solution would have 4 complete revolutions, between an outbound initial flyby and an inbound final flyby. An example of this solution is shown in Figure 4. The flight time for all of the ‘oi’ families is shown in Figure 5. The solar phase angle is much more significant than the initial lunar  $V_\infty$ .

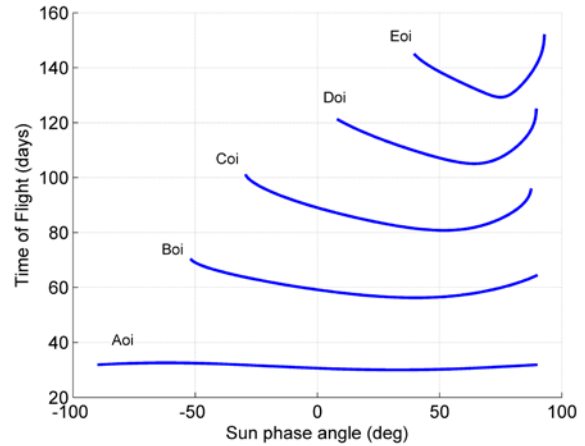


Figure 5: Flight time for the ‘oi’ families vs solar phase angle, for a  $V_\infty$  of 1.2 km/sec.

With all of these trajectories characterized, we can look at the turn angle for the final flyby that results in the maximum C3. In some cases, this will not even produce an escape, or results in an Earth impact, but all the feasible cases have been collected and organized. Of particular interest is the maximum escape C3 vs. heliocentric flight path angle  $\gamma$  of the outbound asymptote. All of these plots are symmetric about the Sun-Earth line, and the  $\gamma$  escape direction is of particular interest for near-Hohmann transfers, although more radial departures are useful for time-constrained Earth-return trajectories and NEO rendezvous trajectories without much phasing error. Figure 6 and 7 show this relationship for the ‘oi’ and ‘ii’ families.

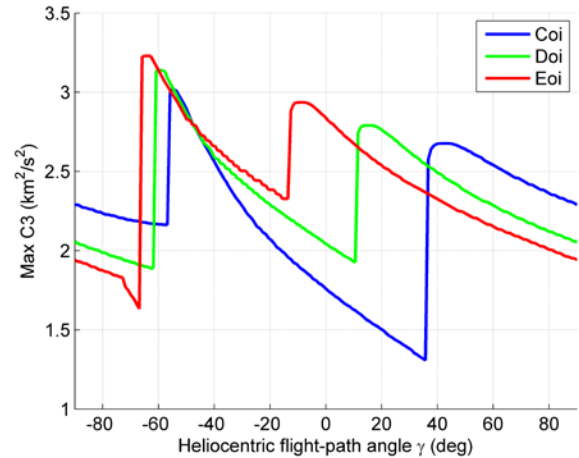


Figure 6: Maximum escape C3 vs heliocentric flight path angle, for 3 of the ‘oi’ families.

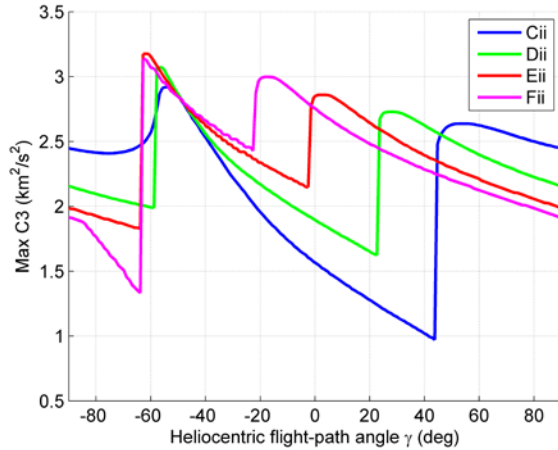


Figure 7: Maximum escape C3 vs heliocentric flight path angle, for 4 of the ‘ii’ families.

The first example of a multiple-lunar flyby trajectory leading to escape is the ICE mission, which used the former ISEE-3 spacecraft to fly through the tail of Comet Giacobini-Zimmer in 1985 [ICE reference]. After spending 4 years in an Earth-Sun L1 halo, the spacecraft embarked on a series of lunar flybys in late 1982 to explore the geomagnetic tail (extending towards the Earth-Sun L2 point), before using a final pair of lunar flybys in late 1983 (October 21 and December 22) to escape from the Earth. In terms of the preceding analysis, this was a trajectory of type Boi, with 2 months between flybys, and no Earth periapsis in between (ie. outbound to inbound), as shown in Figure 8. The escape C3 of 2.8 is more than would be predicted by the simplified analysis above, but still within reason considering the differences caused by using a real ephemeris, and there was also a deterministic 10.5 m/s maneuver near apogee, which also departs from the simplified analysis.

