A Nonlinear Dynamic Model and Free Vibration Analysis of Deployable Mesh Reflectors

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This paper presents a dynamic model of deployable mesh reflectors, in which geometric and material nonlinearities of such a space structure are fully described. Then, by linearization around an equilibrium configuration of the reflector structure, a linearized model is obtained. With this linearized model, the natural frequencies and mode shapes of a reflector can be computed. The nonlinear dynamic model of deployable mesh reflectors is verified by using commercial finite element software in numerical simulation. As shall be seen, the proposed nonlinear model is useful for shape (surface) control of deployable mesh reflectors under thermal loads.

I. Introduction

LARGE deployable mesh reflectors are of continued R&D interest for space applications. Examples include many renowned projects such as ETS VIII for satellite communication, MBSAT for global broadcasting, "NEXRAD in Space (NIS)" mission for remote sensing and climate forecasting and GEO-mobile satellites by Boeing for mobile communications [1-4]. Due to the stringent requirements on surface performance of a reflector to serve signal with high accuracy, high resolution and wide frequency bandwidth, of a large-sized deployable reflectors with small surface RMS errors are in urgent demand, the development of which, however, has already been a challenge for years [5]. Considering the performance limitation in passive structure and the manufacturing tolerance [6, 7], it has been suggested that active surface (shape) control must be introduced on the deployable space reflectors [8].

To develop an active surface control technique, a control-orientated dynamic model of deployable mesh reflectors is necessary. Following the authors' previous effort on the design of initial profile of mesh reflectors [9, 10], this paper presents a method to build up a control-orientated dynamic model. The model is based on the structural dynamic theories [11 - 14], and should be ready for the design of feedback surface control laws in future research. The structure of the mesh reflector is shown in Fig. 1. The reflector is modeled as a nonlinear truss structure, whose elements can only sustain axial tension stress. The structure is fixed on the boundary and the working surface is formed by the truss elements. The tension ties are connected to the nodes, which provide

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the vertical external loads.

The remainder of the paper is arranged as follows. In Section II, a 3-D nonlinear dynamic model of the reflector is presented. In Section III, linearization based on a nonlinear static equilibrium of the reflector leads to a linearized dynamic model of the reflector. The models are validated in numerical simulation in Section IV, and conclusions are made in Section V.

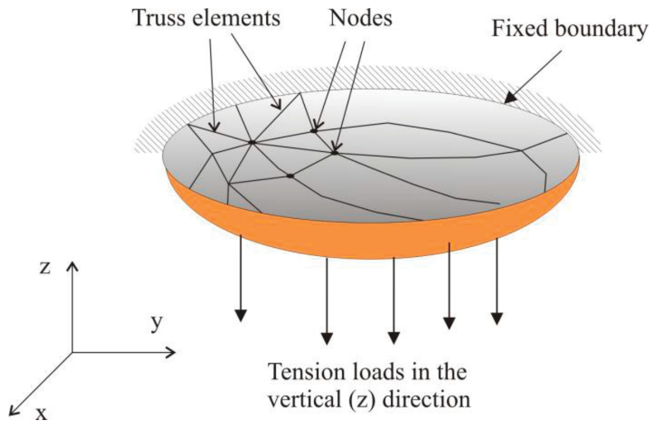


Figure 1. The deployable mesh reflector in consideration

II. Nonlinear Dynamic Model

As it mentioned previously, the deployable mesh reflector in consideration is viewed as a 3-D truss structure, with each element only sustaining axial (tension) stress. For the kth element of the truss of initial length L_k (given undeformed length), let the nodes (ends) of the element after deformation be described by $(x_{i,d}, y_{i,d}, z_{i,d})$ and $(x_{j,d}, y_{j,d}, z_{j,d})$, where i, j are the indexes of the ends of the kth element. The vector of nodal coordinates of the element can be written as

$$\{y_d\}_k = \begin{Bmatrix} x_{i,d} \\ y_{i,d} \\ z_{i,d} \\ x_{j,d} \\ y_{j,d} \\ z_{j,d} \end{Bmatrix} \tag{1}$$

