Optimal Terminal Descent Guidance Logic To Achieve a Soft Lunar Touchdown

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I. Introduction

Altair Lunar Lander is the linchpin in the Constellation Program for human return to the Moon. In the 2010 design reference mission, Altair is to be delivered to low Earth orbit by the Ares V heavy lift launch vehicle, and after subsequent docking with Orion in LEO, the Altair/Orion stack is delivered through trans-lunar injection (TLI). The Altair/Orion stack separates from the Ares V Earth departure stage shortly after TLI and continues the flight to the Moon as a single stack. Fig. 1 depicts one version of the Altair lunar lander.[1,2]

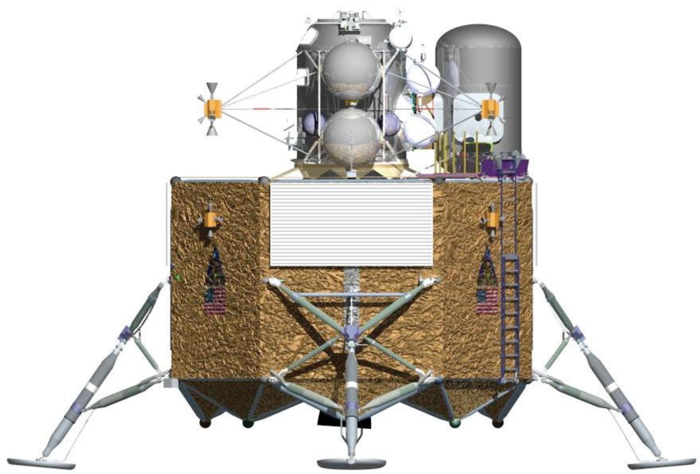

Figure 1. Altair Lunar Lander, Side View. *The Altair Lunar Lander is comprised of an Ascent Module (top, center), an Airlock (top, right), and a Descent Module (bottom, gold Mylar).*

Altair performs the lunar orbit insertion (LOI) maneuver(s), targeting a 100-kilometer (km) circular orbit. This orbit will be a polar orbit for Outpost missions landing near the lunar South Pole and other inclinations for Global Access missions to other points on the lunar surface. After spending approximately 20–116 hours in low lunar orbit (LLO), the lander undocks from Orion and performs a series of small maneuvers to set up for descending to the lunar surface. This descent begins with a small descent orbit insertion (DOI) maneuver, putting the lander on an orbit that has a perilune of 15.24 km (50,000 ft), the altitude where the actual powered descent initiation (PDI) commences.

II. Descent Phase

The descent phase is one continuous burn, beginning at Powered Descent Initiation (PDI). There are three sub-phases comprising the descent phase of the Altair mission: the braking burn, the approach, and terminal (note a short pitch-up maneuver will be executed near the beginning of approach), all shown in Figure 2.

