Proof of Concept of the Quantum Capacitance Detector

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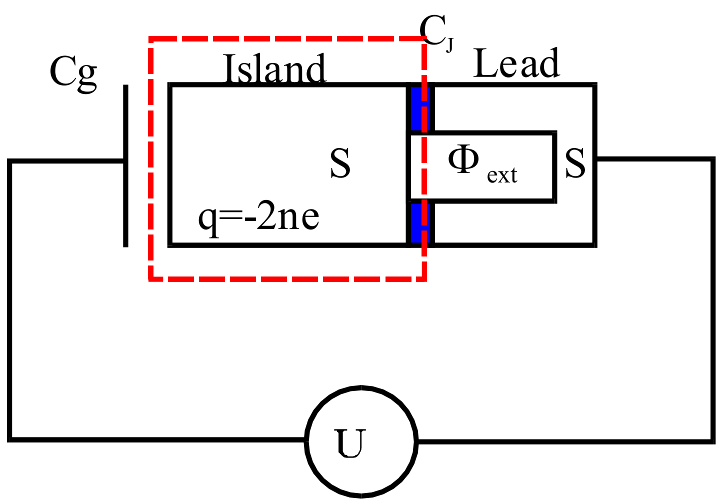
This work was performed at the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration

Single Cooper-Pair Box



Electrostatic gate charge $n_G = \frac{C_G V_G}{2e}$

Charging energy $E_C = \frac{e^2}{2C_\Sigma}$

Josephson coupling $E_J = E_J^{max}\left|\cos\left(\frac{\pi\Phi}{\Phi_O}\right)\right|$

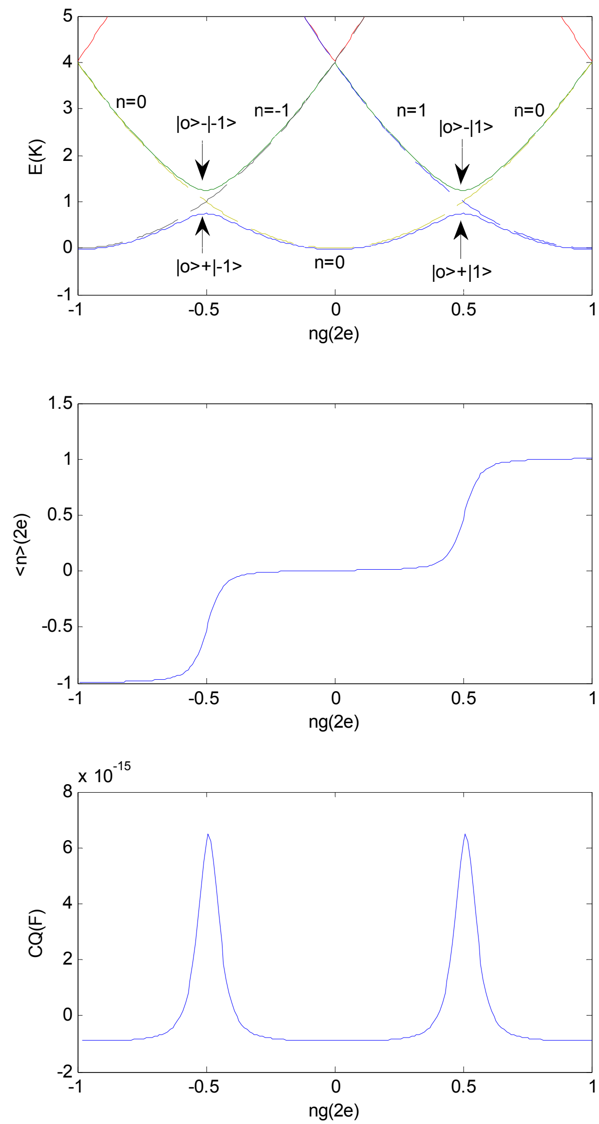
$$H = 4E_C \sum_n (n - n_G)^2 |n\rangle\langle n| - \frac{E_J}{2}\sum_n \left(|n+1\rangle\langle n| + |n\rangle\langle n+1|\right)$$

Energy levels, Coulomb Staircase and Quantum Capacitance

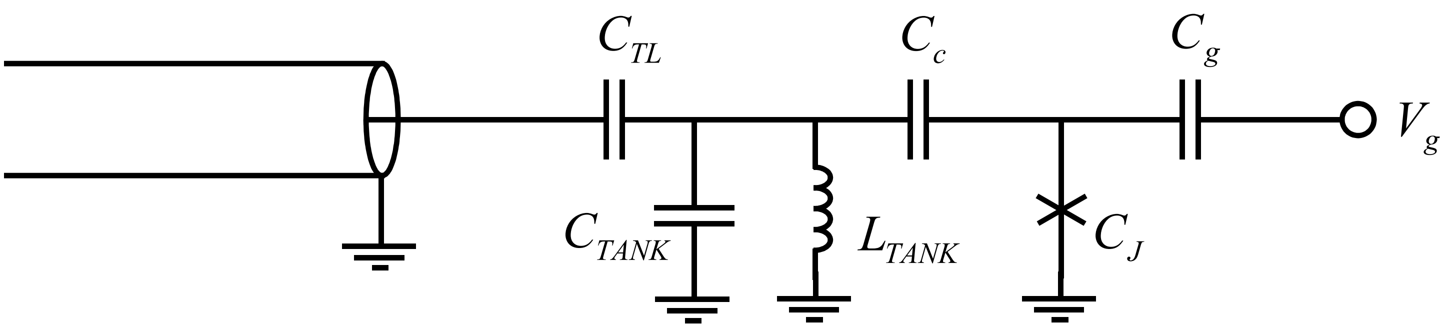
- In the absence of Josephson coupling, Energy is given by parabolas centered at integer values of Cooper Pair Charge

$$E = (Q - 2ne)^2 = (C_g V_g - 2ne)^2$$

- As the gate voltage is increased, Cooper Pairs tunnel to minimize the energy and the charge on the island changes in a stepwise fashion
- The capacitance of the island $C_Q = 2e \frac{d\langle n \rangle}{dV_g}$ spikes up at the degeneracy points where the charge in the island is changing fast
- The Josephson Coupling introduces splittings in the energy levels
- Eigenvectors are symmetrical and anti-symmetrical combinations of the charge states
- The larger Ej, the “rounder” the charge staircase and the smaller the capacitance peaks
- In the absence of tunneling, only one parabola would exist (n=0) and the capacitance would be constant as a function of the gate voltage
- The variable capacitance is due to the quantum nature of the system and is called the quantum capacitance

