Load Balancing for Mobility-on-Demand Systems

Marco Pavone†, Stephen L. Smith‡,
Emilio Frazzoli*, and Daniela Rus*

†NASA Jet Propulsion Lab, California Institute of Technology
‡University of Waterloo
*Massachusetts Institute of Technology

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Personal urban mobility in the 21st century

Facts:

- Urban population will jump from 3.45 bil to 6.37 bil in 40 years
- Personal urban mobility will increase to formidable levels

Physical constraint:

- Available urban land for roads and parking is continuously decreasing

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Towards a "*Car*mageddon"?

- The Los Angeles Carmageddon (from NBC LA, June 2011)

Emerging paradigm: mobility-on-demand (MOD)

- Paradigm shift: from infrastructure augmentation policy to new modes of personal urban mobility

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One-way bike sharing:

Car sharing:

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Mobility-on-demand: merging private and public mobility

One-way bike sharing:

Car sharing:

- Key advantage: from low utilization rates ($< 10\%$) to near constant use of vehicles
- Key challenge: rebalancing problem
  - overnight trucks rebalance the vehicles
  - or, cars to be returned to initial parking lot

Mobility-on-demand with driverless cars

- In the meantime, autonomous driving is becoming a reality (Google car has driven $>$ 200K miles in real traffic)

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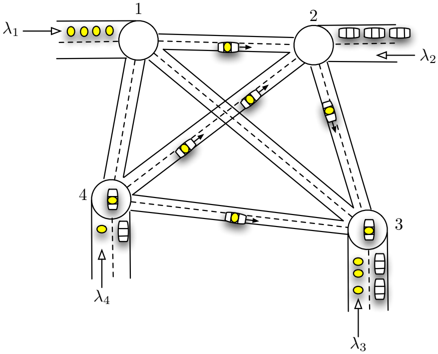
Next generation MOD: one way sharing + car sharing via driverless cars

In this talk:

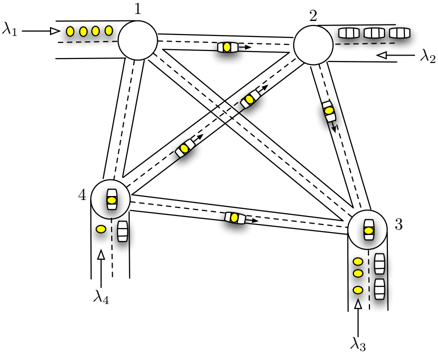
- Effective rebalancing for next-generation MOD systems

- The rebalancing problem for driverless cars:



Rebalancing via driverless cars

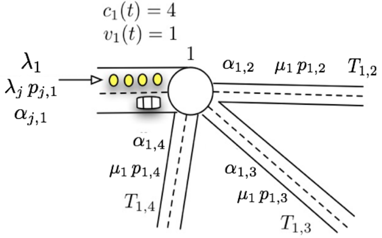
- The rebalancing problem for driverless cars:



Related work:

- Dynamic Traffic Assignment [Peeta and Ziliaskopoulos, 2001]
- Dynamic one-to-one Pick-up and Delivery [Berbeglia et al., 2010]
- Dynamic load balancing in computing systems [V. Cardellini et al., 1999]

Modeling the system



Input data:

- λ_i = arrival rate of customers
- μ_i = departure rate of customers ($> \lambda_i$)
- p_{ij} = prob. customer is destined for station j
- T_{ij} = travel time from i to j

State:

- $c_i(t)$ = customers at station i
- $v_i(t)$ = vehicles at station i

Rebalance control:

α_{ij} = rate at which empty vehicles are sent from i to j

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Fluid approximation:

- Customers and vehicles are continuous quantities
- Akin to taking expectation of stochastic process
- Evolve according to differential equations

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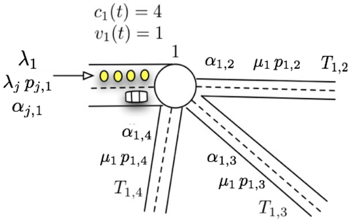
$$\dot{c}_i(t) = \text{in-rate cust.} - \text{out-rate cust.} = f\left(c_i(t), v_i(t), \lambda_i, \mu_i\right)$$

$$\dot{v}_i(t) = \text{in-rate veh}_{\text{full}} - \text{out-rate veh}_{\text{full}} + \text{in-rate veh}_{\text{empty}} - \text{out-rate veh}_{\text{empty}}$$
$$= g\left(c_i(t), v_i(t), \mathbf{c}_{0:t}, \mathbf{v}_{0:t}, \lambda, \mu, p, \alpha\right)$$