By Eq. (1), the strain of the element is of the form

$$\varepsilon_k\left(\{y_d\}_k\right) = \frac{\Delta L_k}{L_k} = \frac{l_k - L_k}{L_k} = \frac{\sqrt{\left(x_{i,d} - x_{j,d}\right)^2 + \left(y_{i,d} - y_{j,d}\right)^2 + \left(z_{i,d} - z_{j,d}\right)^2} - L_k}{L_k} \tag{2}$$

where ΔL_k is the elongation of the element and l_k is the deformed element length. Here it is assumed that the truss structure is only deformed under external loads without any thermal distortion. Since the deployable mesh reflectors are used in space with negligible gravity, the potential energy of an element is purely the elastic energy. The potential energy of the element due to elastic deformation is then

$$V_k = \int_0^L \int_0^{\varepsilon_k} E_k(\varepsilon)\varepsilon A(\varepsilon, s) d(\varepsilon ds) = \int_0^L \int_0^{\varepsilon_k} E_k(\varepsilon)\varepsilon A(\varepsilon, s) d\varepsilon ds \tag{3}$$

where $A(\varepsilon, s)$ is the area of cross section at axis-location, and a nonlinear stress-strain relation $\sigma = E(\varepsilon)\varepsilon$ has been adopted. The kinematic energy is

$$T = T_t + T_r \tag{4}$$

where T_t is the kinetic energy due to translation, and T_r due to rotation about the longitudinal axis of the element. Because the moment of inertia of the element about the longitudinal axis is small, $T_r \approx 0$. The center of mass of the element is

$$s_{c,k} = \frac{\int_0^L m_k(s) s ds}{M_k} = \alpha_{c,k} L \tag{5}$$

where M_k is the total mass of the element. Then the velocity of the center of mass is obtained as

$$\begin{aligned} \dot{x}_{c,k} &= \alpha_{c,k}\dot{x}_{j,d} + (l - \alpha_{c,k})\dot{x}_{i,d} \\ \dot{y}_{c,k} &= \alpha_{c,k}\dot{y}_{j,d} + (l - \alpha_{c,k})\dot{y}_{i,d} \\ \dot{z}_{c,k} &= \alpha_{c,k}\dot{z}_{j,d} + (l - \alpha_{c,k})\dot{z}_{i,d} \end{aligned} \tag{6}$$

which in a matrix format is

$$\{\dot{y}_c\}_k = [\alpha]\{\dot{y}_d\}_k \tag{7}$$

with

$$[\alpha]_k = \begin{bmatrix} 1-\alpha_{c,k} & 0 & 0 & \alpha_{c,k} & 0 & 0 \\ 0 & 1-\alpha_{c,k} & 0 & 0 & \alpha_{c,k} & 0 \\ 0 & 0 & 1-\alpha_{c,k} & 0 & 0 & \alpha_{c,k} \end{bmatrix} \tag{8}$$

$$\{y_c\}_k = \begin{bmatrix} x_{c,k} \\ y_{c,k} \\ z_{c,k} \end{bmatrix} \tag{9}$$

Therefore, the kinematic energy of the element is

$$T_k = T_{t,k} = \frac{1}{2} M_k \left(\dot{x}_{c,k}^2 + \dot{y}_{c,k}^2 + \dot{z}_{c,k}^2\right) \tag{10}$$

Under the virtual work of external nodal forces at the kth element is

$$\delta W_{nc,k} = \delta\{y_d\}_k^T \{F_k\} \tag{11}$$

For the entire structure, we have

$$\delta V = \sum_1^m \delta V_k = \delta\{y_{def}\}^T \sum_1^m \left\{\frac{\partial \varepsilon_k}{\partial\{y_{def}\}}\right\}^T E_k(\varepsilon_k)\varepsilon_k \int_0^L A(\varepsilon_k, s) ds \tag{12}$$

$$\delta T = \sum_1^m \delta T_k = -\delta\{y_{def}\}^T \sum_1^m M_k \left\{\frac{\partial\{y_c\}_k}{\partial\{y_{def}\}}\right\}^T \{\ddot{y}_c\}_k \tag{13}$$

$$\delta W_{nc} = \sum_1^m \delta W_{nc,k} = \delta\{y_{def}\}^T \{Q\} \tag{14}$$

where m is the number of elements of the truss, $\{y_{def}\}$ is the global coordinate vector of all nodes (after deformation), and $\{Q\}$ is the vector of the external forces applied at the nodes. By the extended Hamilton principle

$$\int_{T_1}^{T_2}\left(\delta W_{nc}-\delta V+\delta T\right)dt=0 \tag{15}$$

$$\delta\left\{y_{def}\right\}^T\int_{T_1}^{T_2}\left(\{Q\}-\sum_1^m\left\{\frac{\partial\varepsilon_k}{\partial\left\{y_{def}\right\}}\right\}^T E_k\left(\varepsilon_k\right)\varepsilon_k\int_0^L A\left(\varepsilon_k,s\right)ds-\sum_1^m M_k\left\{\frac{\partial\left\{y_c\right\}_k}{\partial\left\{y_{def}\right\}}\right\}^T\left\{\ddot{y}_c\right\}_k\right)dt=0 \tag{16}$$