The usage of  $\Delta$ -Vs at periapsis is covered in the next subsection, but there is yet an additional class of ballistic lunar flyby trajectories to consider. If the lunar flyby at the end of the solar loop transfers the spacecraft to a retrograde, perhaps hyperbolic trajectory with a fairly low perigee, the spacecraft can re-encounter the Moon on the outbound leg, and perform a final flyby to increase the departure energy. This is referred to as a Triple Lunar Flyby (TLF), and an example is shown in Figure 9. While the geometry is fairly constrained, the maximum C3 can reach 4 to 4.5 in some limited directions, as shown in Figures 10 and 11. Such trajectories are particularly useful for SEP spacecraft that are unable to easily generate large  $\Delta$ -Vs at periapsis. In addition, some of the accessible directions with

maximum overall C3s are close to the Sun line, which permits a fairly quick (<12 months) re-encounter with the Earth after heliocentric thrusting to increase the Earth-relative  $V_\infty$ . It may also be possible to perform small amounts of thrusting around apoapsis to expand the region of accessibility for the outbound asymptote at the same higher C3 values, and it is certainly possible to use such thrusting to expand the range of acceptable solar phase angles, making a continuous launch period easier to achieve.

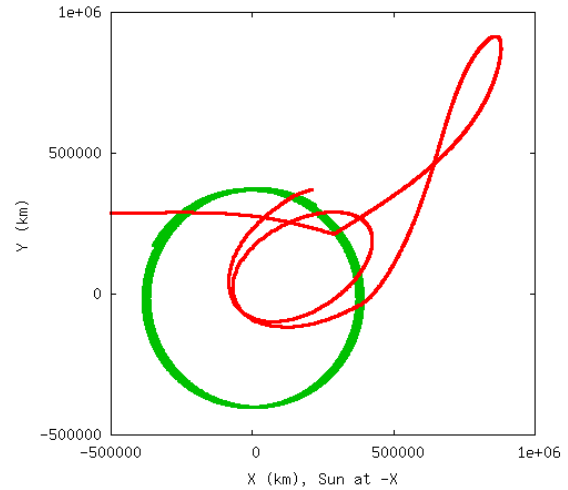


Figure 8: Final stages of ICE Earth departure trajectory, in the Sun-Earth rotating frame. ICE is in red and the Moon is in green. Lunar flyby dates are 9/28/83, 10/21/83, and 12/22/83.

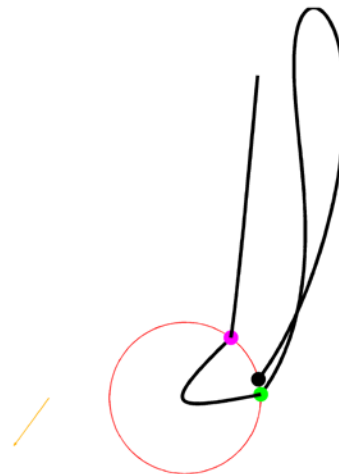


Figure 9: Triple Lunar Flyby (TLF), with 4+ months between the first and second flybys (Doi family).



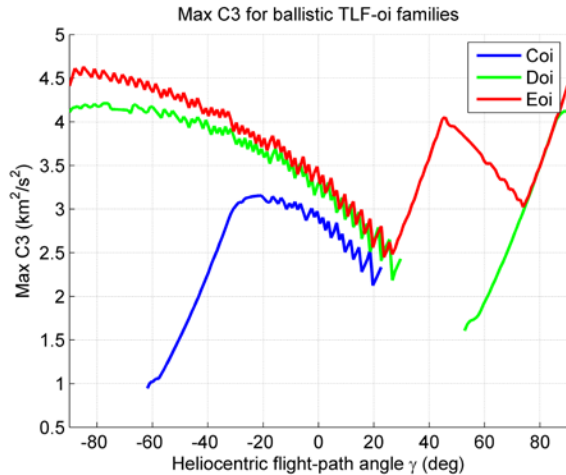


Figure 10: Max C3 vs heliocentric flight-path angle for 'oi' family of TLF trajectories.