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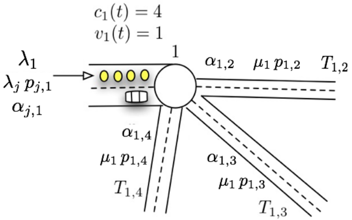


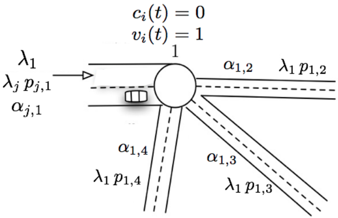
Non-linear, time-delayed, and discontinuous DE:

- Solutions exist in the Filippov's sense

Equilibria

- For the existence of equilibria, we require (balance equations)

$$\underbrace{\lambda_i + \sum_j \alpha_{ij}}_{\text{vehicles out at station } i} = \underbrace{\sum_j \alpha_{ji} + \sum_j \lambda_j p_{ji}}_{\text{vehicles in at station } i}$$



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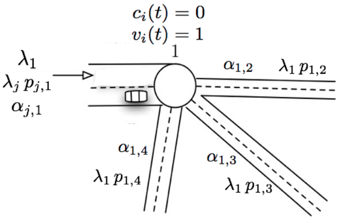

- Minimum number of vehicles for given α

$$V_\alpha^{\min} := \sum_{i,j} \underbrace{\lambda_i p_{ij} T_{ij}}_{\#\text{ of } i \to j \text{ full veh.}} + \underbrace{\alpha_{ij} T_{ij}}_{\#\ i \to j \text{ rebal. veh.}}$$

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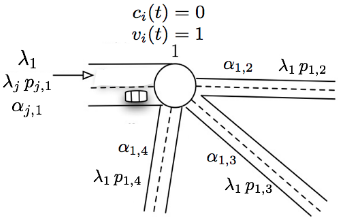

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- For α satisfying balance eqns and $V > V_\alpha^{\min}$, equilibria are

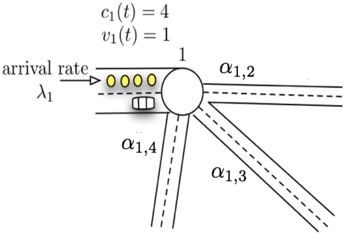
$$\mathcal{E}_\alpha := \left\{ (\mathbf{c}, \mathbf{v}) \mid c_i = 0,\ v_i > 0,\ \text{and} \sum_i v_i = V - V_\alpha^{\min} \right\}$$

The vehicle rebalancing problem

Goal: vehicle rebalancing

Select α_{ij} such that:

1. number of customers at each station remains bounded
2. number of rebalancing trips is minimized



- Solution: choose α to minimize # of rebalance trips (and V_α^{min}):

$$\begin{aligned} \text{minimize} \quad & \sum_{i,j} T_{ij}\alpha_{ij} \\ \text{subject to} \quad & \text{balance equations} \\ & \alpha_{ij} \geq 0 \end{aligned}$$

- If triang. inequality applies, can be formulated with fewer constraints

Stability of equilibria

Stability with respect to changes in number of customers and vehicles?

Stability of equilibrium set

For any $\bar{\alpha}$ satisfying balance eqns, if $V > V_{\bar{\alpha}}^{\min}$, then

- $\mathcal{E}_{\bar{\alpha}}$ is locally asymptotically stable with $\alpha_{ij}(t) = \bar{\alpha}_{ij}$;

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Stability of equilibrium points

For any $\bar{\alpha}$ satisfying balance eqns, if $V > V_{\bar{\alpha}}^{\min}$, then

- $(0, v_i^{\mathrm{desired}})$ is locally asymptotically stable with

$$\alpha_{ij}(t) = \bar{\alpha}_{ij} + \left(v_i(t) - v_i^{\mathrm{desired}}\right)^+.$$

Real-time version of optimization

- Previous policies require the knowledge of p_{ij}, λ_i, and μ_i and operate under the assumption of average behavior

