Design and verification of external occulters for direct imaging of extrasolar planets

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Finding planets

1995: 51 Pegasi b is the first planet found around a Sun-like star (Mayor and Queloz 1995).

Since then: over 500 planets found and counting, mostly hot, close gas giants.

We’d like to start finding objects similar to Earth.
Planet-finding methods

The problem: How do we find and characterize Earth-size planets?

Many approaches to find planets:
- Radial velocity
- Transits
- Astrometry
- Microlensing
- Pulsar timing
- Direct imaging (in particular, from space)
Going to space

Some advantages in space over ground-based telescopes, even very large ones!
- No atmospheric aberrations (Reduced adaptive optics requirements.)
- No atmospheric absorption (In particular, can see UV.)

There are still two problems to overcome, though:

**Diffraction:**

Airy pattern: Point Spread Function (PSF) of circular aperture

**Contrast:**

SAO Solar System Model at 10 PC

\[ \sim 10^{10} \]
How to do direct imaging from space?

There are a few approaches:

- coronagraphs,
- interferometers,
- and occulters.

and I’ll focus on occulters for this talk.
Occulter design
What is an occulter?

Occulter design

An occulter is an optical element which is placed in front of the telescope to block most of the light from a star before it reaches the optics inside, without blocking the planet.

In our case, we use two spacecraft flying in formation:
- First has its edge shaped to cancel the starlight
- Second is the telescope which images the star and planet

Eric Cady (JPL)
Why not a disk for planets?

Problem with a simple disk: while geometric optics predicts complete suppression, wave optics predicts diffraction around the edges.

1. On-axis, creates Poisson’s spot in the center of the shadow, where the intensity is not attenuated at all
2. Off-axis, full diffraction calculation shows we can only suppress the starlight by $\sim 10^3$
Designing an occulter (I)

Following (Vanderbei, Cady, and Kasdin 2007), we start by thinking about Fresnel propagation from a plane wave incident on an apodized aperture with circular symmetry:

If the apodization is given by a function $A(r)$, then after a distance $z$, the electric field is:

$$E_{ap}(\rho) = E_0 e^{ikz} \left( \frac{2\pi}{i\lambda z} \int_0^R e^{\frac{\pi i}{\lambda z} (r^2 + \rho^2)} J_0 \left( \frac{2\pi r \rho}{\lambda z} \right) A(r) r \, dr \right)$$
Designing an occulter (II)

An apodized occulter is the complement of this aperture:

We can write the equation for this using Babinet’s Principle:

\[
E_{\text{occ}}(\rho) = E_0 e^{ikz} \left(1 - \frac{2\pi}{i\lambda z} \int_0^R e^{\frac{\pi i}{\lambda z} (r^2 + \rho^2)} J_0 \left(\frac{2\pi r \rho}{\lambda z}\right) A(r) r \, dr\right)
\]

\[
= E_0 e^{ikz} - E_{\text{ap}}(\rho)
\]
Designing an occulter (III)

Can’t build an apodized occulter from real materials, so we convert it to a binary occulter with $N$ petals:

Choose petal width so electric field mostly unchanged for small $\rho$:

\[
E_{\text{bin}}(\rho, \phi) = E_{\text{occ}}(\rho) - E_0 e^{ikz} \sum_{j=1}^{\infty} \frac{4\pi(-1)^j}{i\lambda z} \cos[jN(\phi - \pi/2)]
\]

\[
\times \int_0^R e^{\frac{\pi i}{\lambda z} (r^2 + \rho^2)} J_{jN} \left( \frac{2\pi r \rho}{\lambda z} \right) \frac{\sin (j\pi A(r))}{j\pi} r \, dr
\]
How do we choose the width of the petals?

1. For profile $A(r)$, we design the petals so that a circle of radius $r$ has a fraction of the circle equal to $A(r)$ blocked by a petal.