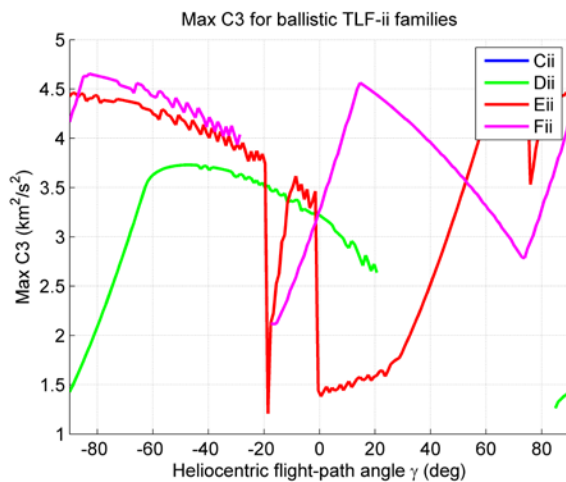


Figure 11: Max C3 vs heliocentric flight-path angle for 'ii' family of TLF trajectories. Note that the peak C3 is available at a  $\gamma$  of  $\pm 90$  and  $\sim 15$ , which is fairly close to the desired value of zero.

For Earth return trajectories, whether they involve SEP thrusting or not, it is possible to add another lunar flyby and further increase the C3 with respect to the Earth. While this has been examined for a particular example that showed some benefit, it hasn't yet been studied systematically to understand the general utility of the technique.

#### Powered Flybys

For departure C3s beyond  $\sim 4$ , the spacecraft must perform a  $\Delta$ -V, functioning in a sense as its own upper stage. The best place to perform the  $\Delta$ -V is at the deepest accessible point in the local gravity well,

where the velocity is highest<sup>§</sup>, which is generally at perigee, although in some limited cases performing a  $\Delta$ -V at perilune may be useful. The achievable C3 is only limited by the acceleration level of the spacecraft, balancing gravity losses against the cost of higher installed thrust levels. For  $\Delta$ -Vs of a few hundred m/s at perigee, the C3 may be boosted to  $\sim 10$ , and acceleration levels of no more than  $0.2 \text{ m/s}^2$  are required to keep the gravity losses under 10%. The maximum C3 vs  $\gamma$  is shown in Figure 12.

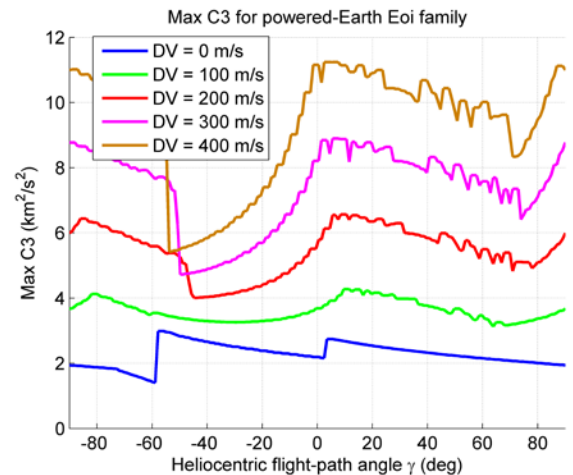


Figure 12: Maximum C3 vs. heliocentric flight-path angle for the Eoi family.

Even without using lunar flybys, there is some advantage in having the spacecraft perform some of the departure delta-V from a highly-elliptical orbit: the staging is improved<sup>\*\*</sup>, both sides of perigee are available for the thrust arc (to reduce the gravity losses), and the departure date can be fixed at the optimum value. Against these benefits are balanced the relatively minor complications of operating for several revolutions in Earth orbit (more on this later), and the need for a moderate acceleration level (which may be needed anyway for orbit insertion at the destination). For a powered flyby after a lunar flyby sequence, there is the added concern that the maneuver is absolutely required to reach the destination in a timely manner (since the spacecraft is already on an Earth-escape trajectory), thus making it a mission-critical event. (By contrast, a perigee burn from an elliptical orbit could be delayed by perhaps one revolution with modest cost).

<sup>§</sup> Per the Oberth effect.

<sup>\*\*</sup> Recall that EELV upper stages have an empty mass of  $\sim 2.5$  to 3 metric tons. In addition dropping that mass, there is no need for a planetary protection bias when the upper stage does not leave Earth orbit.

As with the TLFs, a powered flyby constrains the last flyby to target a low perigee for the burn. The trajectory from the Moon to the Earth is nearly radial, and there are feasible solutions for both posigrade and retrograde perigees. The retrograde option is probably preferred from an energy point-of-view, since it may require a smaller turn angle at the Moon, but retrograde flybys have some small increased risk from the debris flux at LEO. These two solutions occur at least a week apart, so the phasing of the Moon's orbit with respect to the optimal departure time may be the determining factor. In addition, changing the outgoing declination from the Earth costs almost nothing, and so departure asymptotes that would be costly to reach with a direct launch can be used easily (as will be illustrated below) to reduce the departure energy.