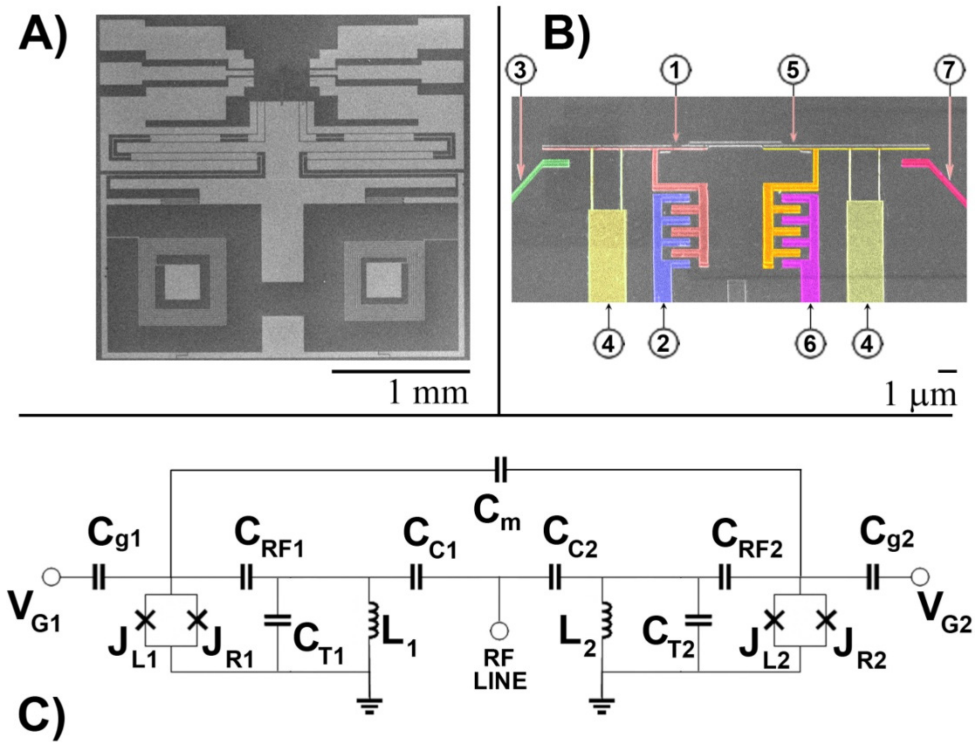


Measurement Technique



- Change in capacitance pulls the frequency of the oscillator
- Resonant frequency (~600 MHz) is far below qubit level spacing (~5 GHz)
- Noise and RF probe isolated from qubit
- Overall circuit capacitance is determined by qubit state
- Enables elegant multiplexing of many qubits
- Phase shift is measured with quadrature mixer

Experimental Setup

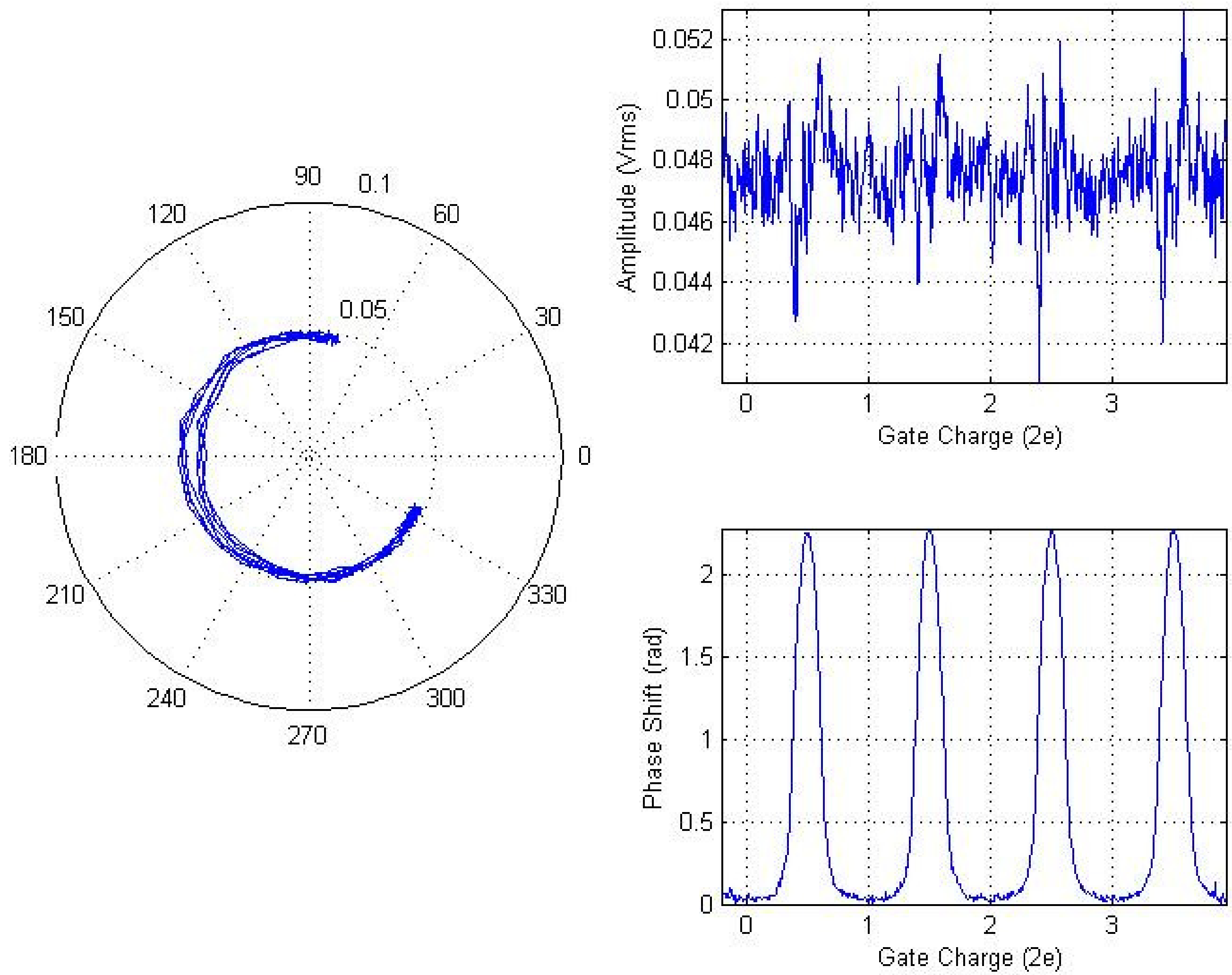


• *SCB qubits are coupled with fixed capacitor*

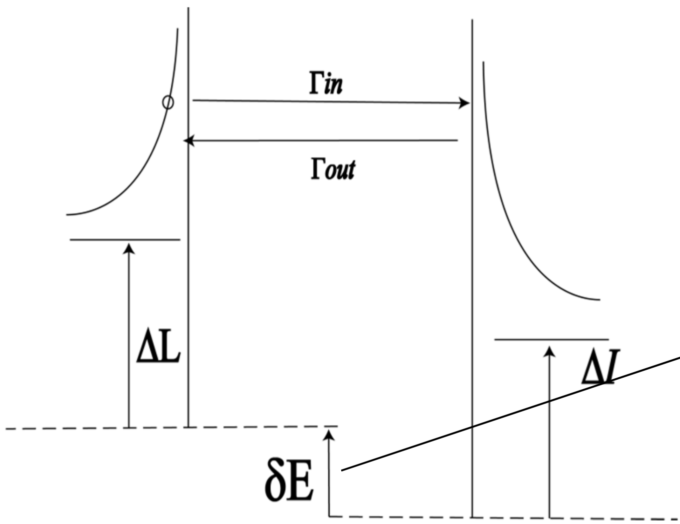
• *Weakly coupled qubits, can be considered independent away from mutual degeneracy point*