2. We repeat the petals $N$ times to place the scattered light outside the aperture.

![Diagram illustrating occulter design](image)
Designing an occulter (V)

Lastly, need to choose $A(r)$ and $N$. We choose $A(r)$ by setting up a linear optimization on the real and imaginary parts of $E_{\text{occ}}$ (Vanderbei et al. 2007, Cady et al. 2008) to constrain the intensity at the telescope pupil:

$$E_{\text{occ}}(\rho; \lambda) = E_0 e^{i k z} \left(1 - \frac{2\pi}{i \lambda z} \int_0^R e^{\frac{\pi i}{\lambda z} (r^2 + \rho^2)} J_0 \left(\frac{2\pi r \rho}{\lambda z}\right) A(r) r \, dr\right)$$

- Strict bounding should be $\text{Re}(E_{\text{occ}}(\rho; \lambda))^2 + \text{Im}(E_{\text{occ}}(\rho; \lambda))^2 \leq c$, with $c$ an upper bound on the intensity; these are quadratic constraints on $A(r)$.
- Bounding the real and imaginary parts independently introduces some slightly conservative assumptions, but assures the optimization remains linear.
- Linear optimizations have globally optimal solutions, so we get good apodization profiles with each run of the optimization rather than getting caught in local minima.
- Even better, we can use $c$ as a variable and put it in the cost function, so we minimize the upper bound on the intensity.
Designing an occulter (VI)

We also add some constraints to ensure center of the occulter is solid (to ensure there is a place for the spacecraft bus) and the petal edges are smooth (as the optimization will tend to produce spiky bang-bang solutions otherwise). Can tweak further if desired.

The full problem:

Minimize: \( c \)

subject to:

\[
\text{Re}(E_{\text{occ}}(\rho; \lambda)) - c / \sqrt{2} \leq 0
\]
\[
- \text{Re}(E_{\text{occ}}(\rho; \lambda)) - c / \sqrt{2} \leq 0
\]
\[
\text{Im}(E_{\text{occ}}(\rho; \lambda)) - c / \sqrt{2} \leq 0
\]
\[
- \text{Im}(E_{\text{occ}}(\rho; \lambda)) - c / \sqrt{2} \leq 0
\]
\[
\forall \rho \leq \rho_{\max}, \lambda \in [\lambda_{\min}, \lambda_{\max}]
\]
\[
A(r) = 1 \quad \forall \quad 0 \leq r \leq a
\]
\[
A'(r) \leq 0, \quad |A''(r)| \leq \sigma \quad \forall \quad 0 \leq r \leq R
\]

Lastly, we choose \( N \) so \( |E_{\text{bin}} - E_{\text{occ}}| \ll c \) for \( \rho \leq \rho_{\max} \).
Optimized shadow

Result: a dark shadow at the telescope aperture.
A note on IWA

Inner working angle (IWA): the closest angle to the optical axis at which we can detect a planet of a given brightness.

- For coronagraphs and interferometers, this value is wavelength-dependent.
- For an occulter, this is approximately set by geometry:
  \[
  \text{geometric IWA} = \arctan \frac{R}{z} \approx \frac{R}{z}
  \]
  and is *not* wavelength-dependent.
- For most occulters, the IWA is in the range of 75 – 150 milliarcseconds (mas).
Point designs
THEIA (Telescope for Habitable Exoplanets and Interstellar/Intergalactic Astronomy) (Kasdin et al. 2009)

- Developed as part of the Astrophysics Strategic Mission Concept Studies
- 4m on-axis UVOIR telescope designed to work jointly with an occulter
- 10m petals on a 40m diameter occulter
- Operates at two distances from the telescope, each providing data in a different spectral band
Scaling the system

Making an occulter work at two distances doesn’t require any changes to shape.

\[ z \rightarrow za \]

\[ \lambda \rightarrow \lambda/a \]

\[ E_{occ}(\rho) = E_0 e^{\frac{2\pi i z}{\lambda}} \left( 1 - \frac{2\pi}{i\lambda z} \int_0^R A(r) J_0 \left( \frac{2\pi r \rho}{\lambda z} \right) e^{\frac{\pi i}{\lambda z} (r^2 + \rho^2)} r dr \right) \]

\[ \downarrow \]

\[ E_{occ}'(\rho) = E_0 e^{\frac{2\pi i za^2}{\lambda}} \left( 1 - \frac{2\pi}{i\lambda z} \int_0^R A(r) J_0 \left( \frac{2\pi r \rho}{\lambda z} \right) e^{\frac{\pi i}{\lambda z} (r^2 + \rho^2)} r dr \right) \]

Result: same electric field, within a constant phase factor; band goes from \([\lambda_L, \lambda_H]\) to \([\lambda_L/a, \lambda_H/a]\). Downside: inner working angle goes from \(\frac{R}{z}\) to \(\frac{R}{za}\). For THEIA, goes from 75mas to 118mas.
Occulting Ozone Observatory: $O_3$ (Kasdin et al. 2010)

- Designed to fit within the cost cap for a probe-class mission
- Uses an off-the-shelf telescope (Nextview) available from ITT for approx. $50 million.
- Occulter is smaller, provides suppression 250 – 550nm
- Designed to find Earth-like planets and look for the presence of ozone
- Fits all in a single rocket fairing; THEIA requires two launches
Intermediate-scale: 1.5m $O_3$ variant

Between these two, can think about designs for mid-sized telescopes.