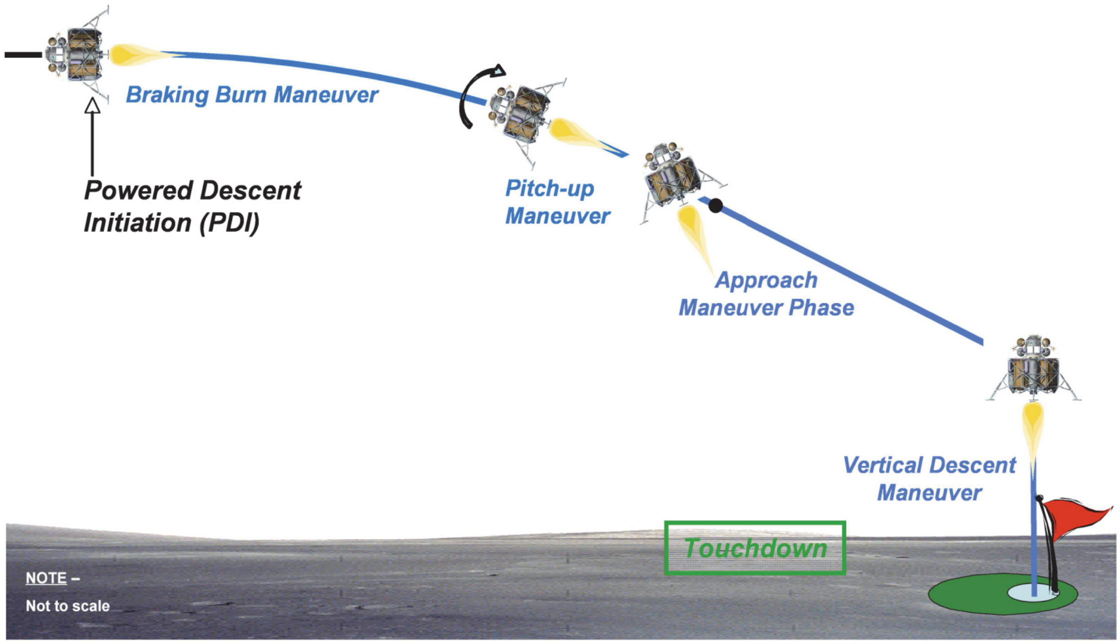

Figure 2. Lunar Descent Subphases. *The descent phase is comprised of a braking burn subphase, a pitch-up maneuver, an approach subphase and a terminal descent subphase.*[1]

Braking Burn

The braking burn starts at the descent orbit perilune altitude of 15.24 km with the descent module (DM) main engine aligned with the lander's velocity vector. It is done to remove the orbital energy at the highest efficiency possible, but not at full thrust to allow for thrust margin between the BB set throttle and the maximum available engine power. Thrust margin is necessary in order be able to remove dispersions during the braking burn.

Pitch-up Maneuver

When the braking burn is completed, the lander will perform the pitch-up maneuver. The "nearly vertical" attitude of the lander will provide the crew with better visibility to detect terrain hazards surrounding the landing site. Re-designation of the landing target can then be performed after the pitch-up is completed.

Approach

During the approach phase, the vehicle descends at a lower throttle (roughly between 60% and 40% full engine thrust) while the landing area is examined for hazards. The magnitude and direction of thrust varies to track the reference trajectory selected for the approach phase. A hazard detection sensor carried onboard is used to assist in identification of the best location in which to land. For the piloted mission, the crew will have to make a decision on the possible need to re-designate to a "safer" landing site. The trajectory design must account for providing a landing approach that enables adequate viewing of the landing area by the both the crew and the hazard detection system. If a new landing location is selected, the target is updated in the GN&C software and the guidance calculates an updated trajectory that delivers the vehicle to the new location. Re-designation can occur multiple times during the Approach Phase. Whether re-designation occurs or not, the approach sub-phase ends at 30 meters vertically above the final (selected) touchdown site.

Terminal

The terminal sub-phase is intended to be a quiescent, controlled, vertical descent for 30 seconds at a constant 1 m/s rate of descent over the last 30 meters of altitude. The DM engine shutdown occurs just prior to touchdown and the shutdown sequence is initiated at 1 meter above the surface. Assuming a free fall from 1-m height at -1 m/s, the expected worst-case touchdown velocity is 2.1 m/s.

III. Optimal Terminal Descent Guidance Logic With Large Initial Horizontal Velocity

In off-nominal fault scenarios, the terminal descent sub-phase might start with a significant horizontal velocity. This undesirable horizontal velocity of the vehicle at the start of the descent should be minimized before touchdown. This will prevent vehicle tip-over at touchdown. Touchdown conditions of the Apollo-11 lander were: Horizontal velocity ≈ 0.45 m/s; Vertical velocity ≈ 0.2 m/s; [pitch, yaw, roll] rates ≈ [-1.5, -6.2, -3.7] deg/s; and [roll, pitch] attitude ≈ [0.04, 0.25] deg. Acceptable touchdown conditions for the Altair's landing gear design are work-in-progress. But regardless of the capability of the landing gear design, it is always desirable to minimize the magnitudes of both the vertical and horizontal velocities of the vehicle at touchdown. Typically, RCS thrusters could be used to tilt the vehicle's attitude slightly so that a component of the large engine thrust could null the horizontal velocity of the lander. Engine shut down will occur just prior to touch down.