It follows that the nonlinear equations of motion of the deployable mesh reflector are in the matrix form

$$\sum_1^m M_k\left\{\frac{\partial\left\{y_c\right\}_k}{\partial\left\{y_{def}\right\}}\right\}^T\left\{\ddot{y}_c\right\}_k+\sum_1^m\left\{\frac{\partial\varepsilon_k}{\partial\left\{y_{def}\right\}}\right\}^T E_k\left(\varepsilon_k\right)\varepsilon_k\int_0^L A\left(\varepsilon_k,s\right)ds=\{Q\} \tag{17}$$

From Eq. (8) and (9), it can be concluded that $\left\{\ddot{y}_c\right\}_k$ is a linear function of $\left\{y_{def}\right\}$,

$$\left\{\ddot{y}_c\right\}_k=[\gamma]_k\left\{\ddot{y}_{def}\right\} \tag{18}$$

where $[\gamma]_k=[\alpha]\dfrac{\partial\left\{y_d\right\}_k}{\partial\left\{y_{def}\right\}}$ and it is a constant matrix. Thus, Eq. (17) can be rewritten as

$$[M]\left\{\ddot{y}_{def}\right\}+\sum_1^m\left\{\frac{\partial\varepsilon_k}{\partial\left\{y_{def}\right\}}\right\}^T E_k\left(\varepsilon_k\right)\varepsilon_k\int_0^L A\left(\varepsilon_k,s\right)ds=\{Q\} \tag{19}$$

with $[M]=\left(\sum_1^m M_k[\gamma]_k^T[\gamma]_k\right)$.

III. Linearized Model and Vibration Analysis

One common way of designing shape controller for a deployable mesh reflector is to derive a linearized model based on the nonlinear equation (19) of motion. Let $\left\{\bar{y}_{def}\right\}$ represent a static equilibrium configuration of the reflector. Here $\left\{\bar{y}_{def}\right\}$ is the solution of the equilibrium equation

$$\sum_1^m\left\{\frac{\partial\varepsilon_k}{\partial\left\{y_{def}\right\}}\right\}^T E_k\left(\varepsilon_k\right)\varepsilon_k\int_0^L A\left(\varepsilon_k,s\right)ds=\{Q\} \tag{20}$$

where is obtained from E. (19) by dropping time-dependent quantities. Equation (20) can be solved by using the nonlinear solver developed in the authors' previous work [9]. Now, consider a small perturbation $\left\{\Delta y_{def}\right\}$ from the equilibrium configuration

$$\left\{y_{def}\right\}=\left\{\bar{y}_{def}\right\}+\left\{\Delta y_{def}\right\} \tag{21}$$

By Taylor's expansion, Eq. (19) is reduced to the linear equation of motion

$$[M]\left\{\Delta\ddot{y}_{def}\right\}+[K]\left\{\Delta y_{def}\right\}=\{P\} \tag{22}$$

where

$$
\begin{aligned}
[K] &= \frac{\partial}{\partial\{y_{def}\}}\left(\sum_{1}^{m}\left\{\frac{\partial\varepsilon_k}{\partial\{y_{def}\}}\right\}^T E_k(\varepsilon_k)\varepsilon_k\int_0^L A(\varepsilon_k,s)\,ds\right)\Bigg|_{\{\bar{y}_{def}\}} \\
\{P\} &= \{Q\}-[M]\{\ddot{y}_{def}\}\Big|_{\{\bar{y}_{def}\}}-\left(\sum_{1}^{m}\left\{\frac{\partial\varepsilon_k}{\partial\{y_{def}\}}\right\}^T E_k(\varepsilon_k)\varepsilon_k\int_0^L A(\varepsilon_k,s)\,ds\right)\Bigg|_{\{\bar{y}_{def}\}}
\end{aligned}
\tag{23}
$$

For the linearized model described by Eq. (22), the eigenvalue problem is

$$\left(-\omega_i^2[M]+[K]\right)\{\mu\}_i=0 \tag{24}$$

where ω_i is the ith natural frequency of the deployable mesh reflector, and $\{\mu\}_i$ is the corresponding mode shape. The natural frequencies (eigenvalues) of the structure are the roots of the characteristic equation

$$\det\left(-\omega_i^2[M]+[K]\right)=0 \tag{25}$$

IV. Numerical Simulation and Discussion

The proposed nonlinear dynamic model and linearized model are applied to two examples in numerical simulation: a six-element truss and a 90-element spherical reflector structure.