The Japanese Nozomi mission to Mars provides an example of a trajectory with a double lunar flyby and a perigee burn, as well as the consequences of not completing the perigee burn as planned. Nozomi launched into phasing loops on July 3, 1998, and performed lunar flybys in September 24 and December 18 (using the Coi family of trajectories) to set up the powered flyby on December 20 [Nozomi ref]. Unfortunately, the perigee burn did not provide enough  $\Delta-V$  due to propulsion anomalies, and the spacecraft did not end up on a Mars-bound trajectory.<sup>††</sup> The planned trajectory in the Earth-Moon system is shown in Figure 13 and 14, and the Earth-relative C3 vs. time is shown in Fig 15 over most of the same time. Note that the Earth-relative trajectory was retrograde at the time of the last lunar flyby, and that the C3 was positive before the perigee burn.

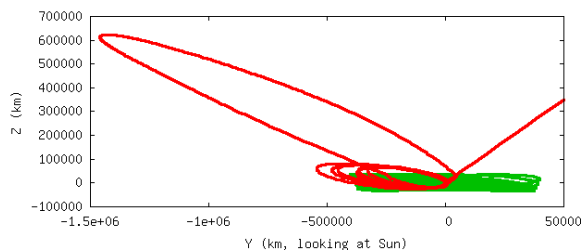


Figure 14: Planned Nozomi Earth departure trajectory, Sun-Earth rotating frame, looking along the Sun line.

<sup>††</sup> The recovery effort led to an innovative trajectory that returned to the Earth 4 years later, performed the first Earth-Earth backflip, and then continued to Mars in the 2003 Type 1/2 trajectory opportunity.

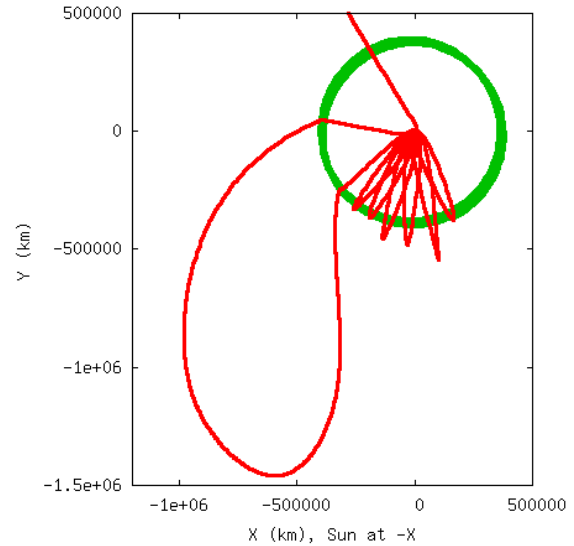


Figure 13: Planned Nozomi Earth departure trajectory, in the Sun-Earth rotating frame. A  $\Delta-V$  of 433 m/s occurs at perigee. Two lunar flybys are used to set up the solar loop, after a series of phasing loops.

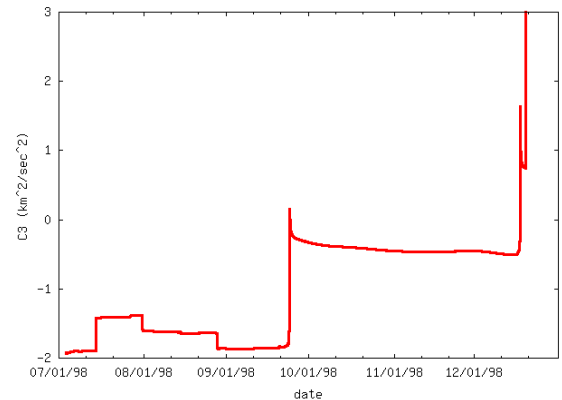


Figure 15: C3 vs time for planned Nozomi trajectory. Final C3 value (not shown for scale) was 9.8. Note the positive C3 value of  $\sim 0.8$  between final lunar flyby and perigee.

The Japanese motivation for using this trajectory certainly included a desire to increase the Earth departure mass above that otherwise available with the M-V launch vehicle. Similarly, a recent study of the 2018 Mars opportunity looked at the available mass increase (over an assumed Atlas V551 launch) for a lunar-assisted departure. The trajectory was developed in an ad hoc manner before the systematic study referred to above. Figure 16 shows both the

integrated trajectory developed for the 2018 launch study, and the corresponding Dii trajectory family from the results discussed above, which matches fairly well except for the ignored out-of-plane direction. This trajectory performs an inbound lunar flyby after a series of phasing loops, and then after a solar loop (where the Earth-relative orbit direction is reversed), encounters the Moon again to return to a prograde direction and set up a perigee  $\Delta$ -V of (ideally) 274 m/s. The 2018 Mars opportunity has large negative outbound declinations, in this case -36 degrees for the departing asymptote, so the Earth equatorial inclination after the final lunar flyby is 46.5 degrees. Compared to a traditional launch, this trajectory produces an ~8% higher payload mass, even accounting for the increased tankage and fairly severe gravity losses (due to a small thruster), and ignoring the additional penalty for outbound declinations higher than the launch site latitude for the traditional launch. Coincidentally, the Moon's phasing was nearly ideal for the prograde case, so a retrograde solution was not investigated.