Example: an occulter for a 1.5m telescope, 250-550nm.

A petal for this occulter design will be tested here at JPL:

- being built out of flight-like materials (ultra-low-CTE composites)
- will performing edge metrology to ensure that it meets tolerances

Tolerances slightly loosened (to $3 \times 10^{-10}$ from edge errors) for first test.

32m diameter, 6m petals. Both $O_3$-type designs can be used at two distances as well.
Occulters with JWST

Other studies look at using JWST (see e.g. Soummer et al. 2009)

- Capable of detection with NIRCam, spectroscopy with NIRSpec.
- Occulters would be 60-70m, in light of the longer wavelengths and larger aperture
- Requires additional starshade hardware for steering, as can’t add hardware to JWST
- Would require changes to optical filters in NIRSpec, possibly NIRCam to reduce out-of-band leak
Spanning the parameter space

The fact the same occulter works at two distances hints that not all of the input parameters to the optimization are independent. We can do a change of variables and rewrite the propagation integral in terms of independent nondimensional parameters:

$$r' \equiv \frac{r}{R}, \quad 0 \leq r' \leq 1$$
$$\rho' \equiv \frac{\rho}{\rho_{\text{max}}}, \quad 0 \leq \rho' \leq 1$$
$$N_o \equiv \frac{R^2}{\lambda z}, \quad N_1 \leq N_o \leq N_2$$
$$N_t \equiv \frac{\rho_{\text{max}}^2}{\lambda z}, \quad N_3 \leq N_o \leq \frac{N_3 N_2}{N_1}$$

$$E(\rho') = 1 + 2\pi i N_o \int_0^1 A'(r') J_0 \left(2\pi \sqrt{N_o N_t} r' \rho'\right) e^{\pi i \left(N_o r'^2 + N_t \rho'^2\right)} r' dr'$$

Three independent parameters bound $N_o$ and $N_t$ for broadband optimization, and let us examine the parameter space of possible occulters. We can mine this data to see what it takes for an occulter to meet desired science requirements.
Occulters for < 2m telescopes

Color gives required occulter diameter.
Occulter parameter space

Occulters for ~4m telescopes

Color gives required occulter diameter.
Occulters for > 6m telescopes

Color gives required occulter diameter.
Testing and verification
Testing occulter performance

Experimentally testing occulter optical performance at full-size is virtually impossible:

- Diameter is tens of meters
- Occulter-telescope distance is tens of thousands of kilometers

Instead, we make scale models to verify optical modeling on the ground and build full-scale petals to verify they meet the required optical tolerances. (This work is ongoing here at JPL and elsewhere.)
Testing and verification

Optical tests

Scaling the system (again)

The general strategy for optical testing: scale the system down by keeping \( \frac{r^2}{\lambda z} \) and \( \frac{\rho^2}{\lambda z} \) constant:

\[
z' \rightarrow \frac{z}{a^2} \quad r' \rightarrow \frac{r}{a} \quad \rho' \rightarrow \frac{\rho}{a}
\]

\[
E_o(\rho) = E_0 e^{\frac{2\pi i z}{\lambda}} \left( 1 - \frac{2\pi}{i \lambda z} \int_0^R A(r) J_0 \left( \frac{2\pi r \rho}{\lambda z} \right) e^{\frac{\pi i}{\lambda z} (r^2+\rho^2)} r \, dr \right)
\]

\[
\downarrow
\]

\[
E_o(\rho) = E_0 e^{\frac{2\pi i z'}{\lambda}} a^2 \left( 1 - \frac{2\pi}{i \lambda z'} \int_0^{R'} A(ar') J_0 \left( \frac{2\pi r' \rho'}{\lambda z'} \right) e^{\frac{\pi i}{\lambda z'} (r'^2+\rho'^2)} r' \, dr' \right)
\]

Result: same electric field, within a constant phase factor. \( a \) is on the order of \( 10^3-10^4 \).
# Overall design

Systems have three parts:

- A source to simulate the star
- A scaled-down occulter with a mount
- A detector (optionally with aperture and lens)

No other optics in the system. Approaches used now include:

**Wire-mounted mask**

Samuele et al. 2010

**Mask with etched mount (made at MDL)**

Cady et al. 2009

Testbeds at Princeton (Cady et al. 2009, 2010), NGAS (Samuele et al. 2009, 2010), University of Colorado (Schindhelm et al. 2007)
Subscale optical tests

Slice through the pupil plane in simulation (top) and experiment (bottom) at 633nm:

Expected contrast in dark hole: $3.98\times10^{-8}$

Actual contrast in dark hole: $5.74\times10^{-6}$

Still looking at causes for extra light in the center.
Mechanical design and deployment: $\text{O}_3$

Petals are wrapped around a central truss, unfurl and then fold down when the truss opens.
Deployment

A proof-of-concept model based on the 1.1m O₃ design was built at JPL, and the deployment was tested:
Metrology tests

Currently, a petal for the 1.5m design is being constructed to test manufacturing tolerances.

Faro arm used to measure edge position
Tolerances for current TDEM study

For the petal currently being manufactured, the following tolerances are specified.

<table>
<thead>
<tr>
<th>Perturbation</th>
<th>Requirement</th>
<th>Units</th>
<th>Mean Contrast</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportional Width</td>
<td>0.0001</td>
<td>n/a</td>
<td>1.0E-10</td>
</tr>
<tr>
<td>Tip Clip</td>
<td>15</td>
<td>mm</td>
<td>4.2E-11</td>
</tr>
<tr>
<td>In-plane quadratic bend</td>
<td>20</td>
<td>mm</td>
<td>1.4E-12</td>
</tr>
<tr>
<td>1 cycle per petal symmetric</td>
<td>141.4</td>
<td>um</td>
<td>5.3E-11</td>
</tr>
<tr>
<td>2 cycle per petal symmetric</td>
<td>50.0</td>
<td>um</td>
<td>1.6E-11</td>
</tr>
<tr>
<td>3 cycle per petal symmetric</td>
<td>27.2</td>
<td>um</td>
<td>3.0E-11</td>
</tr>
<tr>
<td>4 cycle per petal symmetric</td>
<td>17.7</td>
<td>um</td>
<td>6.2E-11</td>
</tr>
<tr>
<td>5 cycle per petal symmetric</td>
<td>12.6</td>
<td>um</td>
<td>6.8E-12</td>
</tr>
<tr>
<td>Symmetric residual &gt; 5 cycles</td>
<td>24.7</td>
<td>um</td>
<td>5.0E-13</td>
</tr>
</tbody>
</table>

Part of an allocation for a $10^{-9}$ occulter.

Only considering terms that can be investigated for a single petal; can’t look at position errors, alignment errors, deployment errors...
Tolerances for THEIA

The following are a subset of the tolerances on position and shape examined for THEIA.

<table>
<thead>
<tr>
<th>Petal Position or Shape Error</th>
<th>Allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>r.m.s shape (1/r^2 power law)</td>
<td>100 µm</td>
</tr>
<tr>
<td>Proportional shape</td>
<td>80 µm at max width</td>
</tr>
<tr>
<td>Length clipping at tip</td>
<td>1 cm</td>
</tr>
<tr>
<td>Azimuthal position</td>
<td>0.003 deg (1 mm at tip)</td>
</tr>
<tr>
<td>Radial position</td>
<td>1 mm</td>
</tr>
<tr>
<td>In-plane rotation about base</td>
<td>0.06 deg (1 cm at tip)</td>
</tr>
<tr>
<td>In-plane bending (r^2 deviation)</td>
<td>5 cm</td>
</tr>
<tr>
<td>Out-plane bending (r^2 deviation)</td>
<td>50 cm</td>
</tr>
<tr>
<td>Cross-track occulter position</td>
<td>75 cm</td>
</tr>
</tbody>
</table>


These are challenging, but not beyond the levels required for large deployable antennas in space now. Moreover, shape errors diffract light which is primarily confined to within the IWA, and can be mitigated further by spinning the occulter.
Above, the pupil and image planes of THEIA without errors. We can introduce errors in occulter position, orientation, and shape, and look at how these errors change the image plane contrast.
Lateral shift (75cm)

Shifts (and sources off the optical axis) can be modeled by modifying the pupil plane.