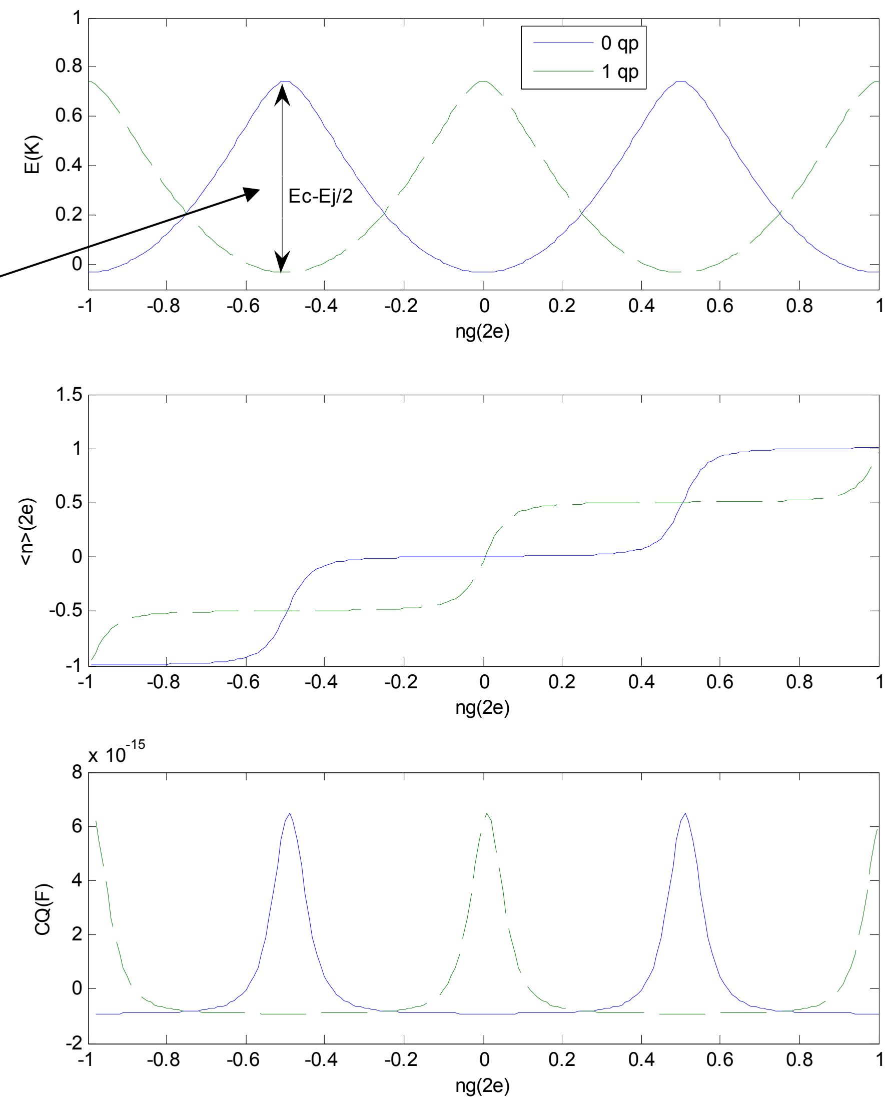
• *Multiplexed quantum capacitance readout scheme*

Experimental measurements



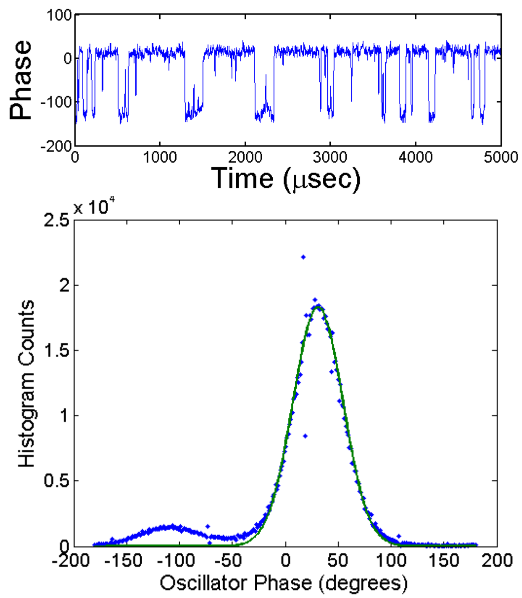
Quasiparticle Poisoning

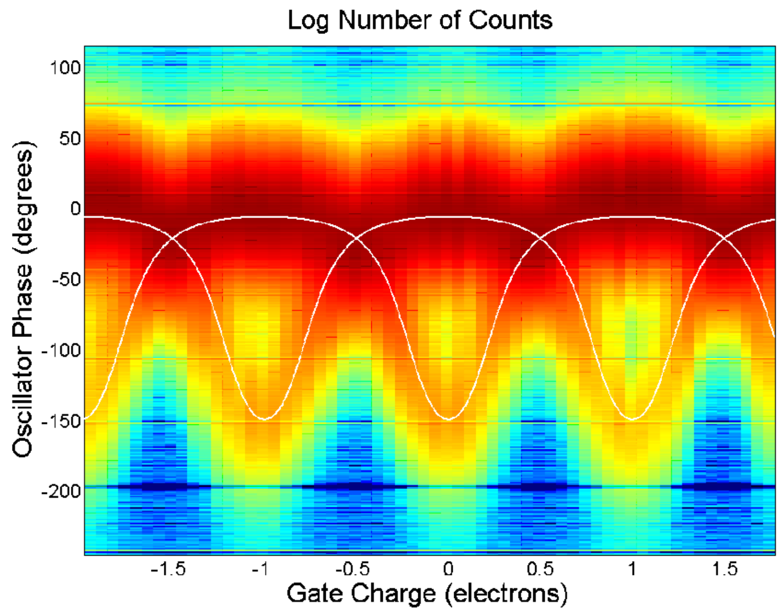




- If there are quasiparticles present in the leads, they could tunnel in and out of the island
- When they tunnel, they shift the effective gate voltage by e/Cg (or ng=0.5)
- Coulomb staircase and quantum capacitance curve shifts by ng=0.5 each time a quasiparticle tunnels in or out.

Tunnel Rate Measurements





- Phase measured at degeneracy point in real time
- One quasiparticle tunnel event shifts phase by ~140 degrees
- Can study statistics and compare with existing theory