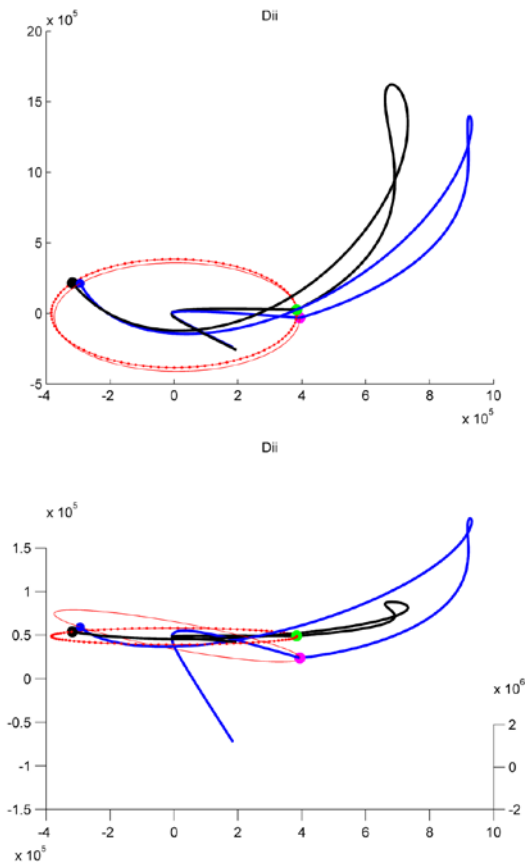


Figure 16: Lunar-assisted powered-flyby Mars 2018 opportunity trajectory (in blue) compared to Dii trajectory (black) for roughly the same outbound direction.

In principle it is possible to combine powered flybys with TLFs. However, maneuver execution errors make having a low lunar flyby less than 2 days after a perigee burn problematic, and generally an underburn would cause a lunar impact. Consequently, it seems prudent to limit powered flybys to the last low periapsis in a departure sequence. When this is a lunar flyby, the benefit of a  $\Delta$ -V is greatly reduced, so it is unlikely to be useful except for marginal cases. Of course,  $\Delta$ -Vs at the last lunar flyby in a DLF case are also possible, and have been systematically cataloged just as with perigee burns. The performance is lower, and there is little if any benefit from lower gravity losses at the same acceleration level. However, for powered flybys that strongly prefer not to be done at a low perigee, perilune  $\Delta$ -Vs might be attractive.

#### IV. GETTING TO THE MOON

As mentioned above, the Moon traverses a large swath of inertial directions over the duration of a typical launch period, while the conditions for the ultimate departure from the Earth-Moon system are nearly instantaneous. Solar system missions making use of the Moon must somehow connect these regimes with an acceptable set of trajectories. Unlike missions to the Moon, the practice of utilizing a launch period with a few launch dates per month or fortnight, over a 3-6 month period, is not acceptable, and not even really all that useful.

Phasing loops represent one typical solution. While holding the lunar flyby sequence fixed, the spacecraft is launched into an orbit period that is less than the one leading into the first lunar flyby. During subsequent perigees, the orbit period is raised in a coordinated manner such that all of the days of the launch period are accounted for. This approach was used for Nozomi and STEREO-A/B (as well as for Chandrayaan 1 and SELENE, both lunar orbiters), and was studied for the Mars 2018 opportunity. The duration from the end of the launch period to the first lunar flyby can be as short as a few days, but typical values start at around 10 days, all depending on the amount of perigee  $\Delta$ -V to be supplied by the spacecraft. These trajectories are generally inapplicable to pure SEP spacecraft, since they would suffer large gravity losses and/or time delays in generating the required period change, but they serve well for spacecraft with moderately high thrust (i.e. those using chemical propellants).<sup>‡‡</sup>

<sup>‡‡</sup> Some care must be taken to avoid perigee drops due to lunar or solar perturbation, and the van Allen



An alternative is a direct launch to the Moon on each day of a (perhaps mostly) contiguous launch period, followed by a flyby sequence that accomplishes the variable duration required to reach a specified departure point. This could be accomplished by using combinations of the initial transfer period to the Moon [Landau], and resonant and non-resonant transfers, with changes in the  $V_\infty$  (due to either launch vehicle targets or spacecraft  $\Delta$ -Vs) to vary the duration of a particular sequence, and alterations in the sequence to produce bigger changes (much as phasing loops will drop complete revolutions for later days in the launch period). However, introducing solar loops just for phasing is more efficient (by keeping the launch energy constant and removing spacecraft  $\Delta$ -Vs) and probably not significantly longer than a sequence that performs all the phasing in a strictly patched conic sense. More work is necessary in this area to quantify the relationship between the launch vehicle C3 variation, spacecraft  $\Delta$ -V, and duration of the pre-departure sequence.