*But* perturbations are very slow; can do feedback control using low-resolution imagery in other bands.
Separation distance shift (1000km)

Changes in occulter-telescope distance require the integral to be recalculated with a different $z$, but tolerances are very high ($\sim 1000\text{km}$) and can be enlarged further by adjusting the optimization bounds:

$$\lambda z = (z + \Delta z) \left( \lambda - \frac{\lambda \Delta z}{z + \Delta z} \right) = (z - \Delta z) \left( \lambda + \frac{\lambda \Delta z}{z - \Delta z} \right)$$
Occulter tilts, to a first approximation, can be modeled by compressing the occulter: taking the projection onto the plane perpendicular to the optical axis. In general, a few degrees has little effect; orientation will be controlled much more finely than that.
Edge error ($100\mu m$ rms, $1/f^2$ power law)

Inexact edges diffract a lot of light—need very precise manufacturing.

*But* not beyond the levels required for large deployable antennas in space now, and primarily confined to within the IWA.

Expect better understanding of the edge errors after current study.
Bending modes

We separate bending modes into two major classifications for error budgeting:

- In-plane bending
- Out-of-plane bending

Exact modes can be determined from finite-element modes of the occulter; currently being modeled here at JPL (see for example Jordan et al. 2009).
Petals have to be well-placed in the plane—if they bend back and forth in the plane, they diffract light into places in the image plane where they could be confused with planets.

Again, this is difficult but not beyond state-of-the-art, and in-plane modes are smaller than out-of-plane modes.
Out-of-plane bend (1m, linear profile)

Petals bending up and down can also change the effective shape, and these are more natural vibrational modes for disks.

However, the out-of-plane tolerances are also much larger.
Future occulter work

Next steps for general occulter work:

- thermal-mechanical-optical simulations
- edge manufacturing and testing
- end-to-end formation flying simulations
- material testing
- further deployment testing...

Thanks to my collaborators and coworkers at JPL, Princeton, and elsewhere.

Thank you for your attention; I’d be happy to take any questions.
Additional slides
A brief history of occulters

First occulters were developed for observations of the solar corona (Evans 1948).

- Simple solid disks located a few meters in front of the telescope to block the entire solar disk
- Used on solar-observing spacecraft such as SOHO and STEREO

1962: Lyman Spitzer at Princeton proposed using an occulter *with a shaped edge*. Investigated many times since; notable mission designs include:

- Big Occulting Steerable Satellite (BOSS): Copi and Starkman 2000
- New Worlds Observer (NWO): Cash 2006
- New Worlds Probe, an occulter to be used with JWST: Soummer *et al.* 2009
- Occulting Ozone Observatory (O₃): Kasdin *et al.* 2009

Last two were designed by me with optimization tools.
THEIA science results

Savransky, Kasdin, and Cady 2010

THEIA ASMCS report

External occulters
This is for illustrative purposes:

A significant amount of stray-light control added:
Before and after

Outside the diffraction from the mask, indistinguishable from camera dark noise.
Hybrid occulters

We proposed using an apodized pupil Lyot coronagraph (APLC) with the occulter, in hopes of reducing the requirements on both. The coronagraph would be placed inside the telescope, behind the occulter’s shadow.
The problem with hybrids

Unfortunately: hybrids do not work as well as we would like:

- The wavefronts from the star can no longer be treated as flat and symmetric; starlight will escape the mask in the image plane.
- The hybrid has stricter alignment tolerances than an occulter alone, if the hybrid is doing a significant portion of the attenuation.
- The image-plane masks are sized to have their radius fall right at the geometric IWA of the occulter at the central wavelength of the band. Oversizing the mask would cut off the planet, while undersizing it would allow more starlight through the APLC. Even so, for a broad spectral band, the hybrid attenuates the planet light as well as the star light, something the occulter does not do.
- Also looked at AIC (achromatic interferometric coronagraph) which doesn’t have this problem—but using it increases requirements on tolerances significantly.
Non-planar wavefronts with APLCs

The star after the occulter is in blue, a flat wavefront of equivalent intensity is in green, and the planet is in red.
Angular spectrum of occulter

The magnitude of the $A(f)$ as a function of $f$. 

The magnitude of the angular spectrum of the THEIA shadow

- 250nm THEIA occulter
- 250nm disk, (THEIA diameter)
- Geometric IWA of occulter
Possible causes of residual light

- Residual stray light
- Nonuniform illumination
- Reflections from sidewalls
- Insufficient mask attenuation
- Manufacturing/etching errors
- Vector sidewall effects