Dependence on Quasiparticle Density

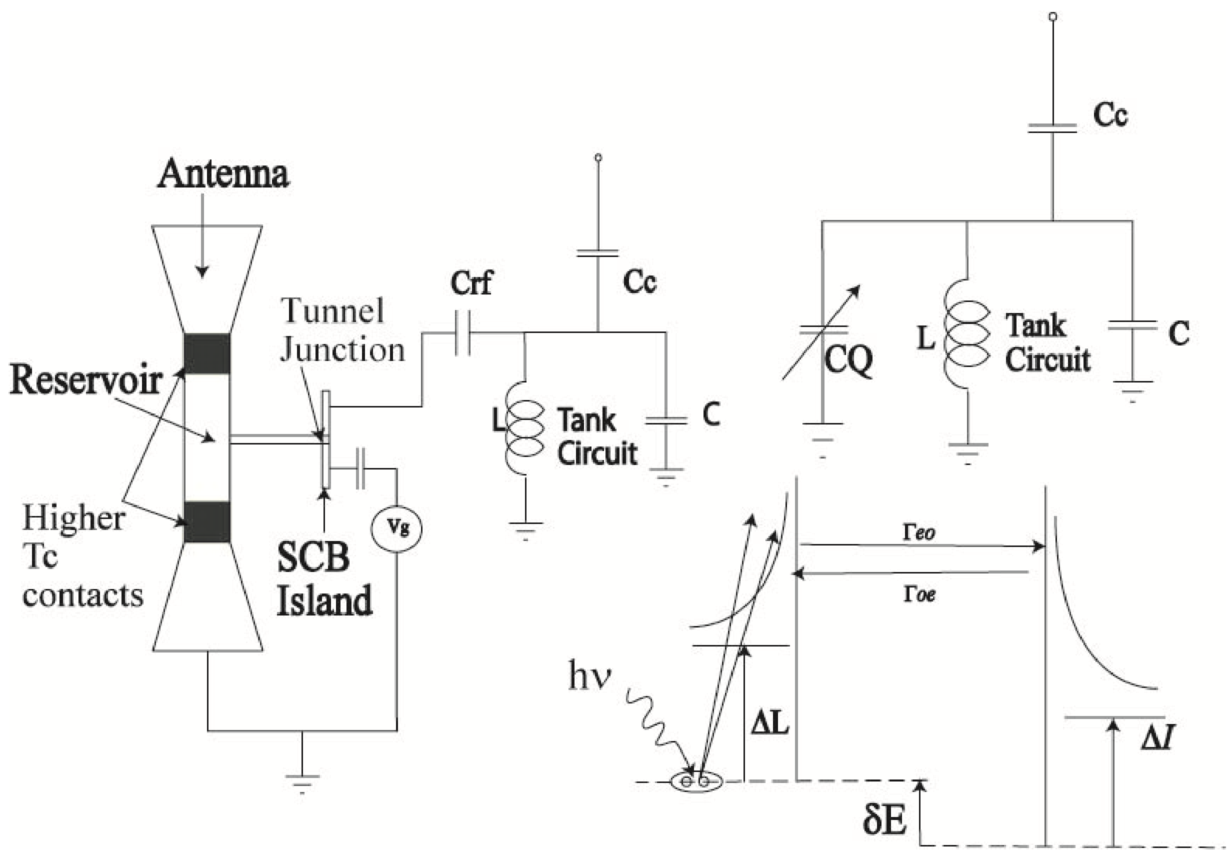
$\Gamma_{eo} \approx K n_{qp}$ Γ_{oe} independent of n_{qp}

$$K = \frac{G_N}{e^2} \frac{e^{\Delta_L/kT}}{N_L} \int_{\max(\Delta_I - \delta E, \Delta_L)}^{\infty} dE \frac{E(E+\delta E) - \Delta_L \Delta_I}{\sqrt{((E+\delta E)^2 - \Delta_I^2)(E^2 - \Delta_L^2)}} e^{-E/kT}$$

- Approximations valid at low temperature
- Simple relationship between rates and QP density is ideal for detector

$$P_{odd} = \frac{\Gamma_{eo}}{\Gamma_{eo} + \Gamma_{oe}}$$

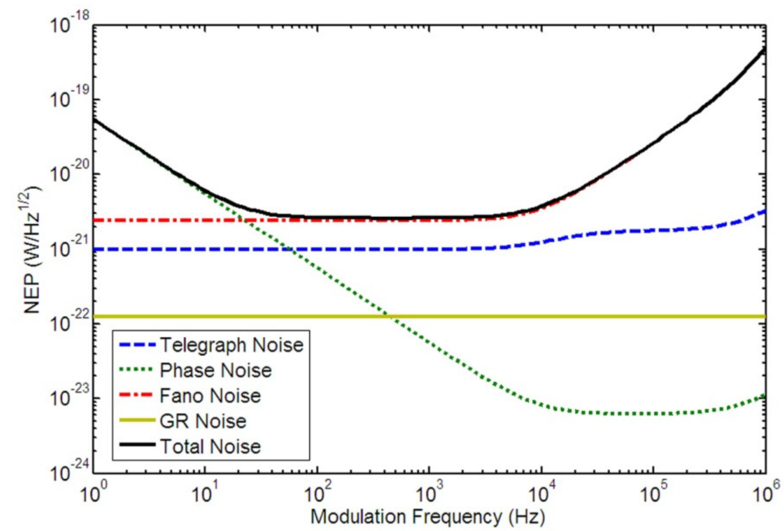
The Quantum Capacitance Detector



- Radiation coupled by an antenna breaks Cooper pairs in the reservoir (absorber)
- Quasiparticles tunnel onto the island with a rate Γ_{eo} proportional to the quasiparticle density in the reservoir
- Quasiparticles tunnel out of the island with a rate Γ_{oe} independent of the number of quasiparticles in the reservoir
- At steady state the probability of a quasiparticle being present in the island is given by $Po(Nqp)=\Gamma eo/(\Gamma eo+\Gamma oe,)$
- The resulting change in the average capacitance will be $CQ= (4EC/EJ)(Cg2/C\Sigma)Po(Nqp)$
- *This change in capacitance will produce a phase shift* $\delta\Phi \sim 2C_Q/(\omega oZoC_C^2)$
- With the existing tank circuit parameters, phase shift per quasiparticle should be 138 degrees, in very good agreement with experiments

Physical Noise Sources

• Phase noise – from phase measurement histogram the rms phase noise is ~ 33 degrees or 1.8×10^{-3} radian/$Hz^{1/2}$ over the 100kHz bandwidth



•Telegraph noise:the tunneling on and off the island is approximated as a Poisson process with rates Γeo and Γoe. At low frequencies the spectral density of noise associated with the process is

$$S_\Phi^{Tele} = \frac{\delta\Phi^2}{\pi}\frac{\Gamma_{eo}\Gamma_{oe}}{\left(\Gamma_{eo}+\Gamma_{oe}\right)^2}$$

•Fano noise: the number of quasiparticles generated by an incoming photon has an uncertainty given by (FNqp)1/2 where F~0.2 is the Fano factor for this system. The associated NEP is

$$NEP^{Fano} = \sqrt{\frac{FP_S\Delta}{\eta}}$$

• Generation-recombination noise: quasiparticles can be thermally excited over the superconducting gap and recombine into Cooper pairs, introducing a fluctuation in the number of quasiparticles in the reservoir The associated NEP is

$$NEP_{GR} = 2\Delta_L\sqrt{\frac{N_{eq}}{\tau_R}}$$

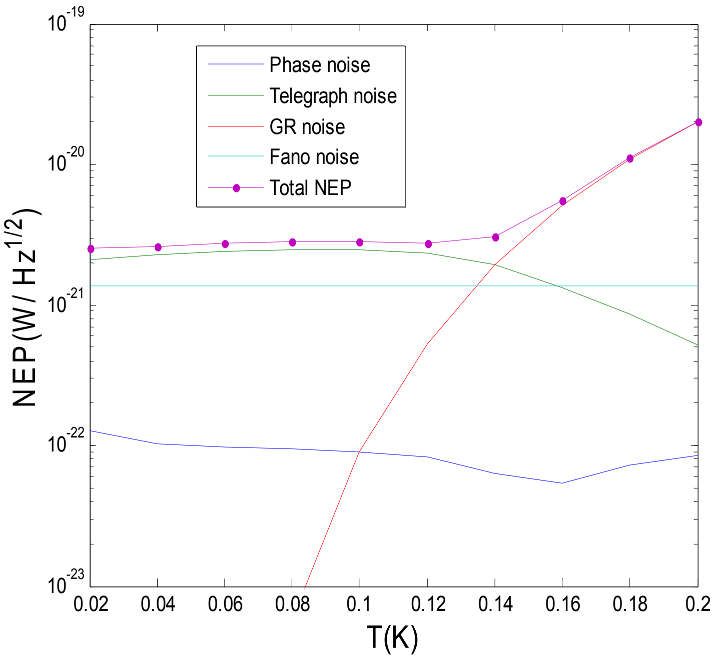
N_{eq} is the number of equilibrium quasiparticles.