The descent and landing of the lander during descent is depicted in Fig. 3. The two-dimensional planner motion of the lander (which is modeled as a point mass) is govern by the following equations of motion.

$$\begin{aligned} \dot{x}_1 &= u_1 \\ \dot{x}_2 &= u_2 - g \\ \dot{x}_3 &= x_2 \end{aligned} \tag{1}$$

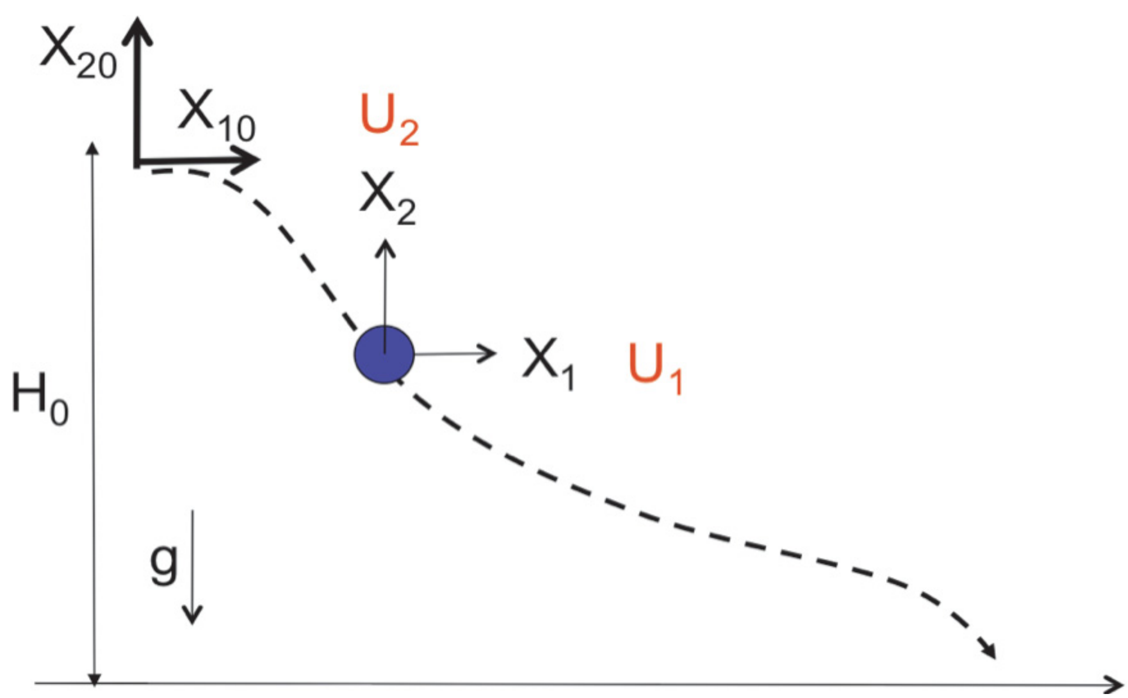

Figure 3. Optimal Control of a Lander to Achieve a "Soft" Touchdown Condition

Here, x_1 is the lander's horizontal velocity (in m/s), x_2 is the lander's vertical velocity (in m/s), and x_3 is the lander's altitude (in m). The positive directions of these variables are depicted in Fig. 3. The horizontal and vertical components of the engine thrust are represented by mu_1 and mu_2, respectively. The units of these thrust components are m/s^2. The positive directions of these thrust components are also depicted in Fig. 3. Finally, g (in m/s^2) denotes the constant acceleration due to gravity of the Moon. The initial conditions of x_1, x_2, and x_3 are denoted by x_{10}, x_{20}, and H_0, respectively. The final condition of x_3, at a pres-selected terminal touchdown time T (in units of s) is $x_3(T)=0$ ("touch down").