Example 1. A six-element truss

Before the nonlinear dynamic model is applied to a mesh reflector, it is necessary to verify its correctness and accuracy. For this reason, we first consider a simple truss structure of six nodes and six elements, which is under external forces in the vertical (z) direction (see Fig. 2). For simplicity, it is assumed that truss elements have linear stress-strain relation and uniform geometry. As shown in the figure, nodes 1, 2 and 3 are fixed, and nodes 4, 5 and 6 are movable. The truss is subject to unity external loads at movable nodes 4, 5, 6 in the vertical down direction (negative z direction). The coordinates of the nodes are given in Table 1. All the elements have the same longitudinal rigidity EA = 10000 N/m^2.

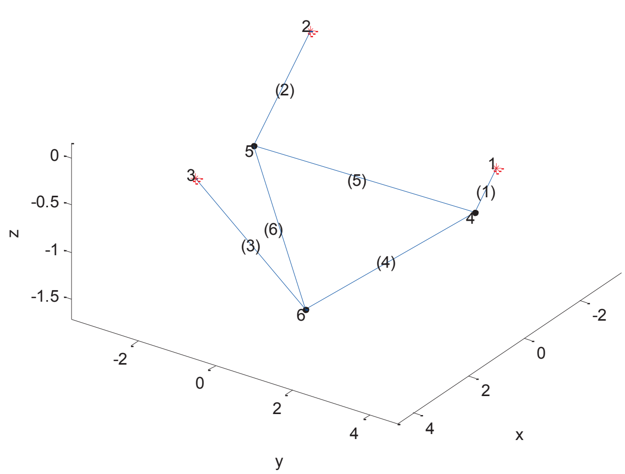


Figure 2 Initial (undeformed) shape of the example structure

Table 1. Nodal coordinates

Node Number	Initial configuration (m)				Deformed configuration (m)		
	$x_{i,ini}$	$y_{i,ini}$	$z_{i,ini}$		$x_{i,def}$	$y_{i,def}$	$z_{i,def}$
1	0	4	0		0	4	0
2	-2.8284	-2.8284	0		-2.8284	-2.8284	0
3	3.8730	-1	0		3.8730	-1	0
4	-2	2	-2.1231		-0.05586	2.5944	-2.9210
5	-1	-3	-1.8729		-1.9808	-2.1293	-2.7113
6	2	0.5	-2.5552		2.4379	-0.8577	-3.4695

The forced response of the truss is computed, to compare the proposed method with the finite element method (FEM). Three unity external loads are applied to the undeformed truss (Fig. 2) at the movable nodes 4, 5, 6 in the negative z direction. To compute the transient response by the proposed model, Eq. (19) is cast into a state equation, which is then solved by numerical integration with Runge-Kutta method. Shown in Fig. 3 is the forced response of the truss at node 4, where the solid lines are the results by the proposed method, the dashed ones by the finite element code ANSYS. Fairly good agreement between the proposed method and FEM is observed.