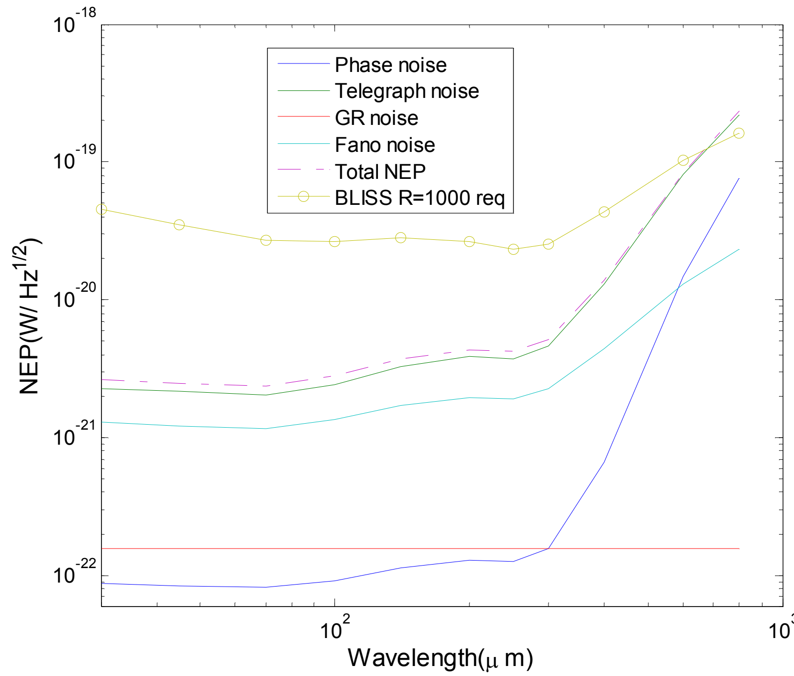
$$N_{eq} = \Omega_L D(E_F)\sqrt{\frac{\pi k_B T\Delta_L}{2}}\exp(-\frac{\Delta}{k_B T})$$

• The noise equivalent power at low frequencies will be given by

$$NEP = \sqrt{\left(\frac{dP_S}{dN_{qp}}\right)^2\left(\frac{d\Phi}{dN_{qp}}\right)^{-2}(S_\Phi^{Phase} + S_\Phi^{Tele}) + NEP_{GR}{}^2 + NEP_{FANO}{}^2}$$

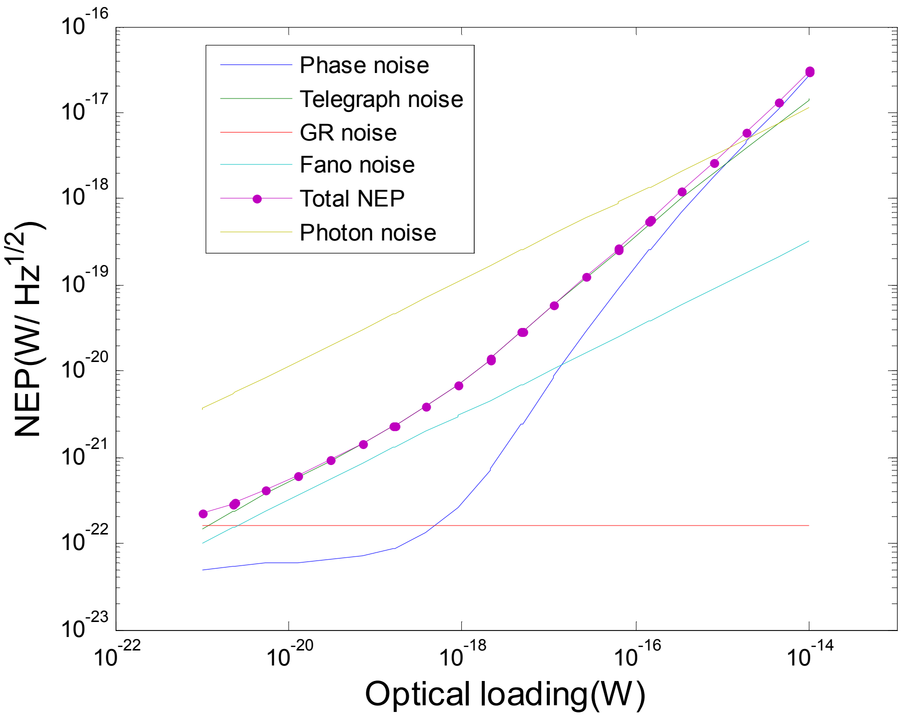
Theoretical Sensitivity





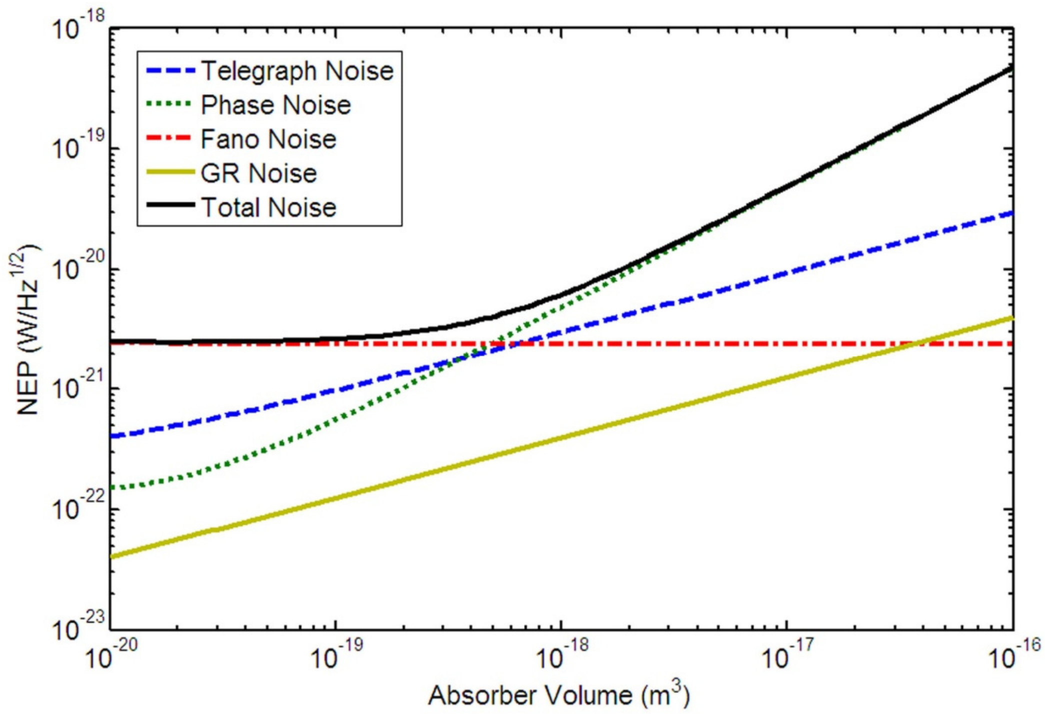
Left: NEPs from various noise sources calculated for devices optimized for λ =100μm, optical loading 10^{-19} W and R=1000 as a function of temperature. Right: NEPs of various noise sources as a function of wavelength as compared to the requirements for a spectrometer with R=1000 and the expected optical loading at L2 for a cold (4.2K) telecope . The operating temperature was chosen to be 0.1K at which the GR noise contribution is negligible.