Biggest two: Vector sidewall effects and random edge errors, both of which should increase the light in the center by a factor of 4.
The inner and outer regions of the “after” image of the, with the number of counts in each. Very noisy in outer region (99.88% of pixels within 10 counts of 0).
Nonuniform illumination (I)

\[ E_h(\rho, \phi) = \frac{2\pi E_2 e^{\frac{2\pi i (z_h + h)}{\lambda}}}{i \lambda z_h h} e^{\left(\frac{\pi i}{\lambda h} - \frac{1}{\sigma^2}\right) r_0^2} e^{\frac{\pi i}{\lambda z_h} \rho^2} \left( \int_0^{R_3} e^{\left(\frac{\pi i}{\lambda h} + \frac{\pi i}{\lambda z_h} - \frac{1}{\sigma^2}\right) r^2} H(r) J_0(wr) r \, dr \right) + \sum_{j=1}^{\infty} 2i^{jN} \cos(jN\theta^*) \int_0^{R_3} e^{\left(\frac{\pi i}{\lambda h} + \frac{\pi i}{\lambda z_h} - \frac{1}{\sigma^2}\right) r^2} J_{jN}(wr) \frac{\sin(j\pi H(r))}{j\pi} r \, dr \]

with

\[ w = -\sqrt{\left[\frac{2\pi \rho}{\lambda z_h}\right]^2 + \left[\frac{2\pi r_0}{\lambda h} + \frac{2i r_0}{\sigma^2}\right]^2 + 2 \left[\frac{2\pi \rho}{\lambda z_h}\right] \left[\frac{2\pi r_0}{\lambda h} + \frac{2i r_0}{\sigma^2}\right] \cos(\phi - \theta_0)} \]

\[ \theta^* = \tan^{-1} \left( -\left[\frac{2\pi r_0}{\lambda h} + \frac{2i r_0}{\sigma^2}\right] \sin \theta_0 + \frac{-2\pi \rho}{\lambda z_h} \sin \phi \right) - \left[\frac{2\pi r_0}{\lambda h} + \frac{2i r_0}{\sigma^2}\right] \cos \theta_0 + \frac{-2\pi \rho}{\lambda z_h} \cos \phi \]
A slice through the shadow from the occulting mask in simulation, with an incident Gaussian set 0.5" from the center of the mask, along the x-axis. Goes to $4.45 \times 10^{-8}$. 
Diverging beam hits sidewalls of mask. Consider 2 cases: fully absorbing sidewalls and fully reflecting sidewalls.

4.7 \times 10^{-8} 

5.9 \times 10^{-8}
Insufficient mask attenuation

The occulter mask is a 100nm titanium coating on a 400µm silicon substrate; there is some question on whether this mask sufficiently attenuates incident light. We model mask attenuation simply by looking at the extinction coefficients of the materials in question—that is, the imaginary part $k$ of the complex index of refraction $\tilde{n} = n + ik$.

$$\frac{E_f}{E_i} = e^{-\frac{4\pi k_{Ti} z_{Ti}}{\lambda}} e^{-\frac{4\pi k_{Si} z_{Si}}{\lambda}}$$

- Titanium at 632.8nm: $k_{Ti} = 3.77$, 100nm thick
- Silicon at 632.8nm: $k_{Si} = 0.019$, 400µm thick
- $E_f/E_i = 1.60 \times 10^{-69}$, with $2.85 \times 10^{-66}$ from the silicon and $5.61 \times 10^{-4}$ from the titanium
Manufacturing/etching errors

Nominal \((4 \times 10^{-8})\)

Electric field across CCD with no errors

Electric field with 2\(\mu\)m RMS edge errors \((1/R^2\) power law)
Sidewalls

We model the occulter mask as an infinitely-thin piece of abstract material. Real mask is a three-dimensional structure whose thickness can be larger than the thin gaps at each tip of the holes in the mask. Result: the mask material can affect the electric field produced by the mask in ways not predicted by the scalar theory.

Ceperley et al. 2006 used finite-element analysis software with models of similar thick silicon masks to determine what effect these have; the contrast increases to $1.78 \times 10^{-7}$. 

![Expected contrast in dark hole: 1.78e-007](image)
How to quantify occulter performance

Everything until now assumes that the occulators are doing exactly what we expect them to. Next, we need quantify how much they can diverge from this ideal before they are no longer sufficient to find a planet.