One simple approach would be to eschew some of the potential energy gain, and launch directly to a C3 of around -0.5. This permits the launch trajectory to reach an altitude where the Sun perturbation is significant, and is analogous to the GRAIL trajectory. While the injection velocity is  $\sim 70$  m/s more, all of the launch period duration can be handled with very small changes in C3, and the duration before the lunar flybys should not need to be more than  $\sim 4$  months. Since the outbound trajectory is not tied to the lunar phasing, it should be possible to reach somewhat different Earth departure directions, which is especially useful for the TLF cases. The total C3 change from -0.5 to  $\sim 4$  still represents a substantial benefit, especially with the relatively short flight time.

So far we have only discussed dedicated launches, but there is another route to the Moon: as a secondary payload. Occasionally, this might be with a primary payload that is on a direct trajectory to the Moon, as was the case with LRO and LCROSS. Connecting such a launch to a solar system departure is possible, but it requires either: 1) a very flexible departure period, 2) a significant planned post-launch loiter in the Earth-Moon system as a protection against launch delays, or 3) a firm commitment to the primary payload launch period. A more likely route would be to hitch a ride with a primary payload bound for geosynchronous orbit, with a launch

vehicle target of geosynchronous transfer orbit (GTO).

While GTO is a good start, the propulsive requirements to reach the Moon are still significant, requiring  $\sim 700$  m/s at perigee to reach lunar altitude. Usually the apogee altitude will have to be even higher, to allow a plane change to be done efficiently, since a GTO instance will seldom line up well with the Moon [Penzo]. For a chemical bi-prop spacecraft, this may be a good option, since even adding  $\sim 1$  km/s to the spacecraft delta-V requirement is not unreasonable, and the staging (due to dropping off the launch vehicle upper stage, with its 2-3 mT empty mass) is favorable. However, it is not useful for SEP spacecraft, due to their inability to perform useful perigee burns.

SEP spacecraft can still make use of a shared launch with a GTO primary payload, without taking the time and delta-V expense of thrusting from that orbit to reach a lunar flyby. By augmenting the launch vehicle (in particular, by adding boosters to the Atlas V family), the payload to GTO may be increased to include not only the secondary payload, but also enough upper stage propellant for another burn to boost apogee. If the secondary payload injection happens 20 minutes after the osculating periapsis, then  $\sim 900$  m/s is required to reach an apogee of around a million km.

From this  $\sim 41$ -day orbit, the SEP spacecraft has time to perform a burn around apogee to reach a lunar flyby, as long as the  $\Delta$ -V is not too high. In order to bound the required  $\Delta$ -V to reach a lunar flyby from such a boosted GTO, a study was conducted to look across all GTO orientations with respect to the Moon, and all possible locations of the Moon at the moment of launch. This assumes that there are no constraints on a simple GTO (of 185 km by 35782 km altitude) launching from Cape Canaveral (inclination 28.5 degrees), and that the primary launches at any time whatsoever. The second upper stage burn is also fixed at 921 m/s, which produces a semi-major axis of 500,000 km.

After this injection, a series of true anomalies near apogee are examined to compute a Lambert solution to points spaced every 5 degrees around the Moon's orbit. The  $\Delta$ -V is then modified by an approximation of the solar gravity perturbation at apogee. This process is repeated for the lunar node (in 15 degree steps) and the solar direction (in 30 degree steps), as well as for the minimum and maximum lunar equatorial inclination, to find the  $\Delta$ -V for each case. Then the  $\Delta$ -Vs are sorted from smallest to largest, and the transfer time and lunar orbit intercept point are examined to find the lowest  $\Delta$ -V value that spans a month range of lunar motion.

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belts cause some minor radiation damage, but none of these effects present more than a minor annoyance.

An example of a worst-case orientation is shown in Figure 17, looking along each axis in turn. The combination of the transfer orbit duration and the location of the lunar intercept point cover a full lunar period, accounting for any possible launch date. The worst-case  $\Delta$ -V across all orientations is 250 m/s, but in light of various approximations, the recommended bounding value is taken as 300 m/s.