Theoretical Sensitivity vs. Signal Power



- *Detector is background limited over a wide range of operation*

Theoretical Sensitivity vs. Absorber Volume



- *Absorber volume is a key parameter*
- *Can be used to trade sensitivity for saturation power*

Single Photon Detection

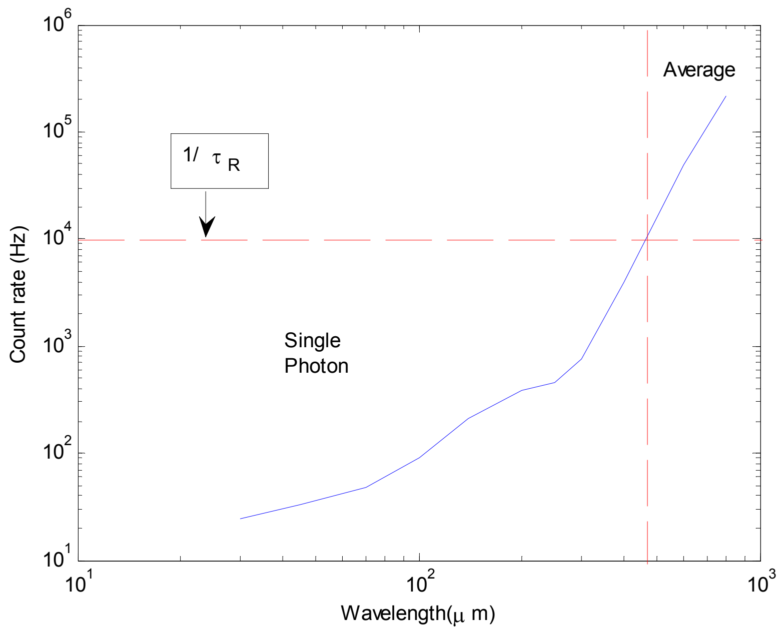
• For photon count rate << $1/\tau_R$ will be a pulse

$$\phi(t) = \frac{US(0)}{\tau_R} e^{-\frac{t}{\tau_R}}$$

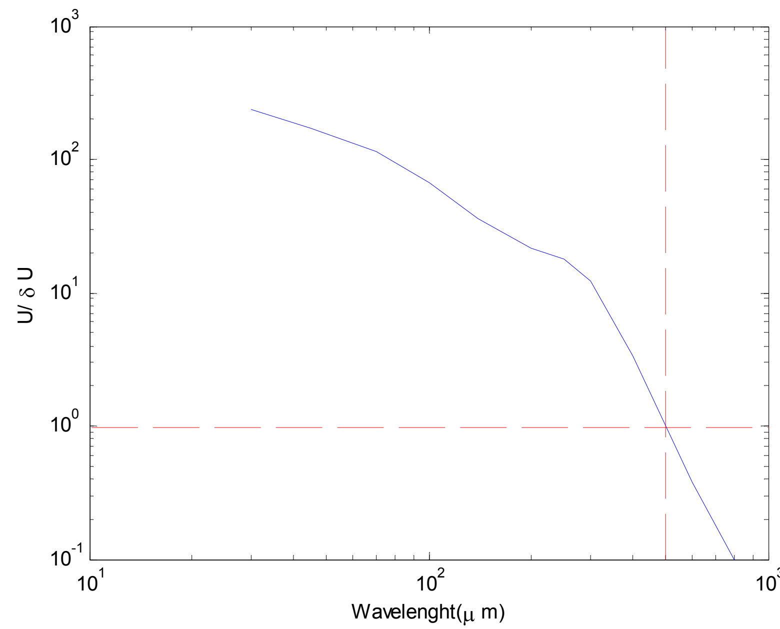
With amplitude proportional to the photon energy *U*. S(0) is the responsivity in radians/Watt

• The energy resolution will be $\delta U = NEP(0)\sqrt{\tau_R}$

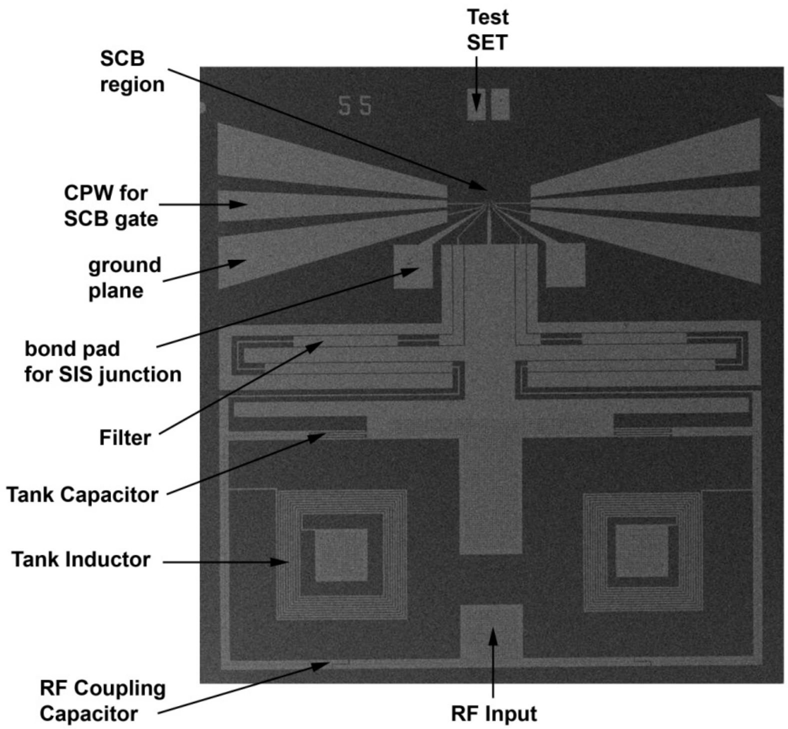
Photon arrival rate for a cold (4K) telescope with an R=1000 spectrometer at L2 as a function of wavelength