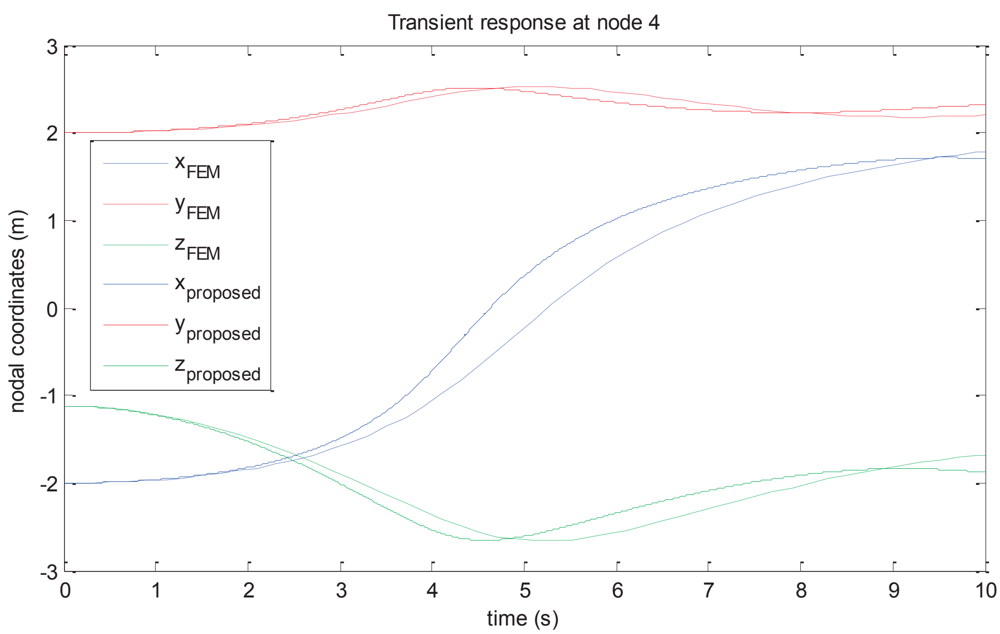
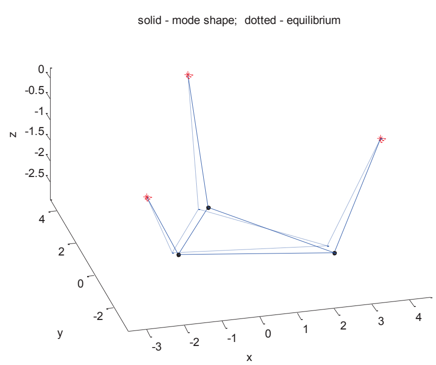


Figure 3. Forced response of the six-element truss at node 4

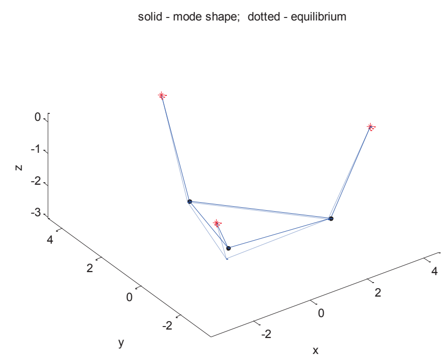
With the linearized model described by Eq. (23) (based on the unity loads described previously), the eigenvalue problem (24) of the truss is solved. The first nine natural frequencies are listed in Table 2, and first four mode shapes of the truss are plotted in Figs. 4.

Table 2. Natural frequencies of linearized model

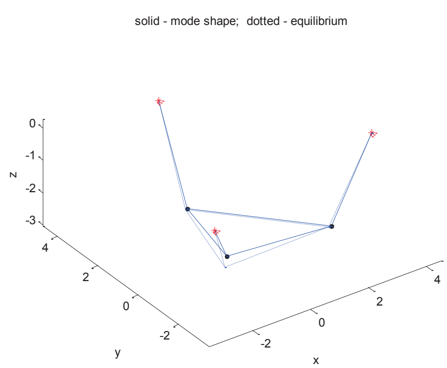
Mode Number	ω_i (rad/s)	Mode Number	ω_i (rad/s)	Mode Number	ω_i (rad/s)
1	0.2987	4	21.0312	7	45.3164
2	0.3536	5	32.5099	8	52.5608
3	0.6261	6	34.9730	9	67.3926



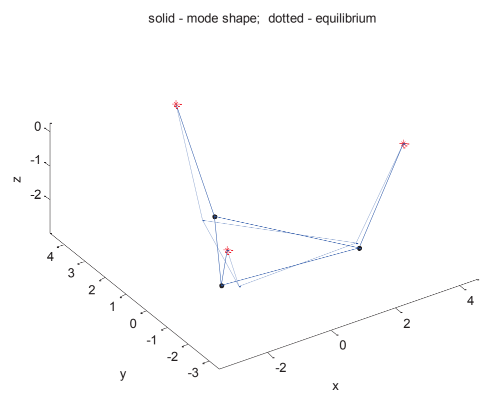
Mode 1



Mode 2



Mode 3



Mode 4

Figure 4. The first four mode shapes of the truss

B. A model of deployable mesh reflector

A deployable mesh reflector is shown in Fig. 5, which is modeled as a truss structure with 37 nodes and 90 members. The reflector is deployed into a spherical surface of diameter D = 30 m and height H = 11.18 m, which is deployed by properly determined external loads that are applied in vertical (negative z) direction. The truss elements of the structure have the nonlinear strain-stress relation $\sigma = E(\varepsilon)\varepsilon$ with

$$E(\varepsilon) = \begin{cases} E_0 & \varepsilon > 0 \\ 0 & \varepsilon \le 0 \end{cases} \tag{26}$$

For the numerical simulation, the Young's modules is $E_0 A$ = 1.1121e+005 N/m^2.