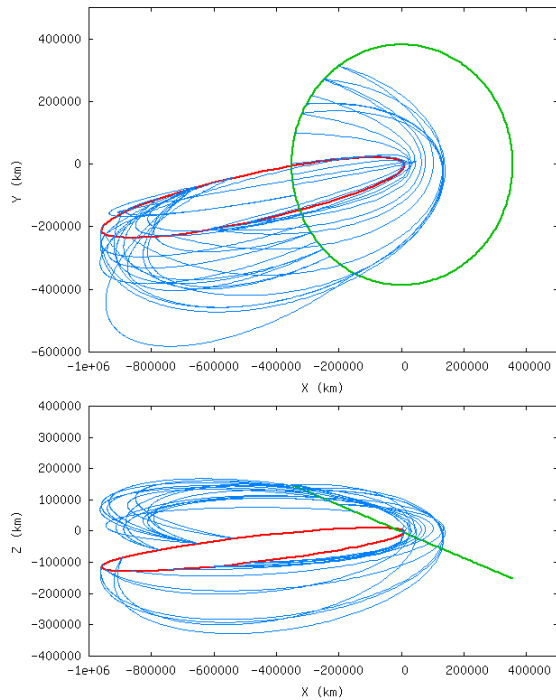


Fig 17. Two views of SEP transfers from boosted GTO orbit to the Moon. The boosted GTO is shown in red, the Moon's orbit is shown in green, and the range of minimum- $\Delta$ -V transfer orbits are shown in blue.

From this first lunar flyby, it seems reasonable to assume that subsequent flybys can achieve a specified departure condition, given several months. This is particularly true if a solar loop is included<sup>§§</sup>. Note that if the time of the primary GTO launch is not constrained, it may be necessary to accept an early launch to reduce the risk of missing a departure window due to primary-payload-induced launch delays, and then wait out extra time in high Earth orbit, with occasional lunar flybys to shape the trajectory.

<sup>§§</sup> The use of lunar flybys and then a solar loop to get somewhere, starting from a different orbit, was demonstrated by Artemis [Artemis ref].

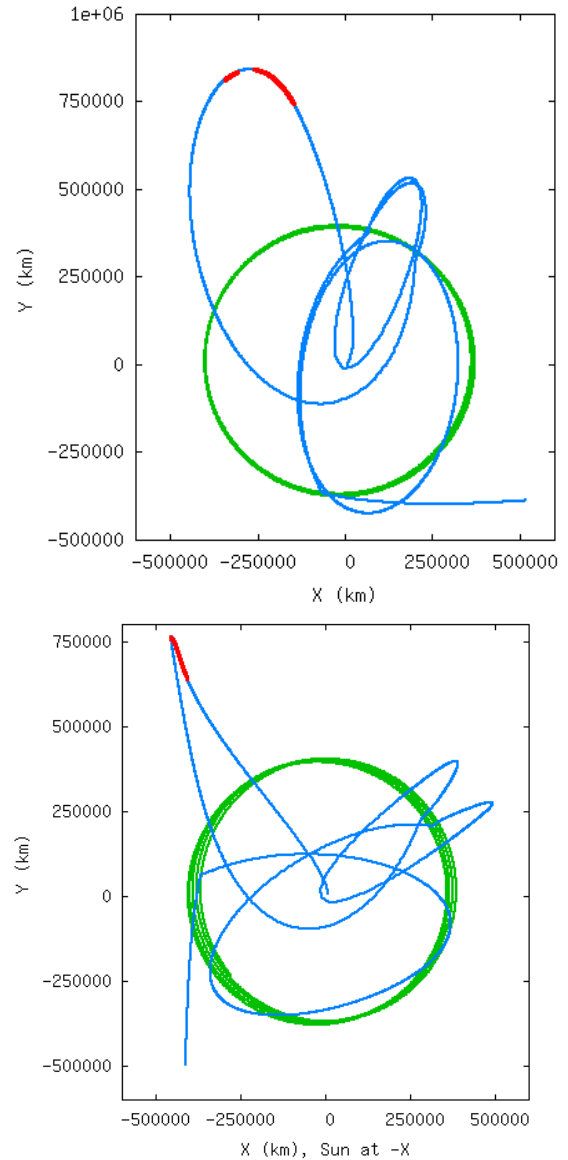


Figure 18. Example trajectory from boosted GTO to Earth escape. Thrust arcs are in red, coast arcs in blue, and the Moon is in green. Top plot shows inertial view, bottom plot shows Sun-Earth rotating view. Three lunar flybys are used to achieve a C3 of 2.3.