Consider the following optimization problem. The thrust components $u_1(t)$ and $u_2(t)$ ($T \geq t \geq 0$) are to be selected to minimize the following cost functional J.

$$J = \frac{1}{2}[x_1^2(T) + x_2^2(T)] + \frac{W}{2}\int_0^T [u_1^2(t) + u_2^2(t)]dt \tag{2}$$

Physically, the first component of J is related to the lander's Touchdown (TD) velocity. The unit of this term is m^2/s^2. The second component of J is related to the fuel cost of both the descend module main propulsion system that is consumed in achieving a "soft" TD condition. The weighting parameter W, in units of seconds, is used to add together the two terms in the cost functional. This optimal control problem could be solved via the classical calculus of variations technique. See, for example, chapter 3 of Reference 3. The resultant optimal control is given by:

$$\begin{aligned} u_1(t) &= -K_1 = \text{constant} \\ u_2(t) &= K_2 t - K_3 = \text{linear variaiton of time} \end{aligned} \tag{3}$$

Here, the constants r, K_1, K_2, and K_3 are given in the following expressions:

$$\begin{aligned} r &= \frac{T}{W} = \text{unitless} \\ \Delta &= \frac{T^3}{3}(1 + \frac{1}{4}r) \\ K_1 &= \frac{x_{10}/W}{\{1 + r\}} \quad \text{(in units of m/s}^2\text{)} \\ K_2 &= \frac{1}{\Delta}\{T(1 + \frac{r}{2})x_{20} + (1 + r)H_0 - \frac{g}{2}T^2\} \quad \text{(in units of m/s}^3\text{)} \\ K_3 &= \frac{T}{\Delta}\{T(1 + \frac{r}{3})x_{20} + (1 + \frac{r}{2})H_0 - \frac{g}{2}T^2(1 + \frac{r}{6})\} \quad \text{(in units of m/s}^2\text{)} \\ K_4 &= K_3 + g \quad \text{(in units of m/s}^2\text{)} \end{aligned} \tag{4}$$

The time histories of the horizontal velocity, vertical velocity, and that of the altitude are given by the following expressions:

$$\begin{aligned} x_1(t) &= -K_1 t + x_{10} \\ x_2(t) &= \frac{1}{2}K_2 t^2 - K_4 t + x_{20} \\ x_3(t) &= \frac{1}{6}K_2 t^3 - \frac{1}{2}K_4 t^2 + x_{20}t + H_0 \end{aligned} \tag{5}$$

The resultant TD states of the lander are given by the following expressions:

$$\begin{aligned} x_1(T) &= \frac{x_{10}}{(1 + r)} \\ x_2(T) &= -\frac{3}{2T(1 + \frac{r}{4})}\{H_0 + T\frac{x_{20}}{3} + \frac{g}{6}T^2\} \\ &\text{Horizontal distance travelled before TD} = \frac{x_{10}T}{2}\{\frac{2 + r}{1 + r}\} \end{aligned} \tag{6}$$

Consider a landing scenario with the following initial conditions: x_{10} = 10 m/s, x_{20} = 0 m/s, and H_0= 30 m. The acceleration due to the lunar gravity g = 1.634 m/s^2. Let the time given to touch down T = 30 s, and the weighting factor W = 1 s (hence, r = T/W = 30). The optimal controls, depicted in Fig. 4, are given by:

$$\begin{aligned} u_1(t) &= -0.303 \quad m/s^2 \\ u_2(t) &= 1.54188 + 0.00255 \times t \quad m/s^2 \end{aligned} \tag{7}$$