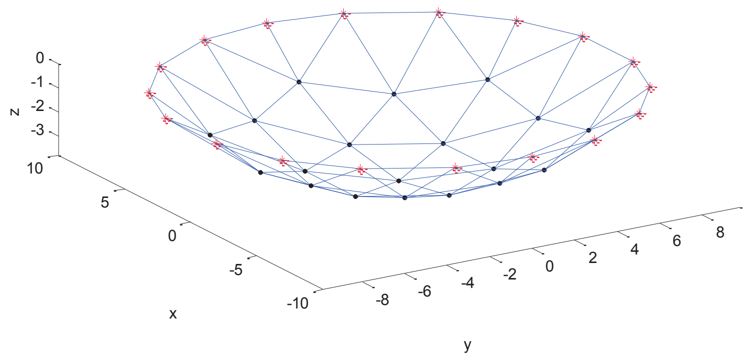


Figure 5. A deployed mesh reflector

The the natural frequencies of linearized model at the equilibrium configuration are calculated. The range of ω_i is from 13.6346 *rad/s* to 195.6108 *rad/s*, with the first four being $\omega_1 = \omega_2 = 13.6346$ rad/s , $\omega_3 = 14.7368$ rad/s, $\omega_4 = 16.0460$ rad/s . Due to the axis symmetry of the reflector, repeated natural frequencies appear in pairs (for instance, the first two) . The first four mode shapes of the reflector are plotted in Figure 6, where the solid lines portray mode shapes and dotted lines represent the equilibrium configuration of the truss.