As a test of this technique, an attempt was made to design a trajectory to match one of the worst-case GTO orientations in the real world, and to progress to series of lunar flybys and then an Apophis rendezvous, using a high-fidelity SEP trajectory optimizer (Mystic). The conic  $\Delta$ -V to reach the Moon dropped by  $\sim$ 100 m/s, so either the optimizer was able to do wonders or this wasn't as bad a case as we thought. In any case, the upper limit of 300 m/s still seems appropriate. After the initial flyby,

two different orbits were used to connect to the departure time, as shown in Figure 18. The first orbit is a non-resonant orbit that does  $\sim 1.67$  revs between encounters, and the second is an inclined orbit that does  $\sim 1.3$  revs in exactly 1.5 months, encountering the Moon again on the other side of the original flyby point. This combination takes up the time until the targeted Apophis departure point, which has a C3 of  $\sim 2.3$ . Even this moderately low value still represents a velocity savings of over 1 km/s compared to an immediate departure at a C3 of zero. Our more current understanding of the benefits of including a solar loop and perhaps 2 lunar flybys at the end would have changed what we tried to do in this case, but Apophis rendezvous is not that difficult for this vehicle, so that wasn't necessary here.

Many of the departure techniques discussed here have a variable duration between the initial and final lunar flyby. Even without using solar loops, there are a set of resonant and non-resonant orbits in the patched conic analysis that allow various amounts of time to be consumed. While resonant orbits return to the Moon at half-period increments regardless of  $V_\infty$ , the non-resonant orbits have flyby intervals that depend on the  $V_\infty$ . The boosted GTO trajectories have a large range of  $V_\infty$  values ( $\sim 1$  to  $\sim 1.7$  km/sec), as do direct launches to a lesser degree, so it is convenient to plot the time between flybys as a function of  $V_\infty$ , as shown in Figure 19. This analysis has been done for a circular lunar orbit at the average distance, but the general trends in the real world will be similar. Each family in Figure 19 is identified by the series of apogees and perigees between flybys, which uniquely specify the trajectory. Several of the families have regions where the perigee is too low for a particular  $V_\infty$ . It is interesting to note that the range of times is particularly narrow around  $V_\infty = 1.45$ , and that it is very broad for values under 1. This suggests that timing changes for higher  $V_\infty$  values are best accomplished by solar loops, rather than by orbits in the patched conic world. Note that there are similar families around 2- and 3- month flyby spacing.

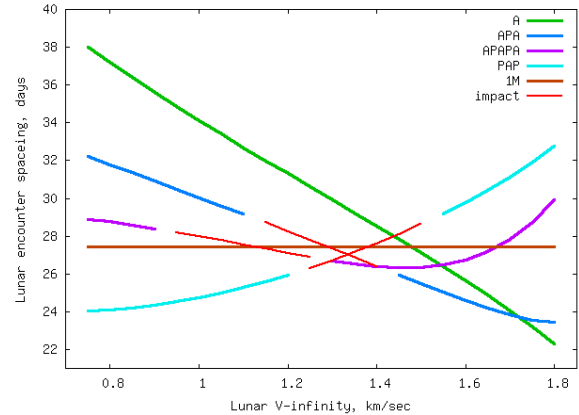


Figure 19: Lunar encounter spacing vs.  $V_\infty$ , for a circular lunar orbit. ‘A’ has only an apogee, ‘APA’ has one rev plus an apogee, etc. All but the 1-month case are non-resonant. Red areas have impacting perigees. To the right of its own red area, the families of trajectories are retrograde, as is the apogee-only case for encounter spacing less than 1 month.

## V. APPLICABILITY

The applicability of this technique is implicit in the examples given above, but a recap is in order as an overall summary.

For missions to Venus, Mars, or beyond, a lunar departure requires additional  $\Delta$ -V and/or time, either during an Earth-Earth loop (for low thrust) or with a periapsis burn (for high thrust). In some cases an Earth-Earth loop may permit an additional lunar flyby at Earth return to provide the necessary Earth-relative  $V_\infty$ .

For missions to NEOs, a lunar departure provides sufficient  $V_\infty$  in many cases. Most potential future NEO missions would be likely to rendezvous with their targets, so substantial deep space propulsions would be required, mitigating any deficiency in the Earth departure  $V_\infty$ .

For heliospheric missions that just want to drift away from the Earth, a lunar departure would be sufficient, and this has already been demonstrated for two different missions, as noted above.

In all of these cases, a longer flight time would be required, but the fairly short period of both the Moon and the Earth make it tolerable in many cases, especially for missions that would already plan to operate for many years.

## VI. CONCLUSIONS

The plethora of lunar departure techniques described above provide the means to efficiently use the Moon to improve mission performance. The

flight time penalty is relatively small, and benefits will become more attractive as missions become more challenging, particularly as regards to launch vehicle capability and cost. Several areas of further research in this area remain to be fully studied.

#### VII. ACKNOWLEDGEMENTS

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#### VIII. REFERENCES

Still need to be added. Would include:

Landau 2012

Penzo

ICE ref

Nozomi ref

STEREO ref

Artemis ref