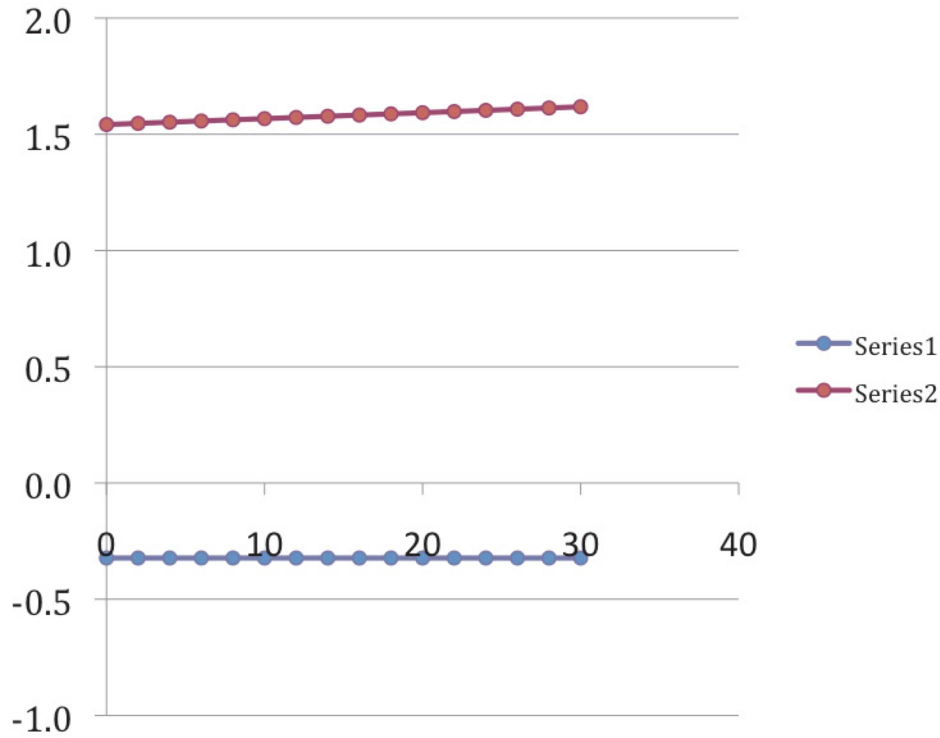

Fig. 4. Time histories of the horizontal (series 1) and vertical (series 2) controls (Ordinate has units of m/s^2 and abscissa has units of seconds)

IV. Optimal Descent Guidance Logic with A "Hard" Terminal Horizontal Landing Distance Constraint

At times, there is a need to constraint the lander to touch down at a horizontal distance "D" m from the initial position of the vehicle. To this end, an additional system "state" must be added to the equations of motion given in equation (1).

$$\begin{aligned} \dot{x}_1 &= u_1 \\ \dot{x}_2 &= u_2 - g \\ \dot{x}_3 &= x_2 \\ \dot{x}_4 &= x_1 \end{aligned} \tag{9}$$

Here x_4 is the horizontal distance travelled before TD. The initial and terminal states of x_4 are: $x_4(0) = 0$ and $x_4(T) = D$. The resultant optimal control for this revised problem could be solved using a similar approach. The optimal control is given by:

$$\begin{aligned} u_1(t) &= L_4 t - L_1 = \text{linear variaiton of time} \\ u_2(t) &= L_2 t - L_3 = \text{linear variaiton of time} \end{aligned} \tag{10}$$

Here, the constants L_1, L_2, L_3, and L_4 are given in the following expressions:

$$L_1 = \frac{x_{10}/W}{(1+\frac{r}{4})} - \frac{T}{\Delta}\{D(1+\frac{r}{2}) - x_{10}T\} \quad \text{(in units of m/s}^2\text{)}$$

$$L_2 = K_2 \text{ in equation (4)} \quad \text{(in units of m/s}^3\text{)} \tag{11}$$

$$L_3 = K_3 \text{ in equation (4)} \quad \text{(in units of m/s}^2\text{)}$$

$$L_4 = \frac{1}{\Delta}\{x_{10}T(1+\frac{r}{2}) - D(1+r)\} \quad \text{(in units of m/s}^3\text{)}$$