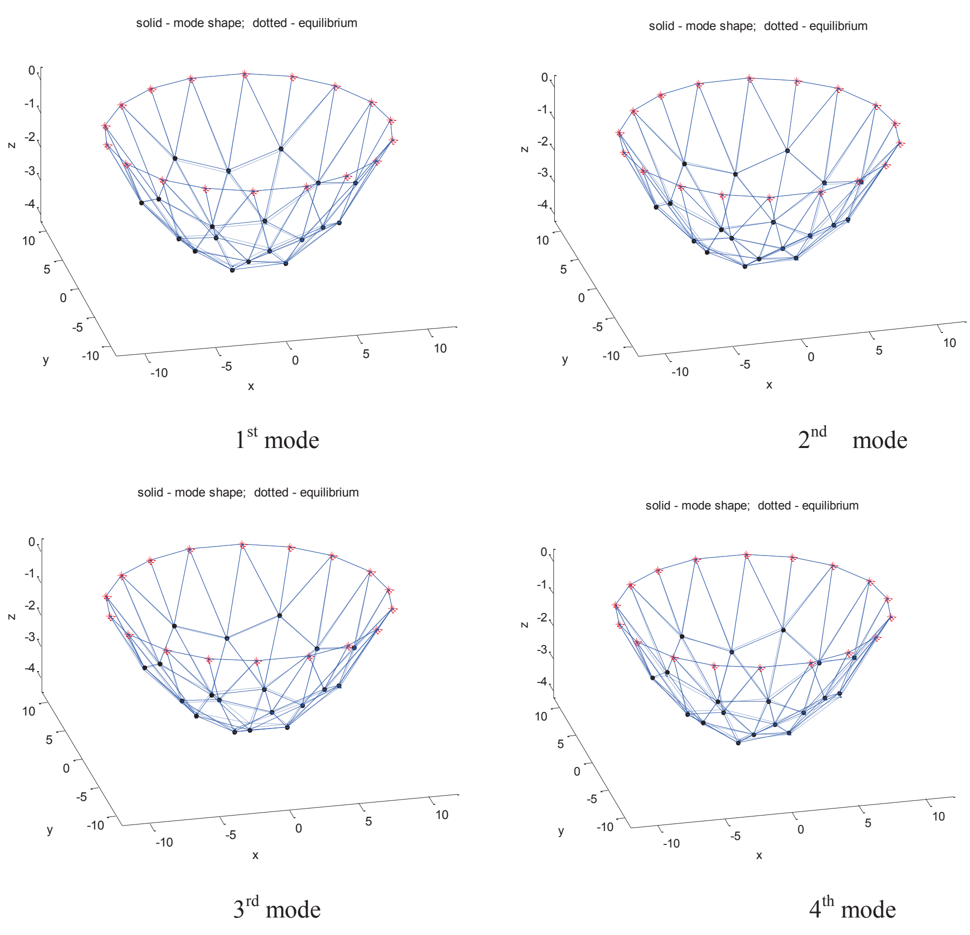


Figure 6. The first four mode shapes of the mesh reflector.

V. Conclusion

A nonlinear dynamic model and a corresponding linearized model for deployable mesh reflectors have been obtained. The numerical predictions by the proposed nonlinear model are in good agreement with the results obtained by commercial finite element software. The linearized model enables free vibration analysis of deployable mesh reflectors, which provides important information on the dynamic properties of the reflector structure. The nonlinear and linearized models will play an important role in shape control of deployable mesh reflectors under thermal loads, which will be addressed in a follow-up investigation.

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References

1 Natori, M.C., Takano, T., Inoue, T., Noda, T., Design and Development of a Deployable Mesh Antenna for MUSES-B Spacecraft, *34th AIAA /ASME /ASCE /AHS /ASC Structures, Structural Dynamics and Materials Conference*, Apr. 19-22, La Jolla, California, 1993

2 Meguro A., Tsujihata, A., Hamamoto, N., The 13 m Aperture Space Antenna Reflectors For Engineering Test Satellite VIII**,** *Antennas and Propagation Society International Symposium, IEEE*, Vol. 3, pp. 1520-1523, Aug. 1999

3 Thomson, M., AstroMesh™ Deployable Reflectors for Ku and Ka Band Commercial Satellites, *20th AIAA International Communication Satellite Systems Conference and Exhibit*, Montreal, Quebec, May 12-15, 2002

4 Im, E., Smith, E.A., Durden, S.L., Huang, J., Rahmat-Samii, Y., and Lou, M., Instrument Concept of NEXRAD In Space (NIS) – A Geostationary Radar For Hurricane Studies, Proceedings of Intl. *Geoscience and Remote Sensing Symposium*, Toulouse, France, 2003

5 Hachkowski, M. R., Peterson, Lee D., A Comparative History of the Precision of Deployable Spacecraft Structures, *University of Colorado, Center for Space Construction*, Paper No. CU-CAS-95-22, December 1995

6 Hedgepeth, J. M., Accuracy Potentials for Large Space Antenna Reflectors with Passive Structure, *Journal of Spacecraft and Rockets*, Vol. 19, No. 3 pp 211-217, 1982

7 Hedgepeth, J. M., Influence of Fabrication Tolerances on the Surface Accuracy of Large Antenna Structures, *AIAA Journal*, Vol. 20, No. 5, pp 680-686, 1982

8 Gawronski, W. K., *Advanced Structural Dynamics And Active Control Of Structures*, Springer, 2004

9 Shi, H., Yang, B., Thomson, M. and Fang, H., Nonlinear Modeling And Optimal Initial Profile Solution For Deployable Mesh Reflectors, *51th AIAA/ASME/ASCE /AHS/ASC Structures, Structural Dynamics and Materials Conference*, Orlando, FL, April. 13 - 16, 2010

10 Yang, B., Shi, H., Thomson, M. and Fang, H, Optimal Mounting of Deployable Mesh Reflectors Via Static Modeling and Quadratic Programming, *50th AIAA/ASME/ASCE /AHS/ASC Structures, Structural Dynamics and Materials Conference*, Palm Spring, CA, May. 4-7, 2009

11 Meirovitch, L., *Principles And Techniques Of Vibrations*, Prentice Hall, 1997

12 Yang, B., *Stress, Strain, And Structural Dynamics*, Elsevier, 2005

13 Pai, P. F., *Highly Flexible Structures: Modeling, Computation, And Experimentation*, AIAA, 2007

14 Nayfeh, A. H. and P. Frank Pai, *Linear And Nonlinear Structural Mechanics*, Wiley, 2004