- Previous equations show electric field right before the telescope aperture; to examine science performance, need to go to the image plane, since that’s the data we'll actually collect.
- We create a figure of merit for performance in the image plane, Q:

\[
Q = \min \frac{\text{Intensity of planet at } (x, y)}{\text{Intensity of star at } (x, y)} \quad \forall (x, y) \text{ outside the IWA} \quad (2)
\]

- We design occulators to suppress the star by $10^{12}$ in the image plane beyond the IWA ($Q = 100$), and let errors in position, orientation, shape, etc. bring the suppression level back up to $10^{10}$ ($Q = 1$).
A simple model for the signal-to-noise ratio (SNR) at the pixel in the image plane of the telescope corresponding to the peak of the planet PSF can be given as (Brown 2005):

\[
\text{SNR} = \frac{C_p}{\sqrt{C_p + C_o + C_b}}
\]

where:

- \(C_p\) is the number of photons from the planet in the pixel
- \(C_o\) is the number of photons from residual starlight in the pixel
- \(C_b\) is the number of background counts from all other sources, including zodiacal and exozodiacal light and camera noise
In this model, \( Q = C_p / C_o \), and so:

\[
SNR = \frac{\sqrt{C_p}}{\sqrt{1 + \frac{1}{Q} + \frac{C_b}{C_p}}}
\]

For \( Q \gg 1 \), \( SNR \approx \sqrt{C_p} / \sqrt{1 + \frac{C_b}{C_p}} \); in other words, the SNR will be virtually independent of the occulter.

Also keep in mind integration time \( \tau \) to achieve a desired SNR goes as \( SNR \sim \sqrt{\tau} \), so \( Q \ll 1 \) can increase \( \tau \) significantly.
Finite stellar size decreases lateral tolerance; for Sun at 10pc with THEIA, would drop by up to 13cm. Points on surface are incoherent; model by uniformly sampling a hemisphere, and then summing resulting PSFs for each point. Generally, little effect on PSF when the occulter is well-aligned.
Changing occulter shapes (I)

As seen before: two requirements to create a profile:

1. For profile $A(r)$, we design the petals so that a circle of radius $r$ has a fraction of the circle equal to $A(r)$ blocked by a petal.

2. We repeat the petals $N$ times to place the scattered light outside the aperture.
Changing occulter shapes (II)

These are the only requirements! Implicit assumptions that are *not* required:

- The petal must be defined by a single profile $A(r)$
- The petal must be symmetric
Key rules

The key rule for multiple profiles:

\[ A(r) = A_1(r) - A_2(r) + A_3(r) - \ldots, \quad A_1(r) \geq A_2(r) \geq A_3(r) \ldots \]

Any set of profiles that meet this rule will keep \( E_{occ} \) the same. (Cady et al. 2010)

The key rule for asymmetric petals:

If the centerline of every petal is shifted by a function \( \beta(r) \), \( E_{occ} \) will be unchanged.

Uses:

- Adding tensioning elements or other structural components between petals
- Moving small gaps between petals or isolating complex regions of the petal edge
- Reducing total number of petals
Structural elements

\[ A_1(r) = \begin{cases} A(r) \\ A(r) + T(r) \end{cases} \]
\[ A_2(r) = \begin{cases} A(r) \\ A(r) + T(r) - W(r) \end{cases} \]
\[ A_3(r) = \begin{cases} A(r) \\ A(r) - W(r) \end{cases} \]

\[ r < r_1 \quad \text{OR} \quad r > r_2 \]
\[ r_1 \leq r \leq r_2 \]
\[ r < r_1 \quad \text{OR} \quad r > r_2 \]
\[ r_1 \leq r \leq r_2 \]
Gaps

\[ A_1(r) = \begin{cases} A(r) & r < r_1 \text{ OR } r > r_2 \\ 1 & r_1 \leq r \leq r_2 \\ 0 & r < r_1 \text{ OR } r > r_2 \\ 1 - A(r) & r_1 \leq r \leq r_2 \end{cases} \]
Isolated regions

\[ A_1(r) = \begin{cases} 
A(r), & r < r_1 \text{ or } r > r_2, \\
S(r), & r_1 \leq r \leq r_2, \\
0, & r < r_1 \text{ or } r > r_2,
\end{cases} \]

\[ A_2(r) = \begin{cases} 
S(r) - A(r), & r_1 \leq r \leq r_2,
\end{cases} \]
Reduced petal number

\[
A_1(r) = \begin{cases} 
A(r) & r < r_1 \\
A(r) + \Delta(r) & r \geq r_1
\end{cases}
\]

\[
A_2(r) = \begin{cases} 
0 & r < r_1 \\
\Delta(r) & r \geq r_1
\end{cases}
\]
Reduced even more!

Can get even fewer petals by running a quadratic optimization on $\beta(r)$ to minimize the first term in the series in $E_{\text{bin}}$. 
Experimental verification
Ideal vs. real

Ideal scaled experiment:
- small occulter
- collimated beam
- occulter floating unattached in space

More realizable/practical to hang the occulter from some outer structure.