The resultant time histories of the horizontal and vertical velocities, and those of the altitude and down-range are given by the following expressions:

$$x_1(t) = \frac{1}{2}L_4t^2 - L_1t + x_{10}$$

$$x_2(t) = \frac{1}{2}L_2t^2 - (L_3 + g)t + x_{20} \tag{12}$$

$$x_3(t) = \frac{1}{6}L_2t^3 - \frac{1}{2}(L_3 + g)t^2 + x_{20}t + H_0$$

$$x_4(t) = \frac{1}{6}L_4t^3 - \frac{1}{2}L_1t^2 + x_{10}t$$

The resultant TD states of the lander are given by the following expressions:

$$x_1(T) = \frac{1}{2(1+\frac{r}{4})}(3\frac{D}{T} - x_{10}) \tag{13}$$

$$x_2(T) = -\frac{3}{2T(1+\frac{r}{4})}\{H_0 + T\frac{x_{20}}{3} + \frac{g}{6}T^2\}$$

Consider the same landing scenario with the following initial conditions: x_{10} = 10 m/s, x_{20} = 0 m/s, and H_0= 30 m. The acceleration due to the lunar gravity g = 1.634 m/s^2. Let the time given to touch down T = 30 s, and the weighting factor W = 1 s (hence, r = T/W = 30). Let D be 100 m. The optimal controls, depicted in Fig. 5, are given by:

$$u_1(t) = -0.6667 + 0.0222 \times t \quad \text{m/s}^2 \tag{14}$$

$$u_2(t) = 1.54188 + 0.00255 \times t \quad \text{m/s}^2$$

Not surprisingly, $u_2(t)$ of the constrained problem is identical to the $u_2(t)$ of the unconstrained optimal control problem (cf. Fig. 4). But unlike the constant $u_1(t)$ of the unconstrained optimal control problem, $u_1(t)$ of the constrained optimal control problem varies linearly with time. For about the first half of the flight time, $u_1(t)$ has larger magnitude. This will help to “brake” the forward motion of the vehicle in order to touch down at a distance of 100 m (instead of the 154.8-m as computed for the unconstrained optimal control problem). The corresponding optimal descent trajectory is depicted in Fig. 6.

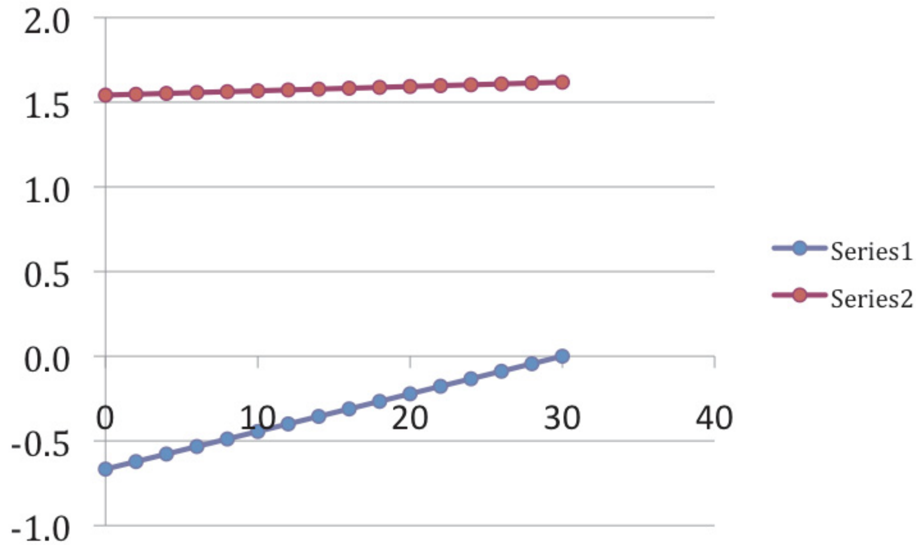

Fig. 5. Time histories of the horizontal (series 1) and vertical (series 2) controls (Ordinate has units of m/s^2 and abscissa has units of seconds)