Rebalance based on current situation

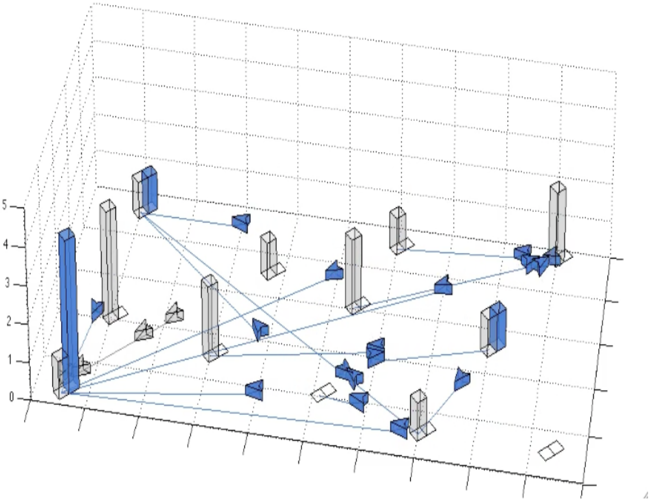
- $\texttt{free}_i = v_i(t) - c_i(t)$
- $\texttt{desired}_i = \dfrac{V - \sum_i c_i(t)}{\#\text{ of stations}}$
- $\texttt{excess}_i = \texttt{free}_i - \texttt{desired}_i$

Optimization based on current data becomes

$$
\begin{aligned}
\text{minimize} \quad & \sum_{i,j} T_{ij}\texttt{num}_{ij} \\
\text{subject to} \quad & \texttt{excess}_i + \sum_j \texttt{num}_{ji} - \sum_j \texttt{num}_{ij} \geq 0 && \forall\, i \\
& \texttt{num}_{ij} \in \{0, 1, 2, \ldots\} && \forall\, i, j
\end{aligned}
$$

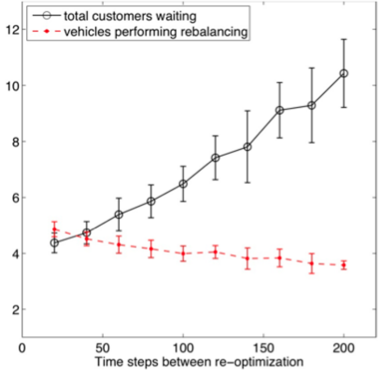
where $\texttt{num}_{ij}$ is number of vehicles sent from i to j

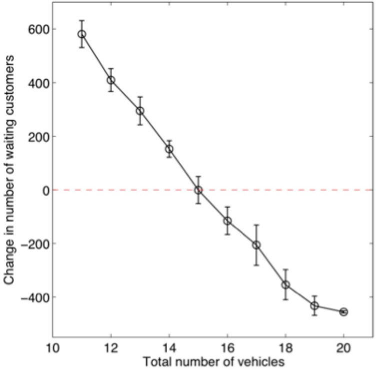
Applying fluid results to stochastic setting



Solves rebalance every 100 time steps

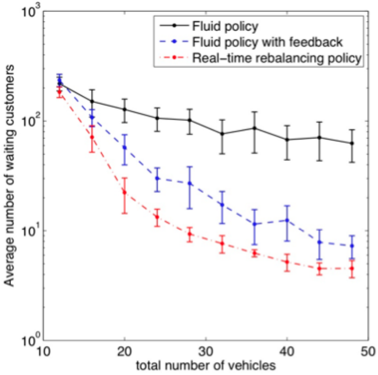
- $V^{\min} \geq 14$

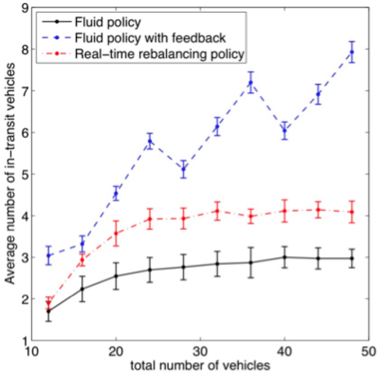

total customers waiting
vehicles performing rebalancing
12
10
8
6
4
2
0
50
100
150
200
Time steps between re–optimization


600
400
200
0
−200
−400
Change in number of waiting customers
10
12
14
16
18
20
Total number of vehicles

Policy comparison

- $V^{\min} \geq 14$





Conclusion

Key message: robotics can enable metropolis-wide MOD systems

Future directions in vehicle rebalancing

- Analysis of stochastic queueing model
- Customers with time windows
- Design of incentive mechanisms

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Hurdles to overcome

- Integration of system-wide coordination and autonomous mobility
- Safety / correctness / certification issues

Opportunities:

- Technology: autonomous driving, localization, traffic estimation, DVR, dynamic pricing, dynamic game theory, business models...
- Market: virtually any large urban area