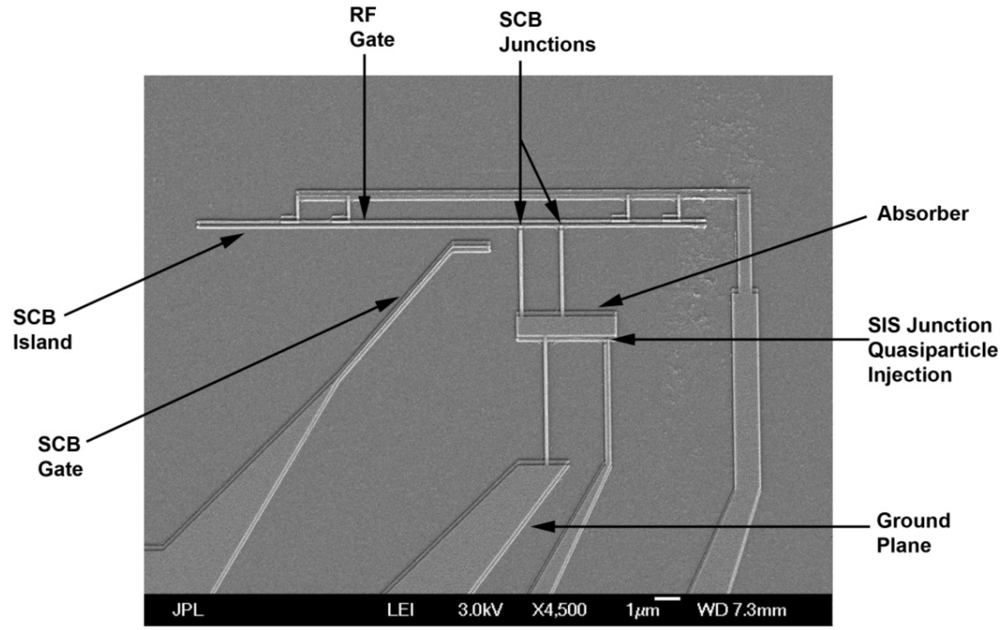


U/δU as a function of wavelength for photon rate shown on left

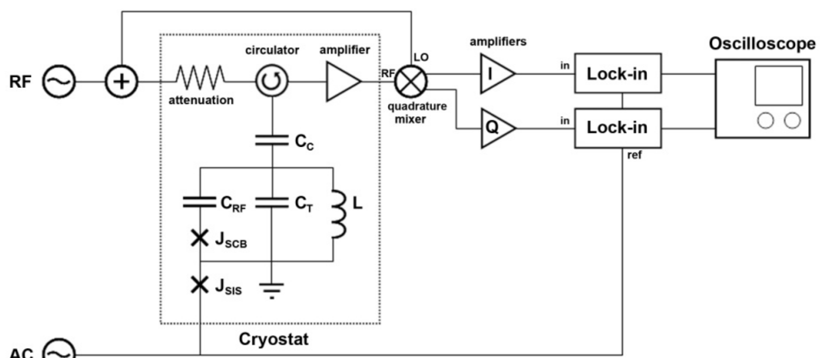


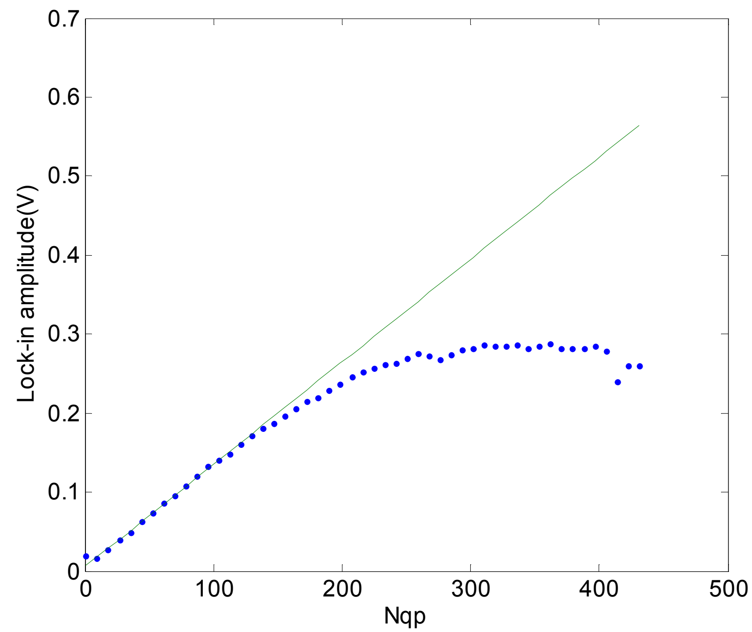
Experimental Confirmation
Quasiparticle Injection with SIS junctions





Experiemental demonstration
Response versus signal



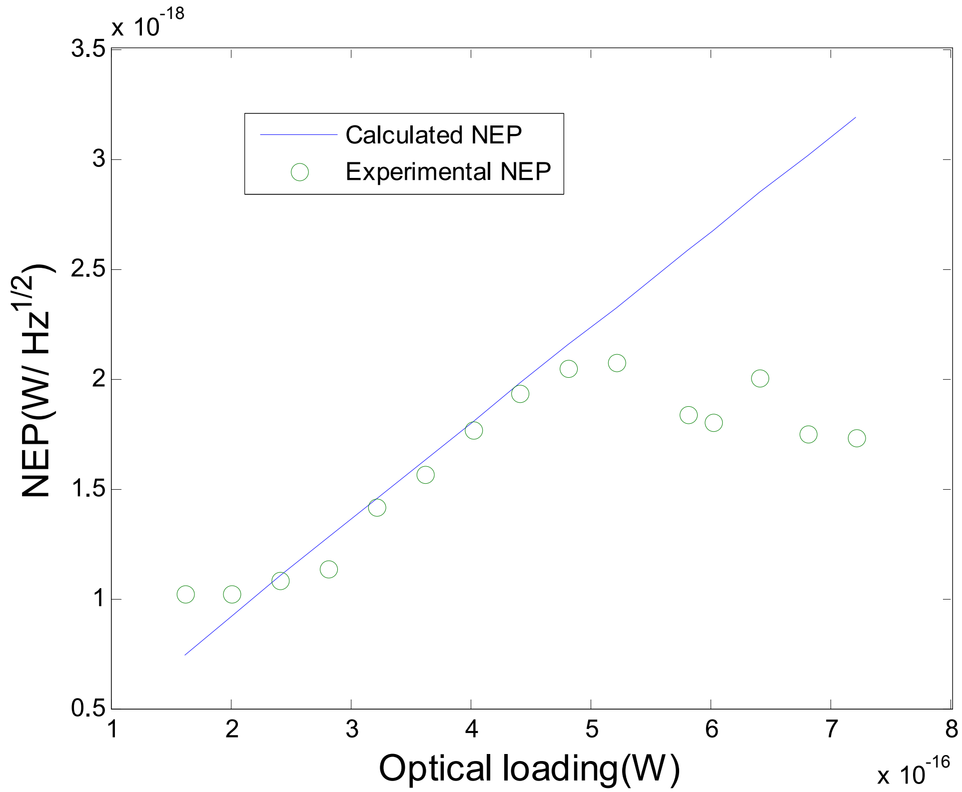


- Ran a current through SIS junction to inject quasiparticles on reservoir
- AC component of current simulates signal and DC optical loading
- τ_D is the time for quasiparticles to diffuse through constriction
- Graph shows lock-in response as a function of number of quasiparticles present in the reservoir.
- The measured noise in number of quasiparticles in the reservoir was δNpq~2.8 qp/Hz$^{1/2}$, which would yield an NEP~ 8x10^{-19}W/Hz
- From loading we expect NEP ~ 9x10^{-19}W/Hz – very good agreement