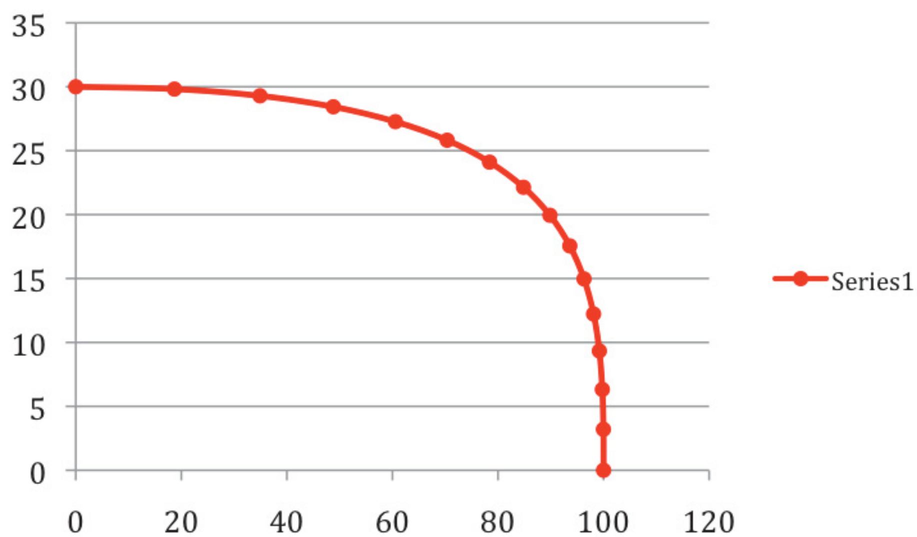

Fig. 6. Optimal descent trajectory (Ordinate is altitude with units of m and abscissa has units of m)

V. Discussions and Conclusions

In this study, the optimal control of a lander to achieve a "soft" touchdown condition is formulated as an optimal control problem. For simplicity, the vehicle is modeled as a point mass and only two-dimensional planner motion is considered. In this study, we use a cost functional that penalizes both the terminal vertical and horizontal velocities of the vehicle as well as the fuel cost of the descent main propulsion system. The resultant optimal control problem could be solved via the classical calculus of variations technique. Analytical expressions for the time history of the optimal control vector, terminal touchdown conditions, and others could be derived for this simplified optimal landing control problem. Similar expressions could also be derived for two variants of this optimal control problem, with "hard" or "soft" constraints on the terminal horizontal touchdown distance. These expressions provide insights on the "physics" of this terminal sub-phase of the "descent and landing" phase of the lander. These insights will provide the systems engineers in achieving a good "balance" between the conflicting needs of a "soft" touchdown condition, the capability of the landing gear design (a capable landing gear system usually has a larger mass penalty), fuel cost of the descent main propulsion system, and others. Results given in this work is in generic. For example, by a simple change in the value of "g" used in these expressions, the achievable touchdown condition of a lander on other planetary bodies (e.g., Mars, asteroid Vesta, others) could also be estimated. The formulated optimal control problem could also be modified to incorporate additional constraints on the kinematics of the lander due to limitations of crew visibility and/or sensor capability.

References

1. Larry Kos, Tara Polsgrove, Ronald Sostaric, Ellen Braden, and Jacob Sullivan, and Thanh Le, "Descent and Ascent Reference Trajectory Design and Initial Dispersion Analyses," AIAA-2010-7717, Proceedings of the AIAA Guidance, Navigation, and Control Conference, August 2–6, Toronto, Ontario, Canada, 2010.
2. Allan Y. Lee, Todd Ely, Ronald Sostaric, Alan Strahan, Joseph E. Riedel, Mitch Ingham, James Wincentsen, and Siamak Sarani, "Preliminary Design of the Guidance, Navigation, and Control System of The Altair Lunar Lander," AIAA-2010-7717, Proceedings of the AIAA Guidance, Navigation, and Control Conference, August 2–6, Toronto, Ontario, Canada, 2010.
3. Bryson, A.E. and Ho, Y.C., "Applied Optimal Control," Hemisphere Publishing Corporation, 1975.