Our approach:
- small occulter
- diverging beam
- small shaped outer ring and struts holding the occulter
Outer ring and struts (1 of 2)

Outer ring and struts allow us to
- control diffraction effects off of the support structure
- eliminate stray light

Outer ring is designed as an occulter, so the light scattering into the center by the outer ring can be made very small.

Invert:

Combine:
Outer ring and struts (2 of 2)

Struts are made by writing the inner and outer rings as a single profile $A^*(r)$ and scaling:

$$A^*(r) \Rightarrow \Rightarrow c_0 A^*(r), \ 0 < c_0 < 1$$

$E_{occ}$ is only scaled by $c_0$, not otherwise modified.
Diverging beam

By using a diverging beam:

- Eliminate the collimating optic, potentially reducing aberrations
- Eliminate noise from other optics, since pinhole acts as spatial filter
- Can adjust the mask and shadow sizes using scaling relations

$$z'' \rightarrow z' \quad R_d \rightarrow R' \sqrt{\frac{h}{h+z'}} \quad \rho'' \rightarrow \rho' \sqrt{\frac{h+z'}{h}}$$

Here we use it to

- make the outer ring fit on a silicon wafer
- make the high-contrast region the size of the CCD
Inside $40' \times 8' \times 4'$ enclosure to isolate from environment

No optics between pinhole and mask

No optics (currently) between mask and camera
Diverging beam optics

- Create diverging beam by focusing a 632.8nm beam through a pinhole
- Optics before pinhole size beam to match pinhole diameter
Alternate method: broadband source

To look at broadband suppression, we use a supercontinuum source:

- Laser is 100mW over 500-1750nm.
- Supercontinuum couples into reflective collimator, passes through filter wheel, recoupled into single-mode fiber.
- Single-mode fiber end mounted in 3-axis stage and tip-tilt holder; this acts as the diverging beam source.
- Twelve filter slots (11 filters and 1 empty hole) can be selected
Occulting mask

- 4” diameter, occulter is inner 2”
- Etched from 400µm wafer at JPL
- Designed for $4 \times 10^8$ contrast
Camera

- 9.1m propagation to camera
- 1100x1024 CCD, 24 µm pixels
- No imaging optics on the camera generally, though lenses can be mounted
- 300mm stage moves camera through different regions of diffraction pattern
Results
Monochromatic images

Slice through the dark region in simulation (top) and experiment (bottom) at 633nm:

Still looking at causes for extra light in the center; currently investigating mask manufacturing.
Possible causes of residual light

In theory, the mask suppresses the beam from intensity 1 to intensity $4.0 \times 10^{-8}$

<table>
<thead>
<tr>
<th>This cause</th>
<th>... reduces the suppression to:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attenuated light passes through the silicon/titanium mask</td>
<td>$4.0 \times 10^{-8}$</td>
</tr>
<tr>
<td>$0.5\mu m$ uniform underetch</td>
<td>$4 \times 10^{-8}$</td>
</tr>
<tr>
<td>Incident beam decentered by 0.5&quot;</td>
<td>$4.5 \times 10^{-8}$</td>
</tr>
<tr>
<td>$0.5\mu m$ uniform overetch</td>
<td>$5 \times 10^{-8}$</td>
</tr>
<tr>
<td>Diverging beam hits sidewalls (fully-absorbing)</td>
<td>$4.7 \times 10^{-8}$</td>
</tr>
<tr>
<td>Diverging beam hits sidewalls (fully-reflecting)</td>
<td>$5.9 \times 10^{-8}$</td>
</tr>
<tr>
<td>Random errors on edges ($2\mu m$ RMS)</td>
<td>$1.5 \times 10^{-7}$</td>
</tr>
<tr>
<td>Finite mask thickness (using FDTD calculations in Ceperley et al. 2006)</td>
<td>$1.78 \times 10^{-7}$</td>
</tr>
</tbody>
</table>

Still can’t explain all of the light in the shadow—may come from coupling between these effects, or from underlying errors (such as deviation from ideal edge shape) being larger than anticipated.
First broadband results

- Using supercontinuum source coupled through single-mode fiber.
- $2 \times 10^{-5}$ suppression in the center.
- Still some residual structure in the center, which moves when the mask is rotated; suggests the mask is limiting here.
- Not clear why performance is worse than the laser alone; might be leak from longer wavelengths with less suppression.