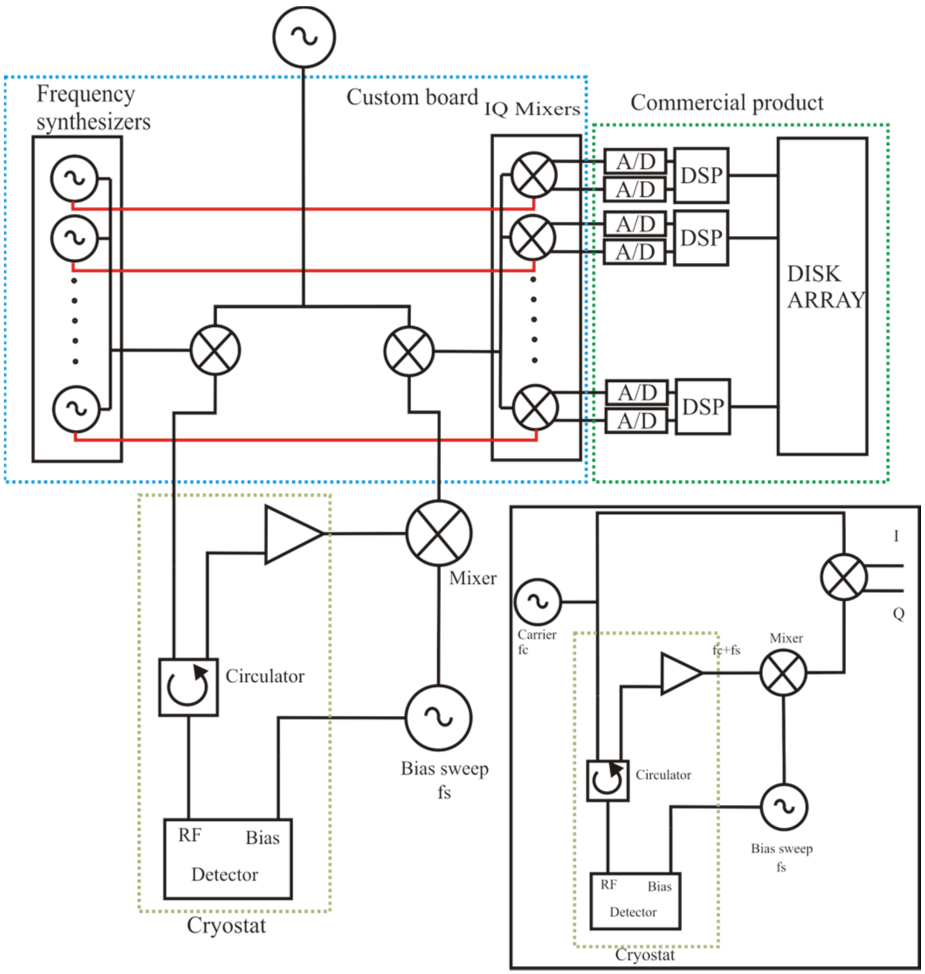
$$N_{qp} = \frac{I}{e}\left(\frac{1}{\tau_R} + \frac{1}{\tau_D}\right)^{-1} \sim \frac{I}{e}\tau_D$$

$$NEP = \frac{\Delta}{\tau_R}\delta Nqp$$

Experimental demonstration
Response x loading

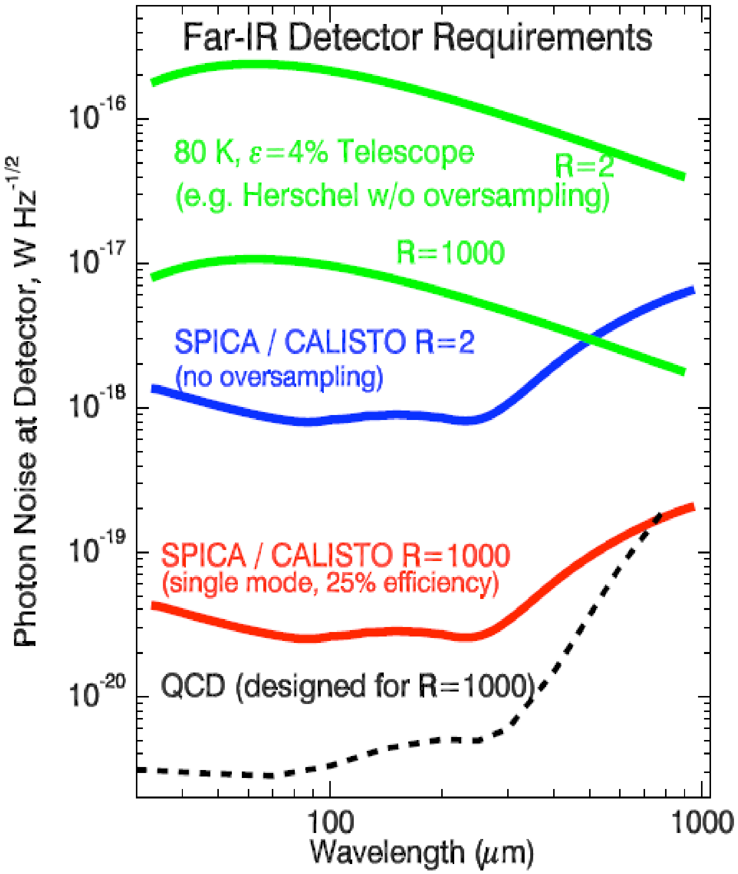


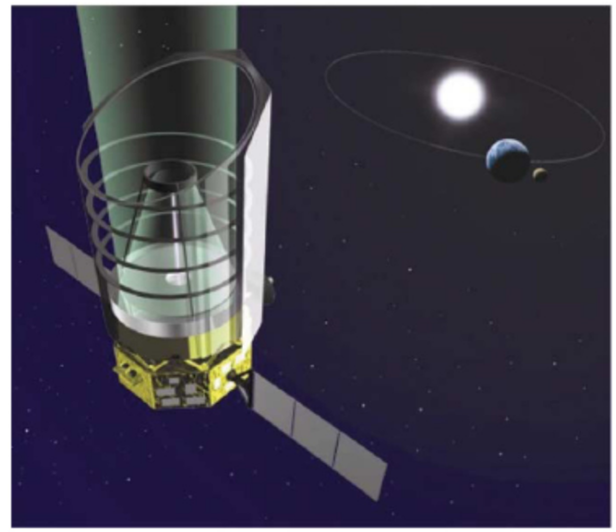
Multiplexing scheme



- Gates are swept with a low frequency *f* signal of amplitude e/Cg
- A mixer demodulates the reflected RF signal to the modulation frequency *f* and a down converter translates the results to DC
- In the multi-pixel readout, A low frequency comb function (0-200MHz) containing several frequency components is produced digitally using a D/A converter and then block up-converted, resulting in a comb of RF carrier frequencies with each frequency corresponding to a particular detector.
- All of the SCB gates are tied together through a common bias line and modulated at the same frequency.
- The reflected RF comb, containing the phase shift information for the entire array, is demodulated at the bias modulation frequency, down-converted to the 0-200MHz band, then digitized and digitally demultiplexed.

Applications in FIR-Submillimeter Astronomy





- Cold (4.2K) telescope at L2 with R=1000 spectrometer

Detector Advantages

- *SCB has extreme sensitivity to the presence of quasiparticles*
- *Sensitivity of QCD rivals MKID and TES*
- *Frequency-domain multiplexing allows scaling to large arrays*
- *Applicable to submillimeter wavelengths for far-infrared astrophysics*
- *Can be easily incorporated with existing technology for MKID arrays*
- *Detector (SCB) is separate from resonator – flexibility of design*
- *NEP and saturation power easily